Lecture 17

Potential Outcomes and Treatment Effects

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Attribution

Today's material is based on lecture notes from Arnaud Maurel (Duke University).

I have adjusted his materials to fit the needs and goals of this course

Plan for the Day

1. What are potential outcomes? What are treatment effects?
2. Challenges to identifying treatment effects
3. Matching & IV
4. Control functions

Potential Outcomes model

- Developed by Quandt (1958) and Rubin (1974)
- Two potential outcomes, (Y_0, Y_1), associated with each treatment status $D \in \{0, 1\}$
- The econometrician only observes:
  - the treatment dummy D
  - the realized outcome $Y = DY_1 + (1 - D)Y_0$
- Note: we could have more than two treatments
- Note: we also assume **SUTVA**: i's treatment doesn't affect j's outcome
  - SUTVA also known as "no interference" in Judea Pearl's world

Objects of interest

Individual-level Treatment Effects:

- $Y_{i1} - Y_{i0}, \quad i = 1, \ldots, N$

Mean treatment effect parameters:

- **Average Treatment Effect (ATE):** $\mathbb{E}(Y_1 - Y_0)$
- **Average Treatment on the Treated (ATT):** $\mathbb{E}(Y_1 - Y_0|D = 1)$
- **Average Treatment on the Untreated (ATU):** $\mathbb{E}(Y_1 - Y_0|D = 0)$
- No covariates in this simple setting, but we could include them fairly easily

Objects of interest (Continued)

- Each treatment parameter answers a different question
- ATT is most related to the effectiveness of an existing program
  - ATT does not account for program's cost
- We can define many other relevant treatment effect parameters:
  - Marginal Treatment Effect
  - Policy Relevant Treatment Effect
- These require imposing some structure on the underlying selection model

Identification challenges

Two main problems arise when identifying the effect of treatment D on outcome Y:

1. **Evaluation problem:** for each i we only observe either Y_0 or Y_1, but never both
2. **Selection problem:** selection into treatment is endogenous, i.e. $(Y_0, Y_1) \not\perp D$

The evaluation problem

- Fundamental observability problem $\implies$ individual TE $Y_{i1} - Y_{i0}$ is not identified
- Thus, we often focus on mean treatment effects, such as the ATE, ATT, ATU
- Or on other parameters that depend on the marginal distributions only; e.g. QTE
- Suppose individuals were randomly assigned across treatment and control groups:

$$\begin{aligned} ATE &= \mathbb{E}(Y_1 - Y_0) \\ &= \mathbb{E}(Y_1|D=1) - \mathbb{E}(Y_0|D=0) \\ &= \mathbb{E}(Y|D=1) - \mathbb{E}(Y|D=0) \end{aligned}$$

- Then the ATE would be directly identified from the data

(as would be the case for the other average treatment effects; they'd all be equal here)

The evaluation problem (Cont'd)

- Direct identification of TE from the data is specific to **mean TE's**
- Why? They only depend on the marginal distributions of Y_0 and Y_1
- Other features of the distribution of TE's depend on the joint distribution of (Y_0, Y_1)
  - e.g. variance, median
- Additional assumptions would be needed for identification of these

The selection problem

- **Major difficulty:** Agents often **choose** to be treated based on characteristics which are related to their potential outcomes
- Canonical model of self-selection is due to Roy (1951)
- Within this framework, selection into treatment is directly based on the TE $Y_1 - Y_0$
- Individuals self-select into treatment iff $Y_1 - Y_0 > 0$
- In this case, $\mathbb{E}(Y_1|D=1) \neq \mathbb{E}(Y_1)$ and $\mathbb{E}(Y_0|D=0) \neq \mathbb{E}(Y_0)$

The selection problem (Continued)

- The ATE cannot be identified directly from the observed average outcomes
- Need to know/assume more about selection rule to identify the TE parameters
- Two alternative approaches: point vs. partial identification
- Tradeoff strength/identifying power of the invoked assumptions

Standard identifying assumptions

Three main alternative assumptions to deal with selection.

1. **Unconfoundedness approach (Matching):** $(Y_0, Y_1) \perp D | X$, where X is a set of observed covariates
2. **IV approach:** $(Y_0, Y_1) \perp Z | X$, where X and Z denote two vectors of covariates affecting the potential outcomes and the treatment status, resp.
3. **Control function approach:** $(Y_0, Y_1) \perp D | X, Z, \nu$ (where ν is an unobserved r.v.), plus some structure on the selection equation.
   Generalization of unconfoundness approach.

(2) and (3) are related in the sense that both hinge on existence of exclusion restrictions

Standard identifying assumptions (Cont'd)

Panel: most popular method to deal with selection is the difference-in-differences approach, which compares the evolution over time in the outcomes of treated vs. untreated individuals:

- $\Delta Y_0 \perp D$ (**parallel trend assumption**), where ΔY denotes the variation in the outcome Y between t_0 and t_1, with the treatment taking place between t_0 and t_1
- Accounts for selection on **time-invariant** characteristics
- One may combine difference-in-differences with matching, which yields identification under weaker conditions (Heckman, Ichimura, Smith, and Todd, 1998)

Matching

- Accounts for selection on observables only
- Main identifying assumption: $(Y_0, Y_1) \perp D|X$
- This is known as the **Conditional Independence Assumption** (CIA), or **Unconfoundedness**
- Conditioning on a set of observed covariates X randomizes treatment D
- Additional assumption: $\mathbb{P}(D = 1|X = x) \in (0, 1)$, for all x in the support of X
- Required to be able to compare the outcomes of treated vs. untreated individuals, for any given value of characteristics $X = x$

Matching (Cont'd)

- Under these assumptions, the ATE is identified:

$$\begin{aligned}\mathbb{E}(Y_1 - Y_0) &= \mathbb{E}(\mathbb{E}(Y_1 - Y_0|X)) \\ &= \mathbb{E}(\mathbb{E}(Y_1|D=1,X)) - \mathbb{E}(\mathbb{E}(Y_0|D=0,X))\end{aligned}$$

- Similar for the other average treatment effect parameters
- However, distributional TE's are not identified without additional restrictions
- Note that the CIA cannot be tested
- The second assumption, on the other hand, is directly testable

IV

The IV approach also deals with selection on unobservables

Key identifying assumptions:

- Exogeneity: $(Y_0, Y_1, (D(z))_z) \perp Z|X$
- Relevance: $\mathbb{P}(D = 1|X, Z)$ is a nondegenerate function of Z given X

Exogenous variation in the instrument Z (conditional on X) generates variation in D

Allows to identify the average treatment effect parameters … under (strong) restrictions on selection into treatment

IV (Cont'd)

Regression representation of the treatment effect model:

$$Y = \alpha + \beta D + U$$

where $\alpha = \mathbb{E}(Y_0)$, $\beta = Y_1 - Y_0$ and $U = Y_0 - \alpha$

- It is useful to consider two important cases:
- Homogeneous treatment effects (β constant)
- Heterogeneous treatment effects, with selection into treatment partly driven by the treatment effects
- Sometimes referred to as a model with **essential heterogeneity** (Heckman, Urzua, and Vytlacil, 2006), aka the **correlated random coefficient model**

IV: homogeneous treatment effects

Unique treatment effect ($ATE = ATT = ATU = \beta$)

- We can apply standard IV method to the previous regression, which identifies the treatment effect $\beta_{\text{IV}} = \frac{Cov(Y,Z)}{Cov(D,Z)}$
- Special case of a binary instrument Z: Wald estimator
- But, assuming homogeneous treatment effects is very restrictive!
- In practice, the effectiveness of social programs tends to vary a lot across individuals

IV: heterogeneous treatment effects

The previous model is a correlated random coefficient model

- Key (negative) result: in general, the instrumental approach does not identify the ATE (nor any standard treatment effect parameters)
- Consider the previous model, where $\overline{\beta}$ is the ATE and $\eta \equiv \beta - \overline{\beta}$. We have:

$$Y = \alpha + \overline{\beta} D + (U + \eta D)$$

- In general, Z is correlated with ηD, and the IV does not identify the ATE $\overline{\beta}$
- The IV approach still works if selection into treatment is not driven by the idiosyncratic gains η, in which case:

$$\begin{aligned} Cov(Z, \eta D) &= \mathbb{E}(Z \eta D) \\ &= \mathbb{E}(ZD\mathbb{E}(\eta | Z, D)) = 0 \end{aligned}$$

IV and Local Average Treatment Effects (LATE)

- However, under an additional **monotonicity** assumption, the IV identifies the LATE (Imbens and Angrist, 1994)

- Monotonicity assumption (Z binary): $D_{Z=1} \geq D_{Z=0}$

- All individuals respond to a change in the instrument Z in the same way (Only sufficient, (de Chaisemartin, 2017))

- Under the previous assumptions, we have:

$$\widehat{\beta}_{IV} \xrightarrow{p} LATE = \mathbb{E}(Y_1 - Y_0 | D_0 = 0, D_1 = 1)$$

- Interpretation: ATE for the subset of individuals who would change their D following a change in Z (**compliers**)

IV and Local Average Treatment Effects (Cont'd)

- In general, if heterogeneous treatment effects, $LATE \neq ATE$ (or ATT)
- Remark: when Z takes more than two values, IV identifies a weighted average of the LATEs
  - corresponding to a shift in Z from z to z'
  - for all z and z' in the support of Z such that $\mathbb{P}(D = 1|Z = z) < \mathbb{P}(D = 1|Z = z')$
- See recent survey by Mogstad and Torgovitsky (2018)
  - discusses extrapolation of IV/LATE estimates to policy-relevant parameters

Control function

- Key idea: use an explicit model of the relationship between D and (Y_0, Y_1) to correct for selection bias

- Main assumption: there exists a variable ν such that the following conditional independence condition holds:

$$(Y_0, Y_1) \perp D \,|\, X, Z, \nu$$

- And some structure is imposed on the selection equation

Control function (Cont'd)

- This is a fairly general framework
- Encompasses many treatment effects models
  - perfect proxy for ν available to the econometrician
  - ν observed with error as is the case for factor models
  - …
- Important special case: seminal selection model of Heckman (1979)

Control function (Cont'd)

- Assume a threshold crossing model for selection into treatment:
  $D = 1\left[g(X, Z) - \nu > 0\right]$
  Discrete choice model: binary logit, etc.
- And additively separable potential outcomes: $Y_k = \psi_k(X) + \varepsilon_k$, with
  $(\nu, \varepsilon_0, \varepsilon_1) \perp (X, Z)$
- $\textbf{ATE}(X) = \psi_1(X) - \psi_0(X)$
  Z needs to meet the exclusion restriction condition.
- $\psi_1(\cdot)$ is identified from:

$$\mathbb{E}(Y_1|X, Z, D = 1) = \psi_1(X) + \mathbb{E}(\varepsilon_1|X, Z, \nu < g(X, Z))$$

Control function (Cont'd)

- Under regularity conditions (absolute continuity and full support) on the distribution of ν: $g(X,Z) = F_\nu^{-1}(\mathbb{P}(D=1|X,Z))$
- Thus, there exists a function $K_1(\cdot)$ (control function) such that:

$$\mathbb{E}(Y_1|X,Z,D=1) = \psi_1(X) + K_1(\mathbb{P}(D=1|X,Z))$$

- This identifies $\psi_1(\cdot)$ **up to location** as long as X and $\mathbb{P}(D=1|X,Z)$ can vary in a sufficiently independent way

  Up to location implies up to an intercept.

  - measurable separability condition, (Florens, Mouchart, and Rolin, 1990; Florens, Heckman, Meghir, and Vytlacil, 2008)

- But, the intercept is crucial to recover the treatment effect parameters!

Control function (Cont'd)

- Solution: address the selection problem **at the limit**, using individuals with treatment probability $\mathbb{P}(D = 1|X, Z)$ approaching 1 (0 for ψ_0) to identify the intercept (Heckman, 1990)

- For these individuals, $K_1(\mathbb{P}(D = 1|X, Z)) \longrightarrow \mathbf{0}$, and therefore: $\mathbb{E}(Y_1|X, Z, D = 1) = \psi_1(X)$, which identifies the intercept.

- Key identifying assumption: $\textbf{Support}\ (\mathbb{P}(D = 1|X, Z)) = [\mathbf{0}, \mathbf{1}]$

- Note that this is quite restrictive!

Control function (Cont'd)

Also identifies the treatment effect on the treated and untreated since:

$$\mathbb{E}(Y_1 - Y_0|X, Z, D = 1) = \mathbb{E}(Y|X, Z, D = 1) - \psi_0(X) - \mathbb{E}(\varepsilon_0|X, Z, D = 1)$$

And it follows from the law of iterated expectations that (denoting by $p = \mathbb{P}(D = 1|X, Z)$):

$$\mathbb{E}(\varepsilon_0|X, Z, D = 1) = -\frac{1 - p}{p} K_0(p)$$

Control function (Cont'd)

- Consistent estimators for (ψ_0, ψ_1) up to location can be obtained
  - e.g. semiparametric regression with linear outcomes (Robinson, 1988)
- Andrews and Schafgans (1998) provide a consistent estimator for the intercept
  - smoothed version of Heckman (1990)

Further reading

- Heckman and Leamer (2007)
- Abadie and Cattaneo (2018)
- Imbens (2015)
- Athey and Imbens (2017)
- Deaton (2010)
- Heckman (2010)
- Imbens and Wooldridge (2009)
- Imbens (2004)
- Mogstad and Torgovitsky (2018)

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