MSAI Statistics & Probability – Week 7 Seminar & HW

Problem 1: Let Ω be countably infinite. Prove that a.s. convergence is equivalent to convergence in probability.

Problem 2: Let $\xi_1, \xi_2, \ldots$ be independent Bernoulli random variables, $\xi_k \sim \mathrm{Bern}(p_k)$. Show that it is necessary and sufficient for $p_n \to 0$ as $n \to \infty$ for the following things to be true:

1. $\xi_n \xrightarrow{P} 0$ as $n \to \infty$ (ξ_n converging to zero in probability)

2. $\xi_n \xrightarrow{L_q} 0$ ($q \geq 1$) as $n \to \infty$ (ξ_n converging to zero in L_q norm with $q \geq 1$. *I just used q instead of p in L_p norm here to avoid confusion with p_k – the probability of success of a trial in Bernoulli's scheme.*)

Problem 3: Let $\{S_n,\ n \in \mathbb{Z}_+\}$ be a simple random walk. Find $P(S_1 \neq 0,\ S_2 \neq 0,\ \ldots)$ (the probability of not getting back to zero after n steps for given n). Find the limit of this probability as $n \to \infty$. *Hint: Stirling's approximation of factorial can be useful here.*