Useful formulas :-

4.1.1) The fourier series for continuous time periodic signals

synthesis $eq^n \rightarrow x_n = \sum_{k=-\alpha}^{\alpha} c_k e^{j2\pi k f_0 t}$

analysis $eq^n \rightarrow c_r = \frac{1}{T_P} \int_{T_P} x(t) e^{-j2\pi k f_0 t} \, dt$

4.1.2) power Density spectrum of periodic signals

$$P_x = \frac{1}{T_P} \int_{T_P} |x(t)|^2 \, dt$$

parsavals relation for power signals

$$P_x = \frac{1}{T_P} \int_{T_P} |x(t)|^2 \cdot dt = \sum_{k=-\alpha}^{\alpha} |C_k|^2$$

4.1.3) The fourier transform for continuous - Time Aperiodic Signals

synthesis $eq^n \rightarrow x(t) = \int_{-\alpha}^{\alpha} X(F) e^{j2\pi F t} \, dF$ (invers transform)

Analysis $eq^n \rightarrow X(F) = \int_{-\alpha}^{\alpha} x(t) \cdot e^{-j2\pi F t} \, dt$ (direct transform)

4.1.4) Energy Density specform of Aperiodic finit energy signals

$$E_x = \int_{-\alpha}^{\alpha} |x(t)|^2 dt = \int_{-\alpha}^{\alpha} |X(F)|^2 \cdot dF$$

4.2.1) The fouriser series for Discrete - time periodic signals

syntesis $eq^n \rightarrow x(n) = \sum_{k=0}^{N-1} C_k e^{j2\pi kn/N}$

analysis $eq^n \rightarrow$

$$C_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}$$

4.2.2) power Density spectrum of periodic Signals

$$P_x = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2$$

4.2.3) The fouriser transform of Discrete - time Aperiodic Signals

Syntesis $eq^n \longrightarrow x(n) = \frac{1}{2\pi} \int_{2\pi} X(\omega) e^{j\omega n} \, d\omega$

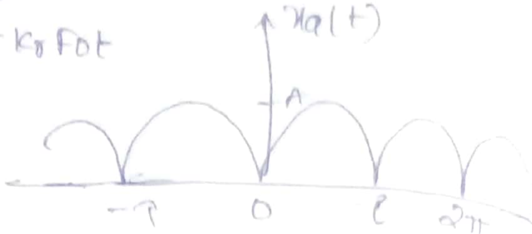
Analysis $eq^n \longrightarrow X(\omega) = \sum_{-\alpha}^{\alpha} x(n) e^{-j\omega n}$

problems:-

1.1) consider the full wave rectified sinusoid

a) determine its spectrum $X_a(F)$

$$x_a(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k F_0 t}$$



let $F_0 = \frac{1}{T}$

$$= \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k \frac{t}{T}}$$

$$c_k = \frac{1}{T}\int_0^T A\sin\left(\frac{\pi t}{T}\right) e^{-j2\pi kF/T} \cdot dt$$

$$= \frac{1}{T}\int_0^T A \cdot \frac{e^{\frac{j\pi t}{T}} - e^{-\frac{j\pi t}{T}}}{2j}\, e^{-j2\pi k\frac{t}{T}}\, dt$$

$$= \frac{A}{2j\cdot T}\int_0^T \left(e^{\frac{j2\pi}{T}} - e^{-\frac{j2\pi}{T}}\right) \cdot e^{-j2\pi k\frac{t}{T}}\, dt$$

$$= \frac{A}{2jT}\int_0^T \left(e^{\frac{j\pi t}{T}} \cdot e^{-j2\pi k\frac{t}{T}}\right) - \left(e^{-\frac{j\pi t}{T}} \cdot e^{-j2\pi k\frac{t}{T}}\right) dt$$

$$= \frac{A}{2jT}\int_0^T e^{j\pi(1-2k)\frac{t}{T}} - e^{-j\pi(1+2k)\frac{t}{T}}\, dt$$

$$= \frac{A}{2j\cdot T}\left[\int_0^T e^{j\pi(1-2k)\frac{t}{T}}\, dt - \int_0^T e^{-j\pi(1+2k)\frac{t}{T}}\, dt\right]$$

$$= \frac{A}{2jT}\left\{\left[\frac{e^{j\pi(1-2k)\frac{t}{T}}}{j\pi(1-2k)\frac{1}{T}}\right]_0^T - \left[\frac{e^{-j\pi(1+2k)\frac{t}{T}}}{-j\pi(1+2k)\frac{1}{T}}\right]_0^T\right\}$$

$$= \frac{A}{2jT}\left[\frac{e^{j\pi(1-2k)} - e^0}{j\pi(1-2k)\cdot\frac{1}{T}} - \frac{e^{-j\pi(1+2k)} - e^0}{-j\pi(1+2k)\frac{1}{T}}\right]$$

$$= \frac{A}{2j\cancel{T}} \cdot \frac{\cancel{T}}{j\pi}\left[\frac{e^{j\pi(1-2k)} - 1}{(1-2k)} + \frac{e^{-j\pi(1+2k)} - 1}{1+2k}\right]$$

$$= \left\{ \frac{A}{2\pi} \left[\frac{-1-1}{(1-2k)} + \frac{-1-1}{1+2k} \right] \right.$$

$$= -\frac{A}{2\pi} \left[-2 \left[\frac{1}{1-2k_\gamma} + \frac{1}{1+2k_\gamma} \right] \right]$$

$$= \frac{A}{\pi} \left[\frac{1}{1-2k_\gamma} + \frac{1}{2k+1} \right]$$

$$= \frac{A}{\pi} \left[\frac{1+\cancel{2k_\gamma}+1-\cancel{2k_\gamma}}{1+\cancel{2k_\gamma}-\cancel{2k_\gamma}-4k^2} \right]$$

$$= \frac{2A}{\pi(1-4k^2)}$$

$$x_A(f) = \int_{-\alpha}^{\alpha} x_q(t)\, e^{-j2\pi f t} \cdot dt$$

$$= \sum_{-\alpha}^{\alpha} \sum_{h=\alpha}^{\alpha} c_{k_\gamma} e^{j2\pi k_\gamma f_0 t} \cdot e^{-j2\pi k t} \cdot dt$$

$$= \sum_{k=-\alpha}^{\alpha} \int_{-\alpha}^{\alpha} c_{k_\gamma} e^{-j2\pi (f-k_\gamma f_0)t} dt$$

$$= \sum_{k=-\alpha}^{\alpha} \int_{-\alpha}^{\alpha} c_{k_\gamma} e^{-j2\pi \left(f-\frac{k_\gamma}{T}\right)t} dt$$

$$= \sum_{k=-\alpha}^{\alpha} c_{k_\gamma} \int_{-\alpha}^{\alpha} c^{-j2\pi \left(f-\frac{k}{T}\right)t} \cdot dt$$

$$= \sum_{k=-\alpha}^{\alpha} c_{k_\gamma}\, \delta\left(f-\frac{k_\gamma}{T}\right)$$

b) compute the power of the signals

$$P_x = \frac{1}{T} \int_0^T x_a^2(t)\, dt$$

$$= \frac{1}{T} \int_0^T \left(A \sin \frac{At}{T}\right)^2 dt$$

$$= \frac{A^2}{T} \int \sin^2 \frac{\pi}{T} \cdot t \cdot dt$$

denomanator

$$= \frac{A^2}{T}\int_0^T \frac{1-\cos^2\left(\frac{\pi}{T}\right).t}{2}$$

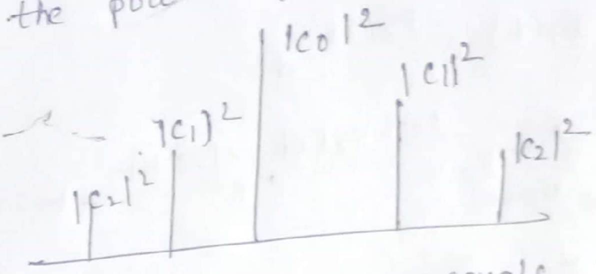
$$= \frac{A^2}{T}\int_0^{\pi} \frac{1}{2} - \frac{\cos 2\left(\frac{\pi}{T}\right)-t}{2}$$

$$= \frac{A^2}{T}\ \frac{T}{2} - \left[\frac{\cos_2(\pi)}{2} - 0\right]$$

$$= \frac{A^2}{\cancel{T}}\ \frac{\cancel{T}}{2} - 0$$

$$= \frac{A^2}{2}$$

c) plot the power spectral density



d) check the validity of parsevals relation for the given signals

$$Px = \sum_{k=-\alpha}^{\alpha} |ck|^2$$

$$= \sum_{k=-\alpha}^{\alpha}\left(\frac{2A}{\pi(1-4k^2)}\right)^2$$

$$= \frac{4A^2}{\pi^2} \cdot \sum_{k=-\alpha}^{\alpha} \cdot \frac{1}{(4k^2-1)^2}$$

$$= \frac{4A^2}{\pi^2}\left[\frac{1}{(4k^2-1)^2}\Big|_{k=0} + 2\sum_{k=1}^{\alpha}\frac{1}{(4k^2-1)}\right]$$

$$= \frac{4A^2}{\pi^2}\left[1+\frac{2}{3^2}+\frac{2}{15^2}+\dots\right]$$

Consider some values

$$= \frac{4A^2}{\pi^2}\left[1.231\right]$$

$$= 0.498\,A^2$$

$$= 0.5\,A^2 \qquad A^2$$

3) compute and sketch the magnitude and phase spectra for the following signals $(a > 0)$

a) $$x_a(t) = \begin{cases} Ae^{-at} & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$x_a(F) = \int_0^{\infty} Ae^{-at}\, e^{-j2\pi Ft}\, dt$$

$$= A \int_0^{\infty} e^{-(a+2\pi F j)t}\, dt$$

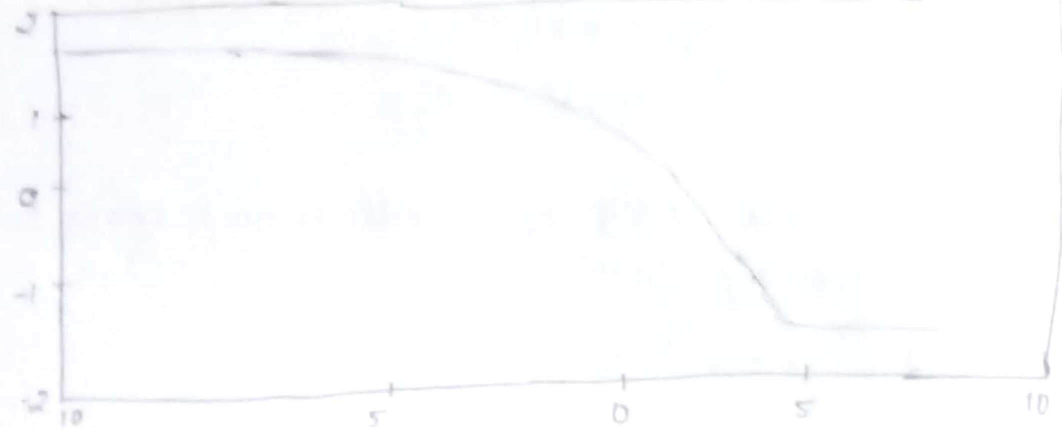
$$= A \left[\frac{e^{-(a+2\pi F j)t}}{-(a+2\pi F j)} \right]_0^{\infty}$$

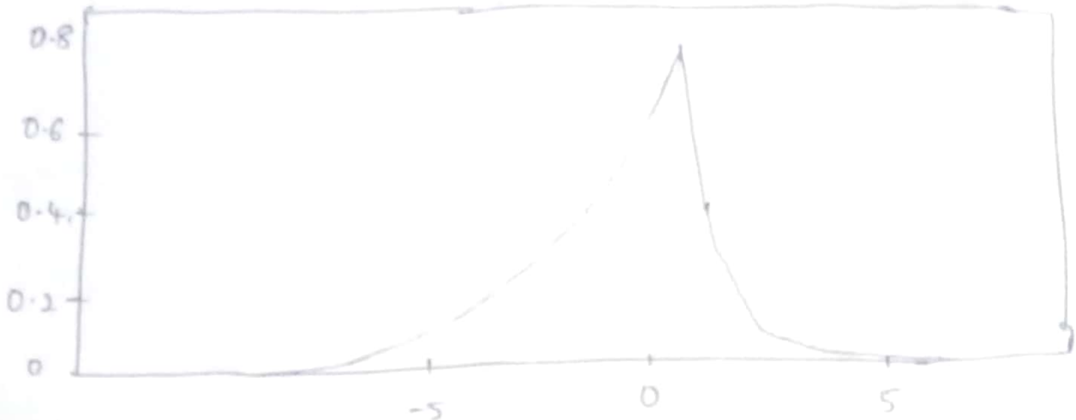
$$= A \cdot \frac{1}{a + j2\pi F}$$

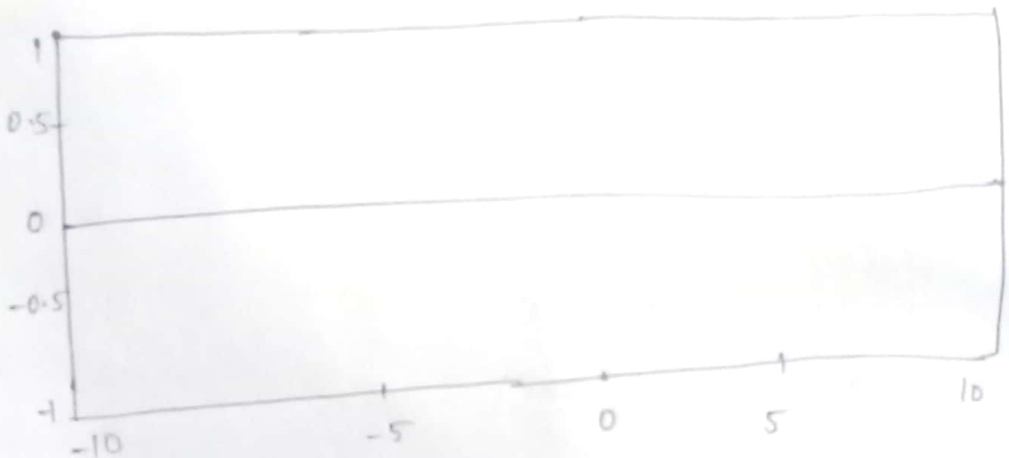
$$= \frac{A}{a + j2\pi F}$$

$$|x_a(F)| = \frac{A}{\sqrt{a^2 + (2\pi F)^2}}$$

$$|x_a(F)| = -\tan^{-1}\left(\frac{2\pi F}{a}\right)$$







b) $x_A(t) = Ae^{-a(t)}$

$$X_A(f) = \int_{-\infty}^{\infty} Ae^{-a(t)} e^{-j2\pi ft} \, dt$$

$$= \int_{0}^{-\infty} Ae^{at} e^{-j2\pi ft} + \int_{0}^{\infty} Ae^{-at} e^{-j2\pi ft}$$

$$= \int_{0}^{-\infty} Ae^{-at} e^{+j2\pi ft} + \int_{0}^{\infty} Ae^{-(a)t} e^{-(j2\pi f)t}$$

$$= A \cdot \int_0^{\infty} e^{-(-j2\pi f + a)t} + A \int_{-\infty}^{0} e^{-(at + j2\pi f)t}$$

$$= A \cdot \left[\frac{e^{-(a-j2\pi f)t}}{-(a-j2\pi f)} \right]_{-\infty}^{0} + A \left[\frac{e^{-(a+j2\pi f)t}}{-(a+j2\pi f)} \right]_0^{\infty}$$

$$= A \left[\frac{1}{a-j2\pi f} \right] + A \left[\frac{1}{a+j2\pi f} \right]$$

$$= \frac{Aa + \cancel{Aj2\pi f} + Aa \cancel{+ Aj2\pi f}}{a^2 - (j2\pi f)^2}$$

$$= \frac{2Aa}{a^2 (2\pi f)^2}$$

$\therefore$ $|xg(F)| = xg(F)$

$$|xg(F)| = \tan^{-1}\left(\frac{0}{\text{semina value}} \right)$$

$$= 0$$

4.3) Consider the signal

$$x(t) = \begin{cases} 1 - \frac{|t|}{T}, & |t| \le T \\ 0, & \text{elsewhere} \end{cases}$$

a) Determine and sketch its Magnitude and pha Spectra $|xa(F)|$.

$$xa(F) = \int_T^0 \left(1 + \frac{t}{T}\right) e^{-j2\pi kt} \, dt + \int_0^T \left(1 - \frac{t}{T}\right) e^{-j2\pi Ft} \, d$$

first we find

$$\text{let } y(t) = \begin{cases} \frac{1}{T}; & T \le t \le T \\ 0 \end{cases}$$

$$Y(F) = \int_{-T}^{0} \frac{1}{T} e^{-j2\pi Ft} \, dt + \int_0^T \frac{-1}{T} e^{-j2\pi ft} \, dt$$

$$= -2 \frac{\sin 2\pi ft}{j\pi ft}$$

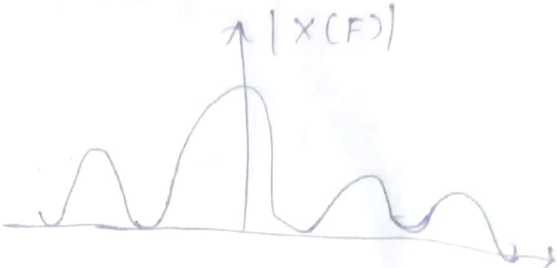
$$= \frac{1}{j2\pi f} Y(F)$$

$$= \tau \left(\frac{\sin \pi f \tau}{\pi f \tau}\right)^2$$

$$|X(F)| = \tau \cdot \left(\frac{\sin \pi f \tau}{\pi f \tau}\right)^2$$

$$\angle x_a(F) = 0$$

$$\tau \cdot \left(\frac{\sin \pi f \tau}{\pi f \tau}\right)^2$$



Let $f = t/\tau$

$$\left(\frac{\sin \pi t}{\pi t}\right)^2 = \sin^2(t)$$

b) Create a periodic signals $x_p(t)$ with fundamental perod $T_p \geq 2\tau$, so that $x(t) = x_p(t)$ for $|t| < T_p/2$. what are the fousier co-fficients $C_{k\tau}$ for the signal $x_p(t)$?

Sol :- $$C_{k\tau} = \frac{1}{T_p} \int_{-T_p/2}^{T_p/2} x(t)\, e^{-j2\pi k t / T_p}\, dt.$$

$$= \frac{1}{T_p} \int_{-\tau}^{0} \left(1 + \frac{t}{\tau}\right) e^{-j2\pi k t / T_p} dt + \int_{0}^{\tau} \left(1 - \frac{t}{\tau}\right) e^{-j2\pi k t / T_p} dt$$

$$= \frac{\tau}{T_p} \left[\frac{\sin \pi k \tau / T_p}{\pi k \tau / T_p}\right]^2$$

c) Using the results in parts a and b show that.

$$C_{k\tau} = \left(\frac{1}{T_p}\right) x_a \left(\frac{C_{k\tau}}{T_p}\right)$$

$$\frac{1}{T_p} \cdot x_a\left(\frac{k_\tau}{T_p}\right)$$

$$\frac{1}{T_p} \cdot \tau \cdot \left[\frac{\sin \pi \cdot \frac{k_\tau}{T_p} \cdot \tau}{\pi \cdot \frac{k}{T_p} \cdot \tau}\right]^2 = C_{k\tau}$$

$$C_{k\tau} = \frac{1}{T_p}\, x_a\left(\frac{k_\tau}{T_p}\right).$$ Hence proved.

1) Consider the following periodic signals

$x(n) = \{\ldots, 1, 0, 2, \underset{\uparrow}{3}, 2, 1, 0, -1 \ldots\}$

a) sketch the signal $x(n)$ and its magnitude and phase spectrum.

1 . 0 . 1 . 2 . 3 . 2 . 1 . 0 . 1

$N = 6$

$$C_{k_r} = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi k n}{N}}$$

$$= \frac{1}{6} \sum_{n=0}^{5} x(n) e^{\frac{-j2\pi k n}{6}}$$

for $n=0 \rightarrow x(0)\, e^{\frac{-j2\pi k_r (0)}{6}} = 3 \times 1 = 3$

$n=1 \rightarrow x(1)\, e^{\frac{-j2\pi k_r(1)}{6}} = 2 e^{\frac{-j\pi 2 k_r}{6}}$

$n=2 \rightarrow x(2) \cdot e^{\frac{-j2\pi k_r}{6}(2)} = e^{\frac{-j2\pi k_r}{3}}$

$n=3 \rightarrow x(3) \cdot e^{\frac{-j2\pi k_r}{6}(3)} = 0$

$n=4 \rightarrow x(4)\, e^{\frac{-j2\pi 4}{6}} = 1 \cdot e^{\frac{-j2\pi k}{3}}$

$n=5 \rightarrow x(5)\, e^{\frac{-j2\pi k \cdot 5}{6}} = 2 \cdot e^{\frac{-j\pi k_r}{6}}$

$$= \frac{1}{6}\left[3 + 2e^{\frac{-j\pi 2k}{6}} + e^{\frac{-j2\pi k}{3}} + 0 + e^{\frac{-j4\pi k}{3}} + 2e^{\frac{-j10\pi k}{6}}\right]$$

for $k_r = 0$

$$= \frac{1}{6}\left[3 + 2 + 1 + 0 + 1 \cdot 2\right]$$

$$= \frac{1}{6}[9] = \frac{9}{6}$$

for $k_r = 1$

$$= \frac{1}{6}\left[3 + 2 \cdot e^{-j\pi \frac{\cancel{2}^{1}}{\cancel{6}_{3}}} + e^{\frac{-j2\pi}{3}} + 0 + e^{\frac{-j4\pi}{3}} + 2e^{\frac{-j10\pi}{6}}\right]$$

$$= \frac{1}{6}\left[3 + 2e^{\frac{-j\pi}{3}} + e^{\frac{-j2\pi}{3}} + 0 + e^{\frac{-j4\pi}{3}} + 2e^{-j\frac{5\pi}{3}}\right]$$

$$= \frac{1}{6}\left[3 + 2\left(\cos\left(\frac{\pi}{3}\right) - j\sin\left(\frac{\pi}{3}\right)\right) + \cos\left(\frac{2\pi}{3}\right) - j\sin\left(\frac{2\pi}{3}\right) + \cos\frac{4\pi}{3} - j\sin\frac{4\pi}{3} + 2\left(\cos\frac{5\pi}{3} - j\sin\frac{5\pi}{3}\right)\right]$$

$$= \frac{4}{6}$$

simillarly

for $k=2$; $C_2 = 0$

$k=3$; $C_3 = 4/6$

$k=4$; $C_4 = 0$

$k=5$; $C_5 = 4/6$

b) using the results in part (a) verify parsevals relation by computing the power is in the time and frequency domains.

$$P_t = \frac{1}{6}\sum_{n=0}^{5} |x(n)|^2$$

$$= \frac{1}{6}\left[1^2+0^2+1^2+2^2+3^2+2^2\right]$$

$$= \frac{1}{6}\left[1+1+4+9+4\right] = \frac{19}{6}$$

$$P_f = \sum_{n=0}^{5} |c(n)|^2$$

$$= \left(\frac{9}{6}\right)^2 + \left(\frac{4}{6}\right)^2 + 0^2 + \left(\frac{1}{6}\right)^2 + 0^2 + \left(\frac{4}{6}\right)^2$$

$$= \frac{114}{36} = \frac{19}{6}$$

4.5) consider the signal

$$x(n) = 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{2} + \frac{1}{2}\cos\frac{3\pi n}{4}$$

a) Determine and sketch its PDS

$$C(n) = 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{2} + \frac{1}{2}\cos\cos\frac{3\pi n}{4}$$

$$= 2 + 2\left(\frac{e^{\frac{j\pi n}{4}} + e^{-\frac{j\pi n}{4}}}{2}\right) + \frac{e^{\frac{j\pi n}{2}} + e^{-\frac{j\pi n}{2}}}{2} + \frac{1}{2}\left[\frac{e^{\frac{j3\pi n}{4}} + e^{-\frac{j3\pi n}{4}}}{2}\right]$$

$$= 2 + e^{\frac{j\pi n}{4}} + e^{-\frac{j\pi n}{4}} + \frac{1}{2}e^{\frac{j\pi n}{2}} + \frac{1}{2}e^{-\frac{j\pi n}{2}} + \frac{1}{4}e^{\frac{j3\pi n}{4}} + \frac{1}{4}e^{-\frac{j3\pi n}{4}}$$

$N = 8$

$$C_{kr} = \frac{1}{8} \sum_{n=0}^{} x(n) e^{-j\pi k \frac{n}{4}}$$

$$x(n) = \left\{ \frac{11}{2}, 2 + \frac{3}{4}\sqrt{2}, 1, 2 + \frac{3}{4}\sqrt{2}, \frac{1}{2}, 2 - \frac{3}{4}\sqrt{2}, 1, 2, \frac{3}{4}\sqrt{2} \right\}$$

$C_0 = 2, C_1 = C_7 = 1, C_2 = C_6 = \frac{1}{2}, C_3 = C_5 = \frac{1}{4}, C_4 = 1$

b) evaluate the power of the signal

$$\sum_{n=0}^{7} x(C_k)^2$$

$$= \left[2^2 + 1^2 + 1^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^2\right]$$

$$= \left[4 + 2 + \frac{1}{2} + \frac{1}{8}\right]$$

$$= \frac{32 + 16 + 4 + 1}{8}$$

$$= \frac{53}{8}$$

4.6) Determine and sketch the magnitude and phase spectra of the followed periodic signals

a) $x(n) = 4 \sin \frac{\pi(n-2)}{3}$

$$4 \left[\frac{e^{j\pi\frac{(n-2)}{3}} - e^{-j\pi\frac{(n-2)}{3}}}{2j}\right]$$

$$4 \left[e^{3j\pi(n-2)} - e^{-3j\pi(n-2)}\right]$$

$N = 6$

$$C_{kr} = \frac{1}{6} \sum_{n=0}^{5} x(n) e^{-\frac{2j\pi kn}{6}}$$

$$= \frac{4}{6} \sum_{n=0}^{5} \left[\frac{e^{j\pi\frac{(n-2)}{3}} - e^{-j\pi\frac{(n-2)}{3}}}{2j}\right] e^{-\frac{2j\pi kn}{6}}$$

$$= \frac{1}{\sqrt{3}} \left[-e^{-j2\pi k/3} - e^{-j\pi k/3} + e^{j\pi k/3} + e^{-j2\pi k/3}\right]$$

$$= \frac{1}{\sqrt{3}}(-j^2) \cdot \left[\sin \frac{2\pi k}{6} + \sin \frac{\pi k}{3}\right] e^{-j2\pi/3}$$

$c = 0,\ c_1 = -j_2 e^{-j2\pi/3},\ c_2 = c_3 = c_4 = 0,\ c_5 = 9$

$\angle c_1 = \frac{5\pi}{6},\ \angle c_5 = \frac{-5\pi}{6},\ \angle c_0 = \angle c_2 = \angle c_3 = \angle c_4 = 0$

b) $x(n) = \cos\frac{2\pi}{3}n + \sin\frac{2\pi}{5}n$; 15

$N = 15$

$\cos\frac{2\pi}{3}n$

$\frac{1}{2}\left[e^{j\frac{2\pi}{3}n} + e^{-j\frac{2\pi}{3}n}\right]$

$e^{-j\frac{2\pi}{3}n} = e^{-2\frac{j\pi k n}{N}}$

$K = \frac{N}{3} = 5$

$15 - 5 = 10$

$$A_K = \begin{cases} 1/2 ; & K = 5, 10 \\ 0 ; & \text{otherwise} \end{cases}$$

$\sin\frac{2\pi}{5}n$

$\frac{1}{2j}\left[e^{j\frac{2\pi}{5}n} - e^{-j\frac{2\pi}{5}n}\right]$

$e^{-j\frac{2\pi}{5}n} = e^{\frac{-2jk\pi n}{N}}$

$K = \frac{N}{5} = \frac{15}{5} = 3$

$15 - 3 = 12$

$$C_{2K} = \begin{cases} \frac{1}{2j} ; & K = 3, 12 \\ -1/2j ; & K = 12 \\ 0 ; & \text{otherwise} \end{cases}$$

$$C_K = C_{1K} + C_{2K} = \begin{cases} \frac{1}{2j}, & K = 3 \\ \frac{1}{2}, & K = 5 \\ \frac{1}{2}, & K = 10 \\ -\frac{1}{2j} & K = 12 \\ 0, & \text{otherwise} \end{cases}$$

c) $x(n) = \cos\frac{2\pi}{3}n \cdot \sin\frac{2\pi}{5}n$

$\cos a \sin b = \frac{\sin(a+b)}{2} - \frac{\sin(a-b)}{2}$

$= \frac{1}{2}\left(\sin\frac{10\pi n + 6\pi n}{15} - \sin\frac{10\pi n - 6\pi n}{15}\right)$

$= \frac{1}{2}\sin\frac{16\pi n}{15} - \frac{1}{2}\sin\frac{4\pi n}{15}$

$= \frac{1}{2}\left(\frac{e^{\frac{j16\pi n}{15}} - e^{-\frac{j16\pi n}{15}}}{2j}\right) - \frac{1}{2}\left(\frac{e^{\frac{4\pi n j}{15}} - e^{-\frac{j4\pi n}{15}}}{2j}\right)$

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$4 = 2K$

$\frac{8}{15} = \frac{18}{M}$ $\quad$ $k=2$

$-\frac{1}{43} \Rightarrow k=2$

$k = 8 \rightarrow \frac{1}{43}$ $\quad$ $15-2 \Rightarrow 13 - \frac{1}{43}$

$$C_K = \begin{cases} \frac{1}{43} ; & 8, 13 \\ -\frac{1}{43} ; & 7, 12 \\ 0 ; & \text{otherwise} \end{cases}$$

d) $x(n) = \{ \ldots\ldots -2, -1, \underset{\uparrow}{0}, 1, 2, -2, -1, 0, 1, 2 \ldots \}$

$N = 5$

$$C_k = \frac{1}{5} \sum_{n=0}^{4} x(n) e^{\frac{-j2\pi kn}{N}}$$

$$= \frac{1}{5} \sum_{n=0}^{4} x(n) e^{\frac{-j2\pi kn}{8}}$$

$$= \frac{1}{5} \left[0 + e^{\frac{-j2k\pi}{5}} + 2e^{\frac{-j4\pi k}{5}} - 2e^{\frac{-j6\pi k}{5}} - e^{\frac{-j8\pi k}{5}} \right]$$

$$= \frac{2j}{5} \left[-\sin\left(\frac{2\pi k}{5}\right) - 2\sin\left(\frac{4\pi k}{5}\right) \right]$$

for porting k values

$k=0$; $C_0 = 0$

$k=1$; $C_1 = \frac{2j}{5} \left[-\sin \frac{2\pi}{5} - 2\sin \frac{4\pi}{5} \right]$

$k=2$; $C_2 = \frac{2j}{5} \left[-\sin \frac{4\pi}{5} - 2\sin \frac{8\pi}{5} \right]$,

$C_3 = -C_2$

$C_4 = -C_1$

e) $x(n) = \{ -1, 2, \underset{\uparrow}{1}, 2, -1, 0, -1, 2, 1, 2 \ldots \}$

$n = 6$

$N = 6$

$$C_k = \frac{1}{6} \sum_{n=0}^{5} x(n) e^{\frac{-j2\pi kn}{N}}$$

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Lly by subtituting from 0 to 5 we get

$$= \frac{1}{6}\left[1+2e^{\frac{-j\pi k r}{3}} - e^{\frac{-j2\pi k r}{3}} - e^{\frac{-j4k}{3}} + 2e^{\frac{-j5\pi k}{3}}\right]$$

$$= \frac{1}{6}\left[1+4\cos\frac{\pi k}{3} - 2\cos\frac{2\pi k}{3}\right]$$

$c_0 = 1/2$; $c_1 = 2/3$; $c_2 = 0$; $c_3 = -\frac{5}{6}$, $c_4 = 0$, $c_5 = 2/3$

f) $x(n) = \{\ldots\ldots 0,0,\underset{n=0}{1},1,0,0,0,\underset{5}{1},1,0,0 \ldots\}$.

$N=5$

$$C_{kr} = \frac{1}{5}\sum_{n=0}^{4} x(n)\, e^{\frac{-j2\pi k n}{5}}$$

$$= \frac{1}{5}\left[1+e^{-\frac{j2\pi k r}{5}}\right]$$

$$= \frac{2}{5}\cos\left(\frac{\pi k}{5}\right) e^{\frac{-j\pi k}{5}}.$$

$\therefore c_0 = \frac{2}{5}$

$c_1 = \frac{2}{5}\cos\left(\frac{\pi}{5}\right) e^{\frac{-j\pi}{5}}$

$c_2 = \frac{2}{5}\cos\left(\frac{2\pi}{5}\right) e^{\frac{-j2\pi}{5}}$

$c_3 = \frac{2}{5}\cos\left(\frac{3\pi}{5}\right) e^{\frac{-j3\pi}{5}}$

$c_4 = \frac{2}{5}\cos\left(\frac{4\pi}{5}\right) e^{\frac{-j4\pi}{5}}$

g) $x(n) = 1$, $-\infty < n < \infty$

$N=1$

$C_{kr} = x(0) = 1$

(or)

$C_0 = 1$

h) $x(n) = (-1)^n$, $-\infty < n < \infty$

$N=2$

$$C_{kr} = \frac{1}{2}\sum_{n=0}^{1} x(n)\, e^{-j\pi n k r}$$

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$= 1/2\,(1 - e^{j\pi k})$

$\therefore c_0 = 0 \; ; \; c_1 = 1$

4.7) Determine the periodic signals $x(n)$ with fundamental period $N = 8$ if these fourier coefficients are given by

a) $c_k = \cos\frac{k\pi}{4} + \sin\frac{3k\pi}{4}$

$$x(n) = \sum_{n=0}^{7} c_k\, e^{\frac{j2\pi nk}{N}}$$

let

$$c_k = e^{\frac{j2\pi pk}{N}}$$

$$\sum_{n=0}^{7} e^{\frac{j2\pi(p+n)k}{N}}$$

It gives

8 ; when $p = -n$

0 ; when $p \neq n$

$$c_k = \frac{1}{2}\left[e^{\frac{j2\pi k}{8}} + e^{\frac{-j2\pi k}{8}}\right] - \frac{1}{2j}\left[e^{\frac{j6\pi k}{8}} - e^{\frac{-j6\pi k}{8}}\right]$$

$x(n) = 4\delta(n+1) + 4\delta(n-1) - 4j\delta(n-1) - 4j\delta(n+3) + 4j\delta(n\ldots$

$-3 \leq n \leq 5$

b) $c_k = \begin{cases} \sin\frac{k\pi}{3}, & 0 \leq k \leq 6 \\ 0, & k = 7 \end{cases}$

$c_0 = 0; \; c_1 = \frac{\sqrt{3}}{2}; \; c_2 = \frac{\sqrt{3}}{2}; \; c_3 = 0; \; c_4 = -\frac{\sqrt{3}}{2}; \; c_5 = -\frac{\sqrt{3}}{2}, \; c_6 = c_7 =$

$$x(n) = \sum_{k=0}^{7} c_k\, e^{\frac{j2\pi nk}{8}}$$

$$= \frac{\sqrt{3}}{2}\left[e^{\frac{j\pi n}{4}} + e^{\frac{j\pi n 2}{4}} - e^{\frac{-j4\pi n}{4}} - e^{\frac{j5\pi n}{4}}\right]$$

$$= \sqrt{3}\left[\frac{\sin\pi n}{2} + \sin\frac{\pi n}{4}\right] e^{\frac{j\pi(3n-2)}{4}}$$

c) $C_{k_r} = \{ \ldots 0, \frac{1}{4}, \frac{1}{2}, 1, 2, 1, \frac{1}{2}, \frac{1}{4}, 0 \ldots \}$

$$x(n) = \sum_{k_r=-3}^{+} C_{k_r} e^{\frac{j2\pi n k_r}{8}}$$

$$= 2 + e^{\frac{j\pi n}{4}} + e^{\frac{-j\pi n}{4}} + \frac{1}{2} e^{\frac{j\pi n}{2}} + \frac{1}{2} e^{\frac{j\pi n}{2}} + \frac{1}{4} e^{\frac{j3\pi n}{4}}.$$

$$= 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{2} + \frac{1}{2}\cos\frac{3\pi n}{4}$$

4.9) compute the fourier transform of the following signals

4.10) Determine the signals having the following fourier transforms.

a) $x(\omega) = \begin{cases} 0, & 0 \le |\omega| \le \omega_0 \\ 1, & \omega_0 < |\omega| \le \pi \end{cases}$

$$x(n) = \frac{1}{2\pi}\left[\int_{-\pi}^{-\omega_0} x(\omega) e^{j\omega n} + \int_{\omega_0}^{\pi} x(\omega) e^{j\omega n} d\omega\right]$$

$$= \frac{1}{2\pi}\left[\int_{\pi}^{-\omega_0} 1 \cdot e^{j\omega n} d\omega + \int_{\omega_0}^{\pi} e^{j\omega n} \cdot d\omega\right] \text{———} ①$$

$$= \frac{1}{2\pi}\left(\left[\frac{e^{-j\omega n}}{jn}\right]_{-\pi}^{\omega_0} + \left[\frac{e^{j\omega n}}{jn}\right]_{\omega_0}^{\pi}\right)$$

$$= \frac{1}{2\pi}\left(\frac{2 \cdot e^{-j\omega_0 n} - e^{j\omega_0 n}}{2jn} + 2\,\frac{e^{j\pi n} - e^{-j\omega_0 n}}{jn}\right)$$

$$= \frac{1}{\cancel{2\pi}}\left[\frac{2}{n} \cdot \sin\omega_0 n + 2 \cdot \frac{\sin\pi n}{2jn}\right]$$

$$= -\frac{\sin\omega_0 n}{n\pi}; \quad n \ne 0.$$

for $n = 0$

from eq ①

$$= \frac{1}{2\pi}(\pi - \omega_0) + \frac{1}{2\pi}(\pi - \omega_0)$$

$$= \frac{(\pi - \omega_0) + (\pi - \omega_0)}{2\pi}$$

$$\Rightarrow \frac{2(\pi - \omega_0)}{2\pi}$$

$$= (\pi - \omega_0) \text{ ; when } n=0$$

b) $X(\omega) = \cos^2\omega$

$$= \left(\frac{e^{j\omega} + e^{-j\omega}}{2}\right)^2$$

$$= \frac{1}{4}\left[(e^{j\omega})^2 + 2\, e^{j\omega} e^{-j\omega} + (e^{-j\omega})^2\right]$$

$$= \frac{1}{4} e^{j2\omega} + \frac{1}{2} + \frac{1}{4} e^{-j2\omega}$$

I.F.T.,

$$= \frac{1}{4}\delta(n+2) + \delta(n)\frac{1}{2} + \frac{1}{4}\delta(n-2)$$

$$= \frac{1}{4}\left[\delta(n+2) + 2\delta(n) + \delta(n-2)\right]$$

c) $x(\omega) = \begin{cases} 1 & ; \omega_0 - \frac{\delta\omega}{2} \le |\omega| \le \omega_0 + \frac{\delta\omega}{2} \\ 0 & ; \text{ eles where} \end{cases}$

$$\omega_0 - \frac{\delta\omega}{2} \le \omega \le \omega_0 + \frac{\delta\omega}{2}$$

$$\omega_0 - \frac{\delta\omega}{2} \le -\omega \le \omega_0 + \frac{\delta\omega}{2}$$

$$-\omega_0 + \frac{\delta\omega}{2} \le \omega \le -\omega_0 - \frac{\delta\omega}{2}$$

Consider limits, $\omega_0 - \frac{\delta\omega}{2} \le \omega \le \omega_0 + \frac{\delta\omega}{2}$

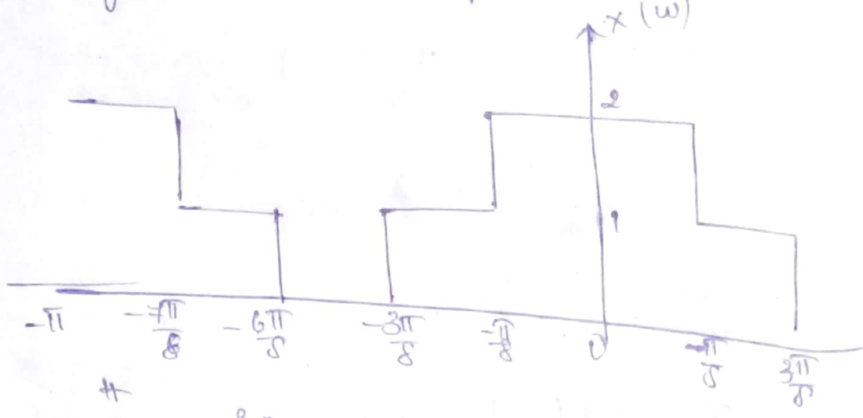
$$\frac{1}{2\pi}\int_{\omega_0 - \frac{\delta\omega}{2}}^{\omega_0 + \frac{\delta\omega}{2}} 1 \cdot e^{j\omega n}\, d\omega$$

$$\frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{\omega_0 - \frac{\delta\omega}{2}}^{\omega_0 + \frac{\delta\omega}{2}}$$

$$\frac{2}{2\pi}\, \frac{e^{j(\omega_0 + \frac{\delta\omega}{2})n} - e^{j(\omega_0 - \frac{\delta\omega}{2})n}}{2jn}$$

$$\delta\omega \left\{ \frac{2}{\pi} \left(\frac{\sin \frac{\delta\omega}{2n}}{\frac{n\delta\omega}{2}} \right) e^{jn\omega_0} \right.$$

d) The signal shown in fig



$$\frac{1}{2\pi} \int_{-\pi}^{\pi} x(\omega) e^{j\omega n} d\omega$$

let consider limits 0 to π

$$\frac{1}{2 \cdot 2\pi} \left[\int_0^{\pi/8} 2e^{j\omega n} d\omega + \int_{\pi/8}^{3\pi/8} e^{j\omega n} d\omega + \int_{6\pi/8}^{7\pi/8} e^{j\omega n} d\omega + \int_{7\pi/8}^{\pi} 2e^{j\omega n} d\omega \right]$$

$$\frac{1}{\pi} \int_0^{\pi/8} 2\cos \omega n \, d\omega + \int_{\pi/8}^{3\pi/8} \cos \omega n \, d\omega + \int_{6\pi/8}^{7\pi/8} \cos \omega n \, d\omega + \int_{7\pi/8}^{\pi} 2\cos \omega n . d\omega$$

$$\frac{1}{\pi} \left[[-2\sin \omega n]_0^{\pi/8} + [\sin \omega n]_{\pi/8}^{3\pi/8} + [-\sin \omega n]_{6\pi/8}^{7\pi/8} + [-2\sin \omega n]_{7\pi/8}^{\pi} \right]$$

$$\frac{1}{\pi} \left[\sin \frac{7\pi}{8} n - \sin \frac{\pi n}{8} + \sin \frac{6\pi}{8} n - \sin \frac{3\pi}{8} n \right]$$

4.11) consider the signal

$$x(n) = \{1, 0, \underset{\uparrow}{-1}, 2, 3\}$$

with fourier transform $x(\omega) = x_R(\omega) + j(x_I(\omega))$.

Determine and sketch and signal $y(n)$ with fourier transform

$$Y(\omega) = \left[x_I(\omega) + x_R(\omega) e^{j2\omega} \right]$$

$$x_e(n) = \frac{x(n)+x(-n)}{2}$$

$$n=0 \;;\; x_e(n) = \frac{x(0)+x(-0)}{2} = \frac{2+2}{2} = 2$$

$$n=1 \;;\; \frac{x(1)+x(-1)}{2} = \frac{3+(-1)}{2} = 1$$

$$n=-1 \;;\; \frac{x(-1)+x(1)}{2} = 1$$

$$n=2 \;;\; \frac{x(2)+x(-2)}{2} = 0$$

$$n=3 \;;\; \frac{x(3)+x(-3)}{2} = \frac{0+1}{2} = 1/2$$

$$n=-2 \;;\; \frac{x(-2)+x(2)}{2} = 0$$

$$n=-3 \;;\; \frac{x(-3)+x(3)}{2} = \frac{1+0}{2} = 1/2$$

$$\therefore x_e(n) = \{1/2, 0, 1, 2, 1, 0, 1/2\}$$

Ily we get

$$x_o(n) = \{1/2, 0, -2, \underset{\substack{\uparrow \\ n=0}}{0}, 2, 0, 1/2\}$$

$$\text{from } x_o(n) = \frac{x(n)-x(-n)}{2}$$

$$X_R(\omega) = \sum_{n=3}^{3} x_e(n) e^{-jn\omega}$$

$$j\, X_I(\omega) = \sum_{n=-3}^{3} x_e(n) e^{-jn\omega}$$

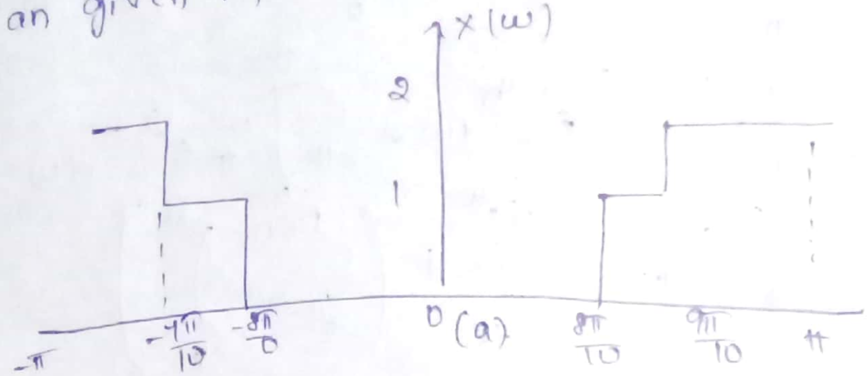
$$Y(\omega) = X_I(\omega) + X_R(\omega) e^{j2\omega}$$

$$= \frac{-j x_o(n)}{j} + x_o(n+2)$$

$$= -j\, x_o(n) + x_e(n+2)$$

$$= \{1/2, 0, 1, -j/2, 2, 1, +j/2, 0, 1/2, 0, 1/2\}$$

4.12) Determine the signal $x(n)$ if its fourier transform is as given in



$$x(n) = \frac{1}{2\pi}\left[\int_{-\pi}^{-9\pi/10} 2\cdot e^{jn\omega}\, d\omega + \int_{-9\pi/10}^{-8\pi/10} 1\cdot e^{jn\omega}\, d\omega + \int_{8\pi/10}^{9\pi/10} e^{jn\omega}\, d\omega + 2\int_{9\pi/10}^{\pi} e^{jn\omega}\, d\omega\right]$$

$$= \frac{1}{2\pi}\left[2\left(\frac{e^{jn\omega}}{jn}\right)_{-\pi}^{-\frac{9\pi}{10}} + \left[\frac{e^{jn\omega}}{jn}\right]_{-\frac{9\pi}{10}}^{-\frac{8\pi}{10}} + \left[\frac{e^{jn\omega}}{jn}\right]_{\frac{8\pi}{10}}^{\frac{9\pi}{10}} + 2\left[\frac{e^{jn\omega}}{jn}\right]_{\frac{9\pi}{10}}^{\pi}\right]$$

$$= \frac{1}{2\pi jn}\left[2\left[e^{-j\pi\frac{8\pi}{10}} - e^{-jn\pi}\right] + e^{jn-\frac{8\pi}{10}} - e^{-jn\frac{9\pi}{10}} + e^{jn\frac{9\pi}{10}} + 2e^{jn\pi} - e^{jn\frac{9\pi}{10}}\right]$$

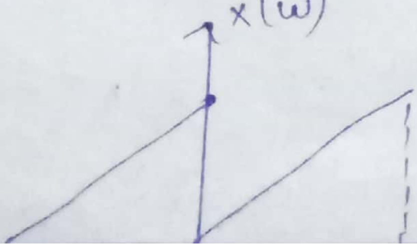
$$= \frac{1}{2\pi jn}\; \cancel{2e^{-j\pi\frac{9\pi}{10}}} - 2e^{-j\pi n} + e^{-j\pi\frac{8\pi}{10}} - \cancel{e^{-jn\frac{9\pi}{10}}} + e + e^{jn\frac{9\pi}{10}} - e^{jn\frac{8\pi}{10}}$$

$$= \frac{1}{2\pi jn}\; e^{-jn\frac{9\pi}{10}} - 2e^{-j\pi n} + 2e^{jn\pi} + e^{-jn\frac{8\pi}{10}} - e^{+jn\frac{8\pi}{10}}$$

$$= \frac{1}{2\pi jn}\left(e^{-jn\frac{9\pi}{10}} - e^{+jn\frac{9\pi}{10}} - 2e^{jn\pi} + 2e^{-jn\pi} + e^{-jn\frac{8\pi}{10}} - e^{jn\frac{8\pi}{10}}\right)$$

$$= \frac{1}{2n\pi}\left[-\sin\left(\frac{9\pi n}{10}\right) - \sin\left(\frac{8n\pi}{10}\right) + \sin n\pi\right]$$

$$= \frac{1}{n\pi}\left[\sin\frac{9\pi n}{10} + \sin\frac{4n\pi}{5}\right]$$

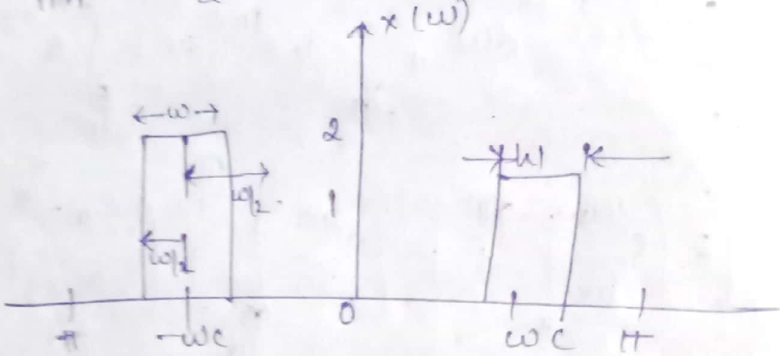


$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{0} x(\omega)\, e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{0}^{\pi} x(\omega)\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{0}\left(\frac{\omega}{\pi}+1\right) e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{0}^{\pi}\frac{\omega}{\pi}\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left(\left(\frac{\omega}{jn\pi}\, e^{j\omega n}\right]_{-\pi}^{\pi} + \left(\frac{e^{j\omega n}}{jn}\right)_{-\pi}^{0}\right]$$

$$= \frac{1}{\pi n}\sin\frac{\pi n}{2}\, e^{-\frac{jn\pi}{2}}$$



$$x(n) = \frac{1}{2\pi}\left[\int_{-\omega_c-\frac{\omega}{2}}^{-\omega_c+\frac{\omega}{2}} 2\cdot e^{jn\omega}\cdot d\omega + \int_{\omega_c-\frac{\omega}{2}}^{\omega_c+\frac{\omega}{2}} 2\, e^{jn\omega}\, d\omega\right]$$

$$= \frac{1}{\pi}\left(\left[\frac{e^{jn\omega}}{jn}\right]_{-\omega_c-\frac{\omega}{2}}^{-\omega_c+\frac{\omega}{2}} + \left[\frac{e^{jn\omega}}{jn}\right]_{\omega_c-\frac{\omega}{2}}^{\omega_c+\frac{\omega}{2}}\right]$$

$$= \frac{1}{jn\pi}\left(e^{jn\left(-\omega_c+\frac{\omega}{2}\right)} - e^{jn\left(-\omega_c-\omega/2\right)} + e^{jn\left(\omega_c+\frac{\omega}{2}\right)}\right.$$

$$= \frac{1}{jn\pi}\; \frac{e^{jn\left(-\omega_c+\frac{\omega}{2}\right)}}{} - \frac{e^{jn\left(-\omega_c-\frac{\omega}{2}\right)}}{} + \frac{e^{-jn\left(-\omega_c-\frac{\omega}{2}\right)}}{} - \frac{e^{-jn\left(\frac{\omega}{2}-\omega\right)}}{}$$

$$= \frac{2}{n\pi}\left[\sin\left(\frac{\omega}{2}-\omega_c\right)n - \sin\left(-\omega_c-\frac{\omega}{2}\right)\right]$$

$$= \frac{2}{n\pi}\left[-\sin\left(\omega_c-\frac{\omega}{2}\right)n - \sin\left(\omega_c+\frac{\omega}{2}\right)\right]$$

$$= \frac{2}{n\pi}\left[-\sin\left(\omega_c-\frac{\omega}{2}\right)n + \sin\left(\omega_c+\omega/2\right)n\right]$$

$= \frac{2}{n\pi}\left[\sin\left(\omega_c + \frac{\omega}{2}\right) - \sin\left(\omega_c - \frac{\omega}{2}\right)n\right]$

4.13) given the fourier transform of the signal

$$x(n) = \begin{cases} 1 & ; -m \le n \le m \\ 0 & ; \text{otherwise} \end{cases}$$

was shown to be

$$X(\omega) = 1 + 2\sum_{n=1}^{m} \cos \omega n$$

then show that the fourier transform of

$$x_1(n) = \begin{cases} 1 & ; 0 \le n \le m \\ 0 & ; \text{otherwise} \end{cases}$$

$$x_2(n) = \begin{cases} 1 & ; -m \le n \le -1 \\ 0 & ; \text{otherwise} \end{cases}$$

$$X_1(\omega) = \sum_{n=0}^{m} 1 \cdot e^{-j\omega n}$$

$$1 + e^{-j\omega} + e^{-j\omega 2} + e^{-j\omega 3} + \cdots - e^{-j\omega(n+1)} + e^{-j\omega m}$$

$$1 - \frac{e^{-j\omega(m+1)}}{1 - e^{-j\omega}} \qquad \frac{e^{j\omega(m+1)}}{1 - e^{-j\omega}}$$

$$X_2(\omega) = \sum_{n=-m}^{-1} e^{-j\omega n}$$

$$\sum_{n-1}^{m} e^{j\omega n}$$

$$= \frac{1 - e^{j\omega m}}{1 - e^{j\omega}} e^{j\omega}$$

$$X(\omega) = X_1(\omega) + X_2(\omega)$$

$$= \frac{1 - e^{-j\omega(m+1)}}{1 - e^{-j\omega}} + \frac{1 - e^{j\omega m}}{1 - e^{j\omega}} \cdot e^{j\omega}$$

$$= \frac{1 + e^{j\omega} - e^{j\omega} - 1 - e^{-j\omega(m+1)} - e^{j\omega(m+1)} + e^{j\omega m} + e^{-j}}{}$$

$$= \frac{2\cos\omega n - 2\cos\omega(m+1)}{2 - 2\cos\omega}$$

$$= \frac{2\sin(\omega m + \frac{\omega}{2}) \cdot \cos\frac{\omega}{2}}{2\sin^2\frac{\omega}{2}}$$

$$= \frac{\sin(m+\frac{1}{2})\omega}{\sin(\frac{\omega}{2})}$$

proved that $1 + 2\sum_{n=1}^{m} \cos\omega n = \frac{\sin(m+\frac{1}{2})\omega}{\sin(\omega/2)}$

4.14) consider the signal

$$x(n) = \{-1, 2, \underset{\uparrow}{-3}, 2, -1\}$$

with fourier transform $X(\omega)$ compute the following quantities, with out explicity computing $X(\omega)$.

a) $X(0)$

$$X(\omega)\Big|_{\omega=0} = \sum_{\omega=0} x(n) e^{-j\omega n}$$

$$X(0) = -3 \cdot e^0 = -3$$

b) $\angle X(\omega) = \pi$ for all 'ω'

c) $\int_{-\pi}^{\pi} X(\omega)\, d\omega$

$$x(0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega)\, d\omega$$

$$\int_{\pi}^{\pi} X(\omega)\, d\omega = 2\pi \cdot x(0)$$

$$= 2\pi \cdot (-3)$$

$$= -6\pi$$

d) $x(\pi) =$

$$x(\pi) = \sum_{n=-\alpha}^{\alpha} x(n)\, e^{-jn\omega}$$

$$= \sum_n e^{-jn\pi} x(n)$$

$$= \sum_n \left[\cos(n\pi) - j\sin[x(n)]\right] x(n)$$

$$= \sum_n (-1)^n x(n)$$

for $n=0$ $\quad (-1)^0 x(n) \Rightarrow 1 \cdot -3 = -3$

$n=1 \quad (-1)^1 x(1) \Rightarrow -1 \cdot 2 = -2$

$n=2 \quad (-1)^2 x(2) \Rightarrow -1 = -1$

$n=-1 \quad (-1)^{-1} x(-1) \Rightarrow -2$

$n=-2 \quad (-1)^{-2} x(-2) \Rightarrow -1$

$\Rightarrow -3-2-1-2-1$

$\Rightarrow -3-4-2$

$\Rightarrow -9$

e) $\displaystyle\int_{-\pi}^{\pi} |x(w)|^2 \, dw$

we know

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} |x(w)|^2 = \sum_n |x(n)|^2$$

$$= (-1)^2 + (2)^2 + (-3)^2 + (2)^2 + (-1)^2$$

$$= 1+4+9+4+1$$

$$= 19$$

$$\int_{-\pi}^{\pi} |x(w)|^2 = 2\pi(19)$$

$$= 38.9$$

(ii) the centre of gravity of a signal is defined as

$$C = \frac{\sum_{n=-\alpha}^{\alpha} n x(n)}{\sum_{n=-\alpha}^{\alpha} x(n)}$$

and provides a measure of time delay of signal

a) express c in terms of $x(\omega)$

we know

$$x(\omega) = \sum_{n=-\alpha}^{\alpha} x(n) e^{-j\omega n}$$

$$x(0) = \sum_{n=-\alpha}^{\alpha} x(n) e^{0}$$

$$x(0) = \sum_{n=-\alpha}^{\alpha} x(n)$$

we know from differentiation of 'ω' domaine multiples 'n' with $x(n)$

$$n x(n) \xleftrightarrow{F.T} j\frac{dx(\omega)}{d\omega}$$

$$-jn x(n) \xleftrightarrow{F.T} \frac{dx(\omega)}{d\omega}$$

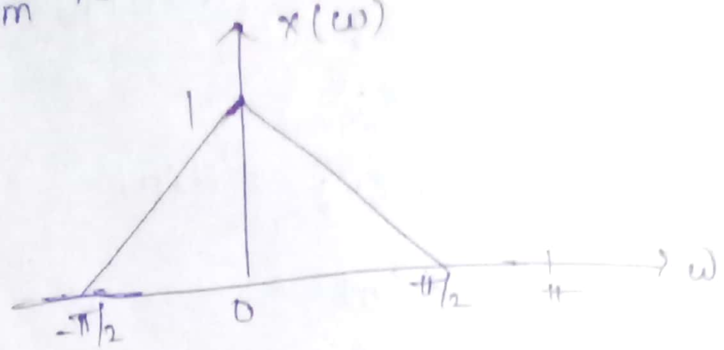
$$\frac{dx(\omega)}{d\omega} = \sum_{n=-\alpha}^{\alpha} -jn x(n) e^{-jn\omega} d\omega$$

$$= -j \sum_{n=-\alpha}^{\alpha} n x(n) e^{-jn\omega} d\omega$$

$$\frac{j\,dx(\omega)}{d\omega} = \sum_{n=-\alpha}^{\alpha} n x(n) e^{-jn\omega} d\omega$$

$$\left.\frac{j\,dx(\omega)}{d\omega}\right|_{\omega=0}$$

c) compute c for the signal $x(n)$ whose fourier transform is shown

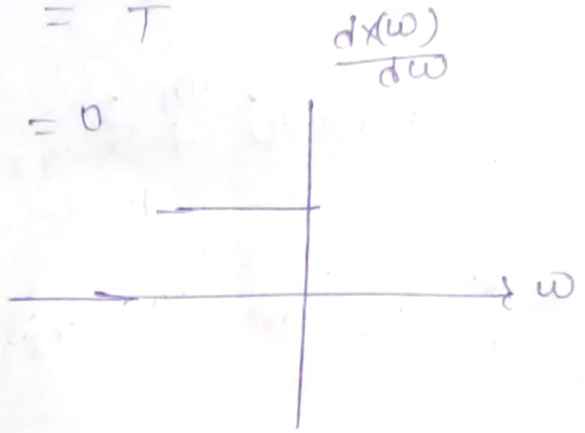


from given figure $X(0) = 1$

$$C = \frac{j\frac{dX(0)}{d\omega}}{X(0)}$$

$$= \frac{0}{1}$$

$$= 0$$



4.16) consider the fourier transform pair

$$a^n u(n) \xleftrightarrow{F.T} \frac{1}{1-ae^{-j\omega}} \quad |a|<1$$

Use the differentiation in frequency theorem and induction to shown

$$x(n) = \frac{(n+l-1)!}{n!(l-1)!} a^n u(n) \xleftrightarrow{F} X(\omega) = \frac{1}{(1-ae^{-j\omega})^l}$$

let $l = k_r + 1$

$$x(n) = \frac{(n+k_r+1-1)!}{n!(k_r+1-1)!} a^n u(n)$$

$$= \frac{(n+k_r)!}{n!k!} a^n u(n)$$

$$= \frac{(n+k)(n k_r - 1)!}{k n! (k_r - 1)!} a^n u(n)$$

let $x_k(n) = \frac{(n+k_r-1)!}{n!(k_r-1)!} a^n u(n)$

$$x_{k_r+1}(\omega) = \frac{n+k_r}{k} x_k(n)$$

$$x_{k_r+1}(\omega) = \sum_{n=-\alpha}^{\alpha} \frac{n+k_r}{k_r} x_{k_r}(n) \cdot e^{-j\omega n}$$

$$= \sum_{n=-\alpha}^{\alpha} \left[\frac{n}{k} x_k(n) + x_k(n) \right] e^{-j\omega n}$$

$$= \frac{1}{k} \sum_{n=-\alpha}^{\alpha} n x_k(n) e^{-j\omega n} + \sum_{n=-\alpha}^{\alpha} x_k(n) e^{-j\omega n}$$

$$= \frac{1}{k} \sum_{n=-\alpha}^{\alpha} n k_r(n) e^{-j\omega n} + x_{k_r}(\omega)$$

$$= \frac{1}{k_r} j \frac{d x_k(\omega)}{d\omega} + x_k(\omega)$$

$$= \frac{a e^{-j\omega}}{(1-ae^{-j\omega})^{k+1}} + \frac{1}{(1-ae^{-j\omega})^{k_r}}$$

4.17) let $x(n)$ be a arbitrary signal, necessary real values with F.T express the fourier transforms of the following signals

a) $x^*(n)$

$$\sum_{n=-\alpha}^{\alpha} x^*(n) e^{-j\omega n}$$

$$\sum_{n=-\alpha}^{\alpha} \left[x(n) e^{-j\omega n} \right]'$$

$$x(-\omega)^*$$

b) $x^{*}(-n)$

$$\sum_{n=-\infty}^{\infty} x^{*}(n) e^{-j\omega n}$$

replace $(-n)$ with (n)

$$\sum_{n=-\infty}^{\infty} x^{*}(n) e^{j\omega n}$$

$$\sum_{n=-\infty}^{\infty} \left(x(n) e^{-j\omega n}\right)^{*}$$

$$X^{*}(\omega)$$

c) $y(n) = x(n) - x(n-1)$

$$= \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} - \sum_{n=-\infty}^{\infty} x(n-1) e^{-j\omega n}$$

$$= X(\omega) - \sum_{n=-\infty}^{\infty} x(n-1) e^{-j\omega n}$$

let $l = n-1$ (dummy variable)

$$= X(\omega) - \sum_{l=-\infty}^{\infty} x(l) e^{-j\omega(l+1)}$$

$$= X(\omega) - \sum_{l=-\infty}^{\infty} x(l) e^{-j\omega l} \cdot e^{-j\omega}$$

$$= X(\omega) - e^{-j\omega} \sum_{l=-\infty}^{\infty} x(l) e^{-j\omega l}$$

Replace ~~an~~ l and n

$$= X(\omega) - e^{-j\omega} \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$= X(\omega) - e^{-j\omega} X(\omega)$$

$$= X(\omega) \left[1 - e^{-j\omega}\right]$$

d) $y(n) = \sum_{k=-\alpha}^{\alpha} x(k)$

$= y(n) - y(n-1)$

$= x(n)$

$x(\omega) = y(\omega)\left[1 - e^{j\omega}\right] \rightarrow$ from (2)

$y(\omega) = \frac{x(\omega)}{1 - e^{-j\omega}}$

e) $y(n) = x(n) - x(n-1)$

$= \sum_{n=-\alpha}^{\alpha} x(n) e^{-j\omega n} - \sum_{n=-\alpha}^{\alpha} x(n-1) e^{-j\omega n}$

$= x(\omega)$.

e) $y(n) = x(2n)$

$y(\omega) = \sum_{n=-\alpha}^{\alpha} x(2n)\, e^{-jn\omega}$

let $l = 2n$

$= \sum_{k=-\alpha}^{\alpha} x(l)\, e^{-j\frac{l}{2}\omega}$

$= \sum_{l=-\alpha}^{\alpha} x(l)\, e^{-jl\frac{\omega}{2}}$

$= x\left(\frac{\omega}{2}\right)$

f) $y(n) = \begin{cases} x\left(\frac{n}{2}\right) ; & \text{'n' even} \\ 0 ; & \text{'n' odd} \end{cases}$

$y(\omega) = \sum_{n} x\left(\frac{n}{2}\right) e^{jn\omega}$

let $n = 2l$

$= \sum_{n} \left(\frac{2l}{2}\right) e^{j2ln\omega}$

$= \sum_{n} x(l)\, e^{jl2\omega}$

$= x(2\omega)$

4.1(?) Determine and sketch the Fourier transform of the following signals

a) $x_1(n) = \{1, 1, 1, 1, 1\}$ (from $n=-2$ to 2, arrow at $n=0$)

$$\sum_{n=-\infty}^{\infty} x_1(n)\, e^{-j\omega n}$$

$$\sum_{n=-2}^{2} x_1(n)\, e^{-j\omega n}$$

for $n=-2$; $1 \cdot e^{j2\omega} = e^{j2\omega}$

$n=-1$; $1 \cdot e^{j\omega} = e^{j\omega}$

$n=0$; $e^0 = 1$

$n=1$; $1 \cdot e^{-j\omega} = \frac{1}{e^{j\omega}}$.

$n=2$; $1 \cdot e^{-2j\omega} = e^{-2j\omega}$

$= e^{j2\omega} + e^{j\omega} + 1 + e^{-j\omega} + e^{-j2\omega}$

$= 2\cos(2\omega) + 2\cos(\omega) + 1$

b) $x_2(n) = \{1, 0, 1, 0, 1, 0, 1, 0, 1\}$ ($n = -4, -3, -2, -1, 0, 1, 2, 3, 4$; arrow at $n=0$)

for $n=-2$; $1 \times e^{j2\omega} = e^{j2\omega}$

$n=-4$; $1 \cdot e^{j4\omega} = e^{j4\omega}$

$n=0$; $1 \times e^0 = 1$

$n=2$; $1 \times e^{-j2\omega} = e^{-j2\omega}$.

$n=4$; $1 \times e^{-j4\omega} = e^{-j4\omega}$

$= e^{j2\omega} + e^{j4\omega} + 1 + e^{-j2\omega} + e^{-j4\omega}$

$= 2\cos(2\omega) + 2\cos(4\omega) + 1$

d) Is these any relation ship between $x_1(\omega)$, $x_2(\omega)$ and $x_3(\omega)$? what is its physical meaning.

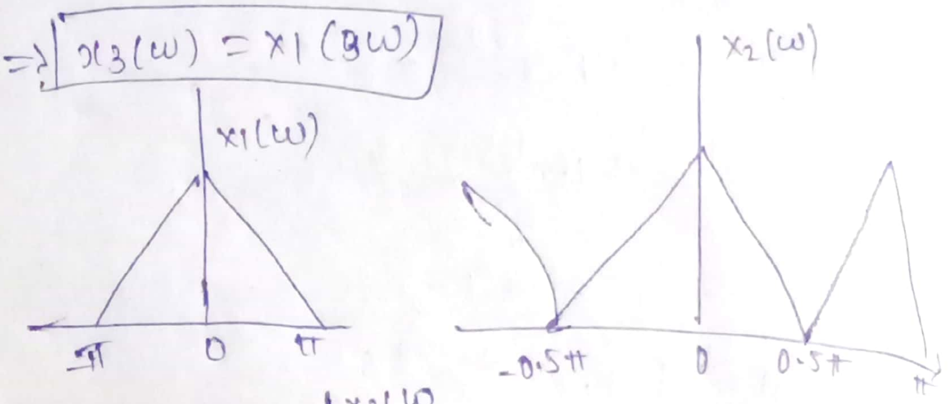
$$x_1(\omega) = 2\cos(2\omega) + 2\cos(\omega) + 1$$

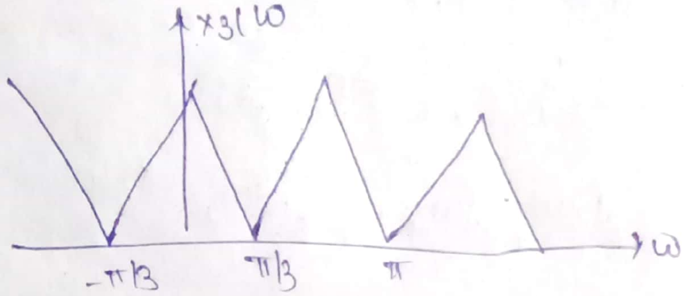
$$x_2(\omega) + 2\cos(2\omega) + 2\cos(4\omega) + 1$$

$$\Rightarrow \boxed{x_2(\omega) = x_1(2\omega)}$$

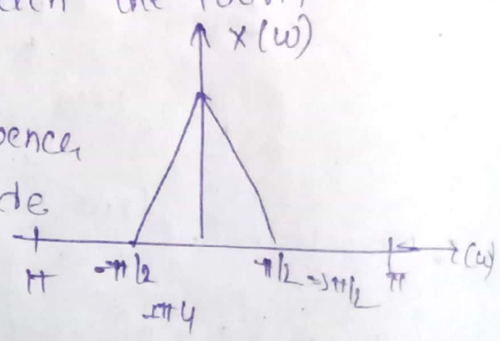
$$x_3(\omega) = 2\cos(6\omega) + 2\cos(8\omega) + 1$$

$$\Rightarrow \boxed{x_3(\omega) = x_1(3\omega)}$$





4.19) let $x(n)$ be a signal with fourier transform a shown determine and sketch the fourier transform of the following signals not that these signal sequence are abtained by amplitude modulation of a $\cos\omega_c n$ by the sequence.

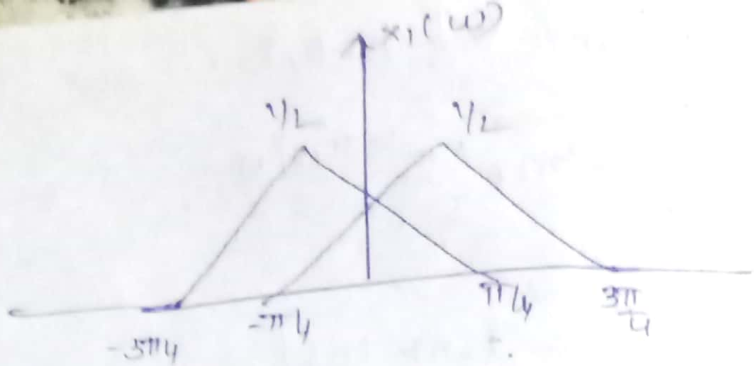


a) $x_1(n) = x(n)\cos\left(\frac{\pi n}{4}\right)$

we know

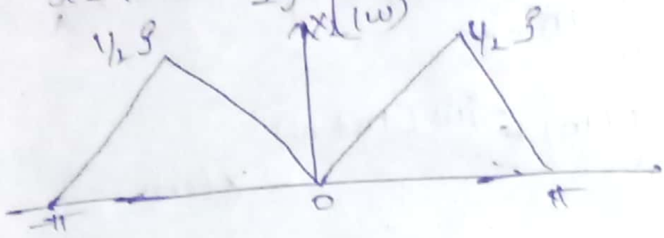
$$x(n)\cos(\omega_0 n) \xleftrightarrow{F.T} \frac{1}{2}\left[X(\omega - \omega_0) + X(\omega + \omega_0)\right]$$

$$X_1(\omega) = \frac{1}{2}\left[X\left(\omega + \frac{\pi}{4}\right) + X\left(\omega - \frac{\pi}{4}\right)\right]$$



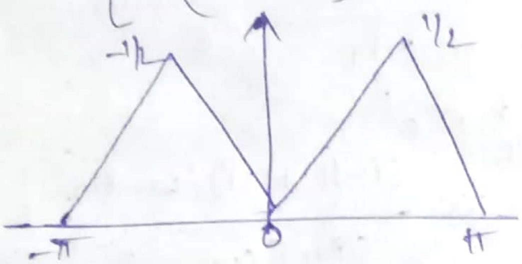
b) $x_2(n) = x(n) \sin n\pi/2)$

$$x_2(\omega) = \frac{1}{2j}\left[X\left(\omega+\frac{\pi}{2}\right) + X\left(\omega-\frac{\pi}{2}\right)\right]$$



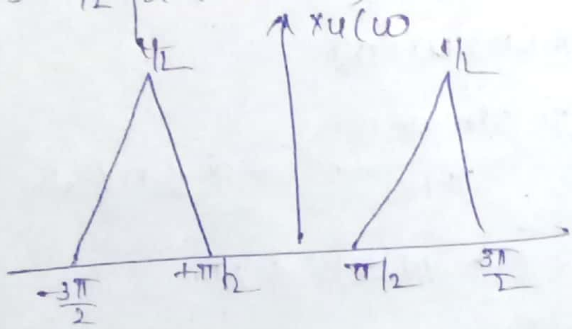
c) $x_3(n) = xn \cos\left(\frac{\pi}{2} n\right)$

$$x_3(\omega) = \frac{1}{2}\left[X\left(\omega - \frac{\pi}{2}\right) + X\left(\omega+\frac{\pi}{2}\right)\right]$$



d) $x_4(n) = x(n) \cos \pi(n)$

$$x_4(\omega) = 1/2\left[X(\omega - \pi) + X(\omega+\pi)\right]$$



4.20) consider an aperiodic signal $x(n)$ with FF $x(\omega)$ show fourier series coefficient C_k 4 as the periodic signal

$$y(n) = \sum_{l=\alpha}^{\alpha} x(n - l N)$$

are given by

$$C_k^y = \frac{1}{N} X\left(\frac{2\pi}{N} k\right) \quad k = 0, 1 \cdots N-1$$

$$C_k^y = \frac{1}{N} \sum_{n=0}^{N-1} y(n) e^{-j2k\pi n/N}$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} \left[\sum_{n=-\infty}^{\infty} x(n - ln) \right]$$

$$= \frac{1}{N} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{N-1-\omega} x(m) e^{-j2\pi k(m+lN)/N}$$

But $$\sum_{l=-\infty}^{\infty} \sum_{m=\omega}^{N-1-\omega} x(m) e^{-j\omega(m+N)} - X(\omega).$$

$$C_k^y = \frac{1}{N} X\left(\frac{2\pi k}{N}\right)$$

4.21) prove that

$$x_N(\omega) = \sum_{n=-N}^{N} \frac{\sin \omega_c n}{n} \cdot e^{-j\omega n}$$

may be expressed as

$$x_N(\omega) = \frac{1}{2\pi} \int_{-\omega c}^{\omega c} \frac{\sin[(2N+1)(\omega - \theta/2)]}{\sin[(\omega-\theta)/2]} d\theta$$

let

$$x_N(n) = \frac{\sin \omega_c n}{\pi n} \quad ; -N \le n \le N$$

$$= x(n)\, \omega(n)$$

where $x(n) = \dfrac{\sin \omega_c n}{\pi n} \quad -\infty \le n \le \infty$

$$\omega(n) = 1 \quad ; -N \le n \le N$$

$$= 0 \quad ; \text{otherwise}$$

$$\frac{\sin \omega c n}{\pi n} \xleftrightarrow{F} X(\omega)$$

$$= 1 \quad ; |\omega| < \omega_c$$

$$x_N(\omega) = x(\omega) \circledast w(\omega)$$

$$= \int_{-\pi}^{\pi} x(\theta) - w(\omega - \theta)\, d\theta$$

$$= \int_{-\omega c}^{\omega c} \frac{\sin(2N+1)(\omega-\theta)/2}{\sin(\omega-\theta/2}\, d\theta$$

4.22) :- A signal $x(n)$ has the following fourier transform

$$x(\omega) = \frac{1}{1-ae^{-j\omega}}$$

Determine the fourier transforms of the following

a) $x(2n+1)$

$$\sum_{n=\alpha}^{\alpha} x(n+2)\, e^{-j\omega n}$$

let $2n+1 = l$

$$\sum_{n=-\alpha}^{\alpha} x(l)\, e^{-j\omega\left(\frac{l-1}{2}\right)}$$

$$\sum_{l=\alpha}^{\alpha} x(l)\, e^{\frac{-j\omega l}{2}}\, e^{\frac{j\omega}{2}}$$

$$e^{\frac{j\omega}{2}} \cdot \sum_{l=-\alpha}^{\alpha} x(l)\, e^{\frac{-j\omega l}{2}}$$

$$e^{\frac{-j\omega}{2}} \sum_{l=-\alpha}^{\alpha} x(n)\, e^{\frac{-j\omega}{2} n}$$

$$e^{\frac{j\omega}{2}} \cdot x\left(\frac{\omega}{2}\right)$$

$$e^{j\omega/2} \cdot \frac{1}{1-ae^{-j(\omega/2)}}$$

$$= \frac{e^{j\omega/2}}{1-ae^{-j(\omega/2)}}$$

b) $e^{\frac{\pi j}{2}} x(n+2)$

$e^{j2\omega} X(\omega - \frac{\pi j}{2})$

$x(n) \longleftrightarrow X(\omega)$

$x(n+2) \longleftrightarrow e^{j2\omega} X(\omega)$

$e^{\frac{\pi j}{2}} x(n+2) \longleftrightarrow e^{j2\omega} X(\omega - \frac{\pi j}{2})$

$e^{j2\omega} X(\omega - \frac{\pi j}{2})$

c) $x(-2n)$

$x(n) \longleftrightarrow X(\omega)$

$x(2n) \longleftrightarrow X(\omega/2)$

$x(-2) \longleftrightarrow X(-\omega/2)$

d) $x(n)\cos(0.3\pi n)$

$x(n)\cos\omega_0 n \longleftrightarrow \frac{1}{2}[X(\omega+\omega_0] + X(\omega-\omega_0)]$

$x(n)\cos(0.3 \cdot \pi n) \longleftrightarrow \frac{1}{2}[X(\omega+0.3\pi] + X(\omega-0.3\pi)]$

e) $x(n) * x(n-1)$

$X(\omega) e^{-j\omega} X(\omega)$

$X^2(\omega) e^{-j\omega}$

f) $x(n) * x(-n)$

$X(\omega) \times X(\omega)$

$\frac{1}{1-a e^{-j\omega}} \cdot \frac{1}{1-a e^{j\omega}}$

$\frac{1}{1-ae^{j\omega} - ae^{-j\omega} + a^2 e^0} = \frac{1}{1+a^2-2\cos\omega}$

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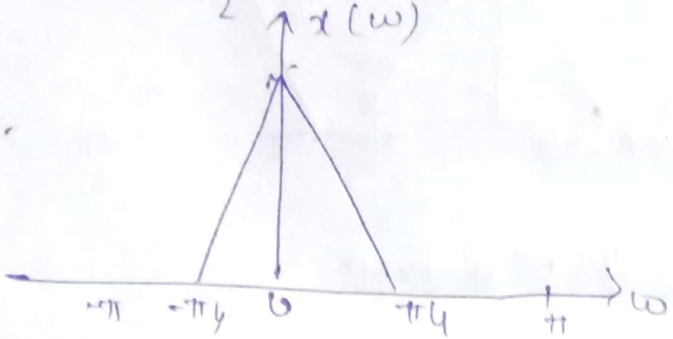
4.23) From a discrete time signal $x(n)$ with fourier transform $x(n)$ show in figure determine and sketch the fourier transform following signals

Note that $y_1(n) = x_n \delta(n)$

where $\delta(n) = \{\ldots 0, 1, 0, 1, 0, 0, 1 \ldots\}$



a) $y_1(n) = \begin{cases} x(n), & 'n' \text{ even} \\ 0, & 'n' \text{ odd} \end{cases}$

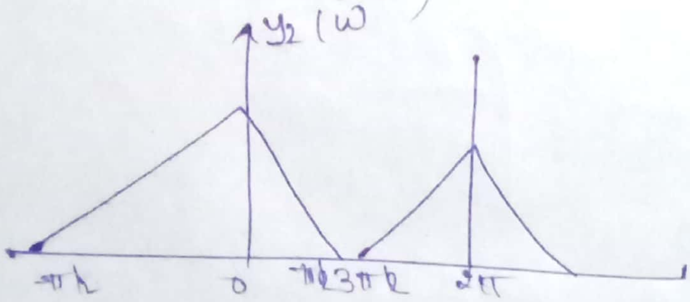
b) $y_2(n) = x(2n)$

$y_2(n) = x(2n)$

$y_2(w) = \sum_n y_2(n) e^{-jwn}$

$= \sum_n x(2n) e^{-jwn}$

$= x\left(\frac{w}{2}\right)$



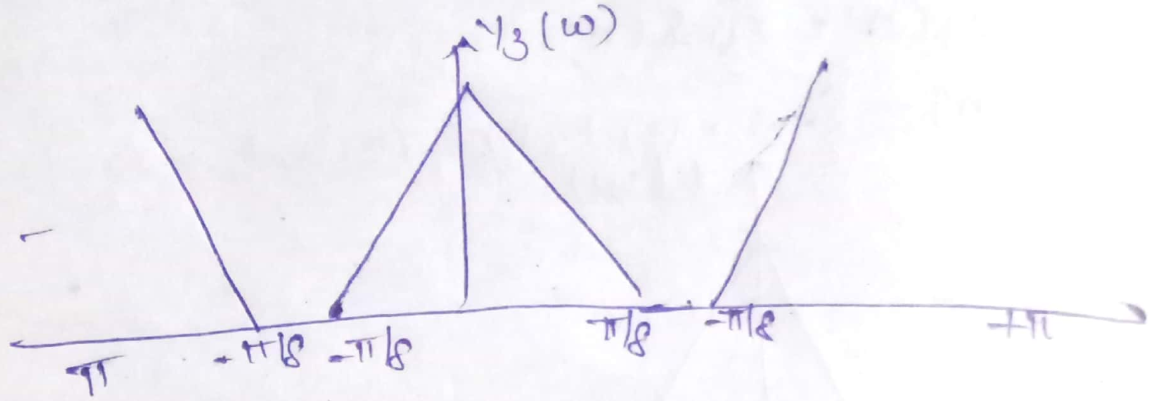
c) $y_3(n) = \begin{cases} x(n/2), & 'n' \text{ even} \\ 0, & 'n' \text{ odd} \end{cases}$

$y_3(w) = \sum_n y_3(n) e^{-jwn}$

$= \sum_{n\,\text{even}} x\left(\frac{n}{2}\right) e^{-jwn}$

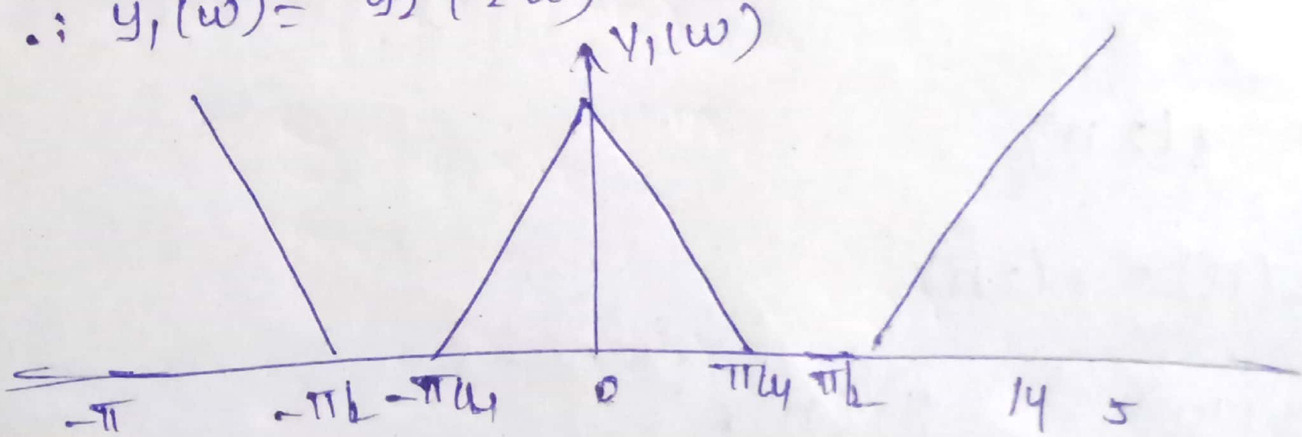
$$= \sum_m x(m) e^{-j2\omega m}$$

$$= x(2\omega)$$



a) $y_1(n) = \begin{cases} y_2\left(\frac{n}{2}\right) ; & \text{'n' even} \\ 0 ; & \text{'n' odd} \end{cases}$

$\therefore Y_1(\omega) = Y_2(2\omega)$



problem :-

find F.T of the signals

a) $x(n) = u(n) - u(n-6)$

$$x(\omega) = \sum_{n=-\alpha}^{\alpha} x(n)\, e^{-j\omega n}$$

$$= \sum_{n=-\alpha}^{\alpha} u(n)\, e^{-j\omega n} - \sum_{n=-\alpha}^{\alpha} u(n-6)\, e^{-j\omega n}$$

$$= \sum_{n=0}^{\alpha} e^{-j\omega n} - \sum_{n=6}^{\alpha} e^{-j\omega n}$$

$$\sum_{n=0}^{\alpha} e^{-j\omega n} = 1 + e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega} + e^{-j34} + \dots$$

$$= \frac{1}{1-e^{-j\omega}}$$

$$\therefore X(\omega) = \frac{1}{1-e^{-j\omega}} - \frac{e^{-j6\omega}}{1-e^{-j\omega}}$$

$$= \frac{1-e^{-j6\omega}}{1-e^{-j\omega}}$$

b) $x(n) = 2^n u(-n)$

$$x(\omega) = \sum_{n=-\alpha}^{\alpha} 2^n u(-n)\, e^{-j\omega n}$$

$$= \sum_{n=-\alpha}^{0} 2^n e^{-j\omega n}$$

$$= \sum_{n=0}^{\alpha} \frac{1}{2^n e^{-j\omega n}}$$

$$= \sum_{n=0}^{\alpha} \frac{e^{j\omega n}}{2^n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{e^{j\omega}}{2} \right)^n$$

$$\sum_{n=0}^{\infty} \left(\frac{e^{j\omega}}{2} \right)^n = 1 + \frac{e^{j\omega}}{2} + \left(\frac{e^{j\omega}}{2} \right)^2 + \left(\frac{e^{j\omega}}{2} \right)^3 + \dots$$

$$= \frac{1}{1 - \frac{e^{j\omega}}{2}}$$

$$= \frac{1}{\frac{2 - e^{j\omega}}{2}}$$

$$= \frac{2}{2 - e^{j\omega}}$$

c) $x(n) = \left(\frac{1}{4}\right)^n u(n+2)$

$$\sum_{n=-\infty} x(n) e^{-j\omega n}$$

$$\sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^n u(n+4)\, e^{-j\omega n}$$

$$\sum_{n=-4}^{\infty} \left(\frac{1}{4}\right)^n e^{j\omega n}$$

$$\sum_{n=0}^{\infty} \frac{1}{4^n} e^{-j\omega n} \left(\frac{1}{4^{-4}} e^{-j\omega(-4)} \right)$$

$$\sum_{n=0}^{\infty} \frac{1}{4^n} e^{j\omega n} \left[4^4 e^{j\omega 4} \right]$$

$$4^4 e^{j\omega 4} \left[1 + \frac{1}{4} e^{-j\omega} + \left(\frac{1}{4} e^{-j\omega} \right)^2 + \dots \right]$$

$$4^4 e^{j\omega 4} \cdot \frac{1}{1 - \frac{1}{4} e^{-j\omega}}$$

$$X(\omega) = \frac{4^4 e^{j\omega 4}}{1 - \frac{1}{4} e^{-j\omega}}$$

d) $x(n) = (\alpha^n \sin \omega_0 n)\, u(n)$, $|\alpha| < 1$

$$\sum_{n=0}^{\infty} \alpha^n \sin \omega_0 n\, e^{-j\omega n}$$

$$\sum_{n=0}^{\infty} \alpha^n \left[\frac{e^{j\omega_0 n} - e^{-j\omega_0 n}}{2j}\right] e^{-j\omega n}$$

$$\sum_{n=0}^{\infty} \alpha^n \left[\frac{e^{j\omega_0 n}\, e^{-j\omega_0 n} - e^{-j\omega_0 n} \cdot e^{-j\omega_0 n}}{2j}\right]$$

$$\sum_{n=0}^{\infty} \frac{1}{2j} \alpha^n e^{j\omega_0 n} e^{-j\omega n} - \sum_{n=0}^{\infty} \frac{1}{2j} \alpha^n e^{-j\omega_0 n} e^{-j\omega n}$$

$$\frac{1}{2j} \sum_{n=0}^{\infty} \left[\alpha e^{j(\omega_0 - \omega)}\right]^n - \frac{1}{2j} \sum_{n=0}^{\infty} \left[\alpha e^{-(\omega_0 + \omega_0)}\right]^n$$

$$\frac{1}{2j}\left[\frac{1}{1 - \alpha e^{-j(\omega - \omega_0)}}\right] - \frac{1}{2j}\left(\frac{1}{1 - \alpha e^{-j(\omega_0 + \omega)}}\right)$$

$$\frac{1}{2j}\left[\frac{1}{1 - \alpha e^{-j(\omega - \omega_0)}} - \frac{1}{1 - \alpha e^{-j(\omega + \omega_0)}}\right]$$

$$\frac{1}{2j}\left[\frac{\cancel{1} - \alpha e^{-j(\omega + \omega_0)} - \cancel{1} + \alpha e^{-j(\omega - \omega_0)}}{(1 - \alpha e^{-j(\omega - \omega_0)})(1 - \alpha e^{-j(\omega + \omega_0)})}\right]$$

numarator

$$\frac{1}{2j}\left[-\alpha e^{-j\omega} e^{-j\omega_0} + \alpha e^{-j\omega} \cdot e^{j\omega_0}\right]$$

$$\frac{\alpha e^{-j\omega} \cdot \left[e^{j\omega_0} - e^{-j\omega_0}\right]}{2j}$$

$$\alpha e^{-j\omega} \sin \omega_0$$

denomanator

e) $x(n) = |\alpha|^n \sin \omega_0 n \quad |\alpha| < 1$

Checking for stability

$$\sum_{n=-\alpha}^{\alpha} |x(n)| < \alpha$$

$$\sum_{n=-\alpha}^{\alpha} ||\alpha||^n \sin \omega_0 n|$$

$$\sum_{n=-\alpha}^{\alpha} |\alpha|^n |\sin \omega_0 n|$$

if $\omega_0 = \frac{\pi}{2}$, $\sin \omega_0 n = 1$

$$\sum_{n=-\alpha}^{\alpha} |\alpha|^n = \alpha$$

$\therefore$ If not satisfies the condn stability

$\therefore$ no. F.T for it

[does not exist]

f) $x(n) = \begin{cases} 2 - (1/2)n & ; |n| \leq 4 \\ 0 & ; \text{elsewhere} \end{cases}$

$$= \sum_{n=-4}^{4} \left[2 - \left(\tfrac{1}{2}\right)n \right] e^{-j\omega n}$$

$$= \frac{2e^{j4\omega}}{1-e^{-j\omega}} - \frac{1}{2}\Big[+4e^{j4\omega} + 4e^{-j4\omega} - 3e^{j3\omega} + e^{-j3\omega} - 2e^{j2\omega} + 2e^{j\omega} \Big]$$

$$= \frac{2e^{j4\omega}}{1-e^{-j\omega}} + j\big[4\sin 4\omega + 3\sin 3\omega + 2\sin 2\omega + \sin\omega\big]$$

g) $x(n) = \{-2, -1, \underset{\uparrow}{0}, 1, 2\}$

Sol:- $\sum_{n=-2}^{2} x(n) e^{-j\omega n}$

$x(-2) e^{j2\omega} + x(-1) e^{j\omega} + x(0) e^{0} + x(1) e^{-j\omega} + x(2) e^{-j2\omega}$

$-2 e^{j2\omega} - e^{j\omega} + 0 + e^{-j\omega} + 2 e^{-j2\omega}$

$-2 \left[e^{j2\omega} - e^{-j2\omega}\right] - \left[e^{j\omega} - e^{-j\omega}\right]$

$-4j \left[\frac{e^{j2\omega} - e^{-j2\omega}}{2j}\right] - 2j \left[\frac{e^{j\omega} - e^{-j\omega}}{2j}\right]$

$-4j \left[\sin(2\omega)\right] - 2j \left[\sin(\omega)\right]$

$-2j \left[2\sin 2\omega + \sin \omega\right]$

h) $x(n) = \begin{cases} A(2m+1-|n|) & ; |n| \le m \\ 0 & ; |n| > m \end{cases}$

$x(\omega) = \sum_{n=-m}^{m} x(n) e^{-j\omega n}$

$= A \sum_{n=-m}^{m} (2m+1-|n|) e^{-j\omega n}$

$= (2m+1)A + A \sum_{k=1}^{n} (2m+1-k)$

$= (2m+1)A + 2A \sum_{k=1}^{m} (2m+1-k) \cos \omega k$