Lecture 09:
Enhancing Policy gradient

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Sber RL course
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References

These slides are deeply based on Practical RL course week09 slides. Special thanks to YSDA team for making them publicly available.

Original slides link: week09_policy_II

Let τ be a trajectory $(s_0, a_0, s_1, a_1, ...)$

Recall:

$$J(\pi) = E_{\tau \sim \pi}\left[\sum_{t=0}^{T} \gamma^t r(s_t)\right]$$ - **goodness** of the π, π depends on θ

Policy gradient theorem:

$$\nabla_\theta J(\pi) \approx E_{(s,a)\sim\pi}\left[Q^\pi(s,a)\nabla_\theta log\, \pi(a|s)\right]$$

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$$\nabla_\theta J(\pi) \approx E_{(s,a)\sim\pi}\left[Q^\pi(s,a)\nabla_\theta log\,\pi(a|s)\right]$$

Problem: Value of learning rate doesn't guarantee degree of policy change.

Let's use smarter optimization method.

Optimization

Suppose we want to optimize $\mathrm{F}(\theta)$ *by* θ in local neighbourhood

We approximate F by $\hat{F}(\theta) \approx F(\theta_0) + \nabla F(\theta)^T(\theta - \theta_0)$

Minimizing $\hat{F}(\theta)$ is the same as minimizing $\nabla F(\theta)^T(d), d = \theta - \theta_0$

We want to find d that

1) minimize $\nabla F(\theta)^T(d)$

2) $\boxed{d^T d} < \epsilon$

Distance

Using Lagrange multipliers, we find that $\mathrm{d}_{opt} \propto -\nabla F(\theta)$

Optimization

Now, let's measure distance using non-identical matrix K:

We want to find d that

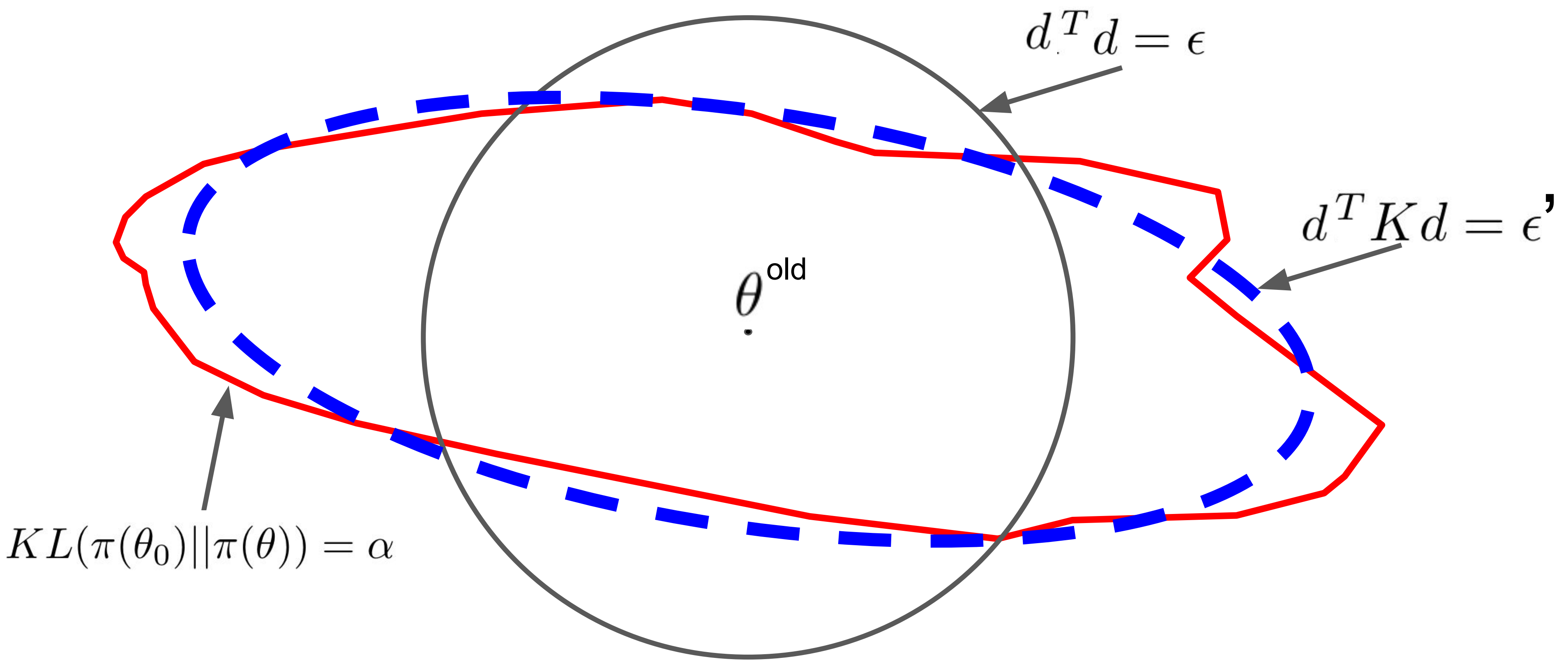
1) minimize $\nabla F(\theta)^T(d)$

2) $\boxed{d^T K d} < \epsilon$

Distance

Using Lagrange multipliers, we find that $\mathrm{d}_{opt} \propto -K^{-1}\nabla F(\theta)$

Space of parameters



Natural Policy Gradient

Suppose:

$$KL(\pi(\theta_0)||\pi(\theta)) \approx 0.5*(\theta-\theta_0)^T K(\theta-\theta_0) = 0.5*d^T K d, K = \nabla_\theta^2 KL(\pi(\theta_0)||\pi(\theta))$$

Solve constrained equation: find vector d that

1) Minimize $\nabla J^T d$

2) $d^T K d < \epsilon$

Solution: $\mathrm{d}_{opt} \propto K^{-1} \nabla J(\theta)$

New update rule: $\theta_{t+1} = \theta_t - \alpha K^{-1} \nabla J(\theta)$

Natural Policy Gradient

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Solve constrained equation: find vector d that

1) Minimize $\nabla J^T d$

2) $d^T K d < \epsilon$

Solution: $\mathrm{d}_{opt} \propto K^{-1} \nabla J(\theta)$

New update rule: $\theta_{t+1} = \theta_t - \alpha K^{-1} \nabla J(\theta)$

Problems: Value of learning rate still doesn't guarantee degree of policy change.
It may be too hard to compute inverse of K.

If we want to find $K^{-1}\nabla J(\theta)$ we may solve $Kx = \nabla J(\theta)$

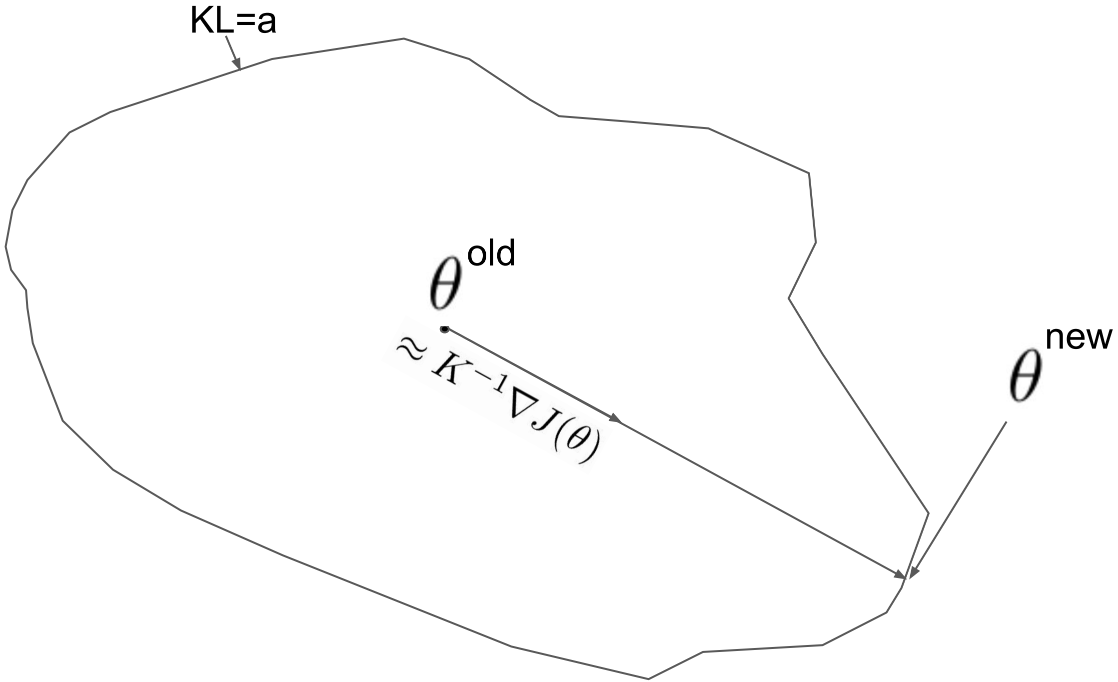
Matrix K is positive-definite so we can use **conjugate gradients**

Number of iterations k allows us to trade-off between precision and time.

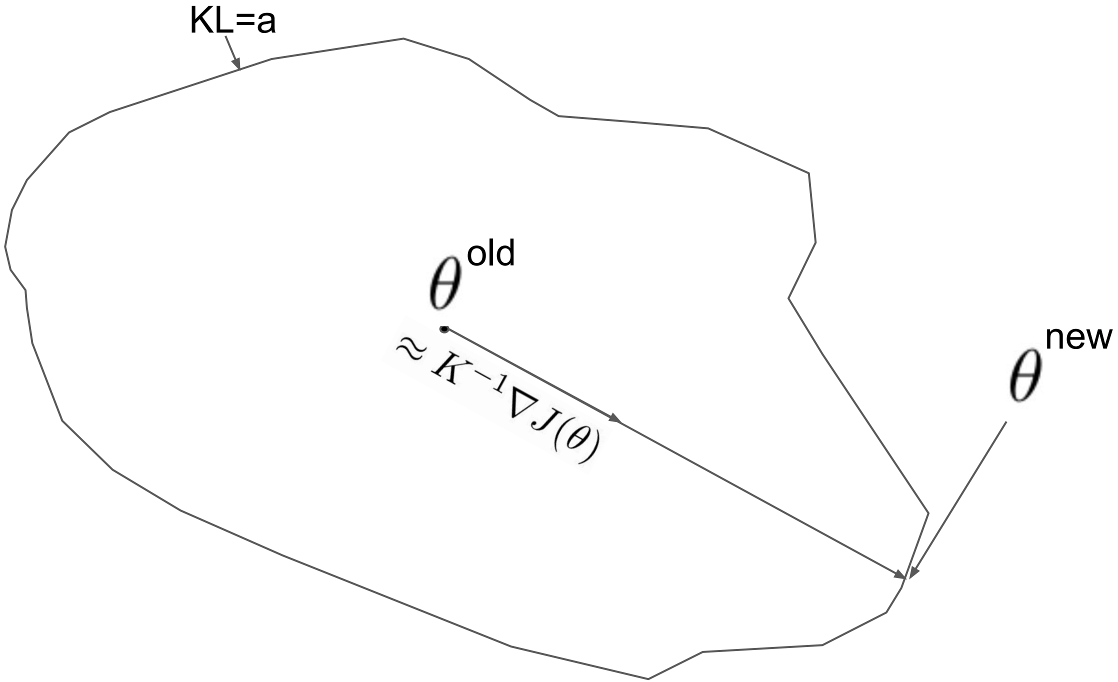
As a result: $\Delta\theta \approx \alpha K^{-1}\nabla J(\theta)$

Last problem: Value of learning rate still doesn't guarantee degree of policy change.

Linear search!

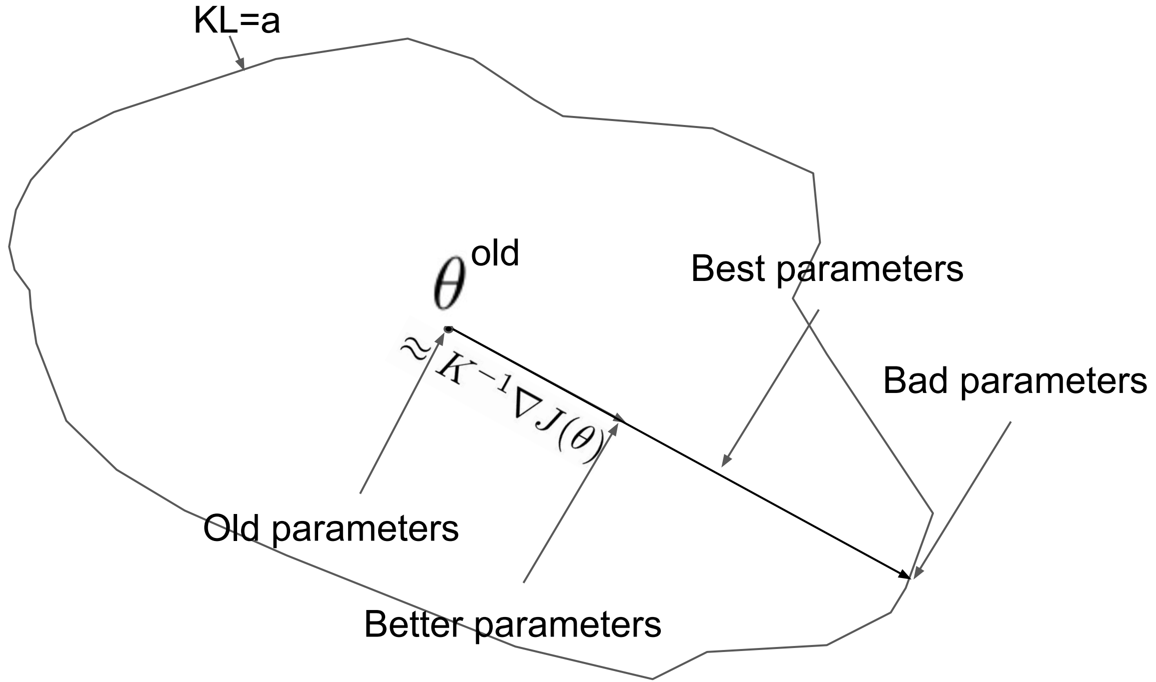


Linear search!

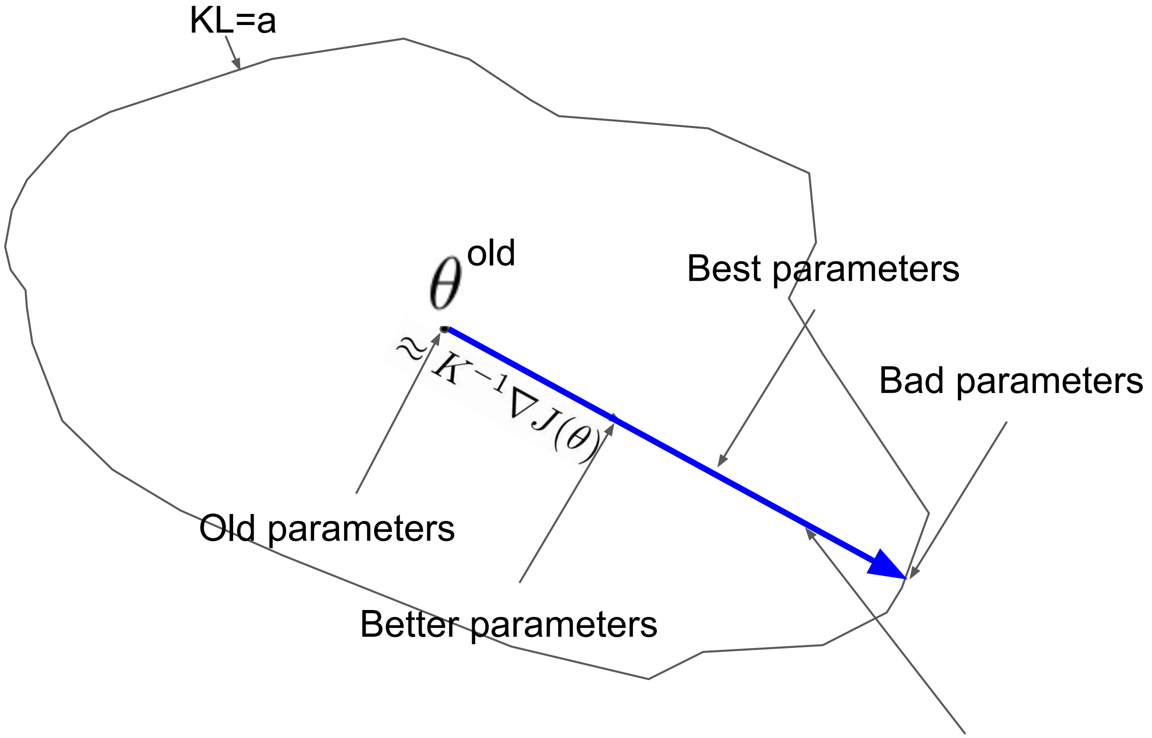


Problem?

Imagine this situation:



Imagine this situation:



We want to compute loss function here!

Recall: $J(\pi) = E_{\tau\sim\pi}\Big[\sum_{t=0}^{T}\gamma^t r(s_t)\Big]$

It can be proven that $J(\tilde{\pi}) = J(\pi) + E_{\tau\sim\tilde{\pi}}\Big[\sum_{t=0}^{T}\gamma^t A_\pi(s_t, a_t)\Big]$

Let's rewrite it this way $J(\tilde{\pi}) = J(\pi) + \sum_s \rho_{\tilde{\pi}}(s) \sum_a \tilde{\pi}(a|s) A_\pi(s, a)$

Trust Region trick:

If $E_s\Big[KL(\pi \,||\, \tilde{\pi})\Big]$ is small,

$$J(\tilde{\pi}) \approx J(\pi) + \sum_s \rho_{\pi}(s) \sum_a \tilde{\pi}(a|s) A_\pi(s, a)$$

Then:

$$J(\tilde{\pi}) \approx J(\pi) + \sum_s \rho_\pi(s) \sum_a \tilde{\pi}(a|s) A_\pi(s,a) =$$

$$= J(\pi) + \sum_s \rho_\pi(s) \sum_a \pi(a|s) * \frac{\tilde{\pi}(a|s)}{\pi(a|s)} * A_\pi(s,a) =$$

$$= J(\pi) + \boxed{E_{(s,a)\sim\pi}[\frac{\tilde{\pi}(a|s)}{\pi(a|s)} A_\pi(s,a)]}$$

Can be computed at every point!

Trust Region Policy Optimization

1) Sample state-action pairs from on-policy distribution

2) Compute $g = \nabla_{\theta'} \hat{J}(\tilde{\pi})$ = $\nabla_{\theta'} \frac{1}{N} \sum_{i=0}^{N} \frac{\tilde{\pi}(s_i, a_i)}{\pi(s_i, a_i)} A_\pi(s_i, a_i)$

$$K = \nabla_{\theta'}^2 \frac{1}{N} \sum_{i=0}^{N} KL(\pi(s_i) \,||\, \tilde{\pi}(s_i))$$

3) Find $\hat{d} = -1 * ConjGrad(Kx = g)$

4) Do linear search in direction of $\hat{d}$, constraint $\frac{1}{N} \sum_{i=0}^{N} KL(\pi(s_i) \,||\, \tilde{\pi}(s_i)) < \alpha$

and simultaneously check value of $\frac{1}{N} \sum_{i=0}^{N} \frac{\tilde{\pi}(s_i, a_i)}{\pi(s_i, a_i)} A_\pi(s_i, a_i)$

TRPO

Advantages

- Very stable training
- Good result

Disadvantages

- Cheap sampling is necessary
- Not easy to implement

KL constraint

Strict: TRPO

$$\underset{\theta}{\text{maximize}} \quad \hat{\mathbb{E}}_t\left[\frac{\pi_\theta(a_t \mid s_t)}{\pi_{\theta_{\text{old}}}(a_t \mid s_t)}\hat{A}_t\right]$$

$$\text{subject to} \quad \hat{\mathbb{E}}_t[\text{KL}[\pi_{\theta_{\text{old}}}(\cdot \mid s_t), \pi_\theta(\cdot \mid s_t)]] \leq \delta.$$

Penalty: PPO-decay

$$\underset{\theta}{\text{maximize}}\, \hat{\mathbb{E}}_t\left[\frac{\pi_\theta(a_t \mid s_t)}{\pi_{\theta_{\text{old}}}(a_t \mid s_t)}\hat{A}_t - \beta\,\text{KL}[\pi_{\theta_{\text{old}}}(\cdot \mid s_t), \pi_\theta(\cdot \mid s_t)]\right]$$

Different losses

Basic objective

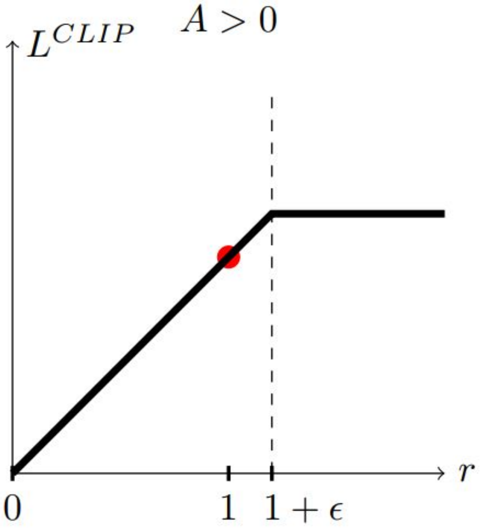
$$r_t(\theta) = \frac{\pi_\theta(a_t \mid s_t)}{\pi_{\theta_{\text{old}}}(a_t \mid s_t)}, \text{ so } r(\theta_{\text{old}}) = 1$$

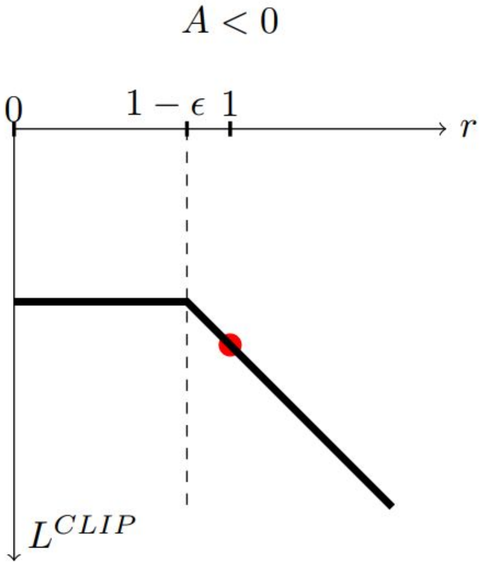
$$L^{CPI}(\theta) = \hat{\mathbb{E}}_t\left[\frac{\pi_\theta(a_t \mid s_t)}{\pi_{\theta_{\text{old}}}(a_t \mid s_t)}\hat{A}_t\right] = \hat{\mathbb{E}}_t\left[r_t(\theta)\hat{A}_t\right]$$

Different losses

Clipped objective

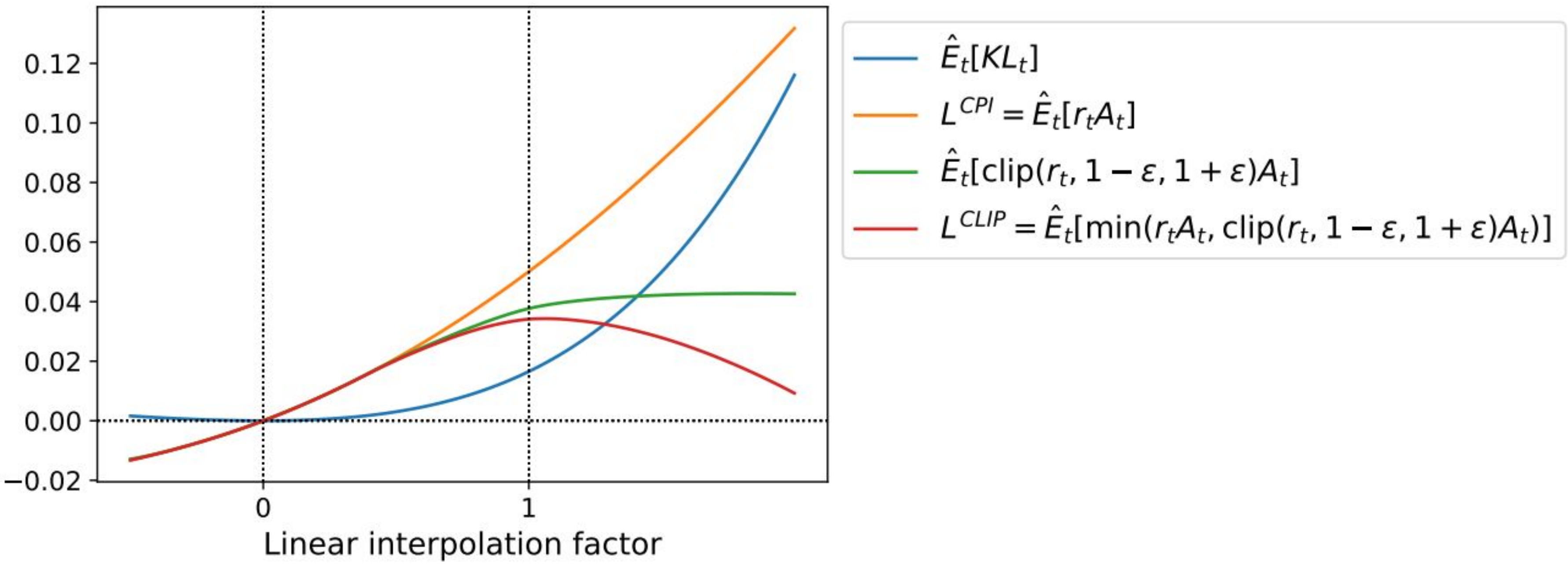
$$L^{CLIP}(\theta) = \hat{\mathbb{E}}_t\left[\min(r_t(\theta)\hat{A}_t, \text{clip}(r_t(\theta), 1-\epsilon, 1+\epsilon)\hat{A}_t)\right]$$





Behaviour of losses

Suppose we interpolate the parameters of the model between updates



PPO-clip

Repeat:

1) Collect data

2) Update policy a few times to maximize $L^{CLIP}(\theta)$

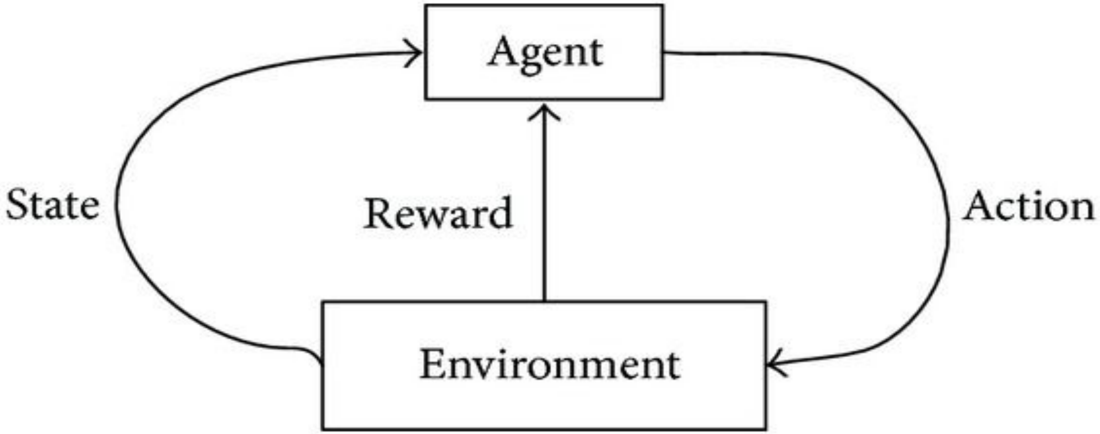
algorithm	avg. normalized score
No clipping or penalty	-0.39
Clipping, $\epsilon = 0.1$	0.76
Clipping, $\epsilon = 0.2$	**0.82**
Clipping, $\epsilon = 0.3$	0.70
Fixed KL, $\beta = 0.3$	0.62
Fixed KL, $\beta = 1.$	0.71
Fixed KL, $\beta = 3.$	0.72
Fixed KL, $\beta = 10.$	0.69

TRPO	PPO
• Works for smaller models • Second-order optimization	• Works for big models • First-order optimization

Continuous action spaces

Regular MDP

$a \in \mathbb{R}^n$



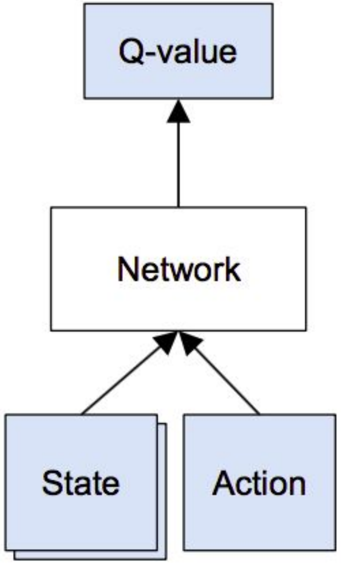
Which methods can we use?

Continuous action spaces

- We can learn critic easily
- The problem is finding

$$a_{opt}(s) = \underset{a}{argmax}\, Q(s,a)$$

Worst case: optimize over neural net!



Normalized advantage functions

- First idea: restrict Q(s,a) so that optimization becomes trivial
- For example, parabola (for 1d action space)

$$Q(s,a)=V(s)+A(s,a)$$

$$A(s,a)=-k_\theta(s)\cdot(a-\mu_\theta(s))^2$$

How to find optimal a?

a_opt = mu(s)

Normalized advantage functions

First idea: restrict Q(s,a) so that optimization becomes trivial
For example, parabola (for 1d action space)

$$Q(s,a)=V(s)+A(s,a)$$

$$A(s,a)=-k_\theta(s)\cdot(a-\mu_\theta(s))^2$$

Q: How does it generalize for n-dimensional **a**?

Normalized advantage functions

First idea: restrict Q(s,a) so that optimization becomes trivial

$$Q(s,a)=V(s)+A(s,a)$$

$$A(s,a)=-0.5\cdot(a-\mu_\theta(s))^T\cdot L(s)\cdot L(s)^T(a-\mu_\theta(s))$$

Where L(s) is a lower-triangular matrix

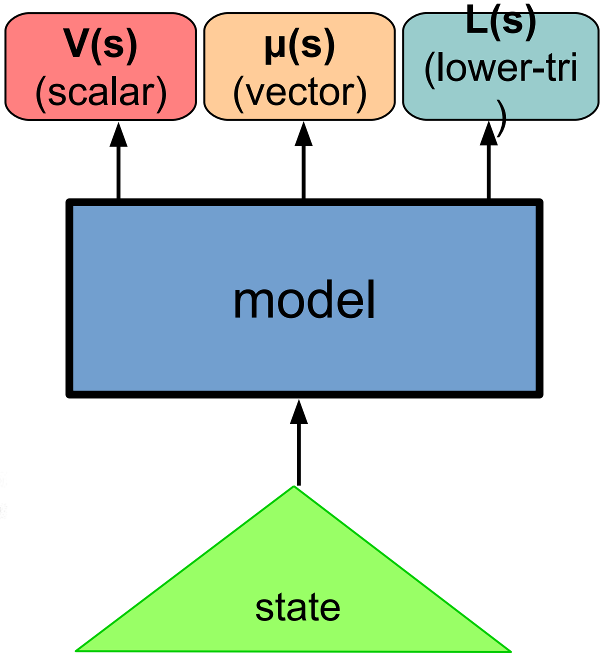
Normalized advantage functions

Network trains end-to-end

$$Q(s,a)=V(s)+A(s,a)$$

$$A(s,a)=\ldots$$

$$\underset{\theta}{argmin}\left(Q(s_t,a_t)-\left[r+\gamma\cdot V(s_{t+1})\right]\right)^2$$



Source: https://arxiv.org/pdf/1603.00748.pdf

Deterministic policy gradient

Second idea: learn a separate network to find a_opt

Train critic $Q_\theta(s, a)$

$$\underset{\theta}{argmin}\left(Q(s_t, a_t) - [r + \gamma \cdot V(s_{t+1})]\right)^2$$

Train actor $a_{opt}(s) \approx \mu_\theta(s)$

How do we get V(s')?

$$\nabla_\theta J = \frac{\partial Q^\theta(s, a)}{\partial a} \frac{\partial \mu(s|\theta)}{\partial \theta}$$

Deterministic policy gradient

Second idea: learn a separate network to find a_opt

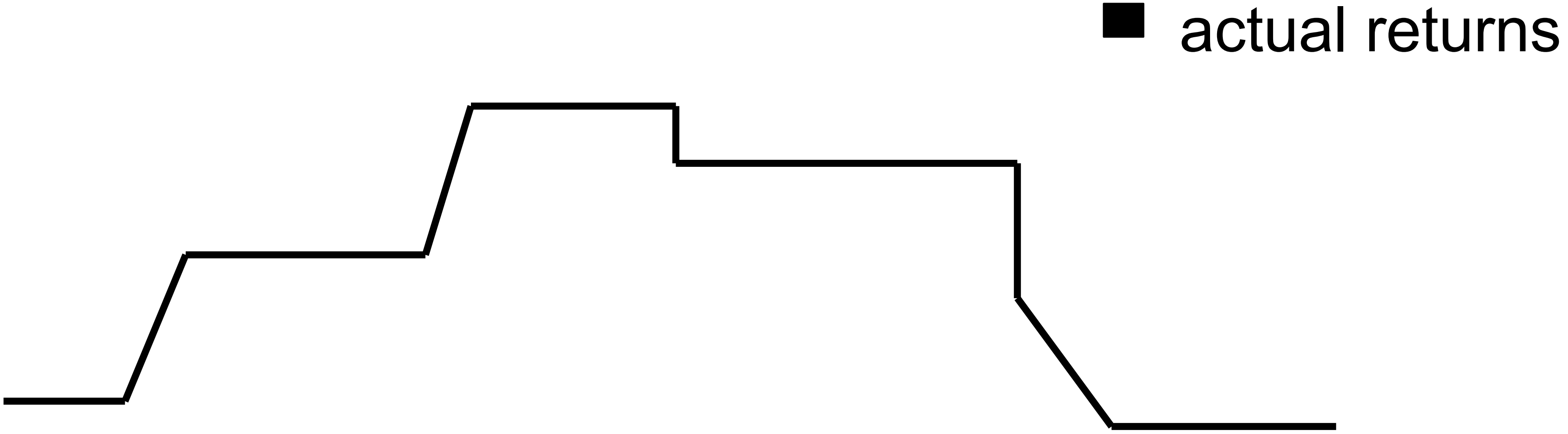
Train critic $Q_\theta(s, a)$

$$\underset{\theta}{argmin}\left(Q(s_t, a_t) - [r + \gamma \cdot Q(s_{t+1}, \mu_\theta(s_{t+1}))]\right)^2$$

Train actor $a_{opt}(s) \approx \mu_\theta(s)$

$$\nabla_\theta J = \frac{\partial Q^\theta(s, a)}{\partial a} \frac{\partial \mu(s|\theta)}{\partial \theta}$$

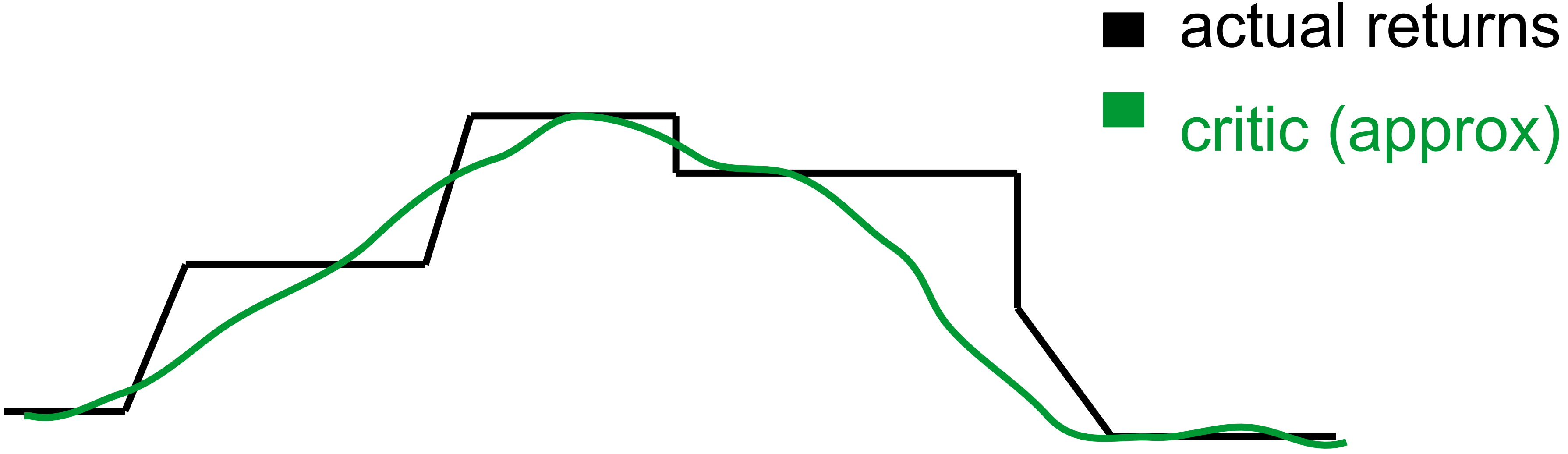
Deterministic policy gradient



Deterministic policy gradient

- Gradient approximation:

$$\nabla_\theta J = \frac{\partial Q^\theta(s,a)}{\partial a}\frac{\partial \mu(s|\theta)}{\partial \theta}$$



Deterministic policy gradient

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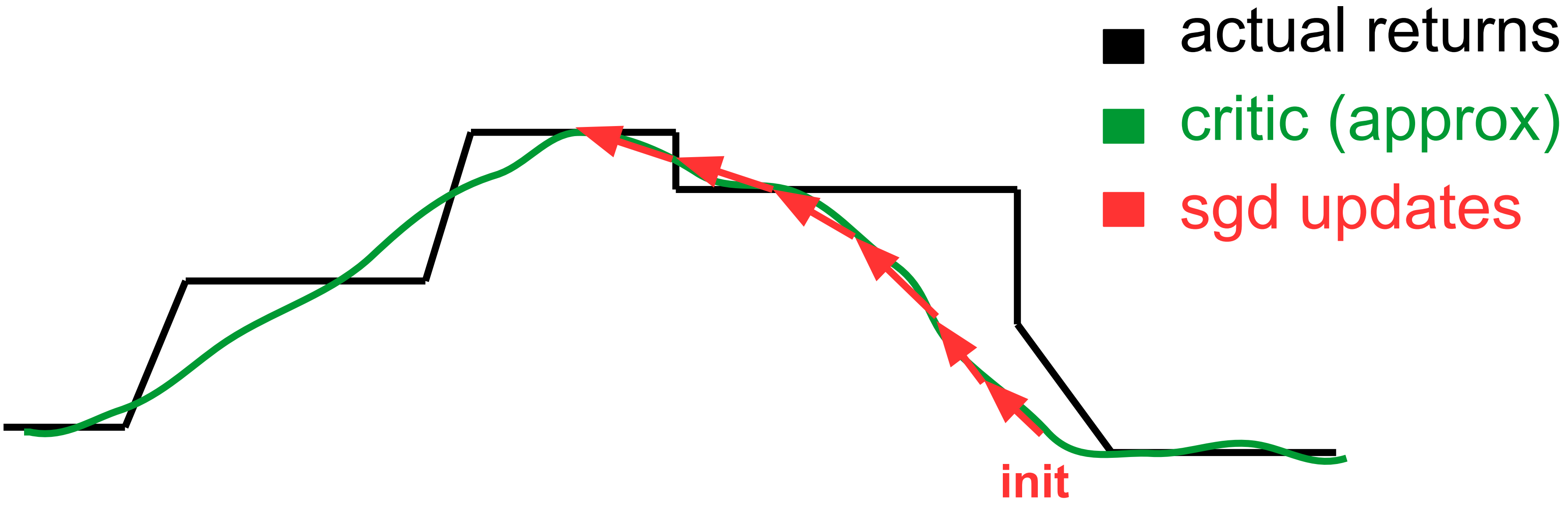
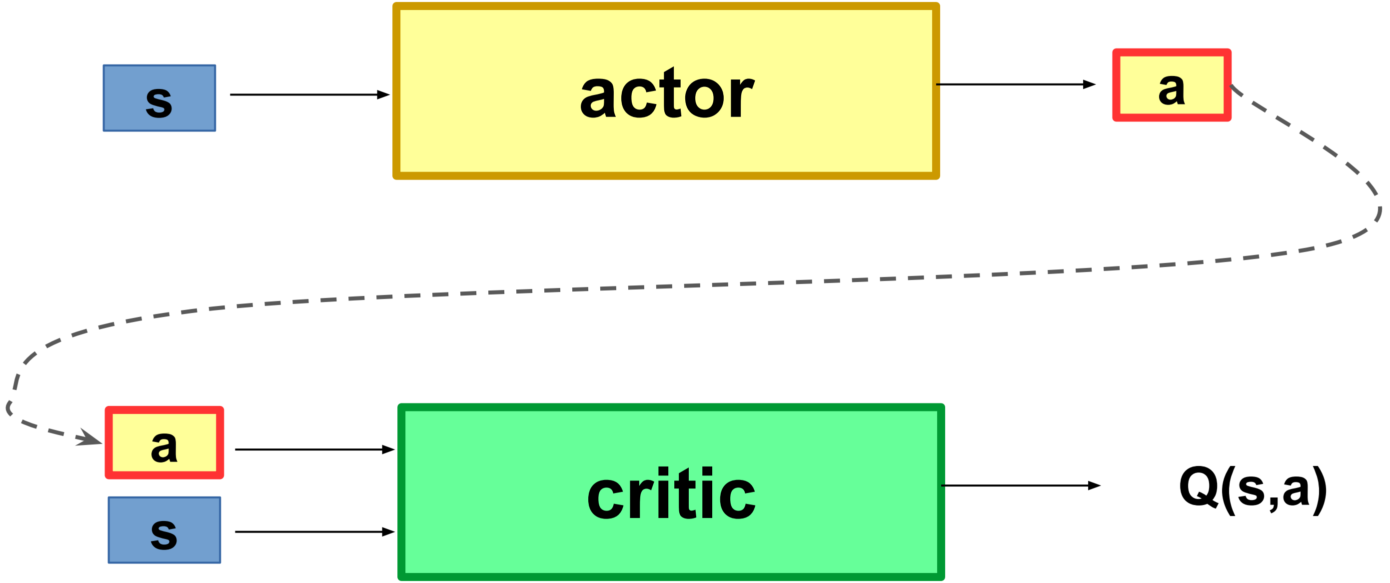


Going neural



Duct tape zone

- In general
  - “Natural” for continuous action spaces
    - Discrete: use gumbel-softmax, bit.ly/2v0Xfpz
  - Approximation is best around current policy
  - Weak critic can introduce bias

- vs. REINFORCE
  - Better off-policy
  - Less variance if reward is smooth
  - (subjectively) harder to tune

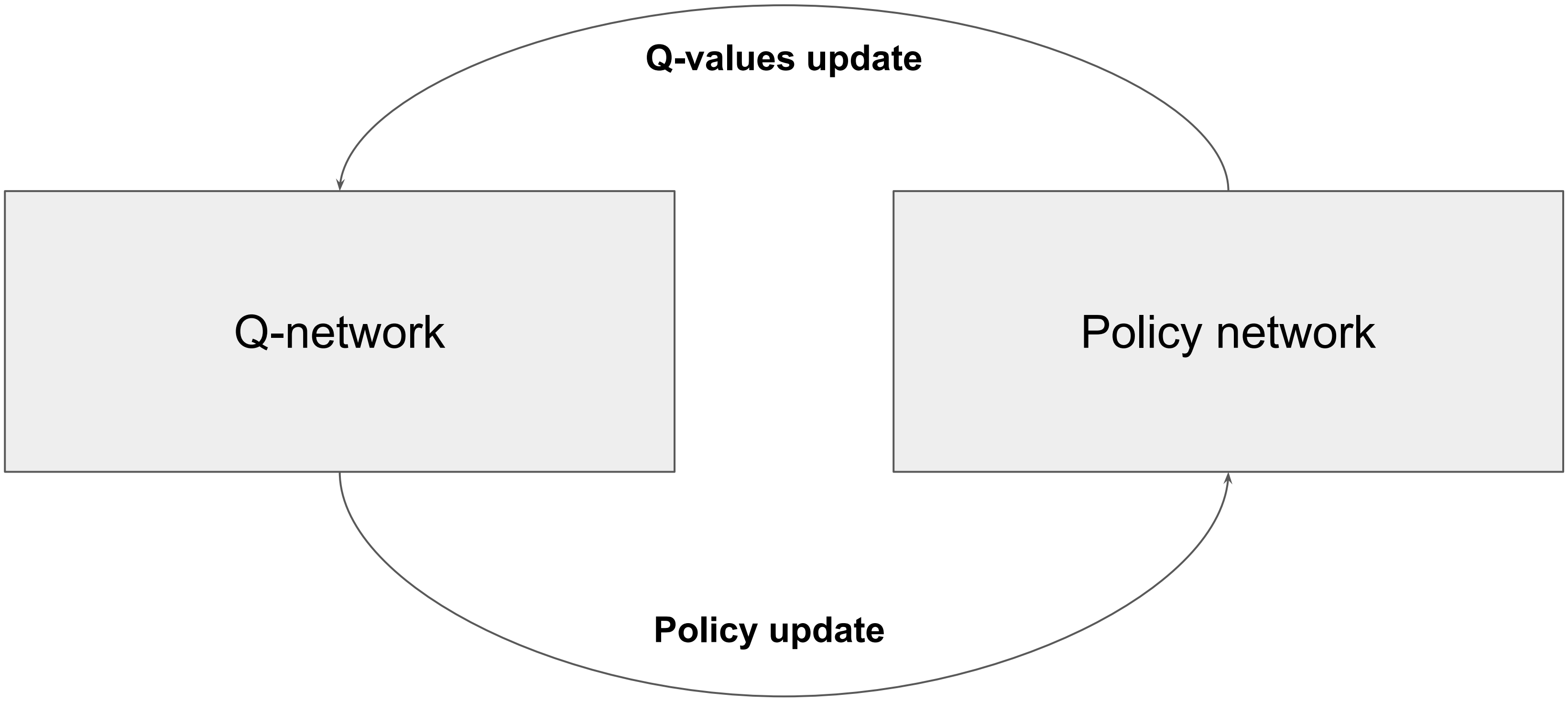
Deterministic policy gradient

- Demo with torcs http://bit.ly/2pXwdKa

Thank you for your attention!

Backup

Policy iteration for neural networks



Q-values update

$$J_Q(\theta) = \mathbb{E}_{(\mathbf{s}_t, \mathbf{a}_t) \sim \mathcal{D}} \left[\frac{1}{2} \left(Q_\theta(\mathbf{s}_t, \mathbf{a}_t) - \hat{Q}(\mathbf{s}_t, \mathbf{a}_t) \right)^2 \right]$$

$$\hat{Q}(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + \gamma \, \mathbb{E}_{\mathbf{s}_{t+1} \sim p} \left[V_{\bar{\psi}}(\mathbf{s}_{t+1}) \right]$$

$$\hat{\nabla}_\psi J_V(\psi) = \nabla_\psi V_\psi(\mathbf{s}_t) \left(V_\psi(\mathbf{s}_t) - Q_\theta(\mathbf{s}_t, \mathbf{a}_t) \right)$$

Policy update

$$\pi_{\text{new}} = \arg\min_{\pi' \in \Pi} \mathrm{D}_{\mathrm{KL}} \left(\pi'(\,\cdot\,|\mathbf{s}_t) \,\middle\|\, \frac{\exp\left(Q^{\pi_{\text{old}}}(\mathbf{s}_t,\,\cdot\,)\right)}{Z^{\pi_{\text{old}}}(\mathbf{s}_t)} \right)$$

Which can be rewritten as:

$$\mathbf{a}_t = f_\phi(\epsilon_t; \mathbf{s}_t)$$

$$J_\pi(\phi) = \mathbb{E}_{\mathbf{s}_t \sim \mathcal{D}, \epsilon_t \sim \mathcal{N}} \left[\log \pi_\phi(f_\phi(\epsilon_t; \mathbf{s}_t)|\mathbf{s}_t) - Q_\theta(\mathbf{s}_t, f_\phi(\epsilon_t; \mathbf{s}_t)) \right]$$

Maximum entropy RL

$$J(\pi) = \sum_{t=0}^{T} \mathbb{E}_{(\mathbf{s}_t, \mathbf{a}_t) \sim \rho_\pi} \left[r(\mathbf{s}_t, \mathbf{a}_t) + \alpha \mathcal{H}(\pi(\cdot | \mathbf{s}_t)) \right]$$

Soft Policy Iteration

$$\mathcal{T}^\pi Q(\mathbf{s}_t, \mathbf{a}_t) \triangleq r(\mathbf{s}_t, \mathbf{a}_t) + \gamma \, \mathbb{E}_{\mathbf{s}_{t+1} \sim p} \left[V(\mathbf{s}_{t+1}) \right],$$

where

$$V(\mathbf{s}_t) = \mathbb{E}_{\mathbf{a}_t \sim \pi} \left[Q(\mathbf{s}_t, \mathbf{a}_t) - \log \pi(\mathbf{a}_t | \mathbf{s}_t) \right]$$

So what's new?

$$\hat{\nabla}_\psi J_V(\psi) = \nabla_\psi V_\psi(\mathbf{s}_t) \left(V_\psi(\mathbf{s}_t) - Q_\theta(\mathbf{s}_t, \mathbf{a}_t) + \log \pi_\phi(\mathbf{a}_t | \mathbf{s}_t) \right)$$

Automatic Tuning of Alpha

$$J(\alpha) = \mathbb{E}_{\mathbf{a}_t \sim \pi_t} \left[-\alpha \log \pi_t(\mathbf{a}_t | \mathbf{s}_t) - \alpha \bar{\mathcal{H}} \right]$$

Automatic Tuning of Alpha

$$J(\alpha) = \mathbb{E}_{\mathbf{a}_t \sim \pi_t} \left[-\alpha \log \pi_t(\mathbf{a}_t | \mathbf{s}_t) - \alpha \bar{\mathcal{H}} \right]$$

This algorithm is called **Soft Actor-Critic**