MSAI Statistics & Probability – Week 2 Seminar & HW

Problem 1: There are R red and G green balls in a box. Alice and Bob pick balls from the box in turns (one ball at a time). The one who is the first to pick a red ball is the winner. Find the probability that Alice wins if she is the first to make a move and 1) they do not put balls back in the box 2) they do put them back. Write a numeric answer for $R = 4$ and $G = 5$.

Problem 2: Are the following statements true?

1. $p(B|A) + p(B|\bar{A}) = 1$
2. $p(B|A) + p(\bar{B}|\bar{A}) = 1$

Problem 3: Two subsets, A_1 and A_2 (they may coincide) are chosen randomly from $\{1, \ldots, n\}$. Find the probability that $|A_1| = l_1$ and $|A_2| = l_2$ under the condition that $A_1 \cap A_2 = \varnothing$. Give a numeric answer for $n = 6,\ l_1 = 2,\ l_2 = 3$.

Problem 4: There are R red and G green balls in a box. Alice is picking balls from the box, one at a time. A_k is the event that, at time k (k-th pick), she picks a red ball. Are the events $A_1, \ldots, A_n$ independent? Alice may be picking balls with replacement or without replacement – if Alice chooses to pick, say, without replacement – then she's making all the picks without replacement. Same for with replacement – provide the answer for both cases.
You may (or may not) need the following notion in your solution: A set of more than 2 events is called mutually (or collectively) independent, if all pairs of events in it are pairwise independent.

Problem 5: Every edge (independently of all others) of K_n – a complete graph on n vertices, is colored in one of k colors. For a set of vertices S of the graph, let A_S be the event that all the edges having both vertices in S have the same color. What are the conditions on S, T for the events A_S and A_T to be independent?