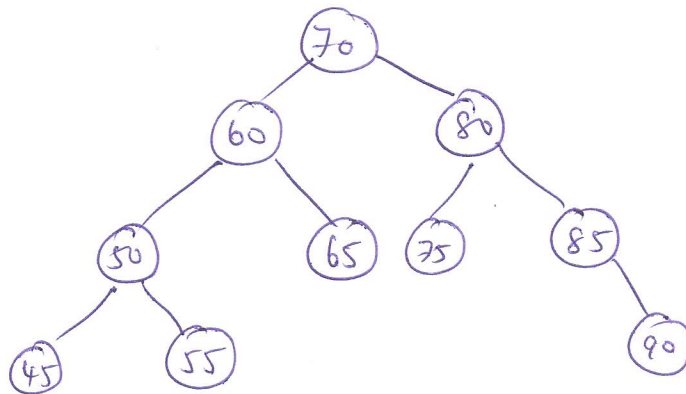
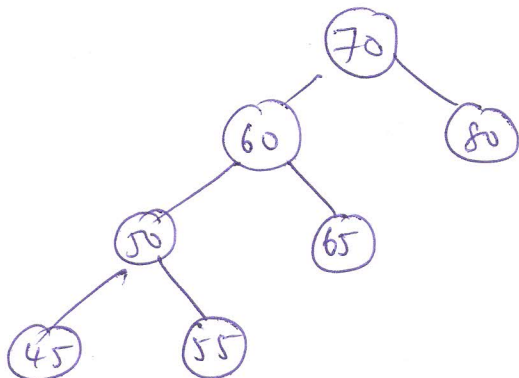


1. Find the root of each of the following binary trees?
  - a. Tree with post order traversal: FCBDG
  - b. Tree with pre order traversal: IBCDFEN
  - c. Tree with in-order traversal: CBI DFGE
  
2. A binary tree has 10 nodes. The inorder and preorder traversal of the tree are shown below:  
Draw the tree.  
preorder: JCB ADEFGH  
inorder: ABCEDFGJHI
  
3. A nearly complete binary tree has nine nodes. The breadth first traversal of the tree is given below.  
Draw the tree  
Breadth FT: JCB ADEFGH
  
4. Draw all possible non-similar binary trees with three nodes. (A, B, C)
  
5. What is the smallest number of levels, a binary tree with 49 nodes can have?
  
6. Draw the expression tree for the following infix expression and find the prefix and postfix expression  
$$(C + D + A * B) * (E + F)$$
  
7. Write an algorithm (pseudo-code) that counts the number of nodes in a binary tree

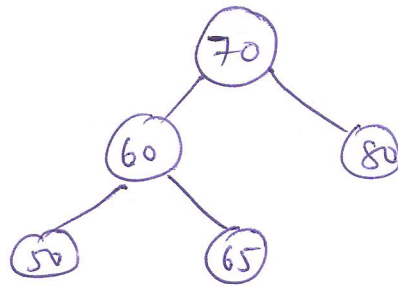
8. Write an algorithm that given a binary tree determines if it is a complete tree
9. Rewrite the binary tree preorder traversal algorithm using a stack instead of recursion.
10. Draw all possible binary search trees for three data elements: 5, 9 and 12
11. a. Create a binary search tree using the following data entered as a sequential set.  
14, 23, 7, 10, 33, 56, 80, 66, 70  
b. insert 46 and 52 in the tree created above
12. a. In the BST shown below, delete the keys 60 and 85



13. Balance the following tree



- (14) Add 49 and 68 to the AVL tree in the following figure. The result must be an AVL tree. Show the balance factors in the resulting tree.



- (15) A full node is a node with 2 children. Prove that the number of full nodes plus one is equal to the number of leaves in a non-empty binary tree.
- (16) Suppose a binary tree has leaves  $l_1, l_2, \dots, l_m$  at depths  $d_1, d_2, \dots, d_m$  respectively. Prove that  $\sum_{i=1}^m 2^{-d_i} \leq 1$  and determine when the equality is true.
- (17) Given input  $\{4371, 1323, 6173, 4199, 4344, 9679, 1989\}$  and a hash function  $h(x) = x \bmod 10$  show the resulting:
- Separate chaining hash table
  - Open addressing hash table using linear probing
  - Open addressing hash table using quadratic probing
  - Open addressing hash table with a second hash function  $h_2(x) = 7 - (x \bmod 7)$
  - Show the result of rehashing the above hash tables.

(18) An alternative collision resolution strategy is to define a sequence,  $F(i) = r_i$ , where  $r_0 = 0$  and  $r_1, r_2, \dots, r_N$  is a random permutation of the first  $N$  integers (each integer appears exactly once)

- Prove that under this strategy, if the table is not full, then the collision can always be resolved
- Would this strategy be expected to eliminate clustering?
- If the load factor of the table is  $\lambda$ , what is the expected time to perform an insert?
- If the load factor is  $\lambda$ , what is the expected time for a successful search?
- Give an efficient algorithm (theoretically as well as practically) to generate the random sequence.  
Explain why the rules for choosing  $P$  are important.

(19) Describe a procedure that avoids initializing a hash table (at the expense of memory)

- (20) a. Draw a 2-3 B-tree of 4 levels  
b. Draw a 2-3-4 B+tree of 3 levels.

x ————— x