MSAI Probability Week 2

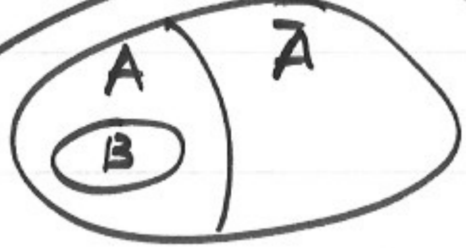
- Conditional Probability
- Maximum Likelihood Estimate
- Naïve Bayes Classifier

P2) 1) $p(B|A)+p(B|\bar{A}) \stackrel{?}{=} 1 \leftarrow$ False

$$p(B|A)p(A) + p(B|\bar{A})p(\bar{A}) = p(B)$$

$$p(A,B) + p(B,\bar{A}) = p(B)$$

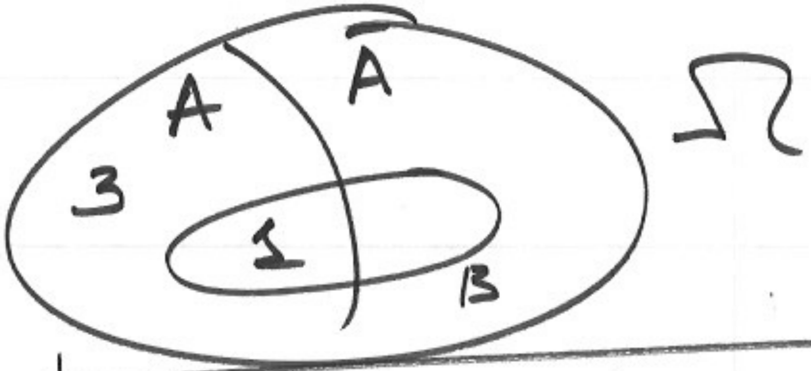
Pick $B=\Omega$ $\quad \underbrace{p(\Omega|A)}_{1}+\underbrace{p(\Omega|\bar{A})}_{1} = 2$ False



2) $p(B|A)+p(\bar{B}|\bar{A}) = 1$

$B \subset A \rightarrow \cancel{p}(\bar{B}|\bar{A}) = p(\bar{A})$

Pick $B=A \rightarrow \underbrace{p(A|A)}_{1}+\underbrace{p(\bar{A}|\bar{A})}_{1} = 2$



$$\boxed{\underbrace{p(B|A)}_{\frac{1}{3+1}} + \underbrace{p(\bar{B}|A)}_{\frac{3}{3+1}} = 1}$$

Bayes' formula

$$p(A,B) = \boxed{p(A|B)\cdot p(B) = p(B|A)\cdot p(A)}$$

$$p(A|B) = \frac{p(B|A)\cdot p(A)}{p(B)}$$

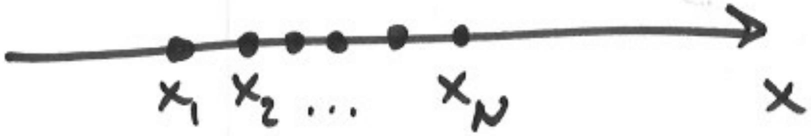
Maximum Likelihood Estimation

$-\mu, \sigma$

$$x \sim \mathcal{N}(\mu, \sigma)$$

$$(2\pi\sigma^2)^{-1/2}$$

$$p(x \mid \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$



assume

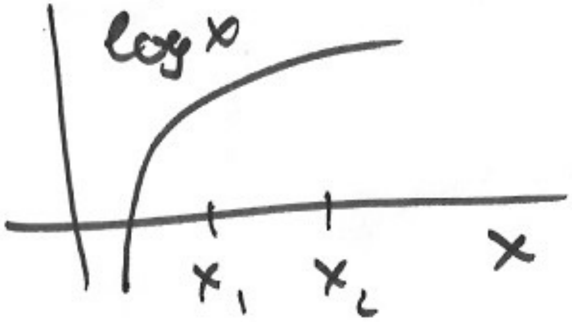
$\tilde{\mu}, \tilde{\sigma}$

$$p(\mu, \sigma \mid \vec{x} = (x_1, x_2, \ldots)) =$$

$$= \frac{p(\vec{x} \mid \mu, \sigma) \cdot p(\mu, \sigma)}{p(\vec{x})}$$

$$\text{posterior (of } \mu, \sigma\text{)} = \frac{\text{likelihood} \cdot \text{prior}}{\text{evidence}}$$

$$\arg\max_{\mu,\sigma} \mathcal{L}(\mu,\sigma) = \arg\max_{\mu,\sigma} \underbrace{\log \mathcal{L}(\mu,\sigma)}$$



$\log \prod_i = \sum_i \log$

$x_2 > x_1$

$\to \log x_2 > \log x_1$

$$\log \mathcal{L} = \sum_{i=1}^{N} \log(2\pi\sigma^2)^{-1/2} + \left(-\frac{(x_i-\mu)^2}{2\sigma^2}\right)$$

$$= -\frac{N}{2}\log(2\pi\sigma^2) - \sum_{i=1}^{N} \frac{1}{2\sigma^2}(x_i-\mu)^2$$

$$\frac{\partial}{\partial\mu}(\dots) = 0 - \sum \frac{1}{\cancel{2}\sigma^2}\cancel{2}(x_i-\mu)\cdot(-1) =$$

$$= \sum_i \frac{1}{\sigma^2}(x_i-\mu) = 0 \to \sum_i x_i = \mu\cdot N$$

$$\tilde{\mu} = \frac{1}{N}\sum_i x_i$$

$$\dots \to \sigma = \sqrt{\frac{1}{N}\sum(x_i-\mu)^2}$$