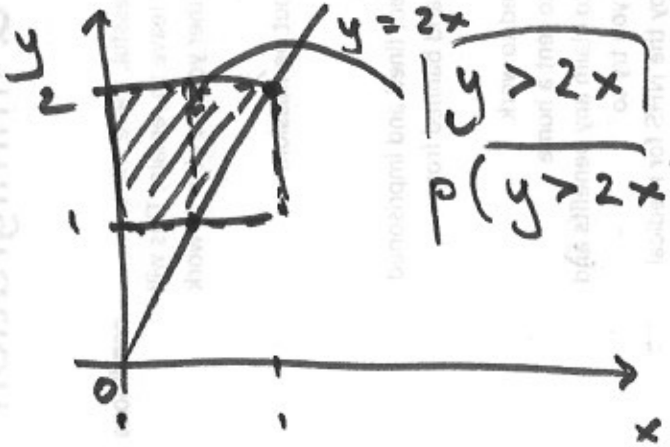
MSAI

1) $x \sim U([0,1])$

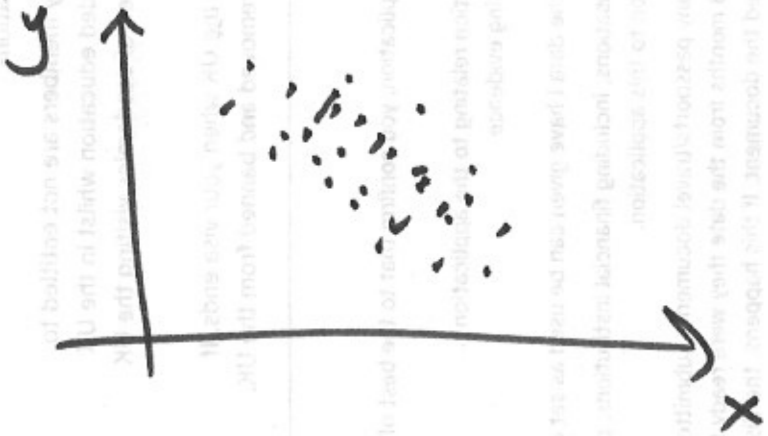
$y \sim U([1,2])$



$\boxed{y > 2x}$

$p(y > 2x) = 3/4$

2) -0.8



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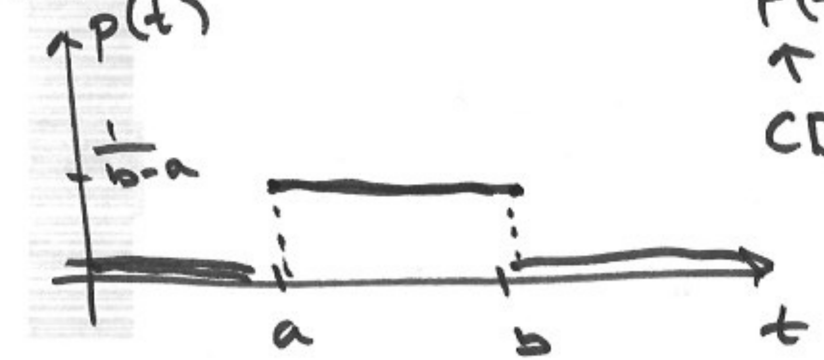
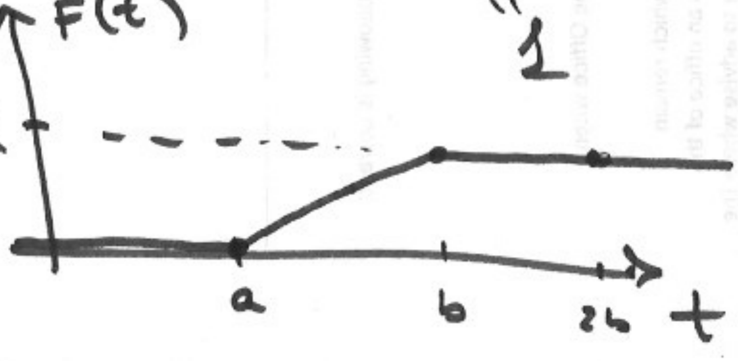
1) $p(t) = \dfrac{I_{[a,b]}(t)}{b-a}$

PDF



$F(t) = \int_{-\infty}^{t} p(t)\,dt$

CDF



$F_1(2b)$

$= 1$

Problem 3)   Poisson distribution

1) $$p(x) = \frac{\lambda^x e^{-\lambda}}{x!} \qquad x = 0, 1, 2, \dots$$

$$P(2\mathbb{Z}_+) \leftarrow \text{even numbers}$$

2) $$\boxed{\begin{gathered} p(x) = p\cdot(1-p)^{x-1} \\ x \in \mathbb{N} \\ x = 1, 2, \dots \end{gathered}}$$

!

$$p(x=2) = \frac{\lambda^2 e^{-\lambda}}{2!}$$

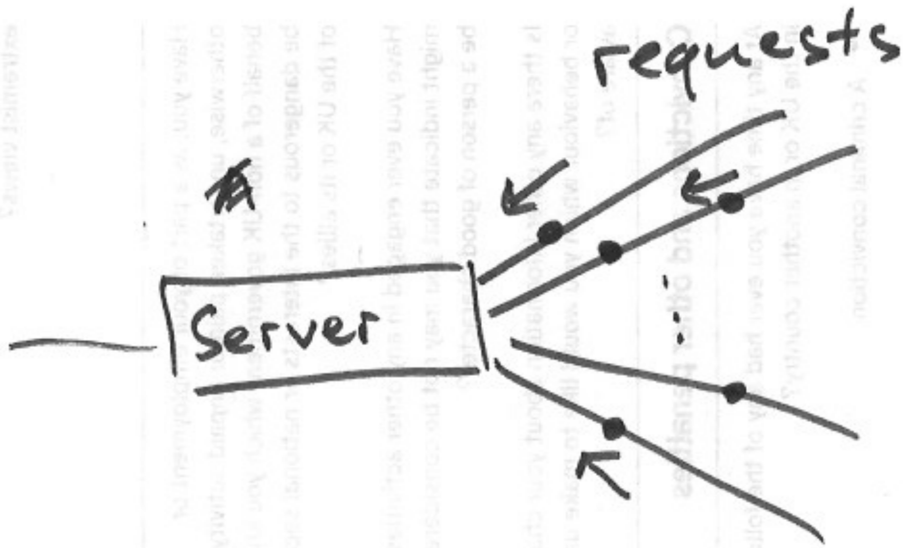
$$\sum_{k=0}^{+\infty} p(k) = \frac{\lambda^0 e^{-\lambda}}{0!} + \frac{\lambda^1 e^{-\lambda}}{1!} + \dots$$

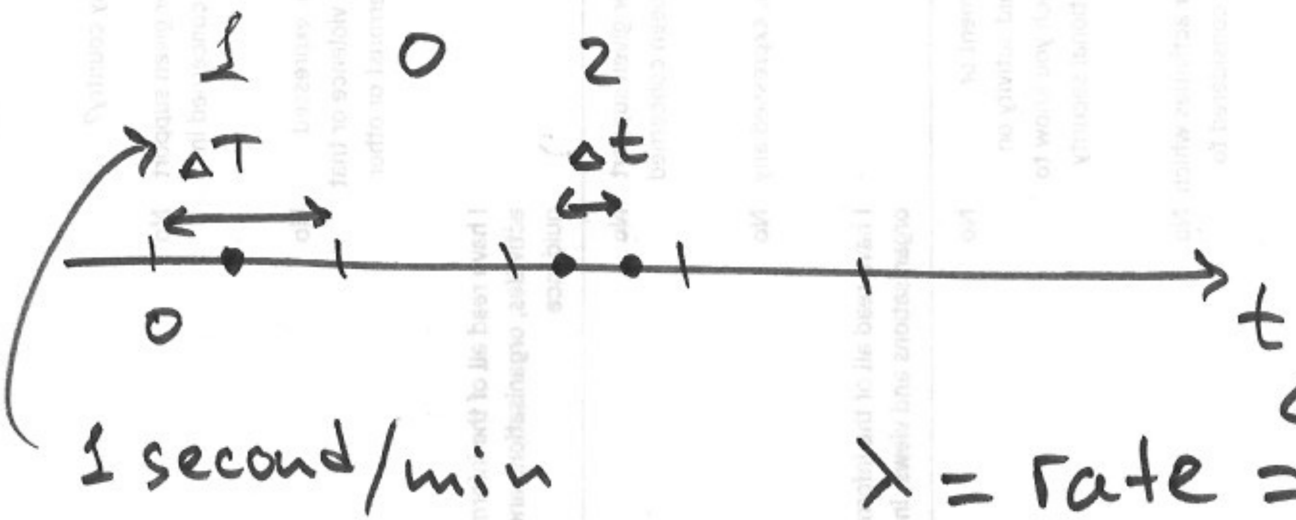
$$= e^{-\lambda}\underbrace{\left(\frac{\lambda^0}{\underset{1}{0!}} + \lambda + \frac{\lambda^2}{\underset{1\cdot 2}{2!}} + \frac{\lambda^3}{3!} + \dots\right)}_{e^{\lambda}} = e^0 = 1$$

Poisson distribution

$$p(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$x = 0, 1, 2, \ldots$

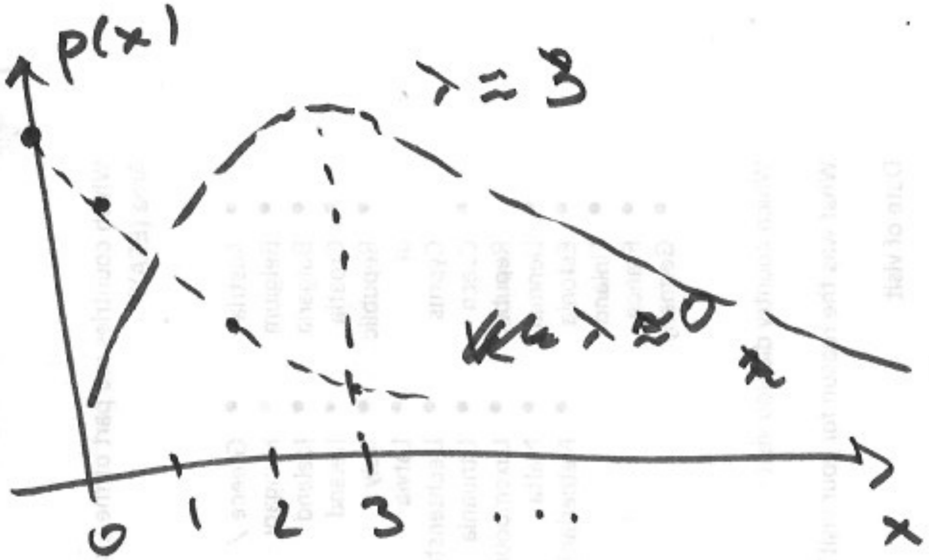




$$\lambda = \text{rate} = \frac{\langle \#\ \text{requests} \rangle}{\Delta T}$$

$\lambda = 2$

Poisson distribution <u>if</u> requests are independent



$t = \begin{cases} 1, & p \\ 0, & 1-p \end{cases}$

Binomial Distribution

0/1

$x = \sum t_i$

$$\underbrace{0 \quad 1 \quad \overset{t_3}{0} \quad \dots\dots \quad 0 \quad 1}_{N}$$

$x = 0, 1, \dots, N$

$\langle x \rangle = p \cdot N$

$$p(x=k) = C_N^k \, p^k (1-p)^{N-k}$$

$\binom{N}{k}$

ΔT

$$\underbrace{0, 1, \ldots, 1, 0}_{N \to \infty}$$

$\lambda = p \cdot N$ – fixed

$$C_N^k = \frac{N!}{k!(N-k)!}$$

$$N! \underset{N \to \infty}{\sim} \left(\frac{N}{e}\right)^N$$

Stierling formula

Poisson:

$$p(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$\langle x \rangle = \lambda$$

$$\boxed{\langle (x - \langle x \rangle)^2 \rangle = \lambda}$$

$$\text{Std}(x) = \sqrt{\lambda}$$

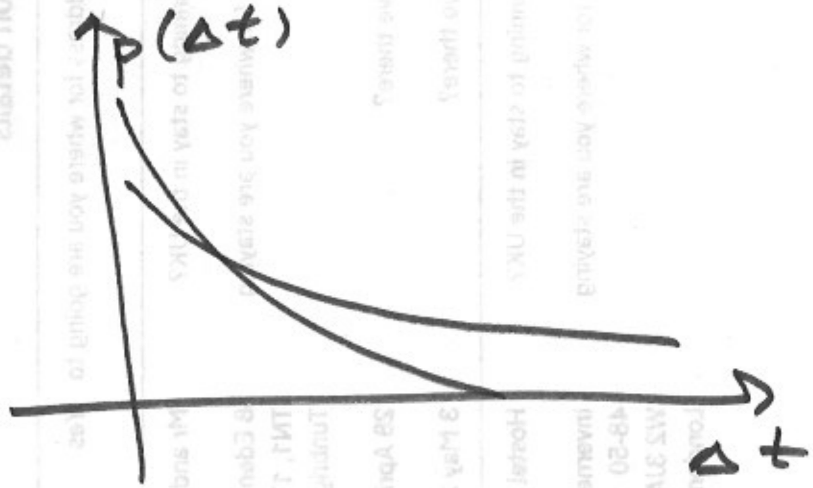
for ~~[illegible]~~

4 requests / min

$4 \pm \underbrace{\sqrt{4}}_{2}$ requests / min

Exponential distribution

Δt (between consecutive requests)



$$\Delta t \sim Exp(\lambda)$$

$$p(\Delta t) = \lambda e^{-\lambda \Delta t}$$

$$[rate] = \frac{1}{sec}$$

$$\langle \Delta t \rangle = \frac{1}{\lambda}$$

1 ± 1 sec

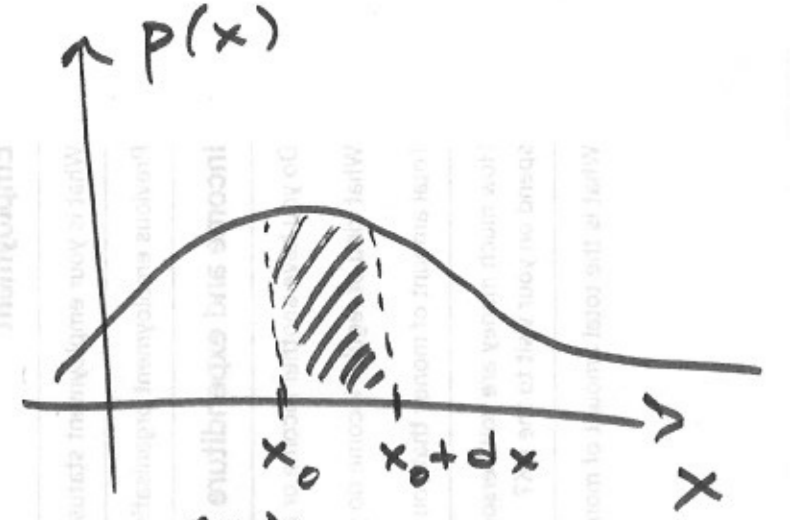
$$Std(\Delta t) = \frac{1}{\lambda}$$

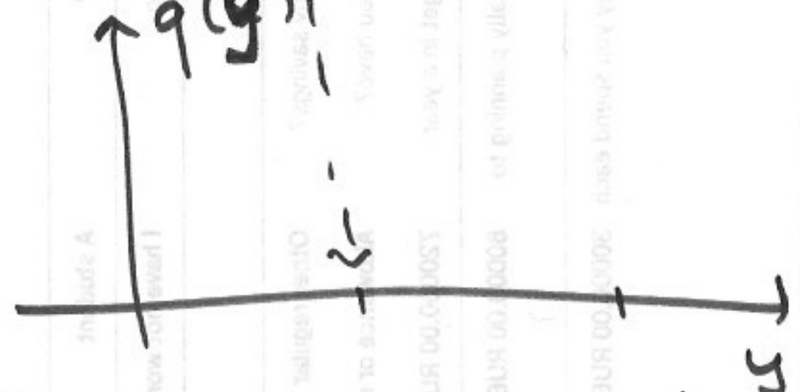
$$\langle \Delta t \rangle = \int_0^{+\infty} x \cdot p(x)\, dx = \int_0^{+\infty} x \cdot \lambda \cdot e^{-\lambda x}\, dx = \frac{1}{\lambda}$$

$$x \sim p(x) = \lambda e^{-\lambda x}$$

$$x^n \sim ?$$

$$1 - e^{-x} \sim ?$$





$$x \sim p_x(x)$$

$$y = f(x) \sim ? \; q(y)$$

monotonic function

$$\cancel{p_y(y) = p_x}$$

$$q(y) = p\left(f^{-1}(y)\right) \cdot \left| \frac{d}{dy}\left(f^{-1}(y)\right) \right|$$

$y =$

$$f(x) = 2x$$

$$f^{-1}(x) = \frac{x}{2}$$

$$\frac{d}{dx} f^{-1}(x) = \frac{1}{2}$$

$$q(y) = p\left(\frac{y}{2}\right) \cdot \frac{1}{2}$$