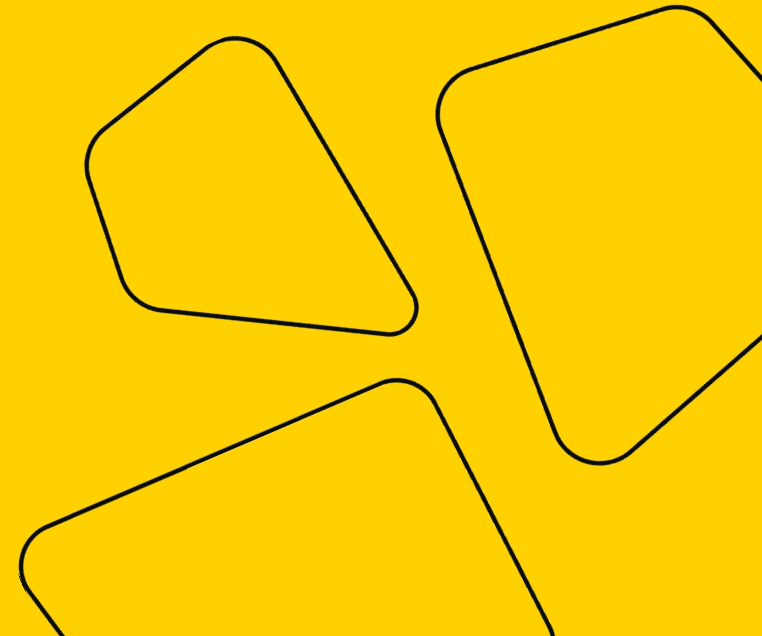


# Mapping

Vladislav Goncharenko  
Materials by Oleg Shipitko  
MIPT, 2022



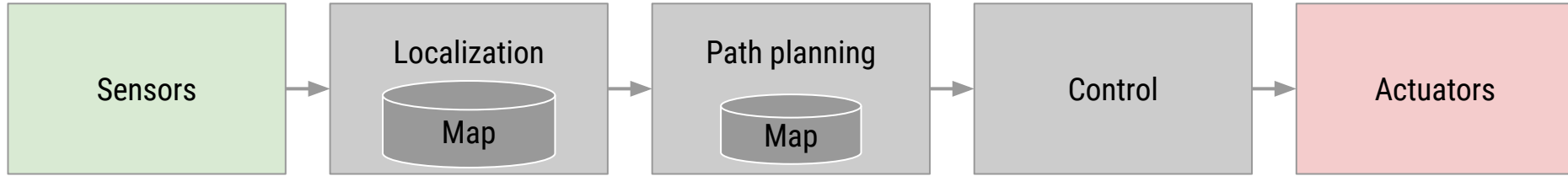
# Outline



1. Mapping problem definition
2. Maps types
3. Topological maps
4. Features/landmarks maps
5. Occupancy grid maps

# (SIMPLIFIED) CONTROL SCHEME OF MODERN MOBILE ROBOT

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# Mapping problem definition

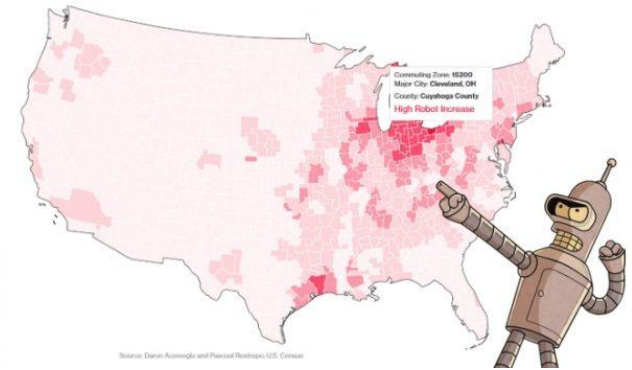
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**girafe**  
**ai**

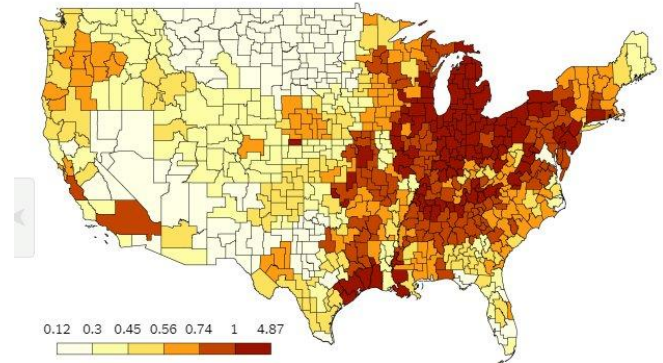
**01**

# WHAT IS MAPPING?

**Mapping** (in robotics) refers to the process of modeling the environment and presenting it in a form that is convenient for further use in navigation (localization, planning and motion implementation).



**Where the Robots Live**  
Robots per one thousand workers.



Sources: Daron Acemoglu, Massachusetts Institute of Technology, and Pascual Restrepo, Boston University

# WHY MAPPING IS TOUGH?

---

- ❑ Sensor measurement errors generate incomplete and/or inconsistent data
- ❑ Errors in determining ego-position (localization) also lead to similar contradictions
- ❑ How to integrate data over time?
- ❑ How to understand that we have already visited a place?

# FORMAL MAPPING PROBLEM

## DEFINITION

---

**Given:**

$\mathbf{X}_{1:t}$  – all previous robot states (poses)

$\mathbf{Z}_{1:t}$  – sensors measurements

**Find:**

$map$  – map of the environment



# Maps types

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**girafe**  
**ai**

02



# WHICH TYPES OF MAPS ARE THERE?

---

## Metric

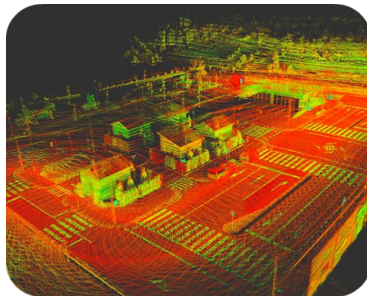
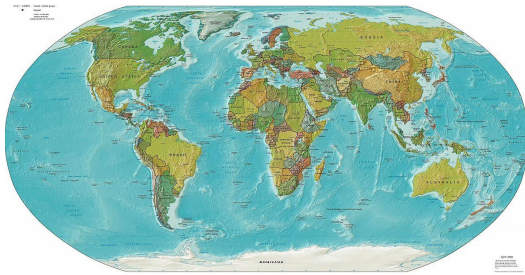
- ❑ Reflect the world in the form of 2D or 3D space
- ❑ Objects are set by their coordinates
- ❑ Distance between objects is measured in meters

## Topological

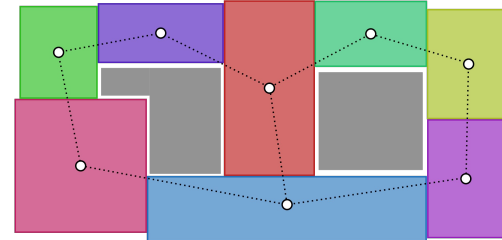
- ❑ Reflect the world in the form of places (locations) and connections (transitions) between them
- ❑ Distances between objects can be stored in links

# WHICH TYPES OF MAPS ARE THERE?

## Metric



## Topological



# WHICH TYPES OF MAPS ARE THERE?

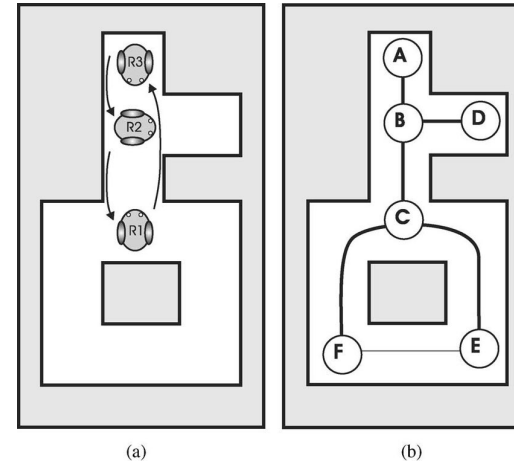
## Metric

- ❑ Occupancy grid maps
- ❑ Feature-based maps, landmark-based
- ❑ (Sometimes) Semantic maps



## Topological

- ❑ Graphs



Source: Peasgood, M., Clark, C. M., & McPhee, J. (2008). A complete and scalable strategy for coordinating multiple robots within roadmaps. IEEE Transactions on Robotics, 24(2), 283-292.  
Sünderhauf N. et al. Place categorization and semantic mapping on a mobile robot //2016 IEEE international conference on robotics and automation (ICRA). – IEEE, 2016. – C. 5729-5736.

# Topological maps

---

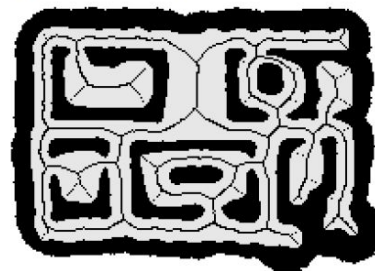
**girafe**  
**ai**

**03**

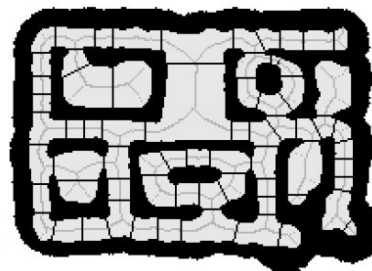
# TOPOLOGICAL MAPS

- ❑ It is a set of locations (**nodes**) and transitions between them (**edges**).
- ❑ **Location** is usually the space in which the robot can be reliably positioned (localized) and / or the point of making a decision about the direction of further motion. For example, a room in the case of a building.
- ❑ The locations are connected with each other by **transitions** containing a certain law of robot control, according to which the transition can be carried out.

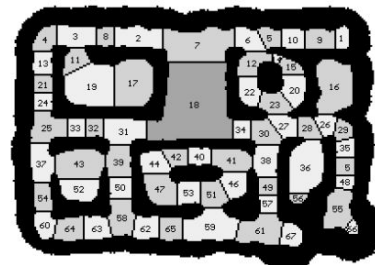
(a) Voronoi diagram



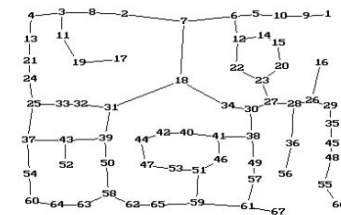
(b) Critical lines



(c) Topological regions



(d) Topological graph



# MAIN DRAWBACK

- ❑ Topological representation does not exist (or is difficult to achieve) for large open spaces



# Features/landmarks maps

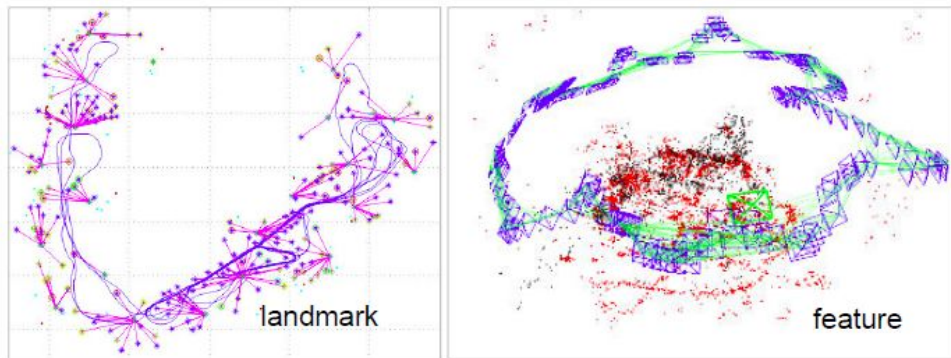
---

**girafe**  
**ai**

**04**

# FEATURE / LANDMARKS MAPS

- ❑ Store features specified by their coordinates in space
- ❑ Anything can be used as feature / landmark: trees, road signs, doors, image keypoints ...
- ❑ Very compact space representation



$(X_1, Y_1)$



$(X_2, Y_2)$



$(X_3, Y_3)$



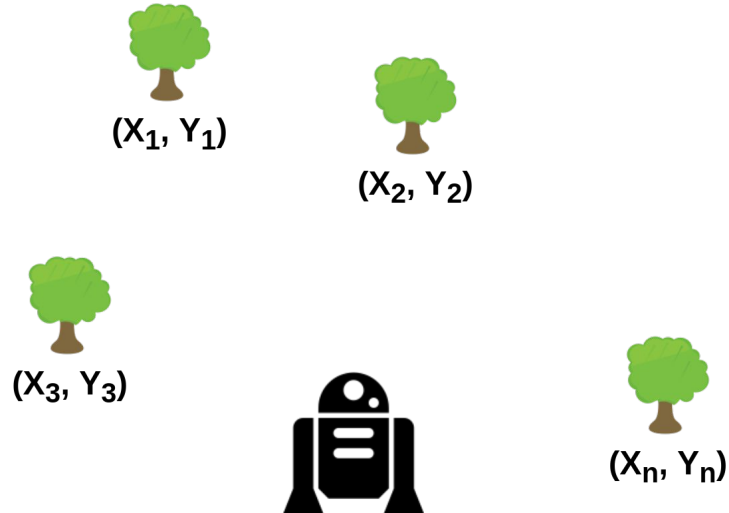
$(X_n, Y_n)$



# FEATURES VS. LANDMARKS

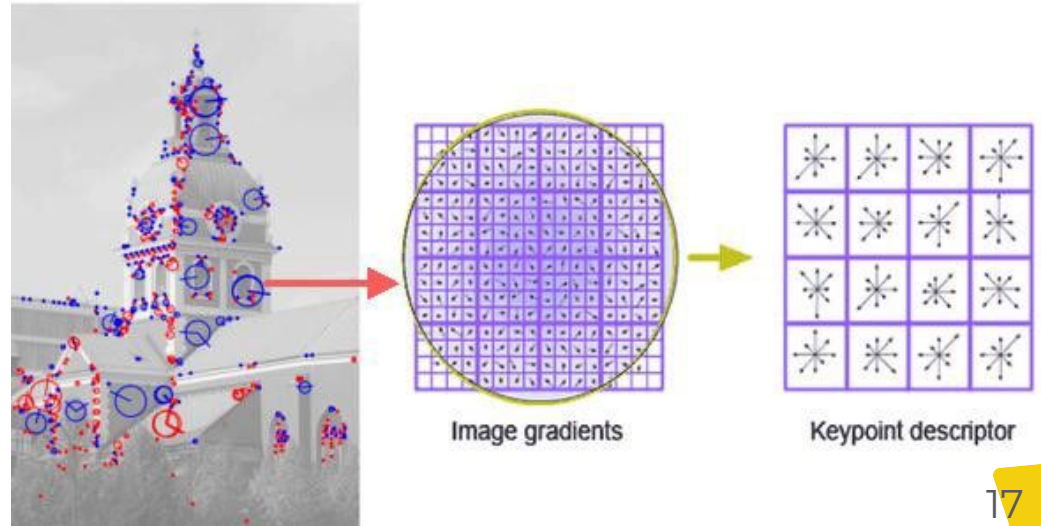
## Landmarks

- ❑ Natural or man-made objects.



## Features

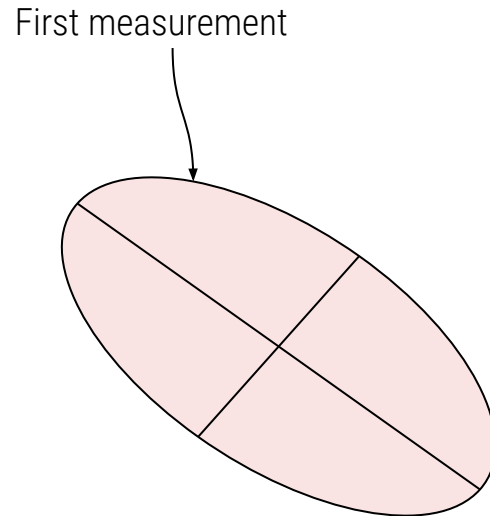
- ❑ Artificially built-up structures. Usually more abstract than landmarks.



# FEATURE / LANDMARKS MAPS GENERATION

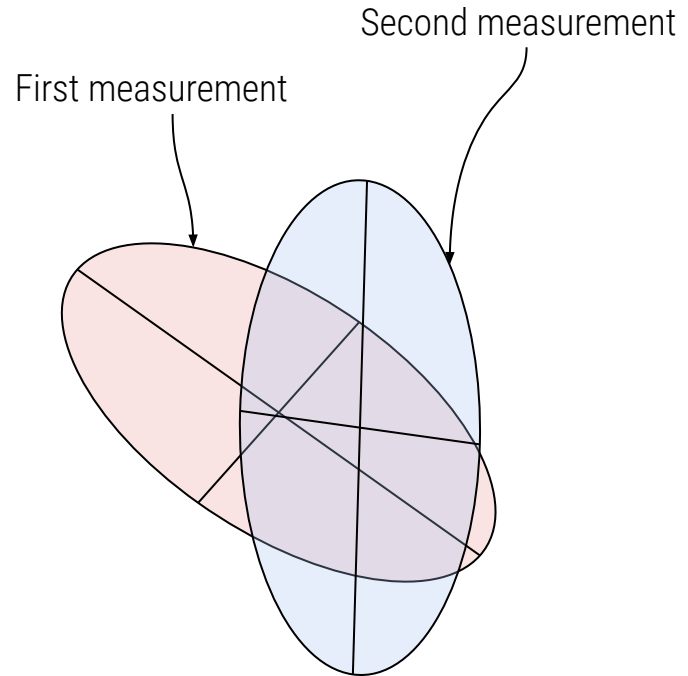
---

- ❑ Most often, the Kalman filter and its modifications are used to build feature maps
- ❑ Each feature is encoded with its own spatial coordinates
- ❑ The estimate of the landmark position is iteratively refined with each new measurement (detection)



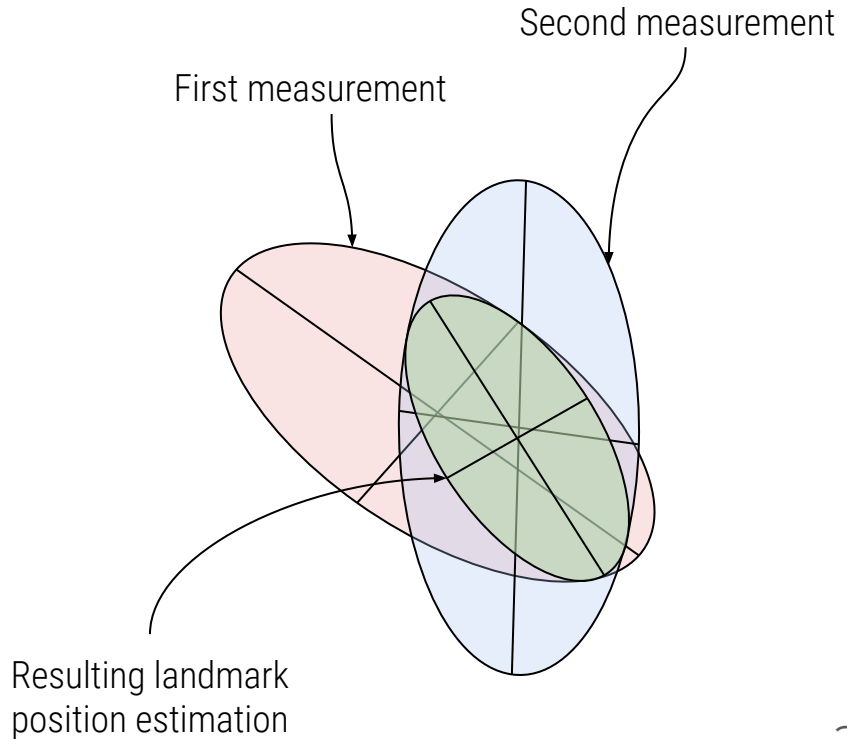
# FEATURE / LANDMARKS MAPS GENERATION

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# FEATURE / LANDMARKS MAPS GENERATION

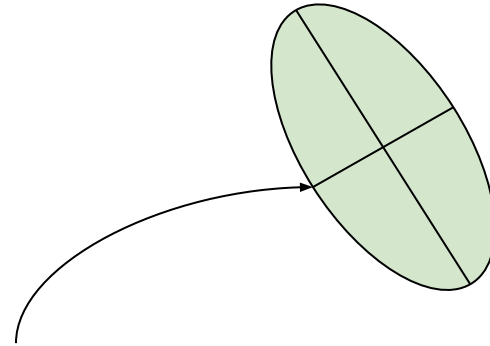
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# FEATURE / LANDMARKS MAPS GENERATION

---

- ❑ Most often, the Kalman filter and its modifications are used to build feature maps
- ❑ Each feature is encoded with its own spatial coordinates
- ❑ The estimate of the landmark position is iteratively refined with each new measurement (detection)



Resulting landmark position estimation

# Occupancy grid maps

---

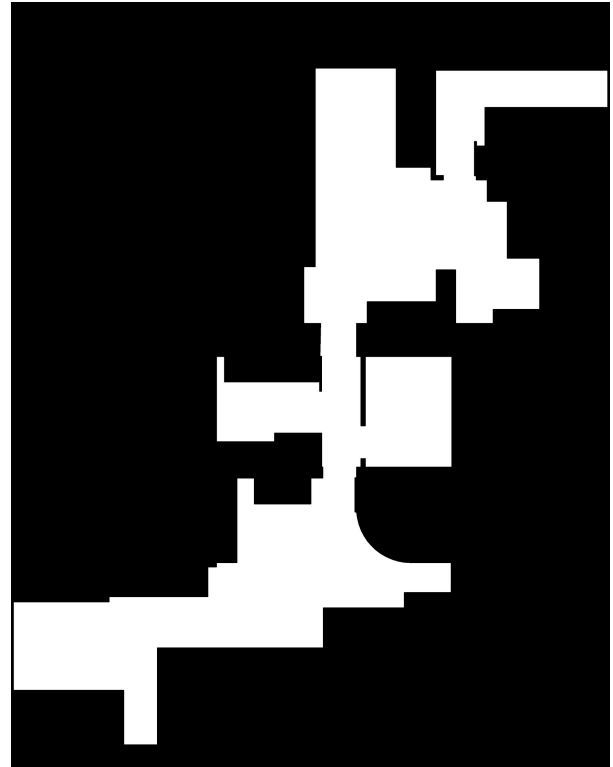
**girafe**  
**ai**

**05**

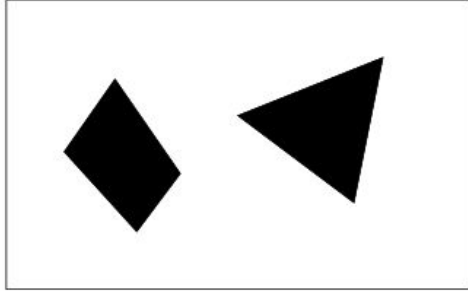
# OCCUPANCY GRID MAPS

---

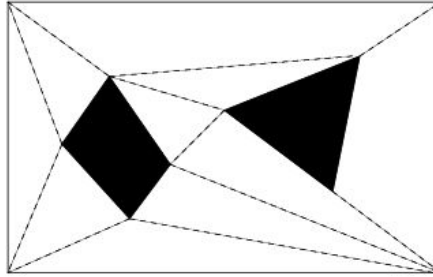
- ❑ The most popular maps format
- ❑ Space is discretized into cells
  - ❑ Usually regular grid is used
- ❑ The probability that the cell is free (passable) or occupied (impassable) is estimated



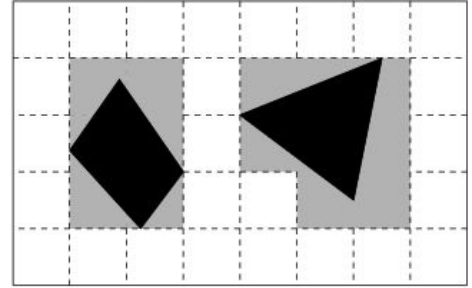
# SPACE DISCRETIZATION APPROACHES



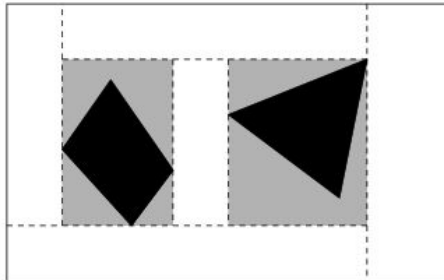
Metric map of the environment



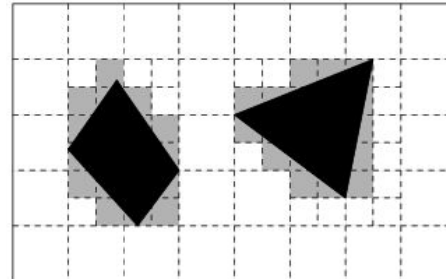
Exact cell decomposition



Regular cell decomposition



Rectangular cell decomposition

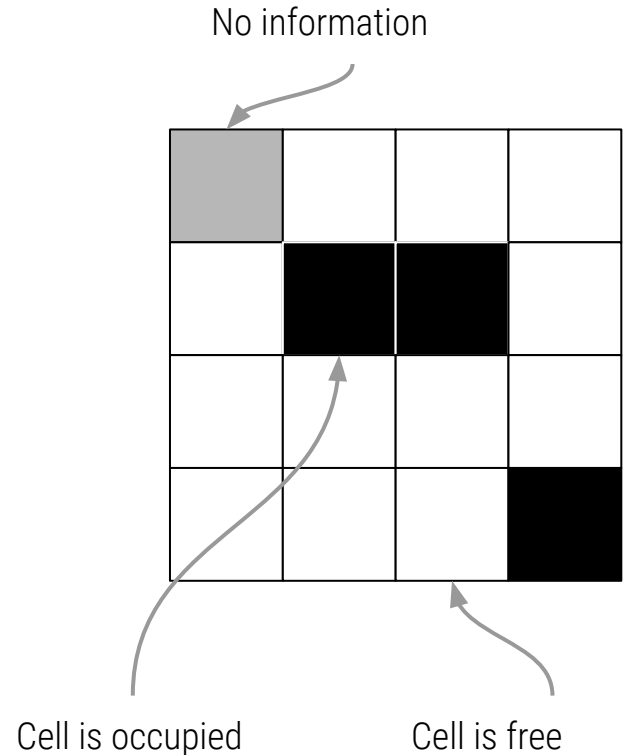


Quadtree decomposition



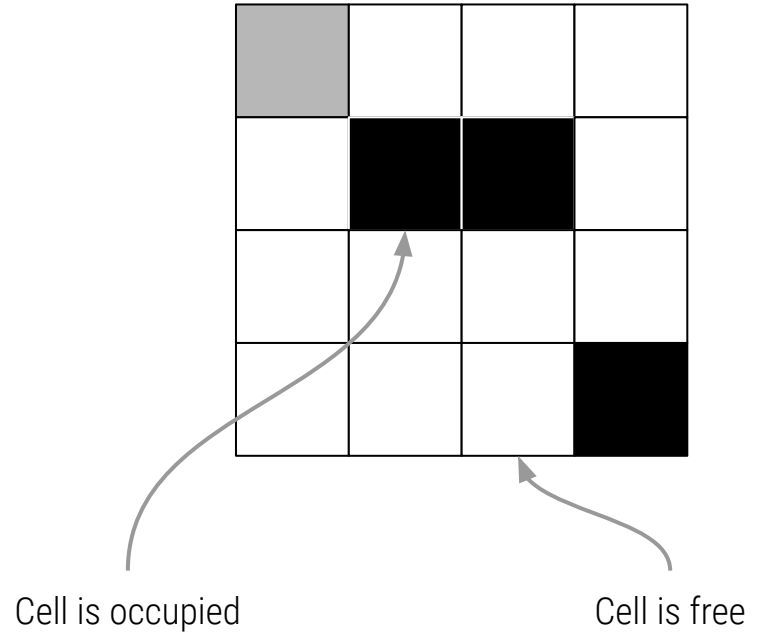
# OCCUPANCY GRID MAPS

- ❑ Each cell is a binary random variable
  - ❑  $p(m_{x,y}) = 1$  – cell is free
  - ❑  $p(m_{x,y}) = 0$  – cell is occupied
  - ❑  $p(m_{x,y}) = 0.5$  – we know nothing on the cell state



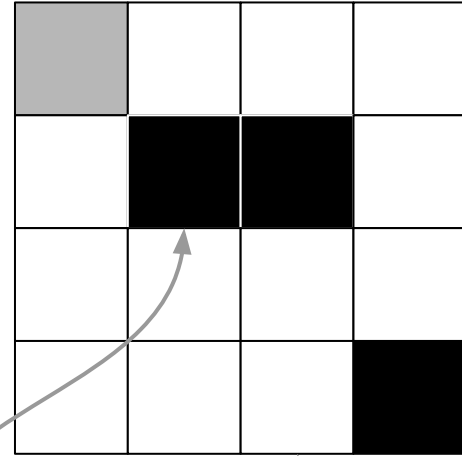
# ASSUMPTIONS

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# ASSUMPTIONS

1. The area described by the cell is entirely occupied or free

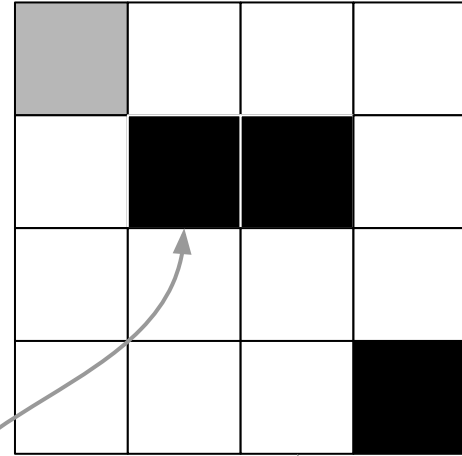


**The whole** cell is occupied

**The whole** cell is free

# ASSUMPTIONS

1. The area described by the cell is entirely occupied or free
2. The world is static

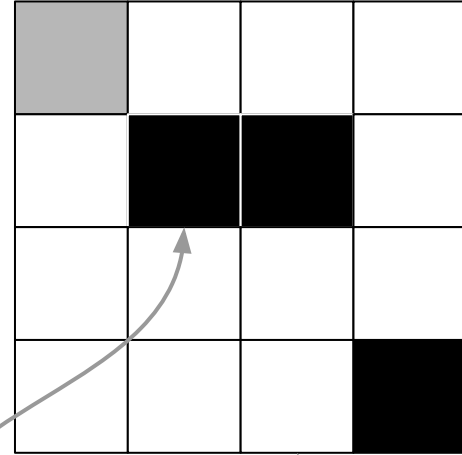


**The whole** cell is **always**  
occupied

**The whole** cell is **always**  
free

# ASSUMPTIONS

1. The area described by the cell is entirely occupied or free
2. The world is static
3. Cells are independent



**The whole cell is always  
occupied (regardless of  
neighboring)**

**The whole cell is always  
free (regardless of  
neighboring)**

# MAP REPRESENTATION

---

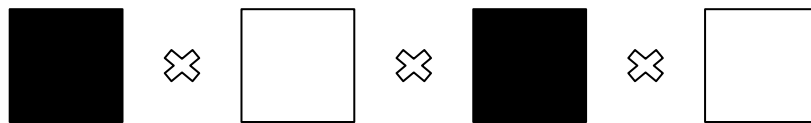
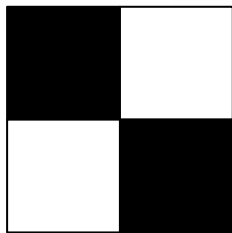
The probability of a map is given by the product of the (independent) probabilities of all its cells.

$$p(\text{map}) = \prod_{x,y} p(m_{x,y})$$

# MAP REPRESENTATION

---

$$p(\text{map}) = \prod_{x,y} p(m_{x,y})$$



# PROBABILISTIC MAPPING PROBLEM

## DEFINITION

---

Given the vector of all consecutive sensors measurements  $\mathbf{z}_{1:t} = \mathbf{z}_0 \dots \mathbf{z}_t$ , and robot (sensor) poses  $\mathbf{x}_{1:t} = \mathbf{x}_0 \dots \mathbf{x}_t$ , it is needed to build / recover the most probable map.

$$p(\text{map} | \mathbf{z}_{1:t}, \mathbf{x}_{1:t}) = \prod_{x,y} p(m_{x,y} | \mathbf{z}_{1:t}, \mathbf{x}_{1:t})$$



# PROBABILISTIC MAPPING PROBLEM

## DEFINITION

---

1. Let's count two numbers:
  - a. How many times have we observed a cell –  $C_{x,y}$
  - b. We will increase or decrease  $O_{x,y}$  by 1 each time we observe an obstacle or a free zone in the cell, respectively
2. Let's calculate the probability of a cell being occupied as:

$$p(m_{x,y}) = \frac{O_{x,y} + C_{x,y}}{2C_{x,y}}$$

# BAYESIAN ESTIMATION

$$p(m_{x,y} | \mathbf{z}_{1:t}, \mathbf{x}_{1:t}) =$$

# BAYESIAN ESTIMATION

$$\begin{aligned} & p(m_{x,y} | \mathbf{z}_{1:t}, \mathbf{x}_{1:t}) = \\ \text{Bayes' rule} \quad & = \frac{p(\mathbf{z}_t | m_{x,y}, \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t}) p(m_{x,y} | \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t})}{p(\mathbf{z}_t | \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t})} = \end{aligned}$$

# BAYESIAN ESTIMATION

$$\begin{aligned} & p(m_{x,y} | \mathbf{z}_{1:t}, \mathbf{x}_{1:t}) = \\ &= \frac{p(\mathbf{z}_t | m_{x,y}, \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t}) p(m_{x,y} | \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t})}{p(\mathbf{z}_t | \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t})} = \\ \text{Markov property} \quad &= \frac{p(\mathbf{z}_t | m_{x,y}, \mathbf{x}_t) p(m_{x,y} | \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t-1})}{p(\mathbf{z}_t | \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t})} = \end{aligned}$$

# BAYESIAN ESTIMATION

$$\begin{aligned} p(m_{x,y} | \mathbf{z}_{1:t}, \mathbf{x}_{1:t}) &= \\ &= \frac{p(\mathbf{z}_t | m_{x,y}, \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t}) p(m_{x,y} | \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t})}{p(\mathbf{z}_t | \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t})} = \\ &= \frac{p(\mathbf{z}_t | m_{x,y}, \mathbf{x}_t) p(m_{x,y} | \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t-1})}{p(\mathbf{z}_t | \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t})} = \\ \text{Bayes' rule} \quad &= \frac{p(m_{x,y} | \mathbf{z}_t, \mathbf{x}_t) p(\mathbf{z}_t | \mathbf{x}_t) p(m_{x,y} | \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t-1})}{p(m_{x,y} | \mathbf{x}_t) p(\mathbf{z}_t | \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t})} = \end{aligned}$$

# BAYESIAN ESTIMATION

$$\begin{aligned} p(m_{x,y} | \mathbf{z}_{1:t}, \mathbf{x}_{1:t}) &= \\ &= \frac{p(\mathbf{z}_t | m_{x,y}, \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t}) p(m_{x,y} | \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t})}{p(\mathbf{z}_t | \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t})} = \\ &= \frac{p(\mathbf{z}_t | m_{x,y}, \mathbf{x}_t) p(m_{x,y} | \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t-1})}{p(\mathbf{z}_t | \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t})} = \\ &= \frac{p(m_{x,y} | \mathbf{z}_t, \mathbf{x}_t) p(\mathbf{z}_t | \mathbf{x}_t) p(m_{x,y} | \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t-1})}{p(m_{x,y} | \mathbf{x}_t) p(\mathbf{z}_t | \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t})} = \\ &= \frac{p(m_{x,y} | \mathbf{z}_t, \mathbf{x}_t) p(\mathbf{z}_t | \mathbf{x}_t) p(m_{x,y} | \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t-1})}{p(m_{x,y}) p(\mathbf{z}_t | \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t})} \end{aligned}$$

Independence  
property

# BAYESIAN ESTIMATION

$$p(\neg m_{x,y} | \mathbf{z}_{1:t}, \mathbf{x}_{1:t}) =$$

Similarly, for the opposite event

$$= \frac{p(\neg m_{x,y} | \mathbf{z}_t, \mathbf{x}_t) p(\mathbf{z}_t | \mathbf{x}_t) p(\neg m_{x,y} | \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t-1})}{p(\neg m_{x,y}) p(\mathbf{z}_t | \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t})}$$

# BAYESIAN ESTIMATION

Let's calculate the probability ratio:

$$\frac{p(m_{x,y} | \mathbf{z}_{1:t}, \mathbf{X}_{1:t})}{p(\neg m_{x,y} | \mathbf{z}_{1:t}, \mathbf{X}_{1:t})} = \frac{\frac{p(m_{x,y} | \mathbf{z}_t, \mathbf{x}_t) p(\mathbf{z}_t | \mathbf{x}_t) p(m_{x,y} | \mathbf{z}_{1:t-1}, \mathbf{X}_{1:t-1})}{p(m_{x,y}) p(\mathbf{z}_t | \mathbf{z}_{1:t-1}, \mathbf{X}_{1:t-1})}}{\frac{p(\neg m_{x,y} | \mathbf{z}_t, \mathbf{x}_t) p(\mathbf{z}_t | \mathbf{x}_t) p(\neg m_{x,y} | \mathbf{z}_{1:t-1}, \mathbf{X}_{1:t-1})}{p(\neg m_{x,y}) p(\mathbf{z}_t | \mathbf{z}_{1:t-1}, \mathbf{X}_{1:t-1})}}$$



# BAYESIAN ESTIMATION

Let's calculate the probability ratio:

$$\frac{p(m_{x,y} | \mathbf{z}_{1:t}, \mathbf{x}_{1:t})}{p(\neg m_{x,y} | \mathbf{z}_{1:t}, \mathbf{x}_{1:t})} = \frac{\frac{p(m_{x,y} | \mathbf{z}_t, \mathbf{x}_t) \cancel{p(\mathbf{z}_t | \mathbf{x}_t)} p(m_{x,y} | \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t-1})}{p(m_{x,y}) \cancel{p(\mathbf{z}_t | \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t})}}}{\frac{p(\neg m_{x,y} | \mathbf{z}_t, \mathbf{x}_t) \cancel{p(\mathbf{z}_t | \mathbf{x}_t)} p(\neg m_{x,y} | \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t-1})}{p(\neg m_{x,y}) \cancel{p(\mathbf{z}_t | \mathbf{z}_{1:t-1}, \mathbf{x}_{1:t})}}}$$

# BAYESIAN ESTIMATION

Let's calculate the probability ratio:

$$\frac{p(m_{x,y}|\mathbf{z}_{1:t}, \mathbf{x}_{1:t})}{p(\neg m_{x,y}|\mathbf{z}_{1:t}, \mathbf{x}_{1:t})} = \frac{p(m_{x,y}|\mathbf{z}_t, \mathbf{x}_t)}{1 - p(m_{x,y}|\mathbf{z}_t, \mathbf{x}_t)} \frac{p(m_{x,y}|\mathbf{z}_{1:t-1}, \mathbf{x}_{1:t-1})}{1 - p(m_{x,y}|\mathbf{z}_{1:t-1}, \mathbf{x}_{1:t-1})} \frac{1 - p(m_{x,y})}{p(m_{x,y})}$$

# BAYESIAN ESTIMATION

Let's calculate the probability ratio:

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The ratio of the probabilities of cell occupancy and freedom under the condition of new measurements

Recursive member (the same ration for previous measurement / moment of time)

Ratio of prior probabilities (e.g.  $p(m_{x,y})=0.5$  if we didn't know anything about the map at the beginning)

# LOGARITHMIC ODDS

---

Let's call an odds:

$$\text{odds}(X) = \frac{p(X)}{1 - p(X)}$$

Let's call a logarithmic odds:

$$\text{logodds}(X) = \log \frac{p(X)}{1 - p(X)}$$

# MAPPING WITH AN INVERSE MODEL IN LOGARITHMIC FORM

---

Let's calculate the logarithmic odds from:

$$\frac{p(m_{x,y}|\mathbf{z}_{1:t}, \mathbf{x}_{1:t})}{p(\neg m_{x,y}|\mathbf{z}_{1:t}, \mathbf{x}_{1:t})} = \frac{p(m_{x,y}|\mathbf{z}_t, \mathbf{x}_t)}{1 - p(m_{x,y}|\mathbf{z}_t, \mathbf{x}_t)} \frac{p(m_{x,y}|\mathbf{z}_{1:t-1}, \mathbf{x}_{1:t-1})}{1 - p(m_{x,y}|\mathbf{z}_{1:t-1}, \mathbf{x}_{1:t-1})} \frac{1 - p(m_{x,y})}{p(m_{x,y})}$$

And we get:

$$\text{logodds}(m_{x,y}|\mathbf{z}_{1:t}, \mathbf{x}_{1:t}) = \text{logodds}(m_{x,y}|\mathbf{z}_t, \mathbf{x}_t) + \text{logodds}(m_{x,y}|\mathbf{z}_{1:t-1}, \mathbf{x}_{1:t-1}) - \text{logodds}(m_{x,y})$$

# MAPPING WITH AN INVERSE MODEL IN LOGARITHMIC FORM


---

Let's calculate the logarithmic odds from:

$$\frac{p(m_{x,y}|\mathbf{z}_{1:t}, \mathbf{x}_{1:t})}{p(\neg m_{x,y}|\mathbf{z}_{1:t}, \mathbf{x}_{1:t})} = \frac{p(m_{x,y}|\mathbf{z}_t, \mathbf{x}_t)}{1 - p(m_{x,y}|\mathbf{z}_t, \mathbf{x}_t)} \frac{p(m_{x,y}|\mathbf{z}_{1:t-1}, \mathbf{x}_{1:t-1})}{1 - p(m_{x,y}|\mathbf{z}_{1:t-1}, \mathbf{x}_{1:t-1})} \frac{1 - p(m_{x,y})}{p(m_{x,y})}$$

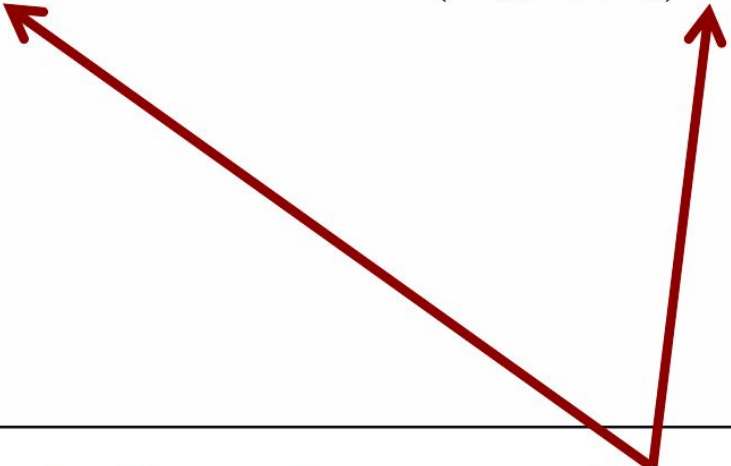
And we get:

$$\text{logodds}(m_{x,y}|\mathbf{z}_{1:t}, \mathbf{x}_{1:t}) = \text{logodds}(m_{x,y}|\mathbf{z}_t, \mathbf{x}_t) + \text{logodds}(m_{x,y}|\mathbf{z}_{1:t-1}, \mathbf{x}_{1:t-1}) - \text{logodds}(m_{x,y})$$

 **Inverse measurement / observation model**

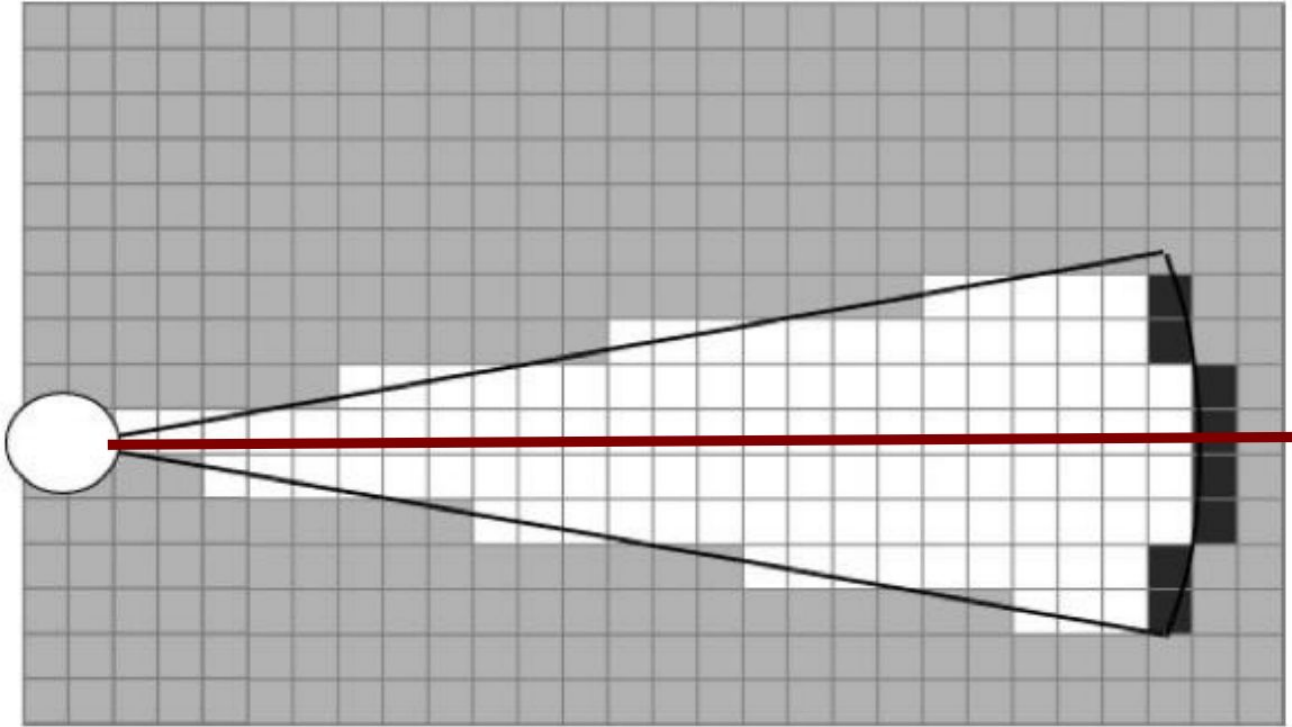
**occupancy\_grid\_mapping( $\{l_{t-1,i}\}, x_t, z_t$ ):**

```
1:   for all cells  $m_i$  do
2:       if  $m_i$  in perceptual field of  $z_t$  then
3:            $l_{t,i} = l_{t-1,i} + \text{inv\_sensor\_model}(m_i, x_t, z_t) - l_0$ 
4:       else
5:            $l_{t,i} = l_{t-1,i}$ 
6:       endif
7:   endfor
8:   return  $\{l_{t,i}\}$ 
```



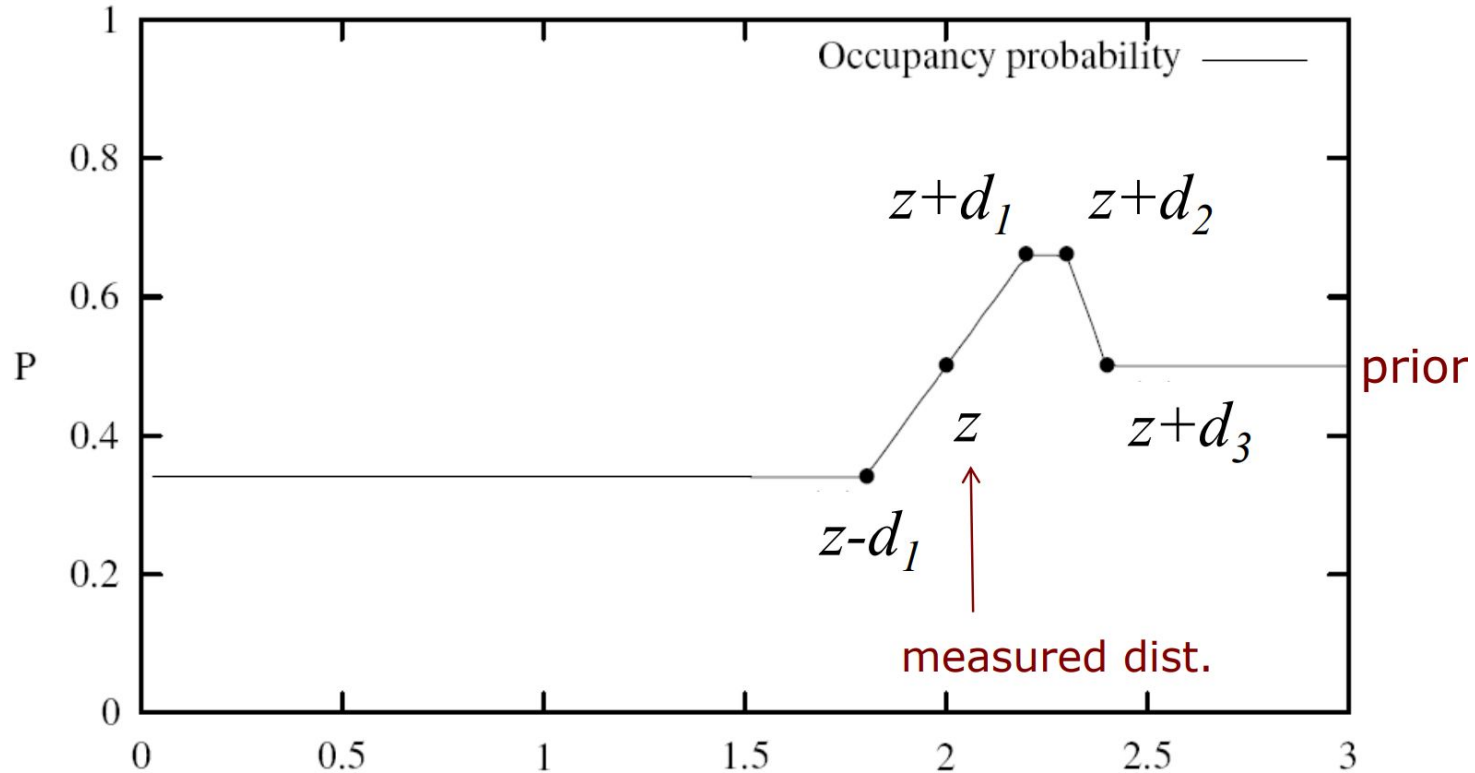
**highly efficient, we only have to compute sums**

# EXAMPLE OF A SIMPLE SONAR MEASUREMENT MODEL



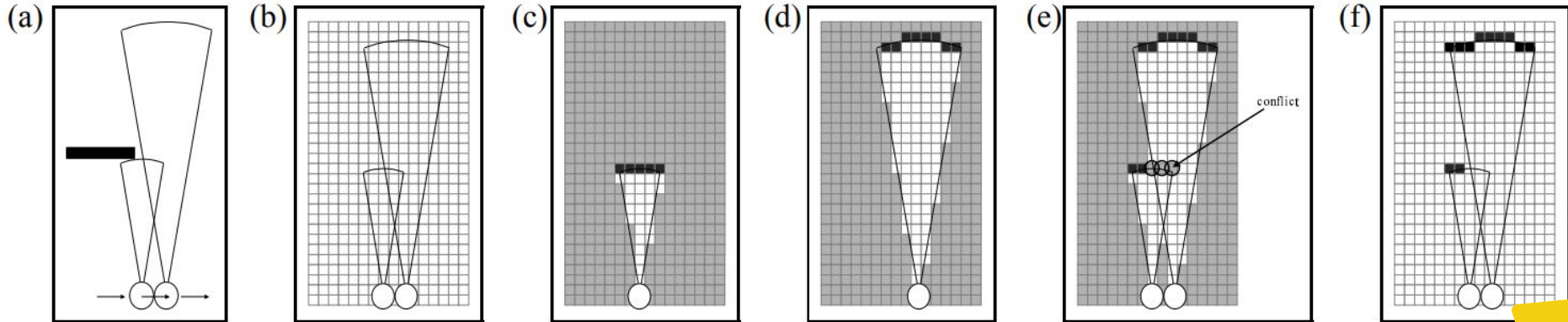


# EXAMPLE OF A SIMPLE SONAR MEASUREMENT MODEL



# WHAT IS THE DISADVANTAGE OF MAPPING WITH A INVERSE MODEL

- ❑ The inverse model considers the cells of the map as **independent** random variables
- ❑ This approach fails to explain contradicting data



# MAPPING WITH FORWARD MODEL

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- ❑ In contrast to mapping with an inverse model, we will consider the search for the **entire map** as an optimization problem in the space of all possible maps
- ❑ We will try to find such a *map* that maximizes the probability of all obtained measurements:

$$\hat{map} = \arg \max_{map} p(z|x, map)$$

# ADDITIONAL RESOURCES



1. [Probabilistic Robotics](#), Chapter 9
2. [Topological Mapping](#). Benjamin Kuipers
3. [Robot Mapping](#). Gian Diego Tipaldi, Wolfram Burgard
4. [Learning Occupancy Grids with Forward Models](#). Sebastian Thrun

# Thanks for attention!

Questions? Additions? Welcome!

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