Human Tracking by a Mobile Robot in Low Illumination Environment

Dr. Jharna Majumdar¹, Sudip Chandra Gupta², Giridhar N R³

¹DEAN R&D Prof. and Head Dept. of MTech Computer Science & Engg., Head Center for Robotics Research

² Research Associate

³ Student, Dept. of Computer Science & Engg. Nitte Meenakshi Institute of Technology, Bangalore – 560 064, India

Abstract: Target tracking at a less illuminated environment is a tedious work. In this paper, Adaptive Enhancement algorithms are used as a pre-processing technique. The performance of these methods on real-time scenario has been tested and using Quality Metrix Parameter their usefulness is proved. Particle Filter Based Tracking [5] is used for the purpose of Target/Human Tracking. A mobile robot is used for tracking in low illumination. A robust Control system is required for the robot to efficiently follow the human using the computer vision algorithm. Hence, Linear Quadratic Integral (LQI) has been implemented on the system for velocity control. The velocity of the robot is monitored such that it doesn't crash into the target.

Keywords: Low Illumination, Tracking, Particle filter, SSMR, Control System, LQI

I. INTRODUCTION

The interest in the domain of mobile robots and its automation has grown in the past few years. There are lots of research on the kinematics and dynamic modelling, and for the control system for the mobilization of the robot, which should satisfy its performance based on time and accuracy. Although several research were conducted on the control system, the implementation on an embedded board has not been mentioned involving memory management and low redundancy. There were several research related to various approaches to achieve the optimum control system and their respective simulation results proved to be optimum, but when used in a system or a robot, the output determines all these models to be sparse.

The use of LQI has been discussed in many papers but the fact of implementation or actual relation to simulated value with real-time value has not been discussed [15[16][17][18]. The optimum control Linear Quadratic (LQ) controller can be performed in by various techniques like LQR, LQG or LQI, but which one is the optimum for a system has not been presented. The main goal LQI controller implemented on a Skid Steer Mobile Robot (SSMR) is to control the velocity of the robot as well as avoid crashing into the target and obstacles.

Skid Steer Mobile Robots (SSMR) are the vehicles that can tract on all terrain condition. To move the

SSMR on all terrain there must be a feedback determining the errors in its predicted parameters and the output parameters, which must be done autonomously by the embedded controller, which provides the actuation signals to all the wheels in terms of pulse-width modulation (PWM). This can be achieved by implementing the control algorithm on an embedded controller Mbed LPC1768. While, the computer vision algorithms provides the tracking reference for the target which is elobrated later in this paper.

Dr. J L Mazher Iqbal et.al [1] have proposed an algorithm which will detect and track objects especially during dim light. They have made use of Thermal Camera to serve their purpose. Object is detected using Foreground segmentation and to remove the speckles they have performed morphological operations. At last the object is tracked using Kalman Filter.

Wenhua Guo et.al [2] for the purpose of Object Tracking at night time, made use of generalized Local Binary Pattern descriptor (LBP) called Double Centre-Symmetric Local Binary Pattern (DCS-LBP) for representation of the object. With the combination of DCS-LBP and color, multiple features of the target model are generated. To reduce the rate of matching mistakes, a posterior probability is introduced.

T. Mahalingam et.al [3] have proposed a three phase algorithm for multiple moving object detection. In the detection phase, Foreground segmentation is done using Adaptive Gaussian Model. Target Tracking is achieved by the blob detection and for the evaluation phase has feature extraction and classification.

Takashi Komuro et.al [4] used a Bayesian Framework to estimate Target Motion and Reference Image simultaneously. The motion parameters that are obtained are regarded as hyper parameters. If the input frames are blurred, they are deblurred and motion estimation is done.

Section II describes the various adaptive enhancement algorithms used. The best algorithm is chosen after analyzing their performance based on QM (Section III). For the mobile robot to efficiently follow the human, LQI controller is used (Section IV) and its

result is shown in Section V. Mobile Robot Design Parameters are shown in Section IV.

II. ADAPTIVE ENHANCEMENT ALGORITHMS

Processing a low contrast image so that the output obtained is more preferable than the original image for a specific application. It sharpens image features such as edges, boundaries, or contrast to make it more helpful for display and analysis. Many adaptive contrast enhancement algorithms have been proposed a brief description of the algorithms two proposed cases given below.

Brightness Preserving Bi-Histogram Equalization (BBHE) [7]: - Input image is split into two sub images based on the mean of the input image. Samples of the input image which are less than or equal to mean forms one sub image, the other sub image consists of samples which are greater than the mean. Each of these sub images are independently equalized based on their respective histograms. The first sub image, containing samples less than or equal to mean, are mapped into the range from the minimum gray level to the input mean. The second sub image, containing samples greater than the mean are mapped into the range from the mean to the maximum gray level.

Dualistic Sub-Image Histogram Equalization (DSIHE) [8]: - In this equalization technique, probability density of the image is calculated. With the help of probability density values, entropy of the grey level distribution is computed which is used as a threshold to split the image into two sub-images. All the image samples which are less than or equal to this threshold forms the first sub-image. Image samples which are greater than the threshold forms the second sub-image. Each sub-image is equalized based on their respective histograms. First sub-image, containing samples less than the threshold are mapped into the range from minimum gray level to the threshold. The second sub-image, containing samples greater than the mean are mapped into the range from the mean to the maximum gray level.

Recursive Mean-Separate Histogram Equalization (RMSHE) [9]: - The main principle of this technique is to preserve more brightness. It is a generalization of Histogram Equalization [10] and Brightness Preserving Bi-Histogram Equalization (BBHE) [7]. If the recursion level is 1, the mean of the input image is computed and image is split to two sub-images. If the recursion level is 2, mean of the input image is computed and two more sub-means are calculated. One sub-mean is the mean of minimum gray level and the input image mean. Second sub-mean is the mean of input image mean and maximum gray level value.

So a total of 4 sub-images are obtained. Samples belonging in between minimum gray level value and first sub-mean, forms first sub-image. Samples belonging in between first sub-mean and input image mean forms second sub-image. Samples belonging in between input-image and second sub-mean forms third sub-image. Samples belonging in between second sub-mean and maximum gray level value forms fourth sub-image. All these sub-images are equalized based on their respective histograms and are mapped to their respective gray level values as done in Sub-Image Histogram Dualistic Equalization (DSIHE).

Adaptively Modified Histogram Equalization (AMHE) [11]: - Probability density function of the input image is calculated. This Probability density function (PDF) is modified into a new Probability density function with uniform distribution property in order to enhance contrast. This modification is done before enhancement. The original shape of the Probability Density Function is preserved. The rate of contrast enhancement can be controlled in AMHE. Using the modified Probability Density Function (PDF), the Cumulative Density Function (CDF) is computed and it is transformed using the Transformation Function.

Weighted and Threshold Histogram Equalization (WTHE) [12]: - Probability density function of the input image is replaced by a weighted Probability Density Function. Two thresholds known as upper and lower threshold are used to clamp the original probability density function. For the values in between these two thresholds, weighted threshold probability density function, is calculated. Using this modified probability density function the same procedure as that off Histogram Equalization (HE), [10], is followed up.

Average Luminance with Weighted Histogram Equalization (ALWHE) [6]: - This enhancement technique preserves and modifies the intensity freely. Mean of the entire image is found out and based on the mean it is split into two parts. Samples less or equal to the mean form one sub-image. Samples greater than the mean form the second sub-image. The probability density function for each of the sub-images are reshaped. From the new probability density function new histogram value for each sub-image is determined and it is mapped to the actual image using the grey level distribution values.

The main objective of this research is to develop a mobile robot would track a human in low illuminated corridors [].Our main aim in this work is to select the best algorithm for the given scenario. Quality Metric

parameters like Entropy Error Rate (ERR), Grey Level Energy (GLE), Global Contrast (GC), Quality Index (QI), Absolute Mean Brightness Error (AMBE) and Relative Entropy (RE) are used. Adaptive enhancement algorithms are first run on Images which

III. LQI CONTROLLER

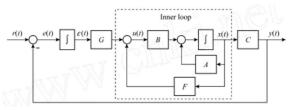


Fig. 1. Block Diagram of LQI Controller

Our previous work [21] deals Linear quadratic Regulator (LQR) where the stability time is not known and it is equivalent to the PD controller where as LQI is LQR controller with additional Integral Block. The integrator are static feedback controller with integral or sum of all the errors that were tracked. This helps in minimizing the cost function of the system and stabilizing the outer loop of the system. Even though the outer loop of the system is stabilized the inner loop has to be stabilized. Instability in case of inner loop will lead to poor dynamic response in the closed loop. Since the outer loop stability is dependent on inner loop stability, the inner loops stability has to be taken care of to remove the performance constrains.

3.1 LQI Design

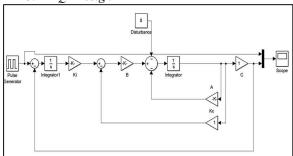


Fig. 2. MATLAB Design of LQI controller

LQI controller can be designed with or without the disturbance factor 'D'. Here the disturbance is assumed to be 0 and the design is made as shown in fig.2

Consider quadratic cost function be

$$J(\bar{x},\bar{u}) = \int_0^\infty (\bar{x}^T Q \bar{x} + u^2) dt$$
 Such that
$$\dot{x} = A\bar{x} + Bu$$

$$\dot{x} = A\bar{x} + Bu$$
$$y = C\bar{x} + Du$$

are less illuminated. After obtaining the best algorithm from the analysis of QM parameters, the same is run on a number of videos captured from the low illuminated corridors

The optimal feedback gain vector can be calculated by:

$$K = R^{-1}B^{T}P$$

Control solution

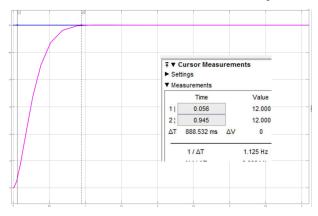
$$u = -R^{-1}B^T P \bar{x}$$

4.2 Implementation of LQI on NXP LPC 1768 microcontroller

LPC 1768 controller has a 32-bit ARM Cortex-M3 core running at 96MHz. It also has 512KB flash, 32KB RAM and lots of I/O pins, which are required for the operation of SSMR. The algorithm and design parameters of the robot for the implementation is mentioned in the appendix I

The Mobile Robot is also shown in Appendix 1

IV. PERFORMANCE RESULT OF LQI



Time Specifications	LQI
Settling Time	0.945
Peak Amplitude	12
Overshoot	0.12%

Table 1: Simulation Results in MATLAB

From the results in Table 1, it is observed that LQI controller has lower overshoot and optimum settling time. The average human walking speed is 1.2 m/s which comes roughly around 120 RPM for our mobile robot that is calculated using the formula in equation no (). The full speed of the motor or the robot is 4.3 m/s which is equivalent to 400 RPM. The motors operates at full speed at 24 V but for convenience the supply voltage is regulated in order to achieve desired speed i.e. 120 RPM which can be easily done through

position-based velocity control which in turn generates the equivalent PWM signal. This can be made clearer in table no 2 as it has been explained as per the simulation.

The LQI Controllers are implemented on the NXP LPC 1768 for the Skid Steering Mobile Robot and the performance observation is noted as mentioned in table no 2.

5.1 Hardware Implementation of LQI on SSMR using NXP LPC 1768

Q4		W	HEELS	
Encoders	LEFT FRONT	LEFT REAR	RIGHT FRONT	RIGHT REAR
VELOCITY (equivalent RPM)	120.147	120.14	120.12	120.14
COUNTS	563	562	561	562

Table 2 – Implementation results on Mobile Robot

The above table shows the result of implementation of LQI Controller on a four wheeled skid steer mobile robot. Position Based Velocity Control was the method implemented to maintain a constant velocity. The total encoder in one rotation is 29000. Control of the velocity is based on the encoder count for which

640 count gives the desired velocity that is 1.2 m/s or 120 RPM. The formula for calculation of the velocity based on encoder count can be seen in Appendix 1 in the calculation. Although the velocity was set at 120 RPM we achieve slightly higher values which is due to the overshoot. The robot had to follow a human target which can move in any directly any time for this the concept of visual servoing [19] has been used where the image or target co-ordinate is tracked along the x-axis of the image and respective control signals are transmitted to the embedded board on the Robot. Since, the camera did not have depth perception an ultrasonic sensor was used to avoid collision with the target as well as prevent the robot from crashing into obstacles.

The transmission of the image co-ordinate is linked with the mobile robot using an interaction matrix [19] given below:

$$\begin{bmatrix} u \\ v \\ v \end{bmatrix} = L\xi = \begin{bmatrix} -\frac{\lambda}{z^c} & 0 & \frac{u}{z^c} & \frac{uv}{\lambda} & -\frac{\lambda^2 + u^2}{\lambda} & v \\ 0 & -\frac{\lambda}{z^c} & \frac{v}{z^c} & \frac{\lambda^2 + v^2}{\lambda} & -\frac{uv}{\lambda} & -u \end{bmatrix} \xi_c^c$$

Where, Z, λ are camera parameters and υ u are the image Cartesian co-ordinate equivalent or representative points on the plane. Using thee above matrix the camera velocity or the target velocity is found which is related to the robot velocity.

V. RESULTS

Sample Images, both from Indoor and Outdoor environment are considered. The six adaptive enhancement algorithms discussed are implemented on them. Based on the QM parameters, <u>best</u> adaptive enhancement algorithm is chosen and the same will be used as a pre-processing technique to the particle filter tracking algorithm [5].

Input Image 1	BBHE	DSIHE	RMSHE	AMHE	WTHE	ALWHE
ERR	-0.00113	0.02285	0.02067	-0.000644	-0.01899	0.0021828
GLE	0.037042	0.03627	0.051924	0.04487	0.024929	0.0389717
GC	8407.24	6924.42	2836.95	3178.72	6539.14	7396.36
QI	3.5094	3.59752	4.29584	5.39643	3.3118	3.8408
AMBE	57.0938	40.9095	13.4577	40.2665	103.604	62.4169
RE	-2.7165	-2.8482	-2.87863	-2.90264	-1.07546	-2.47569

Table 3: Result of Adaptive Enhancement Algorithms and their respective QM parameters on Image1.

Input Image 2	ВВНЕ	DSIHE	RMSHE	AMHE	WTHE	ALWHE
					N.	
ERR	0.026880	0.052213	0.150838	0.022757	-0.00095	0.024671
GLE	0.027038	0.024504	0.0033412	0.013278	0.0101791	0.03054
GC	8927.29	7183.96	2531.92	2640.77	7876.04	8362
QI	2.99315	2.8543	3.39576	2.72317	1.69806	3.19135

AMBE	52.4296	40.2067	10.5605	44.8559	105.261	54.7357
RE	-0.24017	-0.26404	-0.52452	-0.85059	-0.62618	-0.79224

Table 4: Result of Adaptive Enhancement Algorithms and their respective QM parameters on Image2.

Input Image 3	ВВНЕ	DSIHE	RMSHE	AMHE	WTHE	ALWHE
			\ A	N		
ERR	0.005899	0.02255	0.007894	-0.02819	-0.02843	0.020538
GLE	0.024944	0.03895	0.029532	0.020669	0.018577	0.038999
GC	5116.13	4707.75	776.247	502.588	1678.63	4455.71
QI	3.0528	3.99184	8.0966	5.37884	2.68825	3.99422
AMBE	75.8469	60.0074	27.5876	66.2399	158.891	59.8731
RE	0.007029	0.05974	-0.00976	0.063422	0.008851	0.057733

Table 5: Result of Adaptive Enhancement Algorithms and their respective QM parameters on Image3

Input Image 4	BBHE	DSIHE	RMSHE	AMHE	WTHE	ALWHE
The state of	Action & Marie					1000
	## # (N 1 1 1 1 1 1 1 1 1			The same		7 (1 (1)
ERR	7.79399	5.59562	1.42971	0.056337	0.033779	4.00198
GLE	0.26494	0.242751	0.265346	0.048213	0.043448	0.418771
GC	425.463	137.674	5.06528	392.746	2239.43	458.082
QI	10.2075	9.38316	91.5701	10.2799	4.93619	12.7809
AMBE	2.56551	1.181695	0.130365	55.5803	84.6384	3.21909
RE	-0.00109	-0.89462	-0.04453	-0.12455	-2.17057	-0.61121

Table 6: Result of Adaptive Enhancement Algorithms and their respective QM parameters on Image4

For each QM parameter, the best Algorithm is highlighted. From Table 3, 4, 5 & 6, Weighted and Threshold Histogram Equalization (WTHE) [12] shows consistent response. WTHE will be used as a pre-processing algorithm to the particle filter based tracking [5].

Weighted and Threshold Histogram Equalization (WTHE) [12] is used as pre-processing technique to Particle Filter Based Tracking Algorithm [5]. For the mobile robot to track the target, LQI Controller is used. This section shows the results of the same. Fig 4 (a), (b), (c), (d), (e) are the input video frames of video sequence 1 and (f), (g), (h), (i), (j) are the corresponding Target Tracking with the combination of WTHE [12] and Particle Filter Tracking Algorithm [5]. Fig 4 (k), (l), (m), (n), (0) are the respective views from the mobile robot.

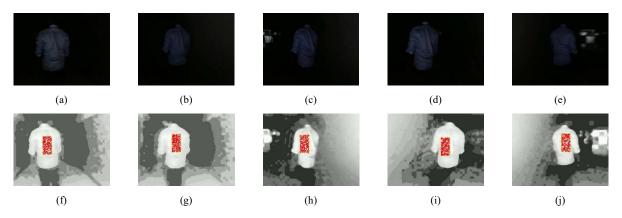


Fig. 4. Human Tracking by a Mobile Robot in Less Illuminated Environment - Video Sequence 1 (a) Frame 46 (b) Frame 47 (c) Frame 48 (d) Frame 49 (e) Frame 50 (f) Weighted Threshold Histogram Equalization and Particle Filter Target Tracking to Frame 46 (g) Weighted Threshold Histogram Equalization and Particle Filter Target Tracking to Frame 47 (h) Weighted Threshold Histogram Equalization and Particle Filter Target Tracking to Frame 48 (i) Weighted Threshold Histogram Equalization and Particle Filter Target Tracking to Frame

Mobile Robot following Human in Less Indoor Illuminated Environment: Weighted Threshold Histogram Equalization (WTHE) [12] + Particle Filter Tracking Algorithm [5] + LQI Controller. Fig 5 (a), (b), (c), (d), (e) shows the same.

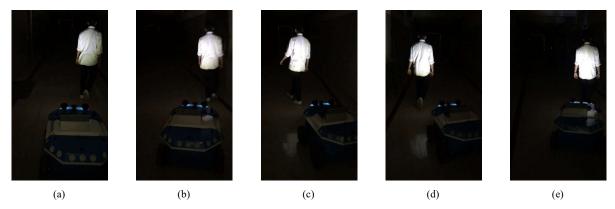


Fig. 5. Human Tracking by a mobile robot in Less Illuminated Indoor Environment – (a) Frame 112 (b) Frame 113 (c) Frame 114 (d) Frame 115 (e) Frame 116.

VI. CONCLUSION

In this paper experiments have been conducted on several adaptive enhancement algorithms and we have found that Weighted and Threshold Histogram Equalization (WTHE) is best suited for enhancement of low illuminated scenes. The robot has to follow a target, this exacts the need of advanced control system with the help of which the robot can navigate its surrounding while maintaining certain velocity and avoiding obstacles or crashing into the target itself. There is lot of future scope for this work as it can be implemented for various applications apart from only mobile robots or robotics applications. It can also be used for surveillance in building, where the live feed will be continuously be transmitted from the robot to the output screen of an end user who is at remote location. This brings the idea of IORT (Internet of Robotic Things). A Cyber Physical System (CPS) is a system that is controlled or checked by Computer based calculations, firmly coordinated with the Internet and its client's. Different post processing algorithm can be developed which can further enhance the image quality.

VII. ACKNOWLEDGEMENT

Our sincere thanks go to the Department of Science and Technology (DST), New Delhi, Govt. of India to acknowledge our research and provide us the financial support to carry out the research at NMIT. We express our sincere thanks to our colleagues at the Centre for Robotics Research, NMIT for providing support.

We would also like to express our thanks to, Mr. Sunil S, Asst. Prof. in the Dept. of Mechanical Eng. NMIT for Design, Analysis and Fabrication of the Mobile Robots.

Finally, our sincere gratitude goes to Prof. N R Shetty, Director NMIT and Dr. H C Nagaraj, Principal NMIT for providing the infrastructure support and whole hearted encouragement to carry out research at NMIT.

REFERRENCES

- [1] Object Detection and Tracking Using Thermal Camera, Dr.J L Mazher Iqbal and K. Iswarya Lakshmi, International Journal of Pure and Applied Mathematics
- [2] Object Tracking Using Local Multiple Features and a Posterior Probability Measure, Wenhua Guo *, Zuren Feng and Xiaodong Ren
- [3] A robust single and multiple moving object detection, tracking and Classification, T. Mahalingamn, M. Subramoniam, Applied Computing and Informatics
- [4] Object Tracking and Image Restoration from Multi- Frame Images Captured in a Dark Environment, Takashi Komuro, HayatoKuroda, ITE Transactions on Media Technology
- [5] Implementation of Real Time Particle Filter Based Tracking Algorithms on Beagleboard-XM, Jharna Majumdar, Kiran, Kiran Kumar, International Journal of Computer Engineering & Technology
- [6] Contrast Enhancement method based on Average Luminance with Weighted Histogram Equalization, Shen-Chuan Tai, Yi-Ying Chang, Kang-Ming Li, Ting-Chou Tsai, 2009 Fifth International Conference on Information Assurance and Security
- [7] Contrast Enhancement Using Brightness Preserving Bi-Histogram Equalization, YEONG-TAEKGI M, 1997 IEEE
- [8] IMAGE ENHANCEMENT BASED ON EQUAL AREA DUALISTIC SUB-IMAGE HISTOGRAM EQUALIZATION METHOD, Yu Wang, Qian Chen, Baomin Zhang, IEEE Transactions on Consumer Electronics, 1999
- [9] Contrast Enhancement using Recursive Mean-Separate Histogram Equalization for Scalable Brightness Preservation, Soong-Der Chen, Abd. Rahman Ramli, 2003 IEEE
- [10] Histogram Equalization,
- [11] Contrast Enhancement Using Adaptively Modified Histogram Equalization, Hyoung-Joon Kim, Jong-Myung Lee, Jin-Aeon Lee, Sang-Geun Oh and Whoi-Yul Kim
- [12] Fast Image/Video Contrast Enhancement Based on WTHE, Qing Wang, Rabab Ward
- [13] "Skid Steering Mobile Robot Modeling and Control" by Osama Elshazly, Hossam's Abbas, Zakarya Zyada and Ahmed Abo-Ismail

at International Conference on Control 9th - 11th July 2014, Loughborough, U.K.

[14]K. Kozlowski and D. Pazderski, "Modeling and control of a 4wheel skid-steering mobile robot," Int. J. Appl. Math. Comput. Sci., vol. 14, no. 4, pp. 477-496, 2004.

[15] J. Yi, D. Song, J. Zhang, and Z. Goodwin, "Adaptive trajectory tracking control of skid-steered mobile robots," in In Proc. of the IEEE International conference on robotics and automation, Italy, 2007, pp. 2605-2610.

[16] L. Caracciolo, A. D. Luca, and S. Iannitti, "Trajectory tracking control of a four-wheel differentially driven mobile robot," in In Proc. of IEEE International Conference on Robotics and Automation, USA, 1999, pp.2632-2638.

[17] S.carriere, S. Caux, M. Fadel, "Optimal LQI synthesis for speed control of Synchronous Actuator under Load Inertia Variation", The international Federation of Automatic control July 6-11,2008

[19] A Tutorial on Visual Servo Control, Seth Hutchinson, Member, IEEE, Gregory D. Hager, Member, IEEE, and Peter I. Corke, Member, IEEE

[20] Missile Attitude Control via a Hybrid LQG-LTR-LQI Control Scheme with Optimum Weight Selection, Kaushik Halder, Saptarshi Das

Appendix I



Fig I. Mobile Robot used for Tracking

Position based Velocity Calculation

Max Speed of Motor: 400rpm

Total count: 28000

Count/10ms = $\frac{(400/60)*28000}{}$ = 1866 100

E.g. For 120rpm Counts/10ms = $\frac{1425*120}{400}$ = 560

Since counts are digital quantity counts/10ms = 560

Algorithm (LQI)

INPUT PARAMETERS - Velocity (Right and Left) OUTPUT - u, PWM

STEPS:

Step 1 Initialize the pins of the microcontroller.

Step 2 Define parameters like gains A, B, C, D, Ki, Kc

Step 3 Input the desired speed

Step 4 Calculate the count per second "t" for the desired speed

Step 5 Assign interrupts for the counts

Step 6 Receive the feedback from the encoder for time

Step 7 Compare the desired speed and feedback for time "t"

Step 8 Calculate the gain matrices A, B, C, D, K, Q, R **Step 9** Calculate the control unit $u = -R^{-1}B^{T}Px$

Step 10 Generate PWM using respective gains and errors as base PWM

Step 11 Run the entire step in loop.

Detail calculation of SSMR Robot is given below:

$$\bullet \quad A = \begin{pmatrix} 0.125998 & 0.015 \\ -0.6576 & 1.9064 \end{pmatrix}$$

•
$$B = \begin{pmatrix} 0.7369 & 0.7369 \\ -7.3499 & 7.3499 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

•
$$D = \begin{pmatrix} -0.3924 \\ -5.45 \end{pmatrix}$$

The gain matrix are obtained using Ricatti equation and matrices obtained from SSMR parameters i.e.

The value of Q matrix for LQI is

$$Q = \begin{pmatrix} 0.1 & 0 \\ 0 & 1 \end{pmatrix}$$

The value of R matrix for LQI is $R = \begin{pmatrix} 0.1 & 0 \\ 0 & 0 \end{pmatrix}$

$$R = \begin{pmatrix} 0.1 & 0 \\ 0 & 0 \end{pmatrix}$$

Therefore after solving the Ricatti Equation, the Gain Matrix is:

$$K = \begin{pmatrix} 1.2187 & -0.6093 \\ 1.2187 & 0.6093 \end{pmatrix}$$

Table I: Final Mechanical parameters of the SSMR robot

Parameters	Values (Dimension)
7 (1) 0 1 0	
R (radius of wheel)	0.1075 (m)
M (Mass of Robot)	46 (kg)
I (Moment of Inertia about COM)	0.006288 (Kg m ²)
R _a (Armature Resistance)	0.317 (Ohms)
K _i (Torque Constant)	0.03 (Nm/A)
K _e (EMF Constant)	0.03 (Vs/rad)
a (Lateral distance of rear wheel center from COM)	0.2275 (m)
b (Lateral distance of front wheel center from COM)	0.2275(m)
c (Lateral distance between COM and the side wheel)	0.2275(m)