1) A_1, A_2 - indep. $\quad \cancel{A_2}, B$

$$p(A_1 \cap A_2 \mid B) \overset{?}{=} p(A_1 \mid B) \cdot p(A_2 \mid B)$$

True

$$p(A_1 \cap A_2) = p(A_1 \mid A_2) \cdot p(A_2)$$

product rule

2) A, B - ind. $\quad \bar{A}, \bar{B}$ - ind ?

$$p(\bar{A} \cap \bar{B}) = p(\overline{A \cup B}) = 1 - p(A \cup B)$$

...

$$\cancel{p(\bar{A} \mid \bar{B})} \cdot p(\bar{B})$$

$$p(\bar{A})\, p(\bar{B})$$

3) t - test, $\quad s$ - sick $\quad \sum_{s, \bar{s}} p(t, s)$

$\quad$ 0, 1 $\qquad$ 0, 1 $\qquad$ marginalising

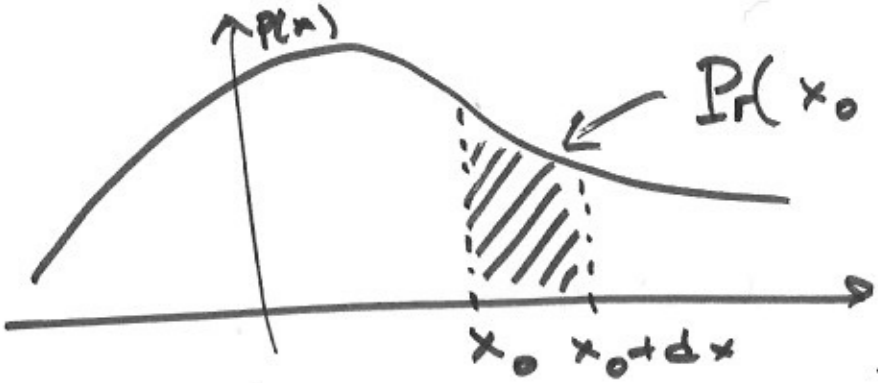
$$p(s \mid t) = \frac{p(t \mid s) \cdot p(s)}{p(t)}$$

$p(t \mid s) = 0.99 \qquad p(s) = 0.01$

$p(\bar{t} \mid \bar{s}) = 0.99$

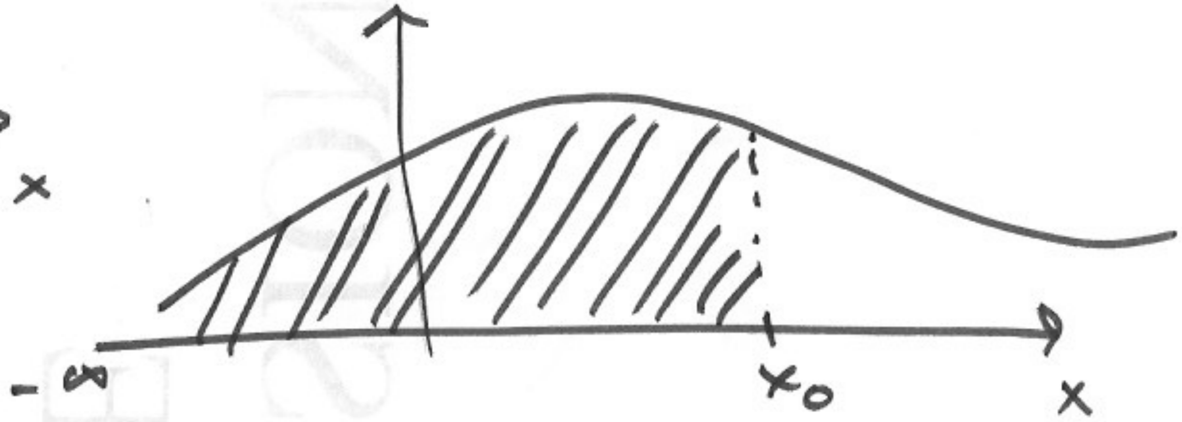
$$= \frac{0.99 \cdot 0.01}{0.99 \cdot 0.01 +} = \frac{1}{2}$$

$$p(t) = p(t \mid s) \cdot p(s) + \underbrace{p(t \mid \bar{s})}_{0.01} \cdot \underbrace{p(\bar{s})}_{0.99}$$



$Pr(x_0 \le x \le x_0 + dx) = p(x) \cdot dx$

propability density function (pdf)



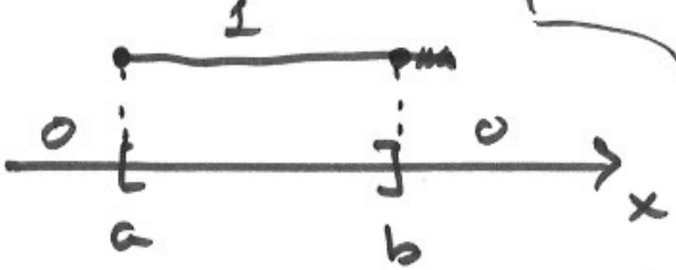
distribution function = cumulative probability density $\underline{P}(x_0) = \int_{-\infty}^{x_0} p(x)\,dx$

$Pr(x \le x_0)$

Problem 2

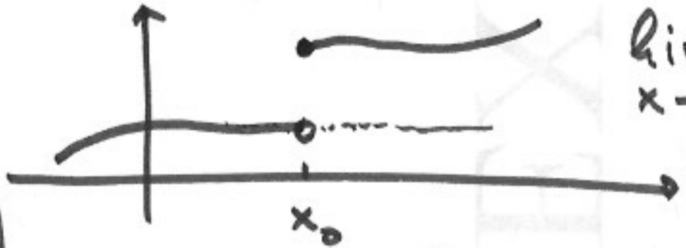
$I_{[a,b]}(t)$

indicator function



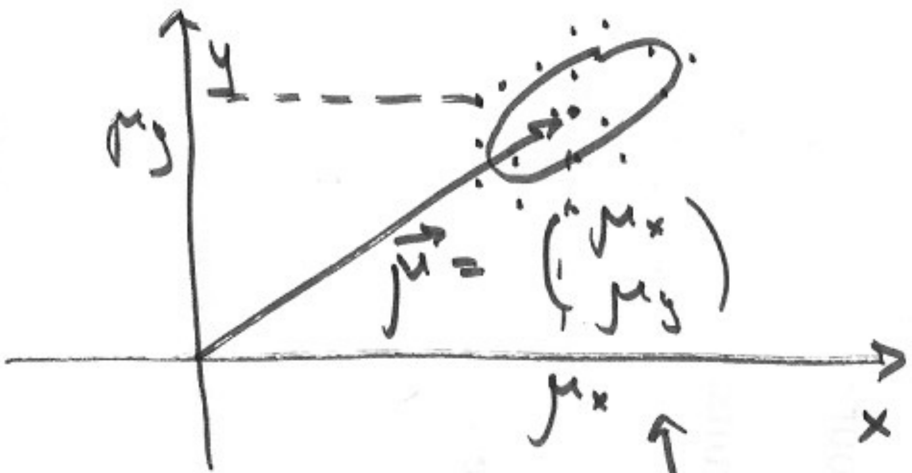
1) $Pr([a,b]) = F(b) - F(a-)$ True

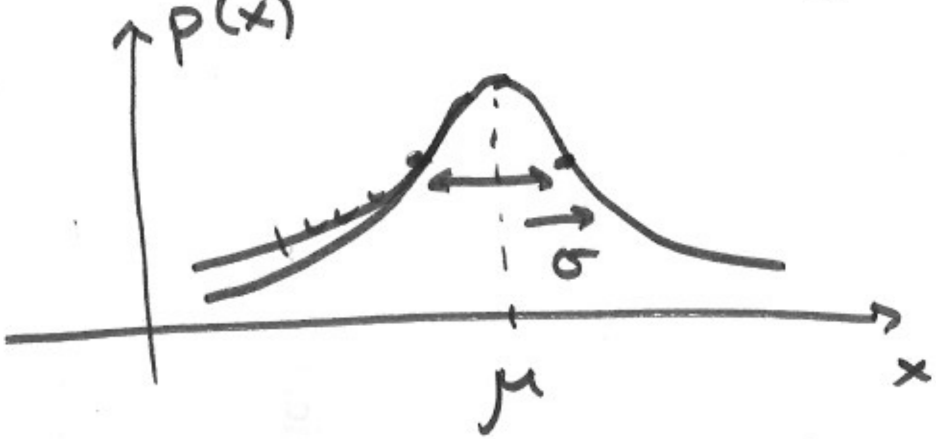
$\lim_{x \to a^-} F(x)$



4) $Pr(\{x\}) = F(x) - F(x-)$ True

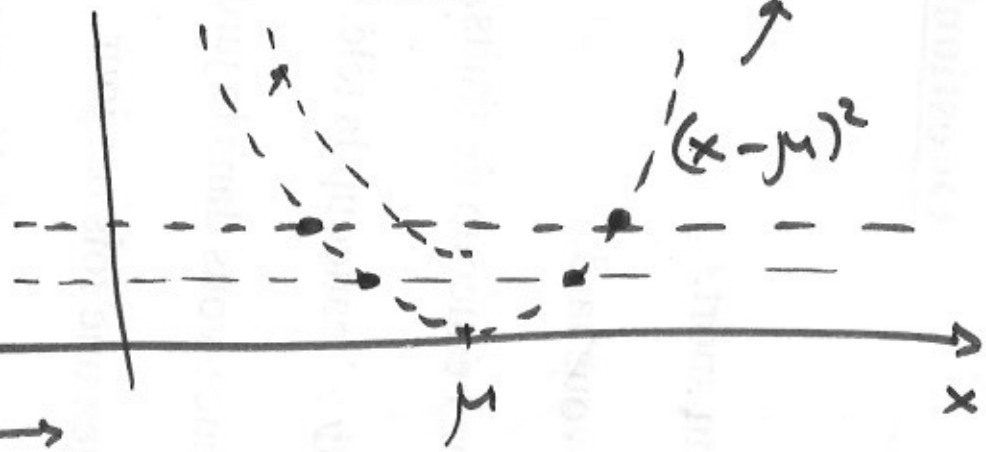
2-dimensional Normal distribution

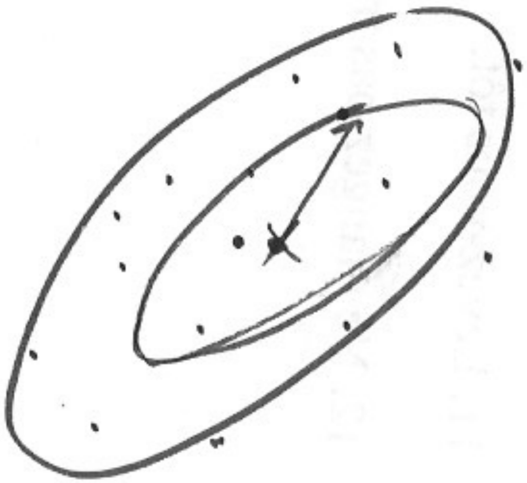




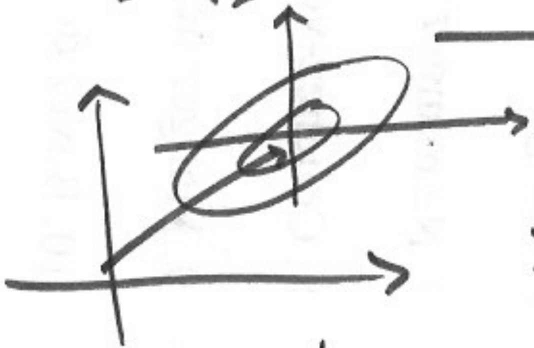
$x, y, z \to x_1, x_2,$

$$p(x \mid \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$





level sets

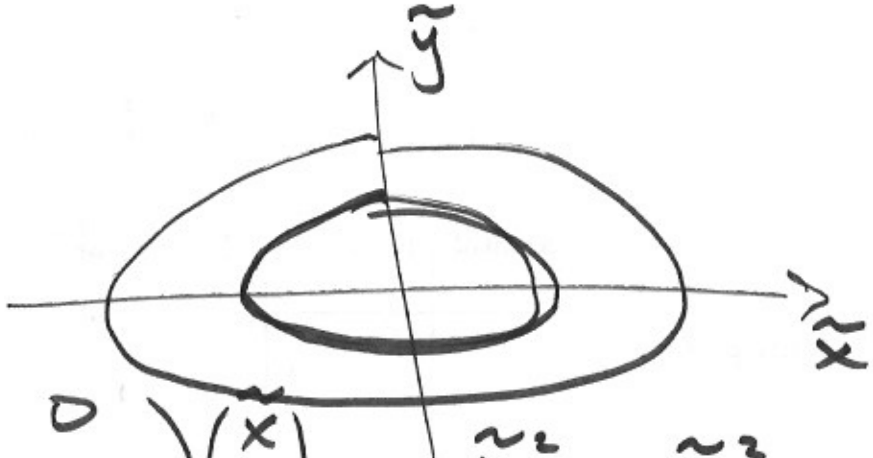


$$x, y \to \tilde{x} = x - \mu_x$$
$$\tilde{y} = y - \mu_y$$

$\mu_x = \frac{1}{N} \sum_i x_i$, $\mu_y = \frac{1}{N} \sum_i y_i$

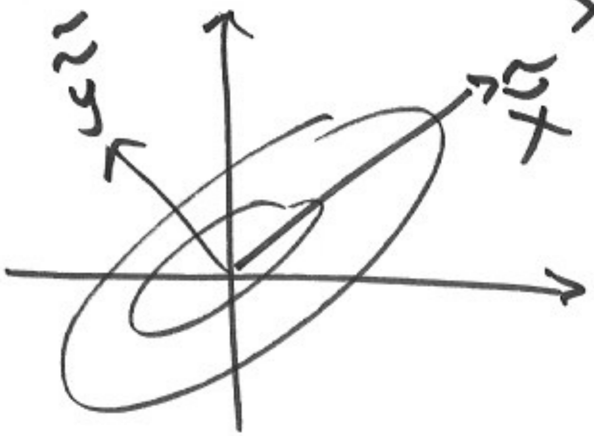
$$-\left(\frac{(x-\mu_x)^2}{2\sigma_x^2}+\frac{(y-\mu_y)^2}{2\sigma_y^2}\right)$$

$f(x,y)$



$$(\tilde x\ \tilde y)\begin{pmatrix}\frac{1}{2\sigma_x^2}&0\\0&\frac{1}{2\sigma_y^2}\end{pmatrix}\begin{pmatrix}\tilde x\\ \tilde y\end{pmatrix}=\frac{\tilde x^2}{2\sigma_x^2}+\frac{\tilde y^2}{2\sigma_y^2}$$

Quadratic form



$$\tilde x^2+2\tilde x\cdot\tilde y+3\tilde y^2=\tilde x^2+\tilde x\tilde y+\tilde y\tilde x+3\tilde y^2$$

cross-term

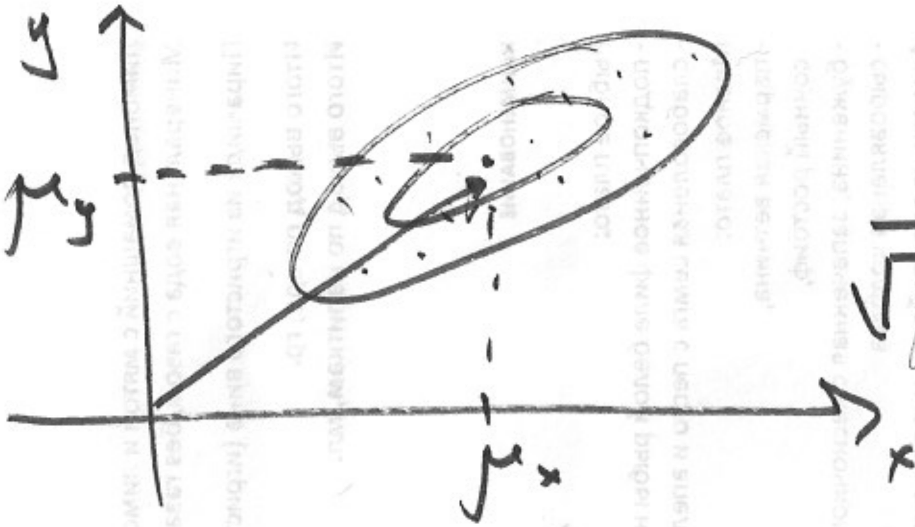
$$=(\tilde x\ \tilde y)\begin{pmatrix}1&1\\1&3\end{pmatrix}\begin{pmatrix}\tilde x\\ \tilde y\end{pmatrix}$$

(in general $\vec x^T A\vec x$)

Symmetric

$\tilde{\tilde x}, \tilde{\tilde y}$

I need a positive-definite quadratic form



$\cancel{\frac{1}{\sqrt{(2\pi)^n}}}$

$$\Sigma = \begin{pmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{pmatrix}$$

covariance matrix

$$p(x,y) = \frac{1}{\sqrt{(2\pi)^2 \, |\Sigma|}} \exp\left(\left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix} \right)^T \Sigma^{-1} \left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix} \right) \right)$$

precision matrix

$$= \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2(1-\rho^2)} \cdot \left[\frac{(x-\mu_x)^2}{\sigma_x^2} + \frac{(y-\mu_y)^2}{\sigma_y^2} - \frac{2\rho(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y} \right] \right)$$

$$p(\vec{x} \mid \vec{\mu}, \Sigma) =$$

$$= \frac{1}{\sqrt{(2\pi)^n \underbrace{|\Sigma|}}} \exp\left(-\frac{1}{2}(\vec{x}-\vec{\mu})^T \Sigma^{-1} (\vec{x}-\vec{\mu})\right)$$

$$\underbrace{\det \Sigma}_{\prod_i \sigma_i^2}$$

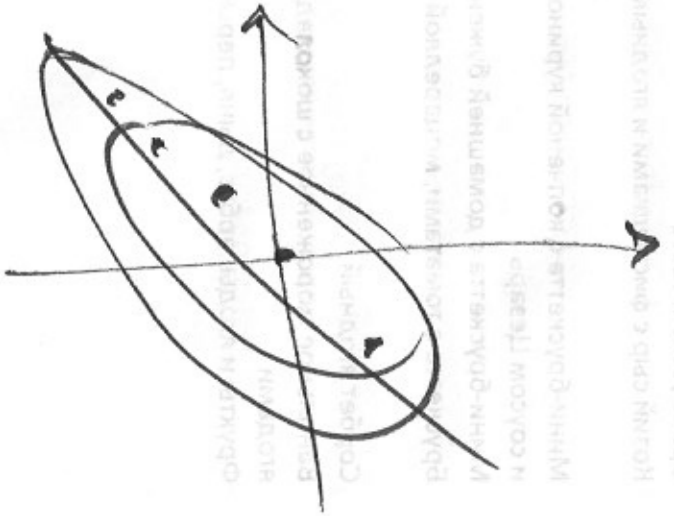
$$\cancel{\begin{pmatrix} x \\ y \end{pmatrix}} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{\sigma^2} \end{pmatrix}$$

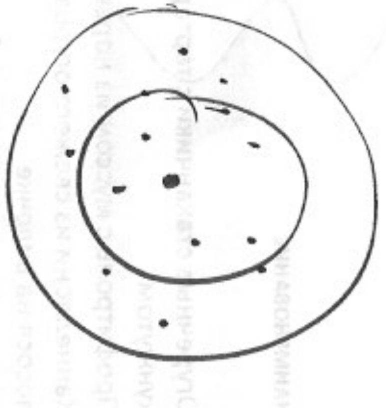
ρ – Pearson correlation coefficient

$$\rho = \frac{\sum_i (x_i - \bar{x}) \cdot (y_i - \bar{y})}{\sqrt{\left(\sum_i (x_i - \bar{x})^2\right) \cdot \left(\sum_i (y_i - \bar{y})^2\right)}}$$

$$-1 \leq \rho \leq 1$$



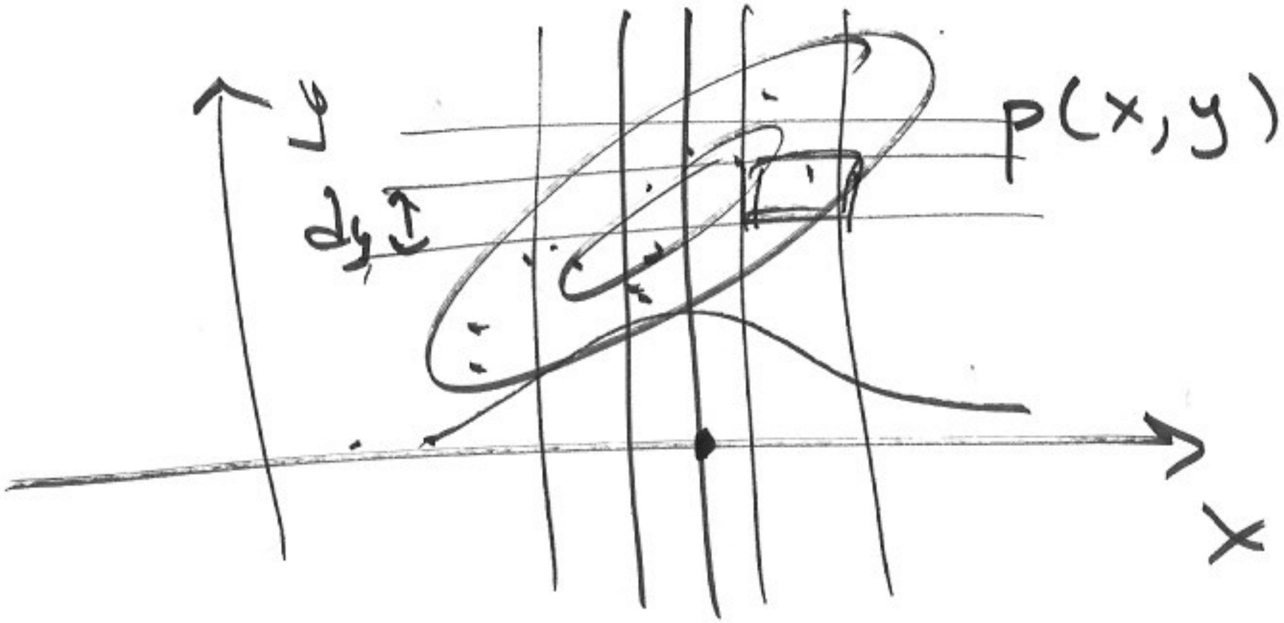
-1



ρ

0

1



1) Marginalise over y

$$\int_{-\infty}^{+\infty} dy \cdot p(x,y) = p(x) \quad \text{(1D)}$$

also normal

2) Take conditional

$$p(x \mid y) = \frac{p(x,y)}{p(y)}$$

(1D)
also normal

$\int dx \, p(x,y)$