Problem 3

$\xi \sim Pois(\lambda)$ $\quad \xi = 0, 1, 2, \dots$

$k = 1, 2, 3, \dots$

$$\mathbb{E}\left(\binom{\xi}{k}\right)$$

$$C_\xi^k = \frac{\xi!}{k!(\xi-k)!}$$

ξ

$$p(\xi = x_n) = \frac{\lambda^{x_n}}{x_n!} e^{-\lambda}$$

$$\sum_{n=0}^{\infty} \cancel{k/k} \binom{n}{k} \cdot p(n) \quad \frac{\lambda^n}{n!} e^{-\lambda}$$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$\binom{\xi}{k} \equiv \binom{n}{k} = \frac{n!}{k!\,(n-k)!} = $$

$n \notin \mathbb{N}$ / $n \in \mathbb{R}$

$n! = n(n-1)\ldots 2 \cdot 1$

$k < n$

$$\binom{\pi}{3}$$

$$\frac{n(n-1)\ldots\ldots(n-k+1)\underline{(n-k)}\\ \underline{(n-k-1)\ldots 2\cdot 1}}{(n-k)!}$$

$$= \frac{n(n-1)\ldots(n-k+1)}{k!}$$

$$\binom{\pi}{3} = \frac{\pi(\pi-1)(\pi-2)}{3!}$$

$n-k+1=$
$k=3$

Gamma-function

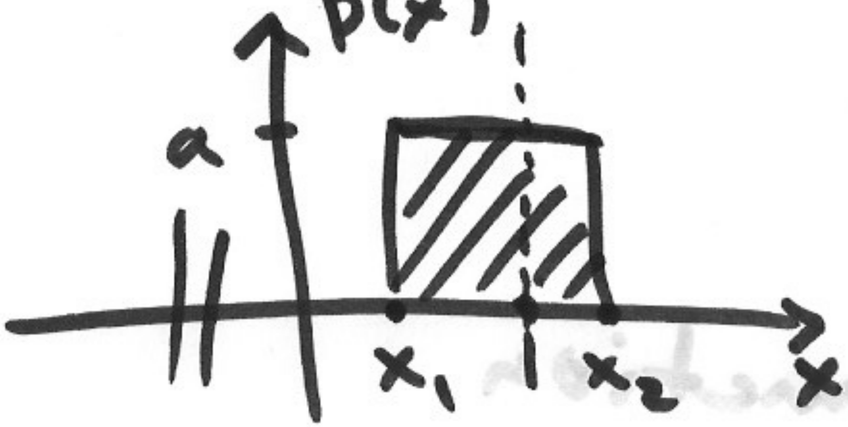
$$\Gamma(x+1) = x!$$
$$x \in \mathbb{N}$$

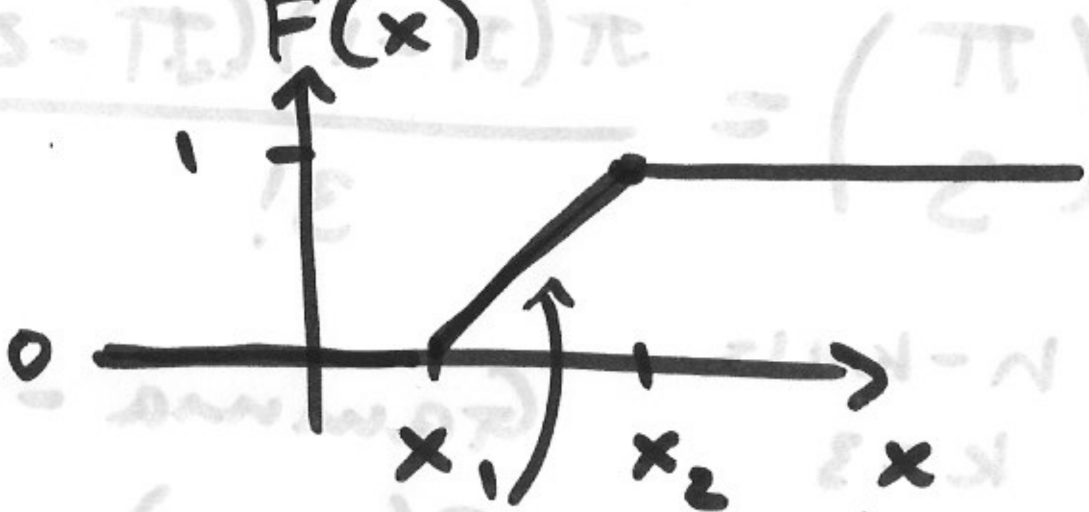
Problem 4

$$F(x) = \begin{cases} 0, & x < -2 \\ 1/5, & -2 \leq x < 1 \\ x^2/4, & 1 \leq x < 2 \\ 1, & x \geq 2 \end{cases}$$

$$F(x_0) = Pr(x \leq x_0) = \int_{-\infty}^{x_0} p(x)\,dx$$

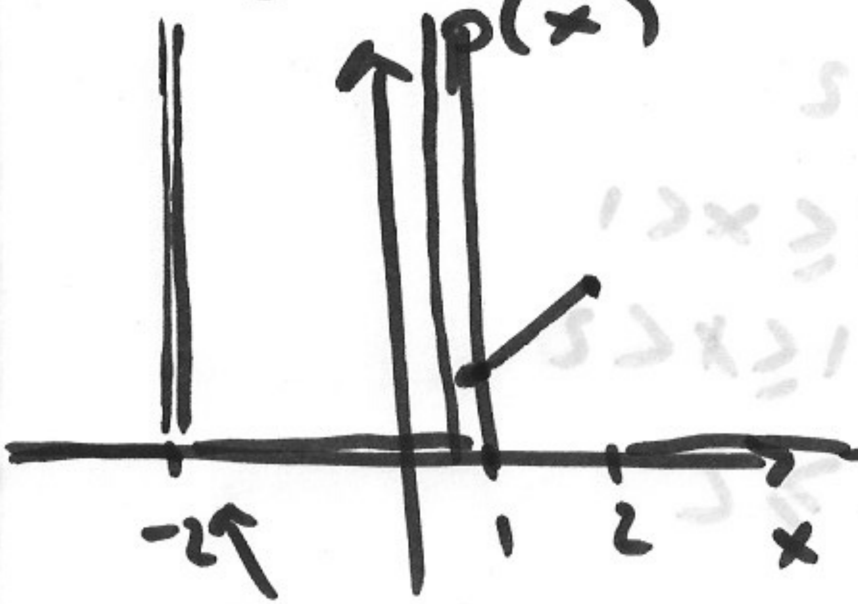
3

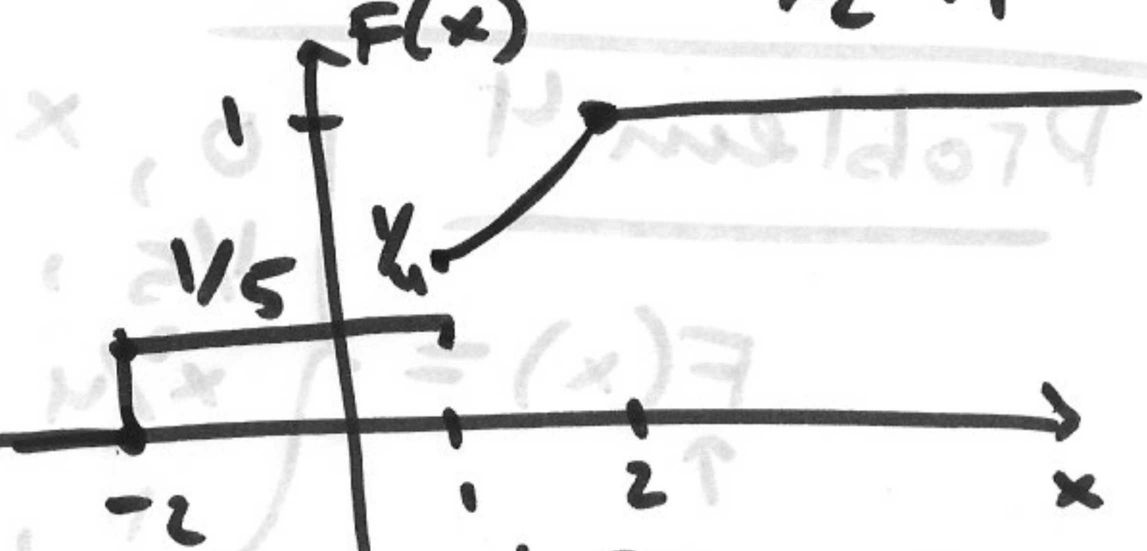




$$\frac{1}{x_2 - x_1}$$

$$\text{Slope} = a = \frac{1}{x_2 - x_1}$$





$$\left(\frac{x^2}{4}\right)' = \frac{2x}{4} = \frac{x}{2}$$

$\frac{1}{5}\delta$-function

$$\mathbb{E}(x) = \int_{-\infty}^{+\infty} x \cdot \underbrace{p(x)\,dx}_{dF(x)} = \int x \cdot dF(x) =$$

$$= x \cdot F(x)\Big|_{-\infty}^{+\infty} - \underbrace{\int_{-\infty}^{+\infty} F(x)\,dx}_{0 + \int_{-2}^{1} \frac{1}{5}\,dx + \int_{1}^{2} \frac{x^2}{4}\,dx + \int_{2}^{+\infty} 1\,dx}$$

$$\int_a^b u\,dv = u \cdot v\Big|_a^b - \int_a^b v\,du$$

$$\cancel{x\Big|^{+\infty}} - \otimes\Big|_{-\infty}$$

$$\underbrace{}_{2}\quad x\Big|_2^{\cancel{+\infty}}$$

5

Problem 5

x $\quad$ y

$z = x + y$

$p(x)$:

$1 - 1/6$

$2 - 1/6$

3

$\vdots$

$6 - 1/6$

$p(x,y) = p(x) \cdot p(y)$

$p(1,5) = 1/6 \cdot 1/6$

$\mathbb{E}(x \mid z) = f(z)$

~~H(x)~~ $H(z) = \alpha \cdot z$

$\tilde{x} \, \shortparallel$

$$\mathbb{E}(x \mid z) = \arg\min_{\alpha} \underbrace{\sum_{x,y} (x - \alpha(x+y))^2 \cdot p(x,y)}_{J(\alpha)}$$

$$\overset{J(\alpha)}{\tilde{\alpha}} : \left. \frac{dJ}{d\alpha} \right|_{\tilde{\alpha}} = 0$$

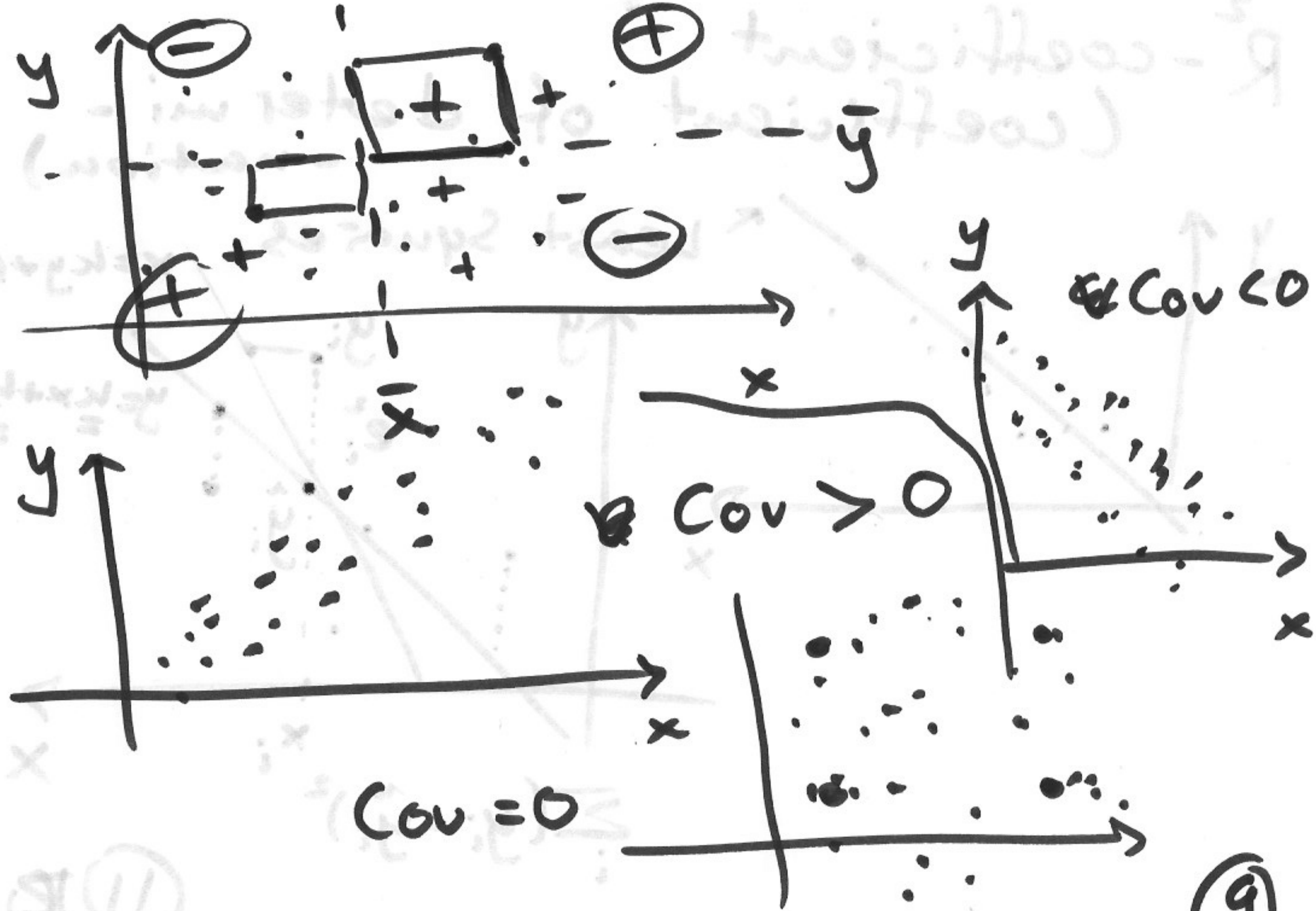
Covariance Correlation

$$\frac{1}{N-1}$$

$$\frac{1}{N}\sum_i (x_i-\bar{x})(y_i-\bar{y})$$

$$\cancel{\mathbb{E}}\ \mathrm{Cov}(x,y) = \overset{x}{\mathbb{E}}(x-\underset{\mathbb{E}x}{\bar{x}})\cdot(y-\bar{y})$$

$$\mathrm{Cov}(x,\cancel{y}x) = \mathrm{Var}(x) = \frac{1}{N}\sum (x_i-\bar{x})^2$$

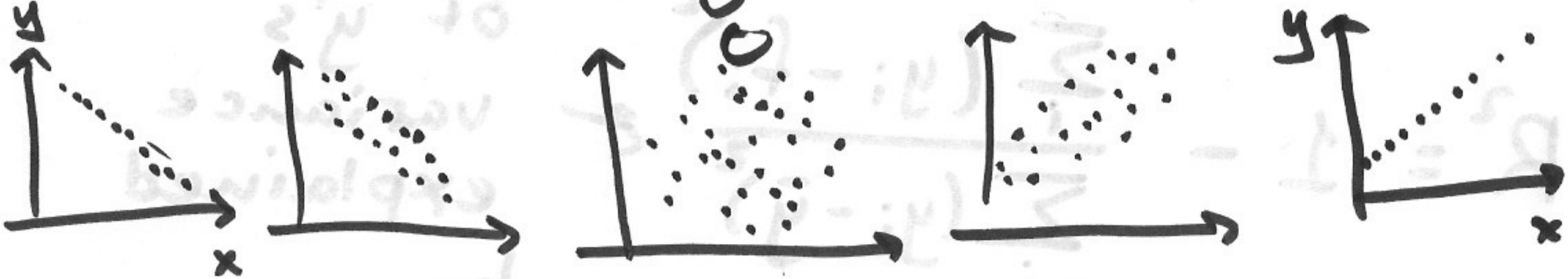

y
x
ȳ
x̄
+
−
Cov > 0
Cov < 0
Cov = 0

Pearson's correlation coefficient

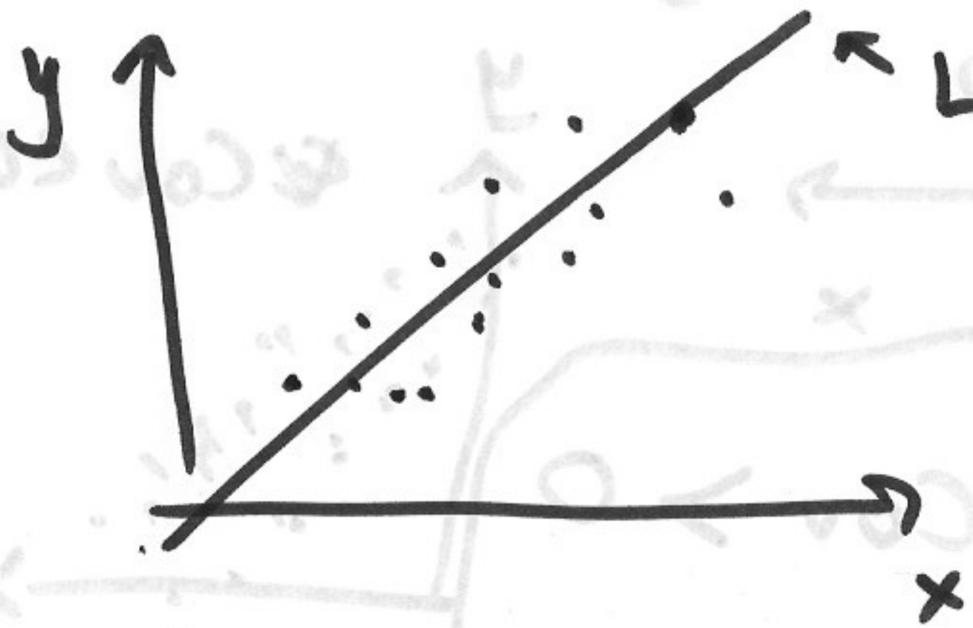
$$\rho = \frac{Cov(x,y)}{Std(x)\,Std(y)} = \frac{Cov(x,y)}{\sqrt{Var(x)\cdot Var(y)}}$$

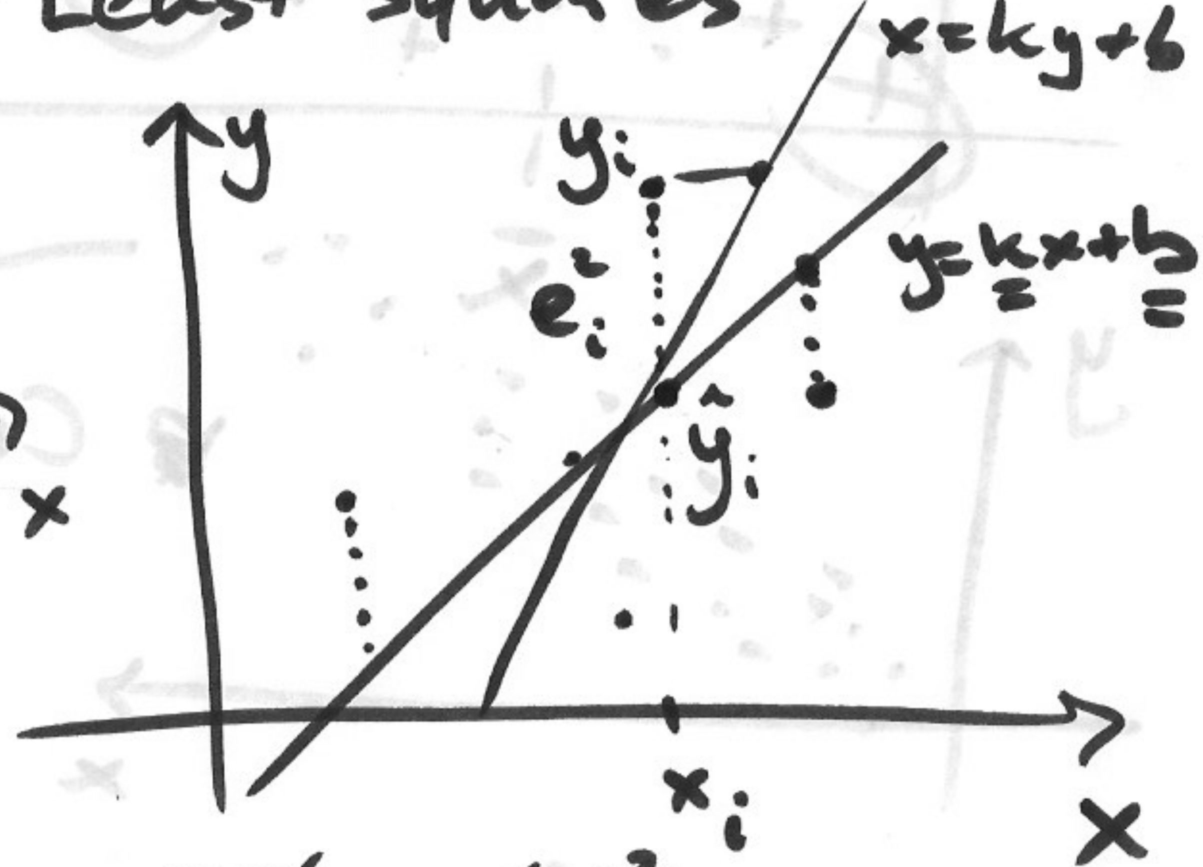
$$-1 \leq \rho \leq 1$$

0



R^2-coefficient (coefficient of determi-
-nation)





$$\sum_i (y_i - \hat{y}_i)^2$$

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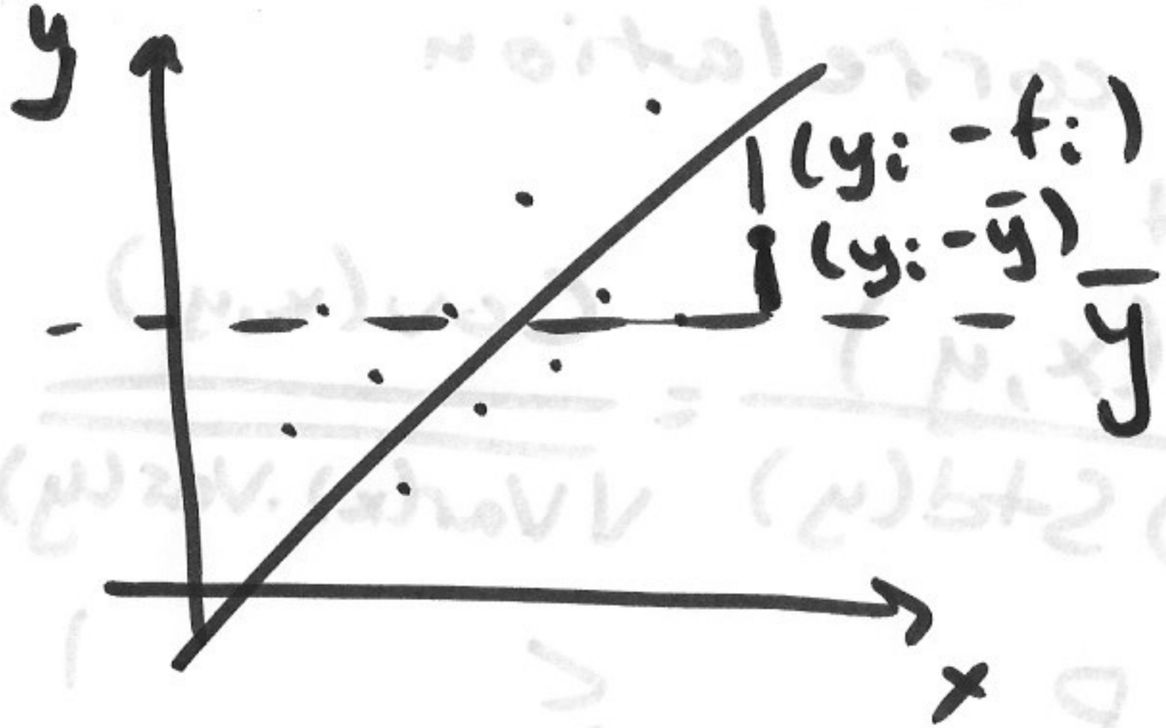
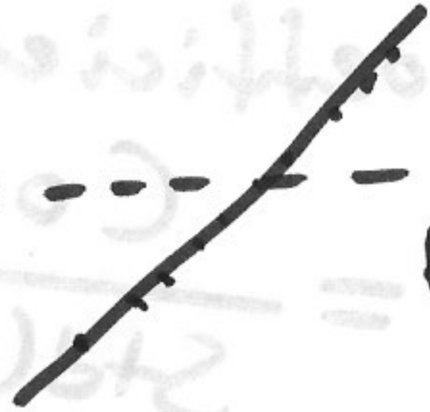


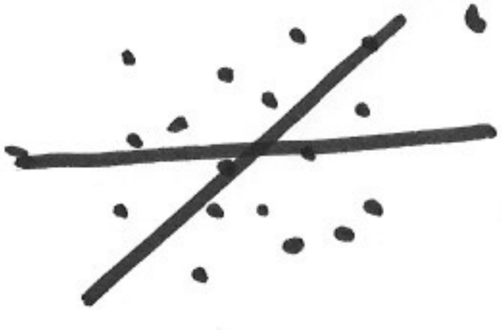
$f(x) = kx + b$



$R^2 = 1$

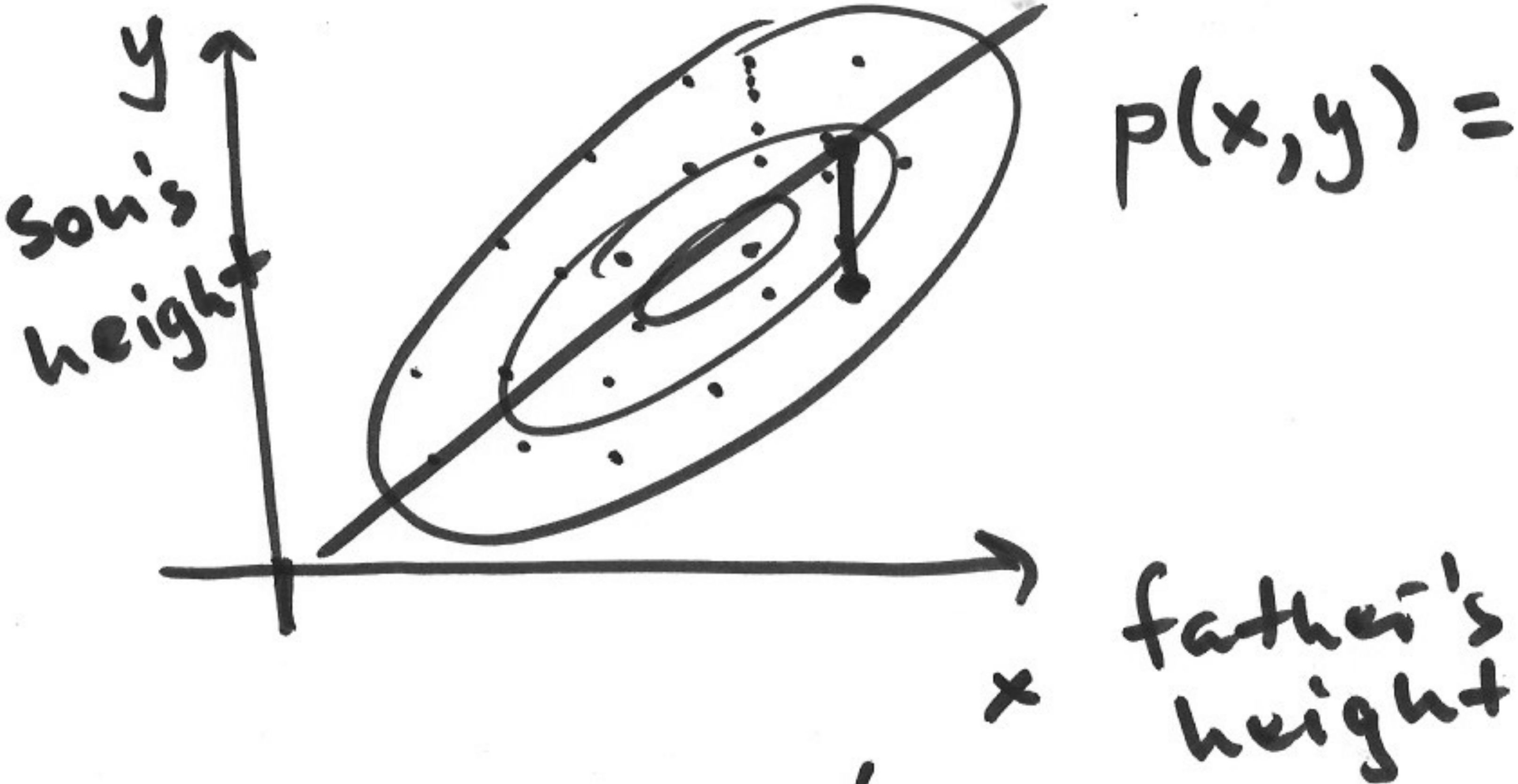
$$R^2 = 1 - \frac{\sum_i (y_i - f_i)^2}{\sum_i (y_i - \bar{y})^2}$$

fraction of y's variance explained by $f(x)$



$R^2 \approx 0$

$R^2 = 0.1$



$$p(x,y) = \mathcal{N}\left(x,y \,\middle|\, \vec{\mu} = \begin{pmatrix}\mu_x \\ \mu_y\end{pmatrix}, \Sigma = (\ldots)\right)$$

$$\underline{\mathbb{E}(y|x)} = \int y \, p(y|x) \, dy$$

$$\mu_y + \frac{\sigma_y}{\sigma_x}\rho \cdot (x - \mu_x)$$