Probability theory

Lecture 3: Probability distribution

Maksim Zhukovskii

MIPT

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2. $\lim_{x\to-\infty} F(x) = 0$, $\lim_{x\to\infty} F(x) = 1$,
3. F is right-continuous.

The proof

From distribution function to probability

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- F *is the distribution function of* P*,*
- *for all* $-\infty \leq a < b < \infty$*,*

$$\mathsf{P}((a, b]) = F(b) - F(a).$$

Examples

$$F(x) = \begin{cases} 0, & x < 0; \\ x, & 0 \leq x \leq 1; \\ 1, & x > 1. \end{cases}$$

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$$\text{for } A \in \mathcal{B}(\mathbb{R}), \quad \mathrm{P}(A) = \begin{cases} 0, & c \notin A, \\ 1, & c \in A, \end{cases}$$

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$$F(x) = \sum_{k\colon x_k < x} p_k$$

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$F : \mathbb{R} \in [0, 1]$,

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is a distribution function.

Proof

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  If $\int_B p(x)dx$ exists,
  then $\mathrm{P}(B) = \int_B p(x)dx$.

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- Cauchy distrib. with parameter $\theta > 0$:

  $p(t) = \frac{\theta}{\pi(t^2+\theta^2)}$.

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- P — probability measure on $(\Omega, \mathcal{F})$ — distribution
- $F : \mathbb{R}^n \to [0,1]$, $F(x_1, \ldots, x_n) = \mathrm{P}((-\infty, x_1] \times \ldots \times (-\infty, x_n])$ — distribution function

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  $F(x_1, \ldots, x_{i-1}, b, x_{i+1}, \ldots, x_n) -$
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Properties

1. For all $a_1 < b_1, \ldots, a_n < b_n$,
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2. $$\lim_{x_1\to\infty,\ldots,x_n\to\infty} F(x_1, \ldots, x_n) = 1,$$
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$$\Delta^1_{a_1,b_1} \ldots \Delta^n_{a_n,b_n} F(x_1, \ldots, x_n) = \mathrm{P}((a_1, b_1] \times \ldots \times (a_n, b_n]).$$

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