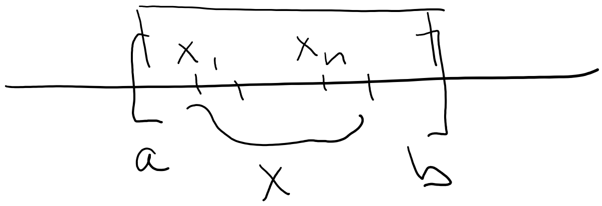
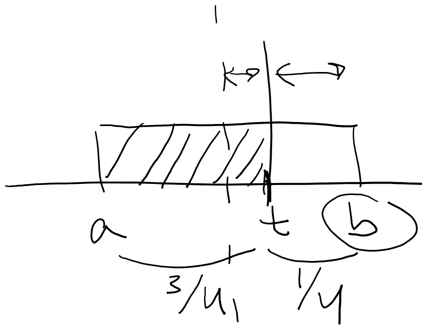
Week 3 Seminar - Parametric inference

28 марта 2022 г. 18:43





$n \to \infty$

$a \approx x_{min}$

$b \approx x_{max}$

$\frac{a+b}{2} \approx \bar{x}$

$t = \frac{a+b}{2} + \frac{1}{4}(b-a)$

$= \frac{1}{4} a + \frac{3}{4} b$

б) $\frac{1}{4} x_{min} + \frac{3}{4} x_{max}$

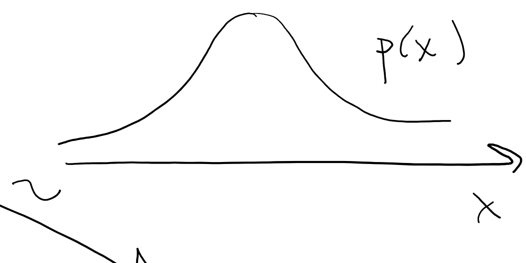
$$\underbrace{\frac{1}{4} a + \frac{1}{4} b}_{} + \frac{1}{2} b$$

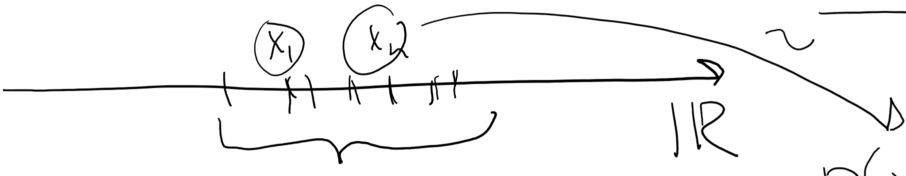
$$\frac{1}{2}\left(\frac{a+b}{2}\right) + \frac{1}{2} b$$

$$\frac{1}{2} \bar{x} + \frac{1}{2} x_{max}$$

Parametric inference

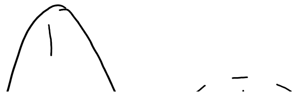
Assumption:

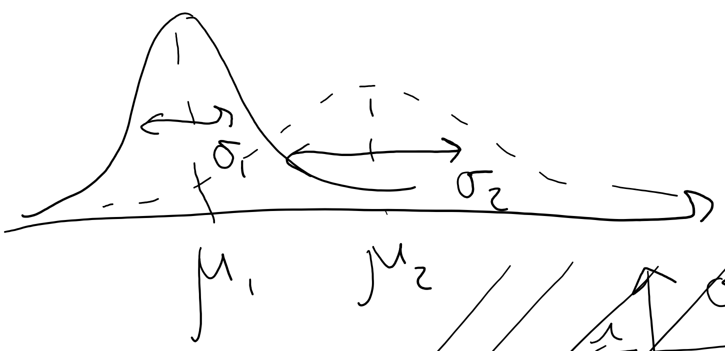




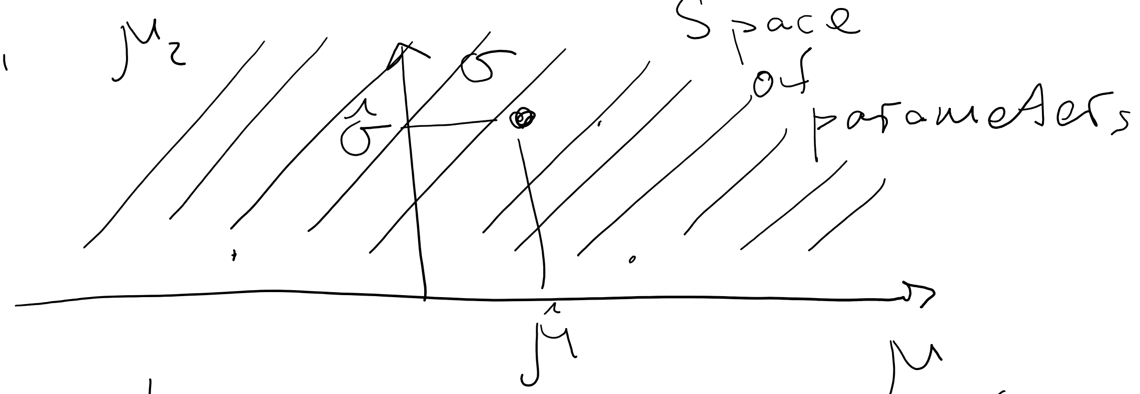
X - data

$$p(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$





$$= \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$



- Maximum Likelihood Estimate (MLE)

$$\boxed{\prod_{i=1}^{n} p(x_i \mid \mu, \sigma)} = \mathcal{L}\text{ikelihood of } X$$

assume independence

$$\mathcal{L}(\mu, \sigma)$$

$$(\hat{\mu}, \hat{\sigma})_{MLE} = \underset{\mu, \sigma}{\arg\max}\, \mathcal{L}(\mu, \sigma)$$

2 eq-s on 2 unknowns μ, σ

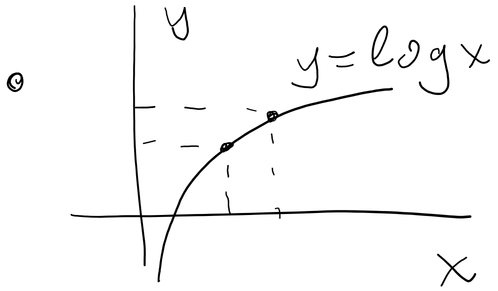
$$\begin{cases} \dfrac{\partial}{\partial \mu} \mathcal{L} = 0 \\[2ex] \dfrac{\partial}{\partial \sigma} \mathcal{L} = 0 \end{cases}$$

2 tricks:

- $\mathcal{L} \to \log \mathcal{L}$ – log-likelihood

$$\log A \cdot B = \log A + \log B$$

$$\to \log \prod_i = \sum_i \log$$



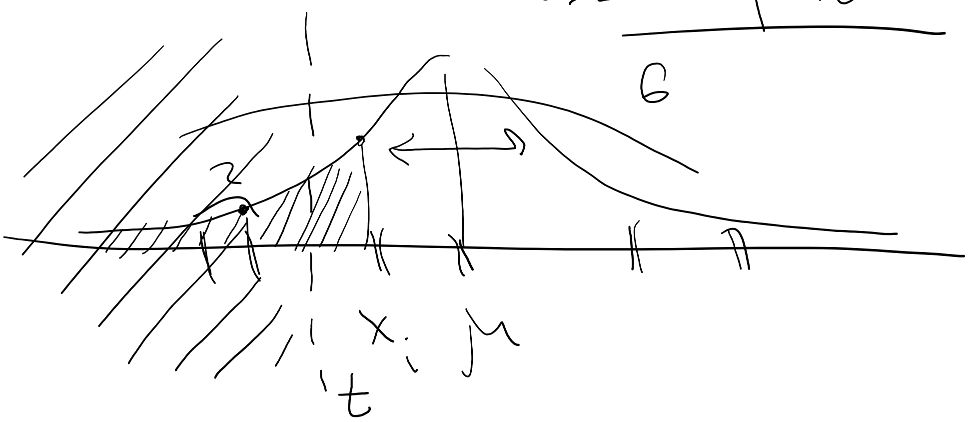
$$\frac{\partial}{\partial \mu} \underbrace{\log \prod \exp\left(\frac{(x-\mu)^2}{2\sigma^2}\right)}$$

$$\frac{\partial}{\partial \mu} \sum - \frac{(x-\mu)^2}{2\sigma^2} = \sum - \frac{\cancel{2}(x-\mu)(-1)}{\cancel{2\sigma^2}} = 0$$

$$\sum_i x = n\mu$$

$$\boxed{\mu_{MLE}} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

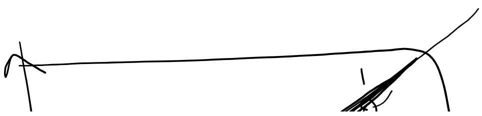
How to justify normality assumption?

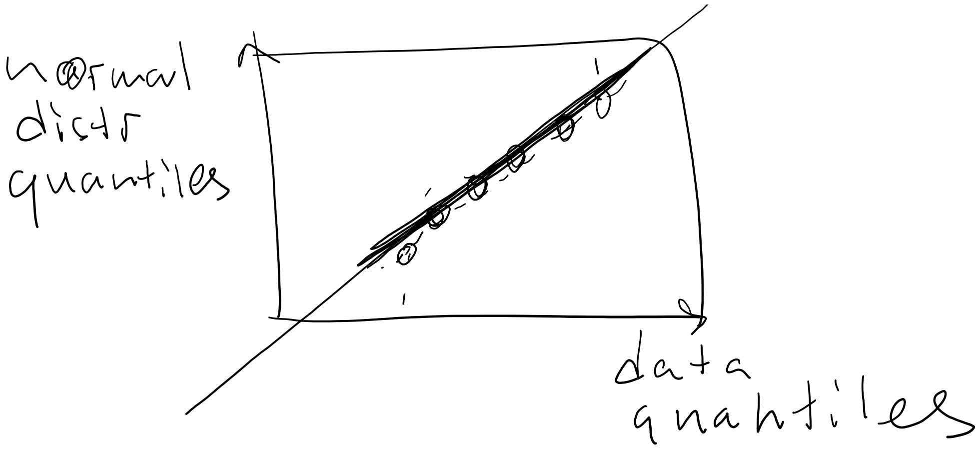


QQ-plot

quantile-quantile

normal

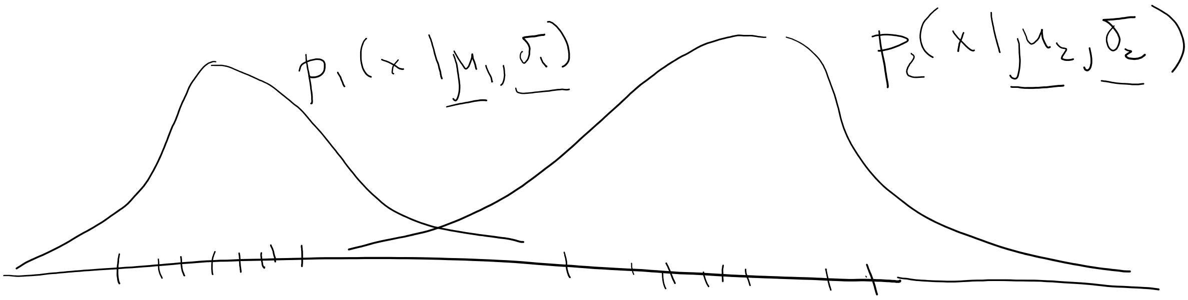




<u>MOM</u>

1) $\frac{1}{n}\sum x_i = \mu$

2) $\frac{1}{n}\sum x_i^2 = \mu^2 + \sigma^2 \longrightarrow \sigma$



model
(assumpt.) : $p(x|\Theta) =$

$= p \cdot p_1(x|\mu\ ) + (1-p) \cdot p_2(x|\dots)$