Proceedings of Machine Learning Research vol 283:1–12, 2025 7th Annual Conference on Learning for Dynamics and Control

Robust Control of Uncertain Switched Affine Systems via Scenario Optimization

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Editors: N. Ozay, L. Balzano, D. Panagou, A. Abate

Abstract

Switched affine systems are often used to model and control complex dynamical systems that operate in multiple modes. However, uncertainties in the system matrices can challenge their stability and performance. This paper introduces a new approach for designing switching control laws for uncertain switched affine systems using data-driven scenario optimization. Instead of relaxing invariant sets, our method creates smaller invariant sets with quadratic Lyapunov functions through scenario-based optimization, effectively reducing chattering effects and regulation error. The framework ensures robustness against parameter uncertainties while improving accuracy. We validate our method with applications in multi-objective interval Markov decision processes and power electronic converters, demonstrating its effectiveness.

Keywords: Uncertain Switched Affine System, Scenario Optimization, Invariant Set of Attraction, Lyapunov Function, Switching Control Law.

1. Introduction

Motivation: Switched affine systems play a critical role in various practical applications, including decision-making in Markov Decision Processes (MDPs), power electronics, and control systems in robotics (Seatzu et al., 2006; Albea et al., 2015; Beneux et al., 2018; Iervolino et al., 2023; Monir et al., 2024). These systems are particularly useful for modeling situations where a system operates under multiple modes governed by distinct switching laws. However, a significant challenge in these applications is managing uncertainty, which can arise from parameter variations, external disturbances, or incomplete model information. This uncertainty greatly affects the stability and robustness of systems, especially in real-world scenarios where precise control and reliable decision-making are essential. Thus, it is crucial to develop robust control strategies that specifically address these uncertainties to ensure optimal and practical implementations in these fields.

Related Works: Switched affine systems have been gaining attention in the literature, particularly regarding their stability and control (Deaecto et al., 2010; Albea Sanchez et al., 2020; Della Rossa et al., 2021; Seuret et al., 2023). Deaecto et al. (2010) focus on stability analysis and control design using quadratic Lyapunov functions and optimization techniques to minimize the volume of the invariant set. However, a common limitation in many existing studies is the assumption that system parameters are precisely known without accounting for uncertainties. To address this issue, Albea Sanchez et al. (2020) propose robust control methods tailored for discrete-time switched affine systems that incorporate parameter uncertainties in system matrices, which may lead to chattering effects and increased regulation error. Seuret et al. (2023) have studied the case where the system model is entirely unknown and employed data-driven methods and experimental data to develop robust switching control laws. Recent studies have also proposed observer-based feedback strategies to improve robustness against uncertainties and disturbances in switched systems (Zhang et al., 2022).

Recently, data-driven methods have emerged as a powerful approach for designing robust control strategies for systems with unknown or uncertain dynamics (De Persis and Tesi, 2019; Van Waarde et al., 2020; Berberich et al., 2020). Scenario-based approaches are data-driven optimizations that have gained traction because they rely minimally on assumptions about data. Unlike other methods that require specific data conditions or model identification, sampled data is used to address robust control and chance-constrained problems, making it ideal for real-world applications. Scenario optimization simplifies robust optimization problems by using sampled scenarios to approximate uncertain constraints (Campi et al., 2009, 2021a,b; Campi and Garatti, 2018a; Garatti and Campi, 2024). For instance, scenario programming have been applied to design stabilizing state-feedback laws for switched linear systems using observed trajectories and Lyapunov-based criteria in Wang et al. (2024). Recent studies have applied these methods in formal verification and synthesis. For instance, Salamati and Zamani (2022) introduced scenario convex programming for safety verification in stochastic systems, enabling the derivation of robust solutions with probabilistic guarantees. Dietrich et al. (2024) extended nonconvex scenario programming to address the reachability problem. These developments demonstrate the versatility and effectiveness of scenario optimization in the face of uncertainty.

Contribution: This paper proposes a novel approach to designing switching control laws for switched affine systems with uncertainties in the system matrices. Our approach utilizes quadratic Lyapunov functions and invariant set of attraction combined with data-driven scenario optimization. This enables us to reduce both chattering effects and regulation errors—common drawbacks of existing methods. Additionally, this paper focuses on designing switching laws without auxiliary control inputs, unlike several prior works that employ both control inputs and switching laws to stabilize the system. We also demonstrate the applicability of our proposed method to various practical systems, including Multi-Objective Interval MDPs (MOIMDPs) and power electronic converter systems, showcasing its effectiveness in addressing real-world challenges.

2. Preliminaries and Problem Setup

Notations: $\mathbb{N}$, $\mathbb{R}$, $\mathbb{R}^n$, and $\mathbb{R}^{n\times m}$ denote, respectively, the sets of natural numbers including zero, real numbers, the n-dimensional Euclidean space, and the set of $n \times m$ real matrices. $\mathbb{N}_{\mathcal{P}}$ denotes the set $\{1, 2, \ldots, \mathcal{P}\}$ and $0_{n\times m}$ indicates the $n \times m$ matrix of zeros. For any $M \in \mathbb{R}^{n\times n}$, the notation $M > 0$ ($M < 0$) means M is symmetric and positive (negative) definite. $\det(M)$ represents the determinant of M. For real vectors or matrices, $\left(^\top\right)$ refers to their transpose. For symmetric matrices, $(*)$ denotes each of their symmetric blocks. The interior of a set is represented by $\mathrm{int}(\cdot)$. The concatenation of two matrices A and B of appropriate dimensions in a row is denoted by $[A, B]$ and in a column by $[A; B]$. The convex combination of matrices with the same dimension is denoted as $X_\lambda = \sum_{i\in\mathbb{N}_{\mathcal{P}}} \lambda_i X_i$ with $\lambda = [\lambda_1, \ldots, \lambda_{\mathcal{P}}] \in \Lambda_{\mathcal{P}}$ where $\Lambda_{\mathcal{P}} := \left\{[\lambda_1, \ldots, \lambda_{\mathcal{P}}] \mid \lambda_i \geq 0, \sum_{i\in\mathbb{N}_{\mathcal{P}}} \lambda_i = 1\right\}$ is the unitary simplex. A square matrix is said to be Schur stable if all its eigenvalues are inside the unit circle on the complex plane.

System Description: Consider the discrete-time switched affine system defined by

$$x_{k+1} = A_\pi x_k + B_\pi, \quad k \in \mathbb{N}, \tag{1}$$

where $x_k : \mathbb{N} \to \mathbb{R}^n$ is the state at time k and $\pi \in \mathcal{N}_p$ is the switching law to be designed and can change depending on the state. The matrices A_π and B_π are uncertain but belong respectively to the intervals $[\underline{\mathrm{A}}_\pi, \overline{\mathrm{A}}_\pi]$ and $[\underline{\mathrm{B}}_\pi, \overline{\mathrm{B}}_\pi]$. Matrices A_π and B_π can be rewritten as $A_\pi = \hat{A}_\pi + \Delta A_\pi$ and $B_\pi = \hat{B}_\pi + \Delta B_\pi$, in which $\hat{A}_\pi$ and $\hat{B}_\pi$ are nominal matrices. $\hat{A}_\pi$ and $\hat{B}_\pi$ can be selected from the intervals $[\underline{\mathrm{A}}_\pi, \overline{\mathrm{A}}_\pi]$ and $[\underline{\mathrm{B}}_\pi, \overline{\mathrm{B}}_\pi]$. Here, we select the midpoint of the interval, i.e. $\hat{A}_\pi = \frac{1}{2}(\underline{\mathrm{A}}_\pi + \overline{\mathrm{A}}_\pi)$, $\hat{B}_\pi = \frac{1}{2}(\underline{\mathrm{B}}_\pi + \overline{\mathrm{B}}_\pi)$.

Remark 1 *The above model class includes switched affine systems with parametric uncertainty. Any such systems can be over-approximated with the system in* (1) *by taking the element-wise optimal values with respect to the uncertain parameters.*

Finally, the following assumption will be considered to define the set of allowable operating points for system (1), which is inspired by (Deaecto and Geromel, 2016; Albea Sanchez et al., 2020).

Assumption 1 *The desired operating point x_e belongs to $\mathcal{F}$, which is defined as*

$$\mathcal{F} := \{x_e \in \mathbb{R}^n,\ \exists \lambda \in \Lambda_{\mathcal{P}},\ (\hat{A}_\lambda - I)x_e + \hat{B}_\lambda = 0\}, \tag{2}$$

where $\hat{A}_\lambda := \sum_{\pi \in \mathbb{N}_{\mathcal{P}}} \lambda_\pi \hat{A}_\pi$, $\hat{B}_\lambda = \sum_{\pi \in \mathbb{N}_{\mathcal{P}}} \lambda_\pi \hat{B}_\pi$ and $\hat{A}_\lambda$ is assumed to be Schur stable.

Error Dynamics: Introducing the error variable $\xi_k := x_k - x_e$, the resulting error dynamics are

$$\xi_{k+1} = A_\pi \xi_k + L_\pi, \tag{3}$$

where $L_\pi = (A_\pi - I)x_e + B_\pi$. Hence, $\underline{\mathrm{L}}_\pi = (\underline{\mathrm{A}}_\pi - I)x_e + \underline{\mathrm{B}}_\pi$ and $\overline{\mathrm{L}}_\pi = (\overline{\mathrm{A}}_\pi - I)x_e + \overline{\mathrm{B}}_\pi$. This can be rewritten as $L_\pi = \hat{L}_\pi + \Delta L_\pi$, where $\hat{L}_\pi = (\hat{A}_\pi - I)x_e + \hat{B}_\pi$ and $\Delta L_\pi = \Delta A_\pi x_e + \Delta B_\pi$.
Invariant Set of Attraction: The goal is to design a state-dependant switching function $\pi(\xi)$ so that ideally, $\xi_k \to 0$ as $k \to \infty$ for any initial condition $x_0 \in \mathbb{R}^n$. However, in the context of uncertain discrete-time systems, achieving this goal is generally impossible. Therefore, we aim to design $\pi(\xi)$ that switches between the elements of $\mathbb{N}_{\mathcal{P}}$ to steer the trajectories of the system toward an *invariant set of attraction* defined next, which encompasses the desired operating point.

Definition 2 ***(Invariant Set of Attraction)*** *A set $\Omega \subset \mathbb{R}^n$ is an invariant set of attraction of the system* (3) *governed by the switching policy $\pi(\xi)$, if there exists a Lyapunov function $V : \mathbb{R}^n \to \mathbb{R}$ such that the following conditions are simultaneously fulfilled: (i) $0_{n\times 1} \in \Omega$, (ii) if $\xi_k \notin \Omega$, then $\Delta V(\xi) := V(\xi_{k+1}) - V(\xi_k) < 0$, and (iii) if $\xi_k \in \Omega$, then $\xi_{k+1} \in \Omega$. These conditions are denoted as specification Ψ for Ω. We denote by $\Omega \models \Psi$ whenever Ω satisfies these three conditions.*

The existence of such a Lyapunov function ensures that, under the switching policy π, the error will converge to the set Ω from any initial error ξ_0 and remain within Ω thereafter. This property classifies the system as *practically stable*. We employ a quadratic candidate Lyapunov function

$$V(\xi) := d + 2h^\top \xi + \xi^\top F \xi, \tag{4}$$

where $h \in \mathbb{R}^n$, $0 < F \in \mathbb{R}^{n\times n}$, and $h^\top F^{-1} h = d \in \mathbb{R}$. Obviously, V is a convex function such that $V(\xi) > 0$ for all ξ except at $\hat{\xi} = -F^{-1}h$, where $V(\xi) = 0$. This naturally leads to the following minimum-type state feedback switching control law

$$\pi(\xi) = \arg \min_{\pi \in \mathbb{N}_\mathcal{P}} V(A_\pi \xi + L_\pi). \tag{5}$$

Furthermore, we consider the level set of V with $\rho > 0$ defined as

$$\Omega(\rho) = \{\xi \in \mathbb{R}^n : V(\xi) \leq \rho\}. \tag{6}$$

We aim to identify a minimal-volume invariant set of attraction to ensure that error trajectories starting outside will eventually enter, including the origin, and prevent solutions from exiting once inside. In the steady state, error trajectories remain within this set, and smaller sets help reduce chattering amplitude. We thus define the following problem:

Problem 3 *Given the error dynamics in (3) and the desired operating point x_e, find a set $\Omega(\rho)$ as in Definition 2 that satisfies the specification Ψ with a confidence of at least $(1-\beta) \in (0,1)$ and the associated control law $\pi(\xi)$.*

3. Data-driven Design of Robust Switching Control Law

3.1. Stability Analysis

The following theorem outlines the ellipsoidal set of attraction with the minimum volume, based on the Lyapunov function in (4), the switching control law in (5), and Definition 2.

Theorem 4 *Consider the error dynamics of the switched affine system in (3). From the optimal solution $F > 0$, $W > 0$, and h of the robust convex programming*

$$\boldsymbol{RCP}^I\!: \inf_{F>0, W>0, h} \Big\{ -\ln(\det(W)) : Q_\lambda > W, \begin{bmatrix} Q_\lambda & \varrho_\lambda \\ * & 1 - c_\lambda \end{bmatrix} > 0, \begin{bmatrix} Q_\lambda & \varrho_\lambda \\ * & 1 \end{bmatrix} > 0,$$
$$\forall A_\pi \in [\underline{A}_\pi, \bar{A}_\pi], \forall L_\pi \in [\underline{L}_\pi, \bar{L}_\pi] \Big\}, \tag{7}$$

with $Q_\lambda = \Sigma_\pi \lambda_\pi (F - A_\pi^\top F A_\pi)$, $\varrho_\lambda = \Sigma_\pi \lambda_\pi (A_\pi^\top h + A_\pi^\top F L_\pi - h)$ and $c_\lambda = \Sigma_\pi \lambda_\pi (2h^\top L_\pi + L_\pi^\top F L_\pi)$, determine the quadratic Lyapunov function $V(\xi)$ as defined in (4). The state-dependent switching control law (5) ensures the existence of the set $\mathcal{X}$ centred at $\xi_o = Q_\lambda^{-1}\varrho_\lambda$ for all $A_\pi \in [\underline{A}_\pi, \bar{A}_\pi]$ and $L_\pi \in [\underline{L}_\pi, \bar{L}_\pi]$ as

$$\mathcal{X} = \{\xi \in \mathbb{R}^n : (\xi - \xi_o)^\top W (\xi - \xi_o) \leq 1\}, \tag{8}$$

which satisfies the first two conditions of Definition 2 with minimum volume. Moreover, the centre $\hat{\xi}$ of the Lyapunov function (4) lies in the interior of (8).

Proof The candidate Lyapunov function (4) is strictly convex and positive, except at $\hat{\xi} = -F^{-1}h$, where it achieves its global minimum value of $V(\hat{\xi}) = 0$. By applying the control switching law to the trajectories of error dynamics (3), it is evident that

$$\Delta V(\xi) = V(\xi_{k+1}) - V(\xi_k) = \min_\pi V(A_\pi \xi + L_\pi) - V(\xi) \leq \underbrace{\Sigma_\pi \lambda_\pi V(A_\pi \xi + L_\pi) - V(\xi)}_{f(\xi)}. \tag{9}$$

The right-hand side of (9) allows us to define the following quadratic function

$$\begin{aligned} f(\xi) &= \Sigma_\pi \lambda_\pi \{\xi^\top (A_\pi^\top F A_\pi - F)\xi + 2(h^\top A_\pi + L_\sigma^\top F A_\sigma - h^\top)\xi + (2h^\top L_\pi + L_\pi^\top F L_\pi)\} \\ &= -\xi^\top \underbrace{\{\Sigma_\pi \lambda_\pi (F - A_\pi^\top F A_\pi)\}}_{Q_\lambda} \xi + 2\{\underbrace{\Sigma_\pi \lambda_\pi (A_\pi^\top h + A_\pi^\top F L_\pi - h)}_{\varrho_\lambda(h)}\}^\top \xi + \underbrace{\Sigma_\pi \lambda_\pi (2h^\top L_\pi + L_\pi^\top F L_\pi)}_{c_\lambda(h)} \\ &= -(\xi - Q_\lambda^{-1}\varrho_\lambda)^\top Q_\lambda (\xi - Q_\lambda^{-1}\varrho_\lambda) + c_\lambda + \varrho_\lambda^\top Q_\lambda^{-1}\varrho_\lambda. \end{aligned} \tag{10}$$

If the robust convex programming problem in (7) admits a solution, then there exist symmetric positive definite matrices F and W such that $Q_\lambda = \Sigma_\pi \lambda_\pi (F - A_\pi^\top F A_\pi) > W > 0$, $\forall A_\pi \in [\underline{\mathrm{A}}_\pi, \overline{\mathrm{A}}_\pi]$. Given that $Q_\lambda > W > 0$ and by applying the second constraint from (7), the set defined in (8) ensures that $\Delta V(\xi) < 0$ for all $\xi \notin \mathcal{X}$. We can demonstrate this based on equations (9), (10) with $-Q_\lambda < -W < 0$ and using Schur complements as follows (Boyd and Vandenberghe, 2004)

$$\begin{aligned} &\Delta V(\xi) < -(\xi - Q_\lambda^{-1}\varrho_\lambda)^\top W (\xi - Q_\lambda^{-1}\varrho_\lambda) + c_\lambda + \varrho_\lambda^\top Q_\lambda^{-1}\varrho_\lambda < 0, \forall \xi \notin \mathcal{X}, \\ \Rightarrow \quad & c_\lambda + \varrho_\lambda^\top Q_\lambda^{-1}\varrho_\lambda < 1 \iff \begin{bmatrix} Q_\lambda & \varrho_\lambda \\ \varrho_\lambda^\top & 1 - c_\lambda \end{bmatrix} > 0, \forall A_\pi \in [\underline{\mathrm{A}}_\pi, \overline{\mathrm{A}}_\pi], \forall L_\pi \in [\underline{\mathrm{L}}_\pi, \bar{\mathrm{L}}_\pi]. \end{aligned} \tag{11}$$

In addition, the third constraints in (7) ensures that origin is inside the set (8). We can show this using Schur complement as follows (Boyd and Vandenberghe, 2004)

$$\begin{aligned} 0 \in \mathcal{X} \iff & (0 - Q_\lambda^{-1}\varrho_\lambda)^\top W (0 - Q_\lambda^{-1}\varrho_\lambda) < (0 - Q_\lambda^{-1}\varrho_\lambda)^\top Q_\lambda (0 - Q_\lambda^{-1}\varrho_\lambda) < 1 \\ & \Rightarrow \varrho_\lambda^\top Q_\lambda^{-1}\varrho_\lambda < 1 \iff \begin{bmatrix} Q_\lambda & \varrho_\lambda \\ \varrho_\lambda^\top & 1 \end{bmatrix} > 0, \forall A_\pi \in [\underline{\mathrm{A}}_\pi, \overline{\mathrm{A}}_\pi], \forall L_\pi \in [\underline{\mathrm{L}}_\pi, \bar{\mathrm{L}}_\pi]. \end{aligned} \tag{12}$$

The constraints (11) and (12) implies that $c_\lambda \in (0, 1)$ as we know that $0 < \varrho_\lambda^\top Q_\lambda^{-1}\varrho_\lambda < 1$. The solution to the problem in (7) ensures the minimization of the volume of the set $\mathcal{X}$ defined in (8). This volume is inversely proportional to $\sqrt{\det(W)}$ (Boyd and Vandenberghe, 2004). Furthermore, we have $\hat{\xi} = -F^{-1}h \in \mathrm{int}(\mathcal{X})$. From (4) and (10), it follows $f(\hat{\xi}) = \sum_{\pi \in \mathbb{M}'} \lambda_\pi V\left(A_\pi \hat{\xi} + L_\pi\right) > 0$, being $V(\hat{\xi}) = 0$ and $V(\xi) > 0, \forall \xi \neq \hat{\xi}$. From (9) and (11), we have

$$f(\hat{\xi}) > 0 \Rightarrow -(\hat{\xi} - Q_\lambda^{-1}\varrho_\lambda)^\top Q_\lambda (\hat{\xi} - Q_\lambda^{-1}\varrho_\lambda) < c_\lambda + \varrho_\lambda^\top Q_\lambda^{-1}\varrho_\lambda < 1.$$

Finally, from the first constraint in (7), it implies that

$$-(\hat{\xi} - Q_\lambda^{-1}\varrho_\lambda)^\top Q_\lambda (\hat{\xi} - Q_\lambda^{-1}\varrho_\lambda) < -(\hat{\xi} - Q_\lambda^{-1}\varrho_\lambda)^\top W (\hat{\xi} - Q_\lambda^{-1}\varrho_\lambda) < 1,$$

which concludes the proof. ■

Remark 5 *In Theorem 4, the set $\mathcal{X}$ in (8) is an ellipsoid centered at $\xi_o = Q_\lambda^{-1}\varrho_\lambda$, with $Q_\lambda = \Sigma_\pi \lambda_\pi (F - A_\pi^\top F A_\pi)$ and $\varrho_\lambda = \Sigma_\pi \lambda_\pi (A_\pi^\top h + A_\pi^\top F L_\pi - h)$. Since the constraints in the optimization problem (7) will be satisfied for all $A_\pi \in [\underline{A}_\pi, \overline{A}_\pi]$ and $L_\pi \in [\underline{L}_\pi, \bar{L}_\pi]$, for any A_π and L_π within the specified intervals, there exists a centre ξ_o that defines the set $\mathcal{X}$ to satisfy the given constraints.*

Theorem 4 helps us identify a set of attraction that meets the first two conditions in Definition 2 by addressing the optimization problem in (7). To establish an invariant set of attraction, we must identify additional conditions to satisfy the final criterion in Definition 2. Thus, the following theorem is presented.

Theorem 6 *Let matrices $F > 0$, $W > 0$, and the vector h denote the solutions to the optimization problem defined in (7). The solution ρ of the following robust convex program*

$$\boldsymbol{RCP}^{II}: \inf_{\rho>0,\tau>0}\left\{\rho: \begin{bmatrix} \rho+\tau\xi_o^\top W\xi_o-\tau & h^\top & -\tau\xi_o^\top W \\ * & F & F \\ * & * & \tau W \end{bmatrix} > 0, \forall A_\pi,\ \forall L_\pi \in [\underline{A}_\pi, \bar{A}_\pi],\ [\underline{L}_\pi, \bar{L}_\pi]\right\}, \quad (13)$$

with $\xi_o = Q_\lambda^{-1}\varrho_\lambda$, where $Q_\lambda = \Sigma_\pi \lambda_\pi(F - A_\pi^\top F A_\pi)$ and $\varrho_\lambda = \Sigma_\pi \lambda_\pi(A_\pi^\top h + A_\pi^\top F L_\pi - h)$, provides the ellipsoidal invariant set of attraction $\Omega(\rho) \subseteq \mathcal{X}$ of minimum volume, expressed according to (6) and satisfying all the three conditions of Definition 2.

Proof We draw inspiration from the steps taken in (Deaecto and Geromel, 2016). Let us consider the Lyapunov function (4) with F, h, and d, calculated by applying Theorem 4. To determine the invariant set of attraction with the minimum volume containing $\mathcal{X}$, we consider the following condition for all $A_\pi \in [\underline{\mathrm{A}}_\pi, \bar{\mathrm{A}}_\pi]$ and $L_\pi \in [\underline{\mathrm{L}}_\pi, \bar{\mathrm{L}}_\pi]$

$$\begin{aligned} \rho &= \max_{\xi\in\mathcal{X}} V(\xi) = \inf_{\rho>0}\{\rho : V(\xi) < \rho, \forall \xi \in \mathrm{int}(\mathcal{X})\} \\ &= \inf_{\rho>0}\left\{\rho : d - \rho + 2h^\top\xi + \xi^\top F\xi < 0, \forall\xi \in \mathbb{R}^n,\ s.t. -1 + (\xi-\xi_o)^\top W(\xi-\xi_o) < 0\right\}, \end{aligned} \quad (14)$$

where $\mathcal{X}$ is the set given in (8). By applying the $\mathcal{S}$-procedure, (14) becomes as follows (Boyd and Vandenberghe, 2004)

$$\rho = \inf_{\rho>0,\tau>0}\{\rho : h^\top(F)^{-1}h + \tau - \tau\xi_o^\top W\xi_o + 2(h+\tau W\xi_o)^\top\xi + \xi^\top(F-\tau W)\xi < \rho, \forall\xi \in \mathbb{R}^n\}. \quad (15)$$

The proof boils down to showing that the problem in (15) is feasible. From the constraint of optimization problem (13), and through the Schur complements, it follows

$$\begin{bmatrix} F & F \\ F & \tau W \end{bmatrix} > 0 \iff F - \tau W < 0,$$

thus the quadratic function in (15), say $g(\xi) = h^\top F^{-1}h + \tau - \tau\xi_o^\top W\xi_o - \rho + 2(h+\tau W\xi_o)^\top\xi + \xi^\top(F-\tau W)\xi$, is strictly concave. Its extreme point occurs for $\xi^* = -(F-\tau W)^{-1}(h+\tau W\xi_o)$, where the function attains its maximum value, $g(\xi^*) = -(h+\tau W\xi_o)^\top(F-\tau W)^{-1}(h+\tau W\xi_o) + h^\top F^{-1}h + \tau - \tau\xi_o^\top W\xi_o - \rho$. From the constraint of optimization problem (13), it is also

$$\begin{bmatrix} (\rho + \tau\xi_o^\top W\xi_o - \tau - (h)^\top(F)^{-1}h) & -(h+\tau W\xi_o)^\top \\ -(h+\tau W\xi_o) & -(F-\tau W) \end{bmatrix} > 0,$$

which, through simple Schur complement calculations, implies that

$$g(\xi^*) = -(h+\tau W\xi_o)^\top(F-\tau W)^{-1}(h+\tau W\xi_o) + h^\top F^{-1}h + \tau - \tau\xi_o^\top W\xi_o - \rho < 0.$$

As a result, it is $g(\xi) < 0, \forall\xi \in \mathbb{R}^n, \forall A_\pi \in [\underline{\mathrm{A}}_\pi, \bar{\mathrm{A}}_\pi]$, and $\forall L_\pi \in [\underline{\mathrm{L}}_\pi, \bar{\mathrm{L}}_\pi]$, and the problem in (15) admits a solution. Finally, it is easy to verify that $\Omega(\rho)$ is an invariant set of attraction, i.e., it satisfies all the conditions in Definition 2, being a $\mathcal{X} \subseteq \Omega(\rho)$ and $V(\xi)$, determined by applying Theorem 6, is a valid Lyapunov function for the system (3) (i.e., its level sets are invariant). ■

Here, the main challenge is to solve two robust convex programming outlined in (7) and (13), which will be addressed in Section 3.2.

Remark 7 *The optimization problems in equations (7) and (13) define a robust convex program with an infinite number of Linear Matrix Inequality (LMI) constraints, as the inequalities must hold for all admissible parameters A_π and L_π within their specified bounds. To simplify this problem, we will use the scenario approach in the next section, which approximates the original problem by applying constraints to a finite set of sampled scenarios.*

3.2. Scenario Optimization to Design the Invariant Set of Attraction

This section addresses proposed robust convex programming problems through a data-driven wait-and-judge scenario approach (Campi and Garatti, 2018b; Campi et al., 2021b), where the robust optimization problems in (7) and (13) can be solved with a certain confidence by taking random samples from the uncertain variables.

The optimization problem (7) can be reformulated as a convex scenario program. We consider a uniform distribution over the space $\mathcal{W} = [\underline{\mathrm{A}}_\pi, \overline{\mathrm{A}}_\pi] \times [\underline{\mathrm{L}}_\pi, \bar{\mathrm{L}}_\pi]$ and obtain N sampled scenarios $\omega^i = (A_\pi, L_\pi)^i = (A_\pi^i, L_\pi^i)$, $i \in \{1, 2, \dots, N\}$. Then, the optimization (7) can be approximated as

$$\boldsymbol{SP_N^I} : \inf_z \left\{ -\ln(\det(W)) : \underbrace{Q_\lambda^i > W, \begin{bmatrix} Q_\lambda^i & \varrho_\lambda^i \\ (\varrho_\lambda^i)^\top & 1 - c_\lambda^i \end{bmatrix} > 0, \begin{bmatrix} Q_\lambda^i & \varrho_\lambda^i \\ (\varrho_\lambda^i)^\top & 1 \end{bmatrix} > 0, F > 0, W > 0,}_{\text{Set of Constraints } \mathcal{Z}} \forall i \in \mathbb{N}_N \right\}, \tag{16}$$

where $Q_\lambda^i = \Sigma_\pi \lambda_\pi (F - (A_\pi^i)^\top F A_\pi^i)$, $\varrho_\lambda^i = \Sigma_\pi \lambda_\pi ((A_\pi^i)^\top h + (A_\pi^i)^\top F L_\pi^i - h)$, $c_\lambda^i = \Sigma_\pi \lambda_\pi (2h^\top L_\pi^i + (L_\pi^i)^\top F L_\pi^i)$ and $z = \{F, W, h\}$ is the set of variables. Denote by $\mathcal{V}_1(z)$ the constraint violation probability with respect to the probability measure put on the uncertainty space $\mathcal{W}$ when the optimization variables are set to z. Let z_N^* represent the optimal solution to equation (16). The solution is considered robust at the robustness level ε_1 if $\mathcal{V}_1(z_N^*) \le \varepsilon_1$.

Similarly, we approximate the optimization (13) with a convex scenario program. Consider the uniform distribution over $\mathcal{W}$ and obtain M new sampled scenarios $w^j = (A_\pi, L_\pi)^j = (A_\pi^j, L_\pi^j)$, $j \in \{1, 2, \dots, M\}$. The optimization (16) can then be approximated as

$$\boldsymbol{SP_M^{II}} : \inf_y \left\{ \rho : \underbrace{\begin{bmatrix} \rho + \tau (\xi_o^j)^\top W \xi_o^j - \tau & h^\top & -\tau (\xi_o^j)^\top W \\ h & F & F \\ -\tau W (\xi_o^j)^\top & F & \tau W \end{bmatrix} > 0, \rho > 0, \tau > 0, \forall j \in \mathbb{N}_M}_{\text{Set of Constraints } \mathcal{Y}} \right\}, \tag{17}$$

with $\xi_o^j = (Q_\lambda^j)^{-1} \varrho_\lambda^j$, where $Q_\lambda^j = \Sigma_\pi \lambda_\pi (F - (A_\pi^j)^\top F A_\pi^j)$, $\varrho_\lambda^j = \Sigma_\pi \lambda_\pi ((A_\pi^j)^\top h + (A_\pi^j)^\top F L_\pi^j - h)$ and $y = \{\rho, \tau\}$ is the set of variables. Denote by $\mathcal{V}_2(y)$ the constraint violation probability with respect to the probability measure put on the uncertainty space $\mathcal{W}$ when the optimization variables are set to y. Let y_M^* represent the optimal solution to equation (17). If $\mathcal{V}_2(y_M^*) \le \varepsilon_2$, the solution is considered robust at the robustness level ε_2.

A scenario ω is considered a ***support scenario*** if its removal changes the solution to optimization problem. Denote by s_{1N}^* and s_{2M}^* the number of support scenarios of (16) and (17), respectively. Following the *wait-and-judge* paradigm described by Campi and Garatti (2018b), we have the following theorem, which describes how to compute robustness levels $\varepsilon_1, \varepsilon_2$ with a certain confidence by identifying the number of support scenarios.

Algorithm 1 Robust Switching Control Law Design

Require: Interval uncertainty $[\underline{A}_\pi, \overline{A}_\pi]$ and $[\underline{B}_\pi, \overline{B}_\pi]$ for system matrices in (1), confidence level β
1: Define $\mathcal{F}$ as in (2) based on the chosen $\hat{A}_\pi, \hat{B}_\pi \in [\underline{A}_\pi, \overline{A}_\pi], [\underline{B}_\pi, \overline{B}_\pi]$
2: Compute $x'_e := \arg\min_{x'}\{\|x' - x\|, x \in \mathcal{F}\}$
3: Define the confidence level for each optimization of $\boldsymbol{SP^I_N}$ and $\boldsymbol{SP^{II}_M}$ in a way that $\beta_1 + \beta_2 = \beta$.
4: Decide on number of samples N based on β_1 and Theorem 8
5: Solve $\boldsymbol{SP^I_N}$ in (16) to get F, W, and h
6: Set F, h, and $d = h^\top F^{-1} h$ in Lyapunov function (4) and get $V(\xi)$
7: Select the number of samples M using β_2 and Theorem 8
8: Solve $\boldsymbol{SP^{II}_M}$ in (17) to get ρ
9: Set ρ in (6) and get invariant set of attraction $\Omega(\rho)$
10: Compute $\mathcal{G}$ as the smallest set such that $\Omega + x'_e \subset \mathcal{G}$ and $x_e \in \mathcal{G}$
11: Set $x_0 = 0$ and $\xi_0 = x_0 - x_e$
12: **for** $k \in \mathbb{N}$ **do**
  Compute $\pi_k(E_k)$ using (5) Update ξ_{k+1} using (3) and by applying upper-bound and lower-bound of uncertainty
  end

Ensure: Set $\mathcal{G}$ and switching control law π

Theorem 8 *Assume the optimizations in (7) and (13) are feasible. For a given confidence value $\beta \in (0,1)$, we have that*

$$\mathbf{P}^{N+M}\left\{\mathcal{V}_1\left(z^*_N\right) > \varepsilon_1\left(s^*_{1N}\right) \text{ and } \mathcal{V}_2\left(y^*_M\right) > \varepsilon_2\left(s^*_{2M}\right)\right\} \leq \beta,$$

where $\varepsilon_i := 1 - \iota_i$, $i = 1, 2$, and ι_1, ι_2 are the unique solutions of the following equation for ι

$$\frac{\eta}{N_c + 1}\sum_{q=k}^{N_c}\binom{q}{k}\iota^{q-k} - \binom{N_c}{k}\iota^{N_c - k} = 0,$$

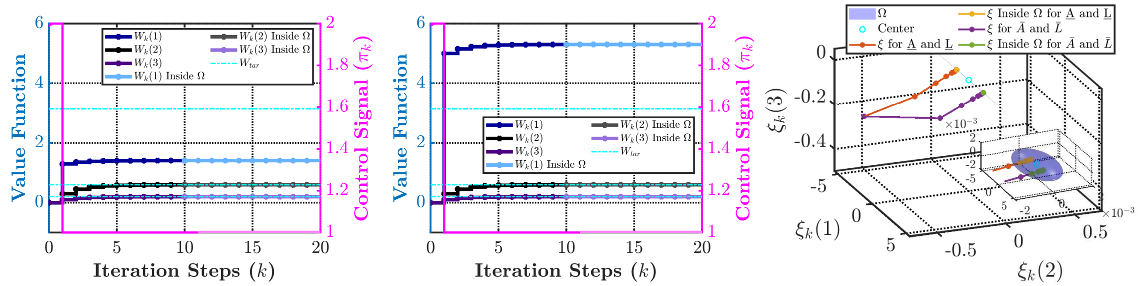
by replacing (η, N_c, k) respectively with (β_1, N, s^*_{1N}) and (β_2, M, s^*_{2M}), and where $\beta = \beta_1 + \beta_2$.

Note that if the robust convex programs in (7) and (13) are feasible, then the corresponding scenario programs in (16) and (17) will also be feasible. The converse is not necessarily true due to the reduction in the number of constraints. The above theorem enables us to have a trade-off between the two confidence values β_1, β_2 attributed to the two constraint violation probabilities. The results of Theorems 4, 6 and 8, along with their scenario programming reformulation in this section, are combined into Algorithm 1 to design a robust switching control law π.

4. Applications

Switched affine systems with interval uncertainty have various applications. This study focuses on two applications: MOIMDPs and power electronic systems. In all the examples provided, the optimization conditions are verified using the CVX package in MATLAB (Grant and Boyd, 2014). The computations are performed on a MacBook with the M2 chip and 16GB of memory.

MOIMDPs. Designing policies to optimize multiple objectives in IMDPs can be modeled with uncertain switched affine systems (Monir et al., 2024), which is performed through value iteration algorithms. The values W that give the objectives as a function of state satisfy $W_{k+1} = A_\pi W_k + B_\pi$, where π is the policy at time k, matrices A_π depend on the transition probabilities, and matrices B_π contain the objectives as a function of states. The target values $W_{tar} := \left[w^1_{tar}; w^2_{tar}; \ldots; w^q_{tar}\right]$ for all objectives can be selected from the desired operating points presented in (2).



(a) (b) (c)

Figure 1: The value function evolution is plotted for IMDP, considering $\underline{\mathrm{A}}_\pi$ and $\underline{\mathrm{L}}_\pi$ in (a), and $\bar{\mathrm{A}}_\pi$ and $\bar{\mathrm{L}}_\pi$ in (b). The switching law is indicated in pink in both (a) and (b). The invariant set of attraction with error trajectories for both lower- and upper-bound matrices is shown in (c).

An example of IMDP is adopted from (Hahn et al., 2019; Monir et al., 2024), with the set of states $S = \{s, t, u\}$, the initial state s, and the set of actions $U = \{a, b\}$. The non-zero transition probability intervals are illustrated (Monir et al., 2024, Fig. 5). Here, we consider two stationary policies $\pi \in \{1, 2\}$, where each policy corresponds to a fixed action selection for all states and remains unchanged over time. The interval transition probability matrices are defined as

$$\underline{\mathrm{P}}(1) = \begin{bmatrix} 0 & \frac{1}{3} & \frac{1}{10} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \bar{\mathrm{P}}(1) = \begin{bmatrix} 0 & \frac{2}{3} & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \underline{\mathrm{P}}(2) = \begin{bmatrix} 0 & \frac{2}{5} & \frac{1}{10} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \bar{\mathrm{P}}(2) = \begin{bmatrix} 0 & \frac{3}{5} & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The interval reward vectors are defined as $\underline{\mathrm{R}}(1) = \left[\frac{13}{10}; \frac{3}{10}; \frac{1}{10}\right]$, $\bar{\mathrm{R}}(1) = \left[5; \frac{3}{10}; \frac{1}{10}\right]$, $\underline{\mathrm{R}}(2) = \left[\frac{1}{2}; \frac{3}{10}; \frac{1}{10}\right]$, and $\bar{\mathrm{R}}(2) = \left[\frac{8}{5}; \frac{3}{10}; \frac{1}{10}\right]$. The discount factor $\gamma = 0.5$ and target W_{tar} corresponding to $\lambda = \left[\frac{9}{10}; \frac{1}{10}\right]$ and the initial values $W_0 = [0; 0; 0]$ are selected. Note that we focus on IMDPs, where transition probabilities are represented as intervals to account for uncertainty. Each policy gives a specific action for each state, but the actual transition probabilities can vary within the specified bounds. Importantly, the upper-bound matrices do not need to be valid probability distributions and may have rows that sum to more than 1. During execution, a valid probability distribution (with rows summing to 1) is selected for each state-action pair within these bounds.

The number of samples taken to solve scenario optimization problems are $N = 700$ and $M = 250$, with the calculated confidence being $\beta = 0.05$. The results obtained via Algorithm 1 are illustrated in Figure 1. To demonstrate our algorithm's applicability to MOIMDPs, we enhance the existing IMDP example by incorporating a second objective, resulting in an IMDP with two objectives. Our new objective is defined via the interval reward vectors $\underline{\mathrm{R}}(1) = \left[\frac{13}{30}; \frac{1}{5}; \frac{2}{5}\right]$, $\bar{\mathrm{R}}(1) = \left[5; \frac{1}{5}; \frac{2}{5}\right]$, $\underline{\mathrm{R}}(2) = \left[\frac{13}{16}; \frac{1}{5}; \frac{2}{5}\right]$, and $\bar{\mathrm{R}}(2) = \left[\frac{19}{12}; \frac{1}{5}; \frac{2}{5}\right]$, as well as a discount factor of $\gamma = 0.7$. Target W_{tar} corresponding to $\lambda = \left[\frac{9}{10}; \frac{1}{10}\right]$ and the initial values $W_0^1 = W_0^2 = [0; 0; 0]$ are selected. The number of samples in the scenario optimization problems is $N = 1500$ and $M = 300$, which gives the confidence $\beta = 0.05$. The results obtained via Algorithm 1 are illustrated in Figure 2.

Power Electronic Systems. The class of switched affine systems encompasses key features of various applications, including power electronic converters (Deaecto et al., 2010; Albea et al., 2015; Mojallizadeh and Badamchizadeh, 2018). Here, we applied Algorithm 1 to Single-Inductor Multiple-Output (SIMO) converter to show the effectiveness of our algorithm. The SIMO boost converter consists of three switches. Depending on the switch positions, the converter can be modeled as a switched affine system. We adopted the model and parameters from

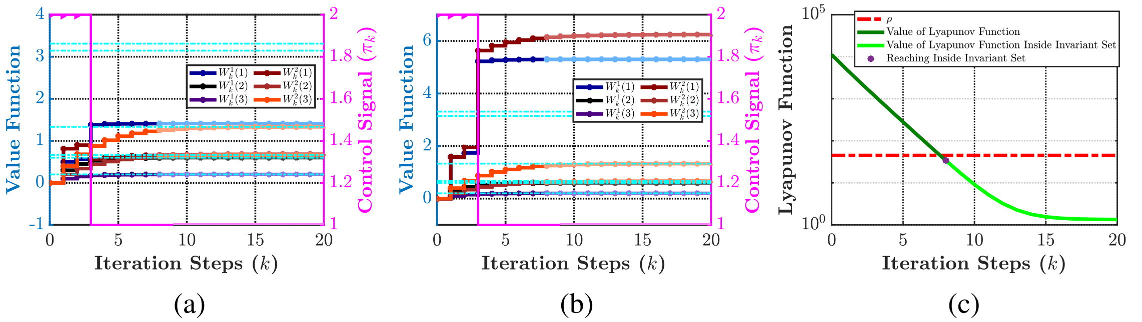


Figure 2: The evolution of the value function for MOIMDP considering $\underline{\mathrm{A}}_\pi$ and $\underline{\mathrm{L}}_\pi$ in (a), and $\bar{\mathrm{A}}_\pi$ and $\bar{\mathrm{L}}_\pi$ in (b). The switching law is indicated in pink in both (a) and (b). The Lyapunov function in (c) shows that the trajectories approach the invariant set of attraction and remain within the set.

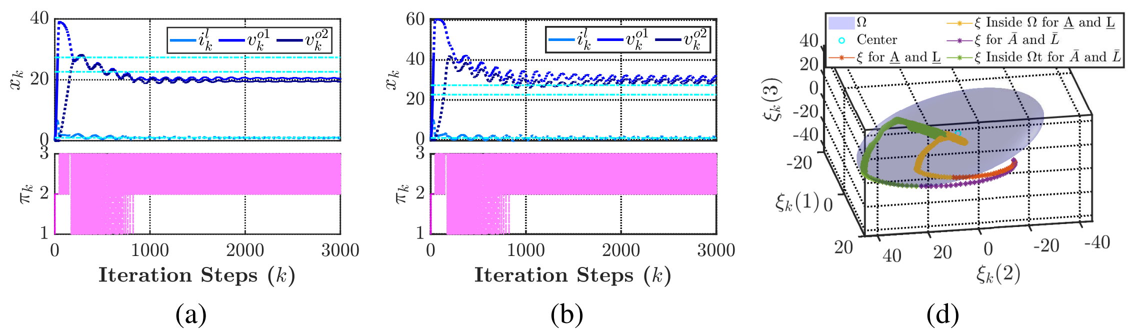


Figure 3: The evolution of x_k for lower- and upper-bound matrices is illustrated in (a) and (b). The desired operating point x_e is indicated by light blue in both (a) and (b). The switching law is shown in pink in both (a) and (b), in steady state it is switching between $\pi = 2$ and $\pi = 3$. The invariant set of attraction with error trajectories for both lower- and upper-bound matrices is expressed in (c).

(Mojallizadeh and Badamchizadeh, 2018) and discretized it using the Euler discretization method with an appropriate sampling step time of $\Delta t = 0.1$ milliseconds (Butcher, 2016). Based on model (1), the state vector is $x_k = [i_k^l; v_k^{o1}; v_k^{o2}]$, where i^l is inductor current; v_k^{o1} and v_k^{o2} are output voltages. The system switching laws are defined as $\pi \in \mathbb{N}_\mathcal{P}$ with $\mathcal{P} = 3$. System matrices are

$$A_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 - \frac{-\Delta t}{\mathcal{R}_1 \mathcal{C}_1} & 0 \\ 0 & 0 & 1 - \frac{-\Delta t}{\mathcal{R}_2 \mathcal{C}_2} \end{bmatrix}, A_2 = \begin{bmatrix} 0 & -\frac{1}{\mathcal{L}} & 0 \\ -\frac{1}{\mathcal{C}_1} & 1 - \frac{-\Delta t}{\mathcal{R}_1 \mathcal{C}_1} & 0 \\ 0 & 0 & 1 - \frac{-\Delta t}{\mathcal{R}_2 \mathcal{C}_2} \end{bmatrix}, A_3 = \begin{bmatrix} 0 & 0 & -\frac{1}{\mathcal{L}} \\ 0 & 1 - \frac{-\Delta t}{\mathcal{R}_1 \mathcal{C}_1} & 0 \\ -\frac{1}{\mathcal{C}_1} & 0 & 1 - \frac{-\Delta t}{\mathcal{R}_2 \mathcal{C}_2} \end{bmatrix},$$

and $B_1 = B_2 = B_3 = \left[\frac{v_i}{\mathcal{L}}; 0; 0\right]$, where $v_i \in [20, 30]\ V$ is input voltage; $\mathcal{R}_1 \in [39, 56]\ \Omega$ and $\mathcal{R}_2 \in [47, 68]\ \Omega$ are resistive loads; $\mathcal{L} = 5\ mH$ denotes the inductance; and $\mathcal{C}_1 = \mathcal{C}_2 = 470\ \mu F$ are the capacitance of capacitors. The parameters are based on the laboratory prototype used in (Mojallizadeh and Badamchizadeh, 2018). Since $\mathcal{R}_1$, $\mathcal{R}_2$, and v_i are uncertain, it is straightforward to determine $[\underline{\mathrm{A}}_\pi, \bar{\mathrm{A}}_\pi]$ and $[\underline{\mathrm{B}}_\pi, \bar{\mathrm{B}}_\pi]$ based on the upper and lower bounds of the uncertain parameters. The operating point x_e corresponding to $\lambda = [0; 0.5; 0.5]$ and the initial values $x_0 = [0; 0; 0]$ are selected. The number of samples for scenario optimization problems are $N = 900$ and $M = 200$, with confidence $\beta = 0.06$. The results obtained via Algorithm 1 are illustrated in Figure 3.

Acknowledgments

The research of N. Monir is supported by the EPSRC EP/W524700/1 and Newcastle University Global Scholarship. The research of S. Soudjani is supported by the following grants: EIC 101070802 and ERC 101089047.

References

C. Albea, G. Garcia, and L. Zaccarian. Hybrid dynamic modeling and control of switched affine systems: application to dc-dc converters. In *2015 54th IEEE Conference on Decision and Control (CDC)*, pages 2264–2269. IEEE, 2015.

C. Albea Sanchez, A. Ventosa-Cutillas, A. Seuret, and F. Gordillo. Robust switching control design for uncertain discrete-time switched affine systems. *International Journal of Robust and Nonlinear Control*, 30(17):7089–7102, 2020.

G. Beneux, D. Astolfi, P. Riedinger, J. Daafouz, and L. Grimaud. Integral action for uncertain switched affine systems with application to dc/dc converters. In *2018 European Control Conference (ECC)*, pages 795–800. IEEE, 2018.

J. Berberich, J. Köhler, M.A. Müller, and F. Allgöwer. Data-driven model predictive control with stability and robustness guarantees. *IEEE Transactions on Automatic Control*, 66(4):1702–1717, 2020.

S. Boyd and L. Vandenberghe. *Convex optimization*. Cambridge university press, 2004.

J.C. Butcher. *Numerical methods for ordinary differential equations*. John Wiley & Sons, 2016.

M.C. Campi and S. Garatti. *Introduction to the scenario approach*. SIAM, 2018a.

M.C. Campi and S. Garatti. Wait-and-judge scenario optimization. *Mathematical Programming*, 167:155–189, 2018b.

M.C. Campi, S. Garatti, and M. Prandini. The scenario approach for systems and control design. *Annual Reviews in Control*, 33(2):149–157, 2009.

M.C. Campi, A. Carè, and S. Garatti. The scenario approach: A tool at the service of data-driven decision making. *Annual Reviews in Control*, 52:1–17, 2021a.

M.C. Campi, A. Carè, and S. Garatti. The scenario approach: A tool at the service of data-driven decision making. *Annual Reviews in Control*, 52:1–17, 2021b. ISSN 1367-5788.

C. De Persis and P. Tesi. Formulas for data-driven control: Stabilization, optimality, and robustness. *IEEE Transactions on Automatic Control*, 65(3):909–924, 2019.

G.S. Deaecto and J.C. Geromel. Stability analysis and control design of discrete-time switched affine systems. *IEEE Transactions on Automatic Control*, 62(8):4058–4065, 2016.

G.S. Deaecto, J.C. Geromel, F.S. Garcia, and J.A. Pomilio. Switched affine systems control design with application to dc–dc converters. *IET control theory & applications*, 4(7):1201–1210, 2010.

M. Della Rossa, Z. Wang, L.N. Egidio, and R.M. Jungers. Data-driven stability analysis of switched affine systems. In *2021 60th IEEE Conference on Decision and Control (CDC)*, pages 3204–3209. IEEE, 2021.

E. Dietrich, A. Devonport, and M. Arcak. Nonconvex scenario optimization for data-driven reachability. In *6th Annual Learning for Dynamics & Control Conference*, pages 514–527. PMLR, 2024.

S. Garatti and M.C. Campi. Non-convex scenario optimization. *Mathematical Programming*, pages 1–52, 2024.

M. Grant and S. Boyd. CVX: Matlab software for disciplined convex programming, version 2.1. https://cvxr.com/cvx, March 2014.

E.M. Hahn, V. Hashemi, H. Hermanns, M. Lahijanian, and A. Turrini. Interval Markov decision processes with multiple objectives: From robust strategies to Pareto curves. *ACM Transactions on Modeling and Computer Simulation (TOMACS)*, 29(4):1–31, 2019.

R. Iervolino, M. Tipaldi, and A. Forootani. A lyapunov-based version of the value iteration algorithm formulated as a discrete-time switched affine system. *International Journal of Control*, 96 (3):577–592, 2023.

M.R. Mojallizadeh and M.A. Badamchizadeh. Hybrid control of single-inductor multiple-output converters. *IEEE Transactions on Industrial Electronics*, 66(1):451–458, 2018.

N. Monir, O. Schön, and S. Soudjani. Lyapunov-based policy synthesis for multi-objective interval mdps. *IFAC-PapersOnLine*, 58(11):99–106, 2024.

A. Salamati and M. Zamani. Data-driven safety verification of stochastic systems via barrier certificates: A wait-and-judge approach. In *Learning for Dynamics and Control Conference*, pages 441–452. PMLR, 2022.

C. Seatzu, D. Corona, A. Giua, and A. Bemporad. Optimal control of continuous-time switched affine systems. *IEEE transactions on automatic control*, 51(5):726–741, 2006.

A. Seuret, C. Albea, and F. Gordillo. Practical stabilization of switched affine systems: Model and data-driven conditions. *IEEE Control Systems Letters*, 7:1628–1633, 2023.

H.J. Van Waarde, J. Eising, H.L. Trentelman, and M.K. Camlibel. Data informativity: A new perspective on data-driven analysis and control. *IEEE Transactions on Automatic Control*, 65 (11):4753–4768, 2020.

Zheming Wang, Guillaume O Berger, and Raphaël M Jungers. Data-driven control of unknown switched linear systems using scenario optimization. *IEEE Transactions on Automatic Control*, 2024.

Lingyu Zhang, Xuyang Lou, and Zhan Wang. Output-based robust switching rule design for uncertain switched affine systems: Application to dc–dc converters. *IEEE Transactions on Circuits and Systems II: Express Briefs*, 69(11):4493–4497, 2022.