MSAI Statistics & Probability – Week 1 Seminar & HW

Problem 1: Two dice, each having M faces, are thrown. Each die has numbers from 1 to M written on its faces. Find the probability that the sum of two numbers equals i. Provide the answer for $M = 10$, and $i = 15$.

Problem 2: A random subset is chosen from the set $\{1, \ldots, N\}$. Find the probability that the cardinality (number of elements) of thus subset is an even number.

Problem 3: Two subsets, A_1 and A_2, of $\{1, \ldots, n\}$ are chosen randomly (they may coincide). Find the probability that $A_1 \cap A_2 = \emptyset$

Problem 4: An algebra is a set of subsets of Ω that 1) contains Ω, 2) is closed under complement (if A is in the algebra – $\bar{A}$, its complement, is also) and union (if A and B are in the algebra – $A \cup B$ is as well). Can these two operations from point 2) be replaced with

1. Δ (symmetric difference) and $\setminus$ (set difference, also called relative complement)?
2. $\overline{A \cap B}$ (just one operation, complement of the intersection)?

Problem 5: Let $\mathcal{F}_1,\ \mathcal{F}_2$ be σ-algebras. Is it true that 1) $\mathcal{F}_1 \cap \mathcal{F}_2$ and 2)$\mathcal{F}_1 \cup \mathcal{F}_2$ are σ-algebras?