MSAI Robotics course

Lecture 4

Probabilistic observation models

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Materials by Oleg Shipitko
MIPT, 2022

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Outline

1. Why probabilistic models?
2. Probabilistic models for distance sensors
   a. Ray-casting model
   b. Beam-end model
3. Probabilistic models for landmarks detection

RECURSIVE BAYESIAN POSE ESTIMATION

$$p(\mathbf{x_t}|map, \mathbf{z_t}, \mathbf{u_t}) = C \cdot p(\mathbf{z_t}|\mathbf{x_t}, map) \int_S p(\mathbf{x_t}|\mathbf{u_t}, \mathbf{x_{t-1}}) p(\mathbf{x_{t-1}}|map, \mathbf{z_{t-1}}, \mathbf{u_{t-1}}) d\mathbf{x_{t-1}}$$

C — normalization coefficient

S — the probabilistic space of robot poses

$p(\mathbf{z_t}|\mathbf{x_t}, map)$ — observation (measurement) model

$p(\mathbf{x_t}|\mathbf{u_t}, \mathbf{x_{t-1}})$ — motion model

$p(\mathbf{x_{t-1}}|map, \mathbf{z_{t-1}}, \mathbf{u_{t-1}})$ — previous system state (robot pose)

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Why probabilistic observation models?

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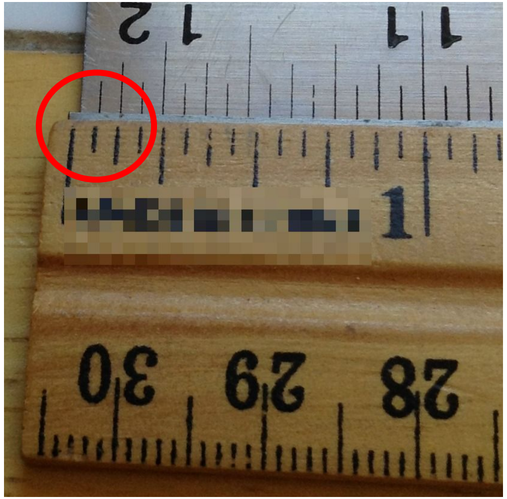
WHY DO WE NEED PROBABILISTIC OBSERVATION MODELS?

- Sensors are not perfect. Their measurements are error prone.



- The world is also not perfect. The imperfection of the world introduces additional errors in measurements.

SENSORS TYPES

- **Distance sensors**
  - LIDARs
  - IR distance sensors
  - Ultrasound
  - RADARs
- **Visual sensors**
  - Cameras
    - Monocular
    - Depth cameras
- **Satellite navigation systems**
- **Contact sensors**
  - Buttons/bumpers
- **Proprioceptive motion sensors**
  - Encoders
  - Gyroscopes
  - Accelerometers
  - Magnetometers
  - Altimeter

SENSORS DATASHEETS

Specifications

Reach RS+ Datasheet
569 kb

Mechanical

Dimensions	145x145x85 mm
Weight	690 g
Operating t°	-20...+65 °C
Ingress protection	IP67 (water and dust)

Electrical

Autonomy	Up to 30 hrs
Battery	LiFePO4 3.2 V
External power input	-40 V
Charging	MicroUSB 5 V
Certification	FCC, CE

Positioning

Static horizontal	5 mm + 1 ppm
Static vertical	10 mm + 2 ppm
Kinematic horizontal	7 mm + 1 ppm
Kinematic vertical	14 mm + 2 ppm

Connectivity

LoRa radio	
Frequency range	868/915 MHz
Distance	Up to 8 km
Wi-Fi	802.11b/g/n
Bluetooth	4.0/2.1 EDR
Ports	RS-232, MicroUSB

Data

Corrections	NTRIP, RTCM3
Position output	NMEA, LLH/XYZ
Data logging	RINEX with events with update rate up to 14 Hz
Internal storage	8 GB

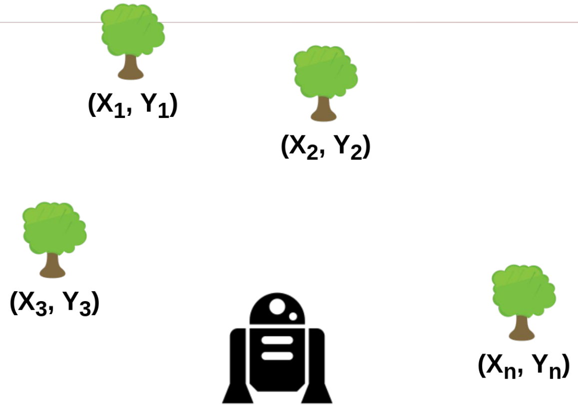
GNSS

Signal tracked	GPS/QZSS L1, GLONASS G1, BeiDou B1, Galileo E1, SBAS
Number of channels	72
Update rates	14 Hz / 5 Hz
IMU	9DOF

___-TO-___ MATCHING

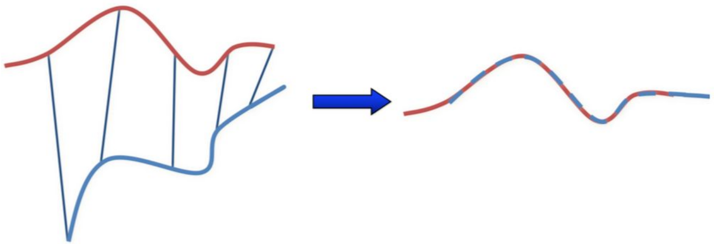
What can be matched:

- ❑ Scan to map
- ❑ Scan to scan
- ❑ Map to map
- ❑ Landmarks to map landmarks
- ❑ ….



How to match:

- ❑ Correlation
- ❑ Likelihood Maximization
- ❑ RANSAC
- ❑ Iterative Closest Point (ICP)
- ❑ Normal Distribution Transform (NDT)
- ❑ …



Probabilistic models for distance sensors

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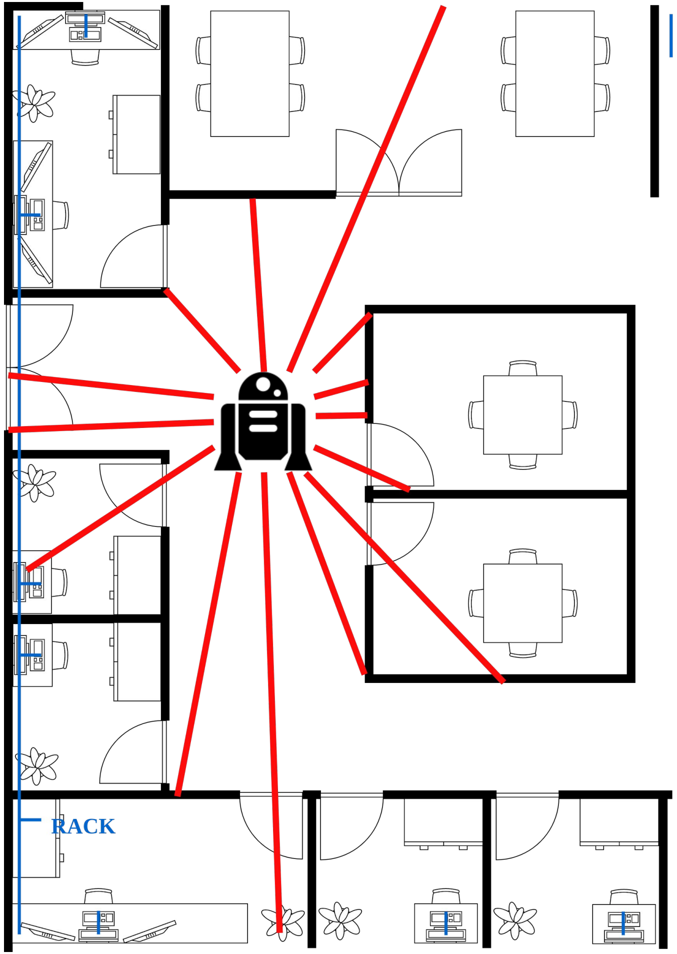
DISTANCE SENSORS

- Most often, models of multi-beam rangefinders are considered (for example, LIDAR or an array of ultrasonic sensors) because they are easier to use than other sensors



DISTANCE SENSOR MODEL

Our task is to estimate the probability of measurement given a fixed position and a map (compact representation of the world):

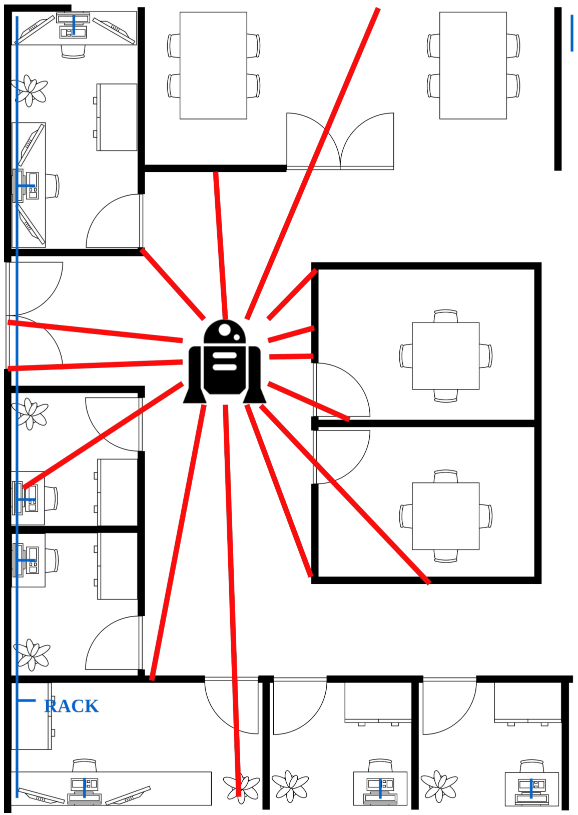
$$p(z|x,m)$$

Each measurement z consists of $\mathbf{k}$ measurements (beams):

$$z = \{z_1, z_2, ..., z_k\}$$

We will assume that each measurement is independent, then the total probability is the product of the probabilities of each individual measurement:

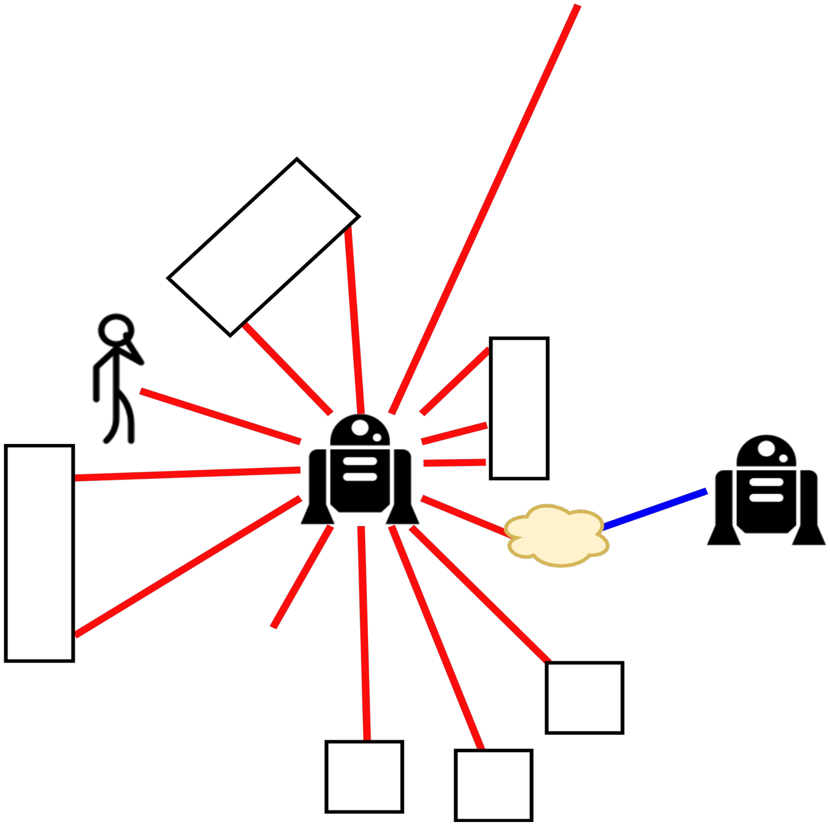
$$p(z|x,m) = \prod_{k=0}^{K} p(z_k|x,m)$$



ERROR SOURCES

When measuring, the following alternatives are possible:

- Reflection of a beam from a static (mapped) obstacle
- Reflection of a beam from a dynamic obstacle (which is not on the map)
- Interference with another sensor of a similar nature
- Random measurement (sensor error)
- Maximum distance measurement (no obstacles)



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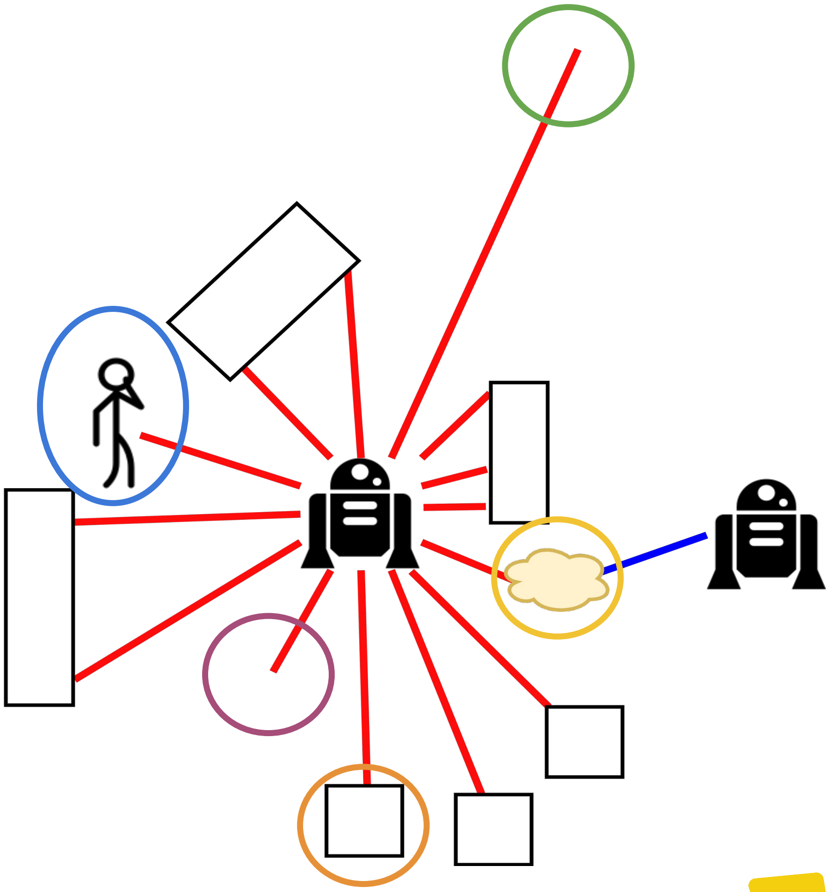
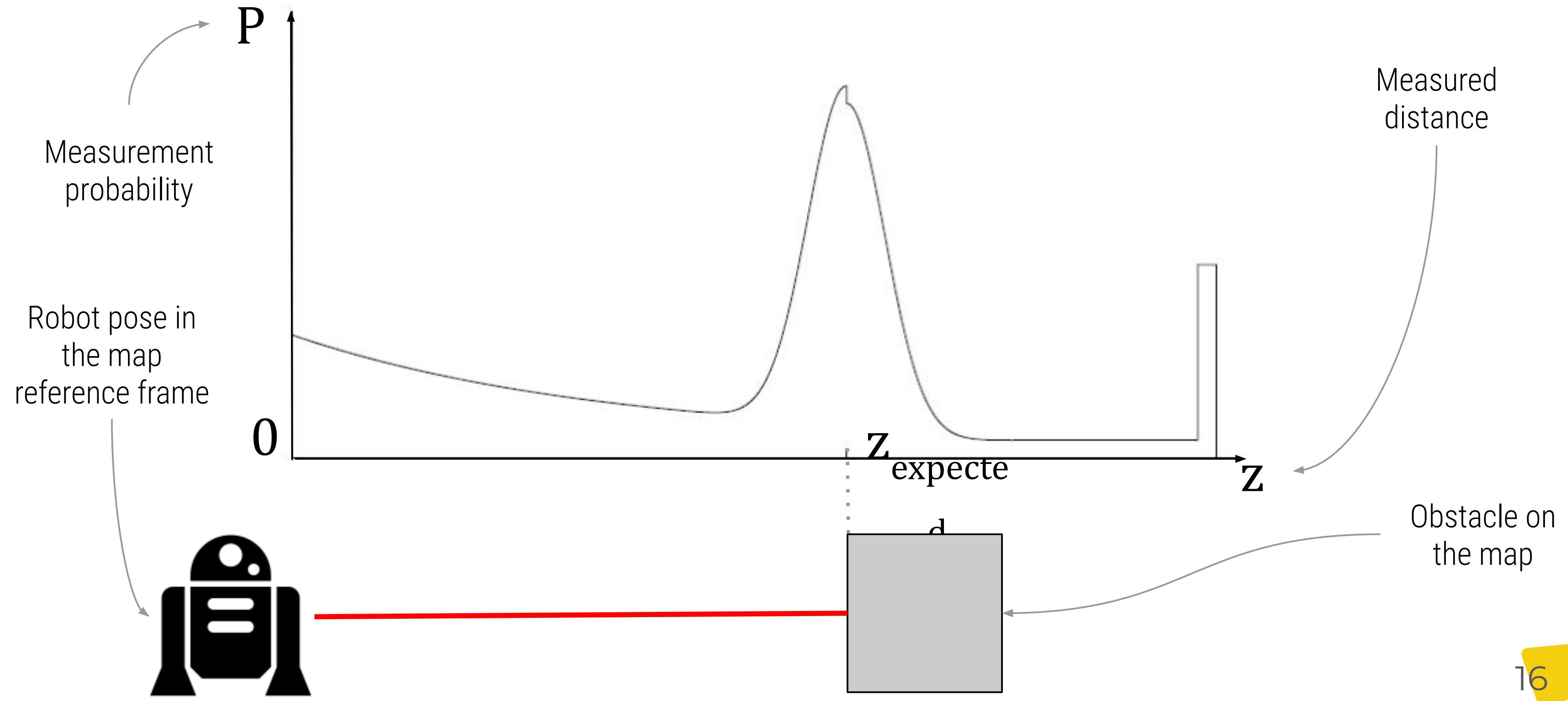


DISTANCE SENSORS MODELS

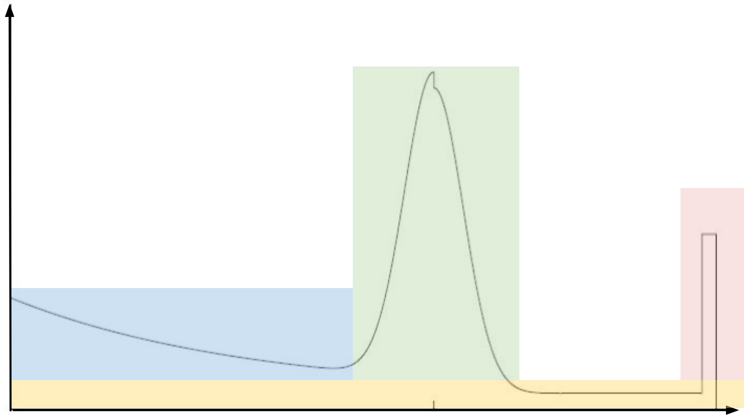
The main types of probabilistic models for distance sensors are:

- **Beam-based**
  - Models various physical reasons for obtaining a particular measurement
  - Assumes independence of measurement causes
  - Assumes the independence of individual beams
- **End-point based (scan-based)**
  - Ignores the physical properties of the beam
  - Assumes independence of measurement causes
  - Assumes the independence of individual beams
- **Scan-matching**
  - Correlation-based model

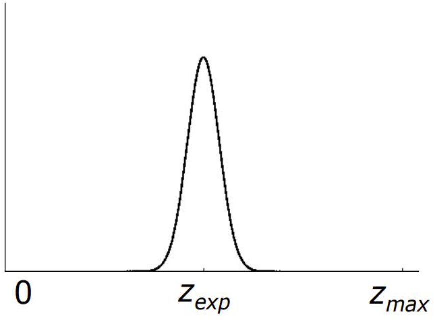
BEAM-BASED MODEL



BEAM-BASED MODEL

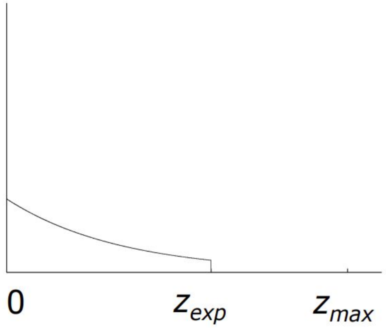


Measurement error (noise)



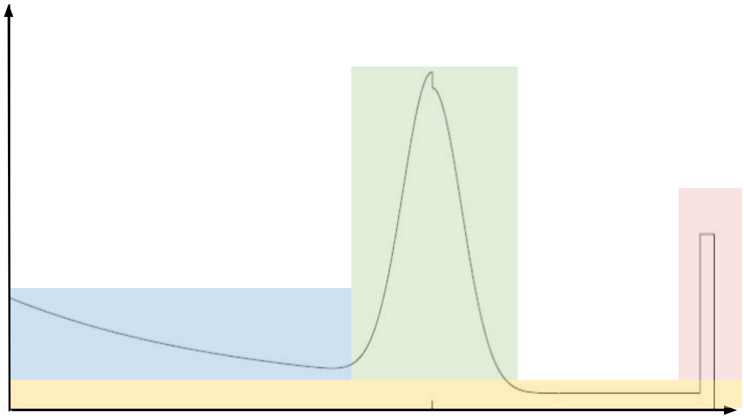
$$P_{hit}(z \mid x, m) = \eta \frac{1}{\sqrt{2\pi b}} e^{-\frac{1}{2}\frac{(z - z_{exp})^2}{b}}$$

Dynamic obstacles



$$P_{unexp}(z \mid x, m) = \begin{cases} \eta\ \lambda\ e^{-\lambda z} & z < z_{exp} \\ 0 & otherwise \end{cases}$$

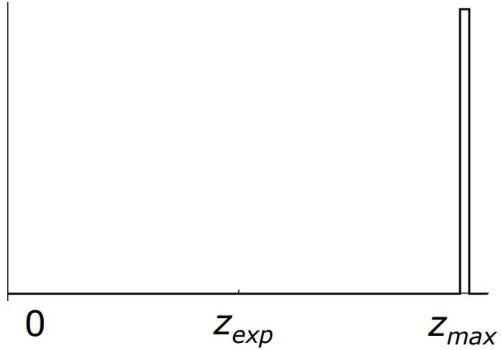
BEAM-BASED MODEL



Random measurement

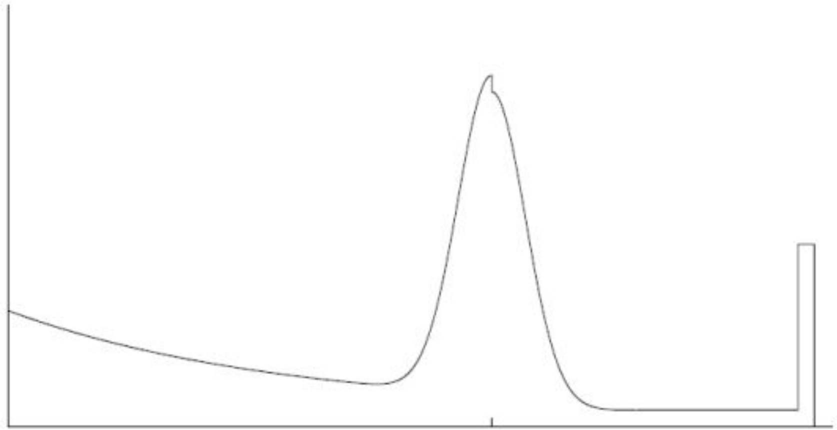
$$P_{rand}(z \mid x, m) = \eta \frac{1}{z_{max}}$$

Maximum measurement (absence of obstacles)



$$P_{max}(z|x,m) = \begin{cases} 1 & z = z_{max} \\ 0 & \text{otherwise} \end{cases}$$

BEAM-BASED MODEL

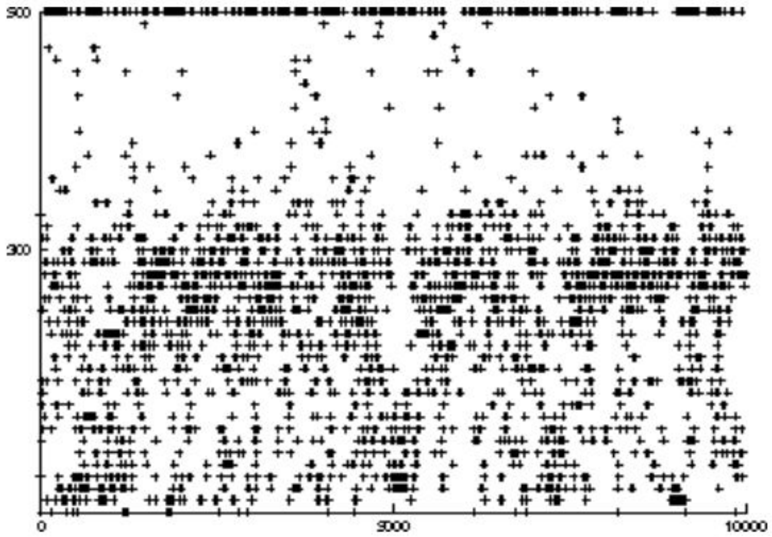


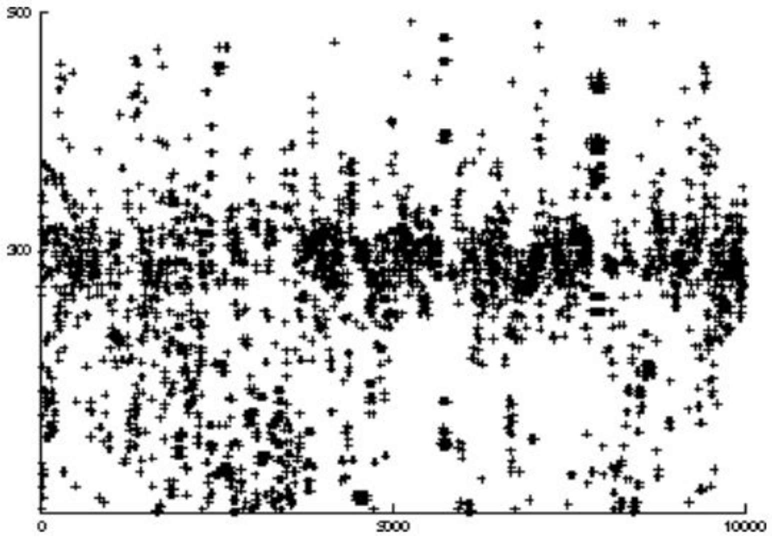
$$P(z \mid x, m) = \begin{pmatrix} \alpha_{\text{hit}} \\ \alpha_{\text{unexp}} \\ \alpha_{\text{max}} \\ \alpha_{\text{rand}} \end{pmatrix}^T \cdot \begin{pmatrix} P_{\text{hit}}(z \mid x, m) \\ P_{\text{unexp}}(z \mid x, m) \\ P_{\text{max}}(z \mid x, m) \\ P_{\text{rand}}(z \mid x, m) \end{pmatrix}$$

How to define model parameters?

MODEL PARAMETERS

Model parameters are often determined experimentally.



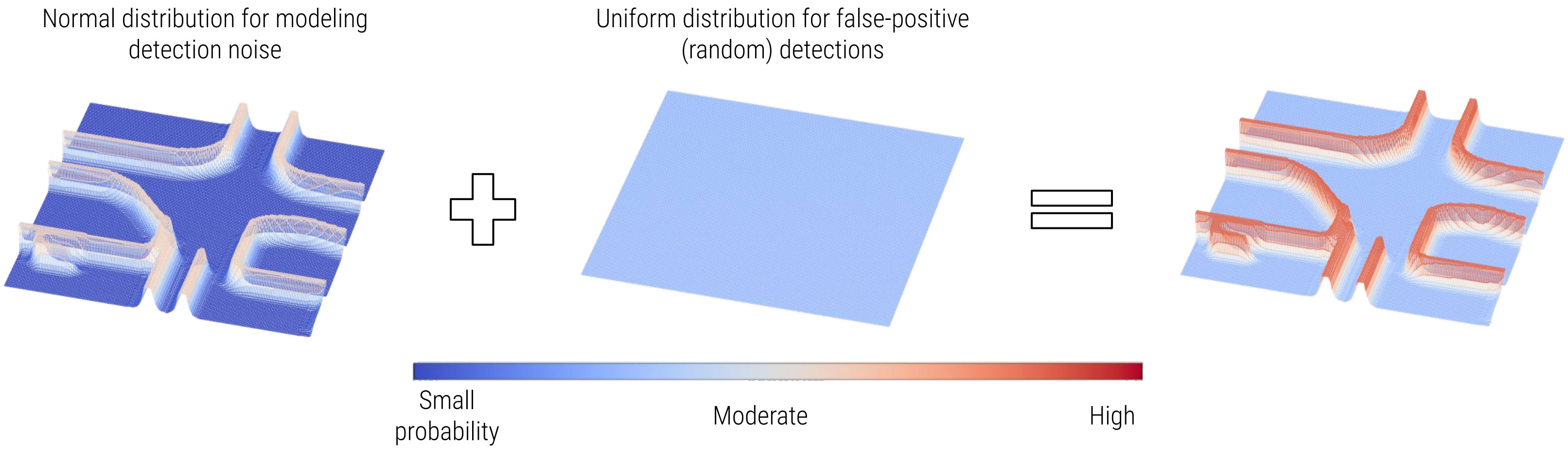


Experimental measurements for ultrasound sensor and LIDAR. The obstacle is located at a distance of *300* cm.

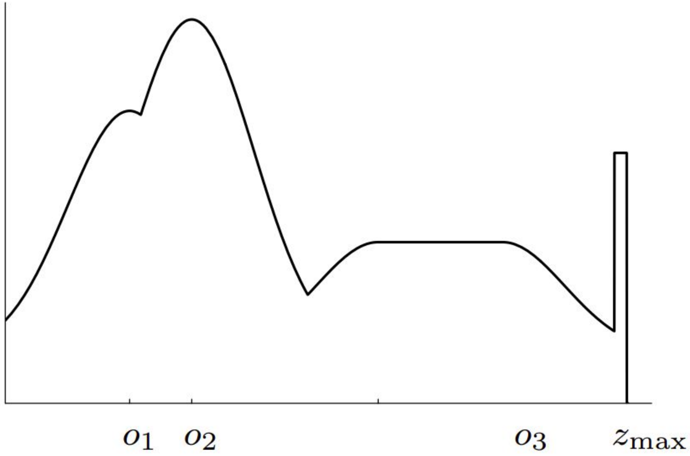
END POINT-BASED MODEL

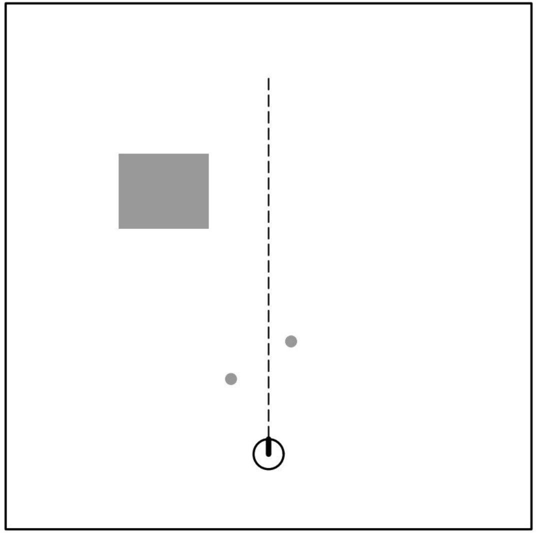
Basic idea: instead of following along the ray, you can only analyze its end point.
Probability is a combination of several distributions:

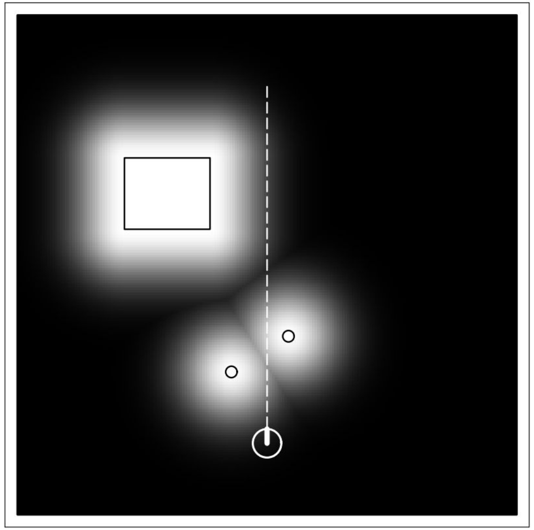
- ❑ **Normal distribution** for obstacle detection
- ❑ **Uniform distribution** for false-positive (random) detections



END POINT-BASED MODEL (likelihood field model)







$$p(z_k|x_t, m) = z_{hit} * p_{hit} + z_{rand} * p_{rand} + z_{max} * p_{max}$$

$$z_{hit} + z_{rand} + z_{max} = 1$$

CORRELATION-BASED MODEL

We "overlay" the local map to the global map, trying to maximize the correlation:

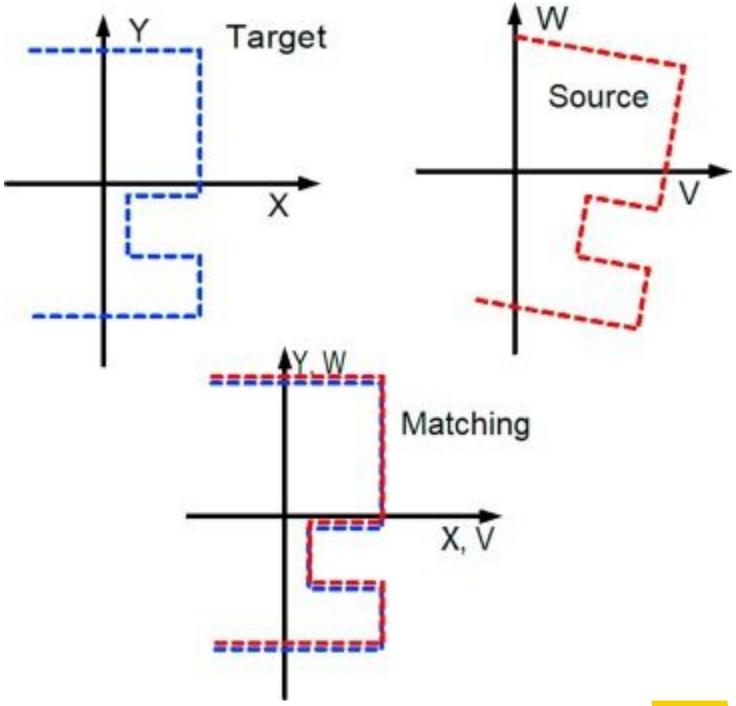
$$\rho_{m,m_{\text{local}},x_t} = \frac{\sum_{x,y}(m_{x,y} - \bar{m}) \cdot (m_{x,y,\text{local}}(x_t) - \bar{m})}{\sqrt{\sum_{x,y}(m_{x,y} - \bar{m})^2 \sum_{x,y}(m_{x,y,\text{local}}(x_t) - \bar{m})^2}}$$

$m_{x,y}$ — global map cell

$m_{x,y,\text{local}}$ — cell of the local map, "collected" from several scans

$$\bar{m} = \frac{1}{2N}\sum_{x,y}(m_{x,y} + m_{x,y,\text{local}})$$

— the average value of the cells of both maps



Probabilistic models for landmarks detection

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MODEL FOR LANDMARKS DETECTION

What are the landmarks:

- ❑ Active (GPS, radio-, ultrasound-beacons)
- ❑ Passive (reflective film, visually detectable features e.g. keypoints)

What is the measurements:

- ❑ Distance to the landmark
- ❑ Bearing to the landmark
- ❑ Distance + bearing

How the position is estimated based on landmarks:

- ❑ Triangulation
- ❑ Trilateration

POSTERIOR PROBABILITY OF LANDMARKS DETECTION

1. Algorithm **landmark_detection_model**(z,x,m):

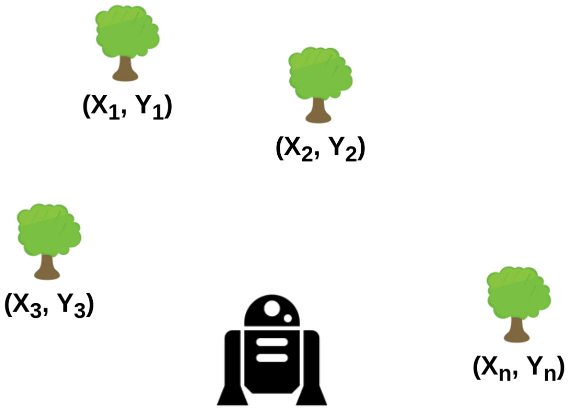
$$z = \langle i, d, \alpha \rangle, x = \langle x, y, \theta \rangle$$

2. $\hat{d} = \sqrt{(m_x(i) - x)^2 + (m_y(i) - y)^2}$

3. $\hat{\alpha} = \text{atan2}(\ m_y(i) - y, m_x(i) - x) - \theta$

4. $p_{\text{det}} = \text{prob}(\ \hat{d} - d, \varepsilon_d) \cdot \text{prob}(\ \hat{\alpha} - \alpha, \varepsilon_\alpha)$

5. Return p_{det}



POSE SAMPLING BASED ON LANDMARKS OBSERVATION MODEL

Algorithm sample_landmark_model_known_correspondence(f_t^i, c_t^i, m**):**

$j = c_t^i$

$\hat{\gamma} = \mathrm{rand}(0, 2\pi)$

$\hat{r} = r_t^i + \mathbf{sample}(\sigma_r^2)$

$\hat{\phi} = \phi_t^i + \mathbf{sample}(\sigma_\phi^2)$

$x = m_{j,x} + \hat{r}\cos\hat{\gamma}$

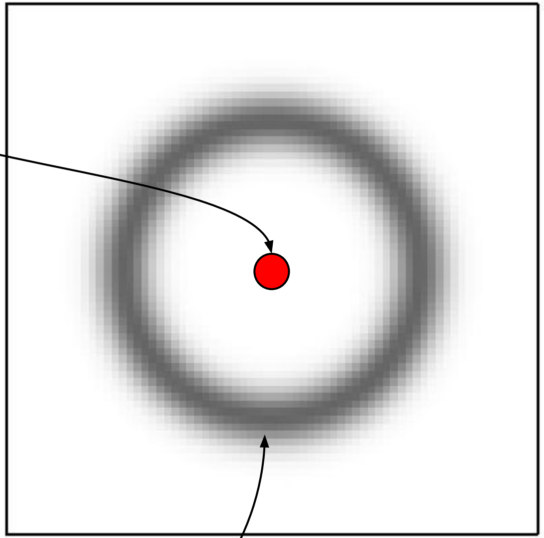
$y = m_{j,y} + \hat{r}\sin\hat{\gamma}$

$\theta = \hat{\gamma} - \pi - \hat{\phi}$

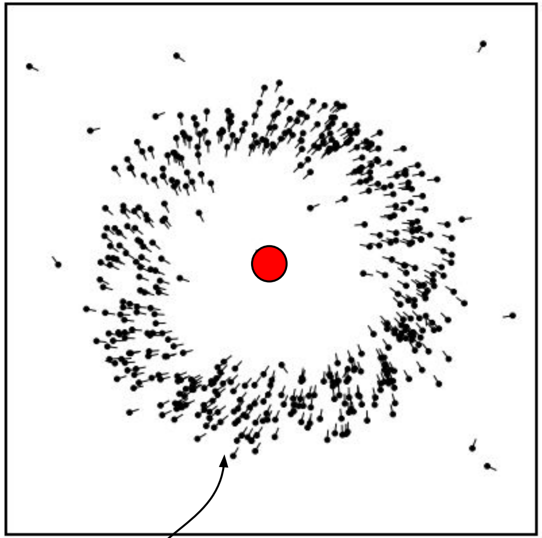
return $(x \;\; y \;\; \theta)^T$

POSE SAMPLING BASED ON LANDMARKS OBSERVATION MODEL

Observed landmark



Example of posterior pose distribution of measurement



Sampling a pose from a landmark observation model

POSE SAMPLING BASED ON LANDMARKS OBSERVATION MODEL

- Explicit inclusion of probabilities in algorithms is the key to robustness.
- The probability (likelihood) of a measurement is estimated by “probabilistic comparison” of the expected measurement with the obtained one.
- The probabilistic observation model most often can be constructed in the following way:
  - Define a “noise-free” process model
  - Estimate noise sources
  - Add a noise model to the process model
- This also works for the motion models discussed in the previous lecture.

ADDITIONAL RESOURCES

1. Probabilistic Robotics (in Notion). Chapter 6.
2. Probabilistic Sensor Models. Marina Kollmitz, Wolfram Burgard

Thanks for attention!

Questions? Additions? Welcome!

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