MSAI Statistics & Probability – Week 3 Seminar & HW

Problem 1: Let P be a distribution on $\mathbb{R}$ and F be its distribution function. Are the following equalities true?

1. $P([a,b]) = F(b) - F(a-)$

2. $P((a,b)) = F(b-) - F(a)$

3. $P([a,b)) = F(b-) - F(a-)$

4. $P(\{x\}) = F(x) - F(x-)$

Problem 2: Let P have density p. Find its distribution, F, if

1. $p(t) = \frac{I_{[a,b]}(t)}{b-a}$

2. $p(t) = \lambda e^{-\lambda t} I_{(0,\infty)}(t)$

3. $p(t) = \frac{\theta}{\pi(t^2+\theta^2)}$

Find the product of $F_1(2b)F_2(\ln 2/\lambda)F_3(0)$.

Problem 3: Let P be a discrete distribution on $\mathbb{R}$, $p(x) = P(\{x\})$

1. $p(x) = \frac{\lambda^x e^{-\lambda}}{x!}$, $x \in \mathbb{Z}_+$ (Poisson distribution). Find $P(2\mathbb{Z}_+)$.

2. $p(x) = p(1-p)^{x-1}$, $x \in \mathbb{N}$ (geometric distribution). Find the distribution function and $P(2\mathbb{Z}_+)$.

Find numerical values for both distributions if $\lambda = \ln 2/2$, $p = 1/3$

Problem 4: Let P be the uniform distribution on the quarter disk $D = \{(x,y) : x^2+y^2 < 1,\ x > 0,\ y > 0\}$. Let $P_1(x)$ be its *marginal distribution* (marginalized y). Find the density function $p_1(x)$ of it, as well as the distribution function F_1. Find the numerical value of $p_1(\sqrt{1 - (\pi/4)^2})$.