MSAI Statistics Home Assignment 3

Problem 1. Let $X_1, \ldots, X_n \sim \text{Poisson}(\lambda)$. Find:

1. (1 point) the method of moments estimator of λ
2. (1 point) the maximum likelihood estimator of λ
3. (1 point) and the Fisher information $I(\lambda)$

Problem 2. Let $X_1, \ldots, X_n \sim \text{Uniform}(a, b)$ where a and b are unknown parameters such that $a < b$.

1. (1 point) Find the method of moments estimators for a and b
2. (1 point) Find the maximum likelihood estimators for a and b

Problem 3. (1 point) Prove that KL-divergence is non-negative: that for any two probability densities $p(x)$ and $q(x)$, $\text{KL}(p||q) \geqslant 0$. Hint: Jensen's inequality.

Problem 4.

1. (2 points) Show that Gamma distribution belongs to the exponential family. Find the sufficient statistics and natural parameter.
2. (2 points) Show that Laplace belongs to the exponential family. Find the sufficient statistics and natural parameter.

Problem 5. Let $X_1, \ldots, X_n \sim \mathcal{N}(\mu, 1)$. Let $\theta = e^{\mu}$ and let $\hat{\theta} = e^{\overline{X}}$ be the MLE. Create a dataset (using $\mu = 5$ and `numpy.random.seed(42)`) consisting of $n = 100$ observations. Use:

1. (1 bonus point) delta method
2. (1 bonus point) parametric bootstrap
3. (2 bonus point) nonparametric bootstrap

to get $\hat{\text{se}}$ and a 95-percent confidence interval for θ.