

Work sheet 2

Sajeev G P

1/14/2019

```
# Symbolic Expressions and Simplification
```

```
x, y, z = var('x, y, z') ; q = x*y + y*z + z*x  
bool(q(x=y, y=z, z=x) == q), bool(q(z=y)(y=x) == 3*x^2)  
(True, True)
```

```
#To replace an expression more complex than a single variable, the \  
substitute
```

```
#method is available:
```

```
y, z = var('y, z'); f = x^3 + y^2 + z  
f.substitute(x^3 == y^2, z==1)  
2*y^2 + 1
```

```
#Transforming Expressions
```

```
x, y = SR.var('x,y')  
f(x)=(2*x+1)^3
```

```
f.expand()
```

```
x |--> 8*x^3 + 12*x^2 + 6*x + 1
```

```
y = var('y'); u = sin(x) + x*cos(y)
```

```
v = u.function(x, y); v
```

```
(x, y) |--> x*cos(y) + sin(x)
```

```
w(x, y) = u; w
```

```
(x, y) |--> x*cos(y) + sin(x)
```

```
p = (x+y)*(x+1)^2
```

```
p2 = p.expand(); p2
```

```
x^3 + x^2*y + 2*x^2 + 2*x*y + x + y
```

```
#collect method groups terms together according to the powers of a \  
given variable:
```

```
p2.collect(x)
```

```
x^3 + x^2*(y + 2) + x*(2*y + 1) + y
```

```
((x+y+sin(x))^2).expand().collect(sin(x))
```

```
x^2 + 2*x*y + y^2 + 2*(x + y)*sin(x) + sin(x)^2
```

```
#Usual Mathematical Functions
```

```
(x^x/x).simplify()  
x^(x - 1)
```

```
f = cos(x)^6 + sin(x)^6 + 3 * sin(x)^2 * cos(x)^2  
f.simplify_trig()  
1
```

```
#Expressions containing factorials can also be simplified:  
n = var('n'); f = factorial(n+1)/factorial(n)  
f.simplify_factorial()  
n + 1
```

```
# Solving Equations  
z, phi = var('z, phi')  
eq = z**2 - 2/cos(phi)*z + 5/cos(phi)**2 - 4 == 0; eq  
z^2 - 2*z/cos(phi) + 5/cos(phi)^2 - 4 == 0
```

```
#We can extract the left-hand (resp. right-hand) side with the lhs (\  
    resp. rhs)  
eq.lhs()  
z^2 - 2*z/cos(phi) + 5/cos(phi)^2 - 4  
  
eq.rhs()  
0
```

```
#then solve it for z with solve:  
solve(eq, z)  
[z == -(2*sqrt(cos(phi)^2 - 1) - 1)/cos(phi), z == (2*sqrt(cos(phi)^2 - 1) + 1)/cos(phi)]
```

```
#solve the equation x^2+5x+6=0  
x=var('x')  
eq=x^2+5*x+6==0  
solve(eq, x)  
[x == -3, x == -2]
```

```
#The roots of the equation can be returned as an object of type \  
    dictionary  
  
solve(eq, x, solution_dict=True)  
[{x: -3}, {x: -2}]
```

```
# Question
```

```
# Solve the generic quadratic equation  $ax^2 + bx + c = 0$  for following \
    a, b and c
# i) 1, 25, 150   ii) 3, -10, -35
```

```
# Solving system of equations
```

```
x,y=var('x', 'y')
eq1= x+y==6
eq2= 2*x-4*y==0
solve([eq1,eq2],x,y)
[[x == 4, y == 2]]
```

```
#Question
```

```
# Solve  $\cos(x) \sin(x) = 1/2$ ,  $x+y = 0$ 
```

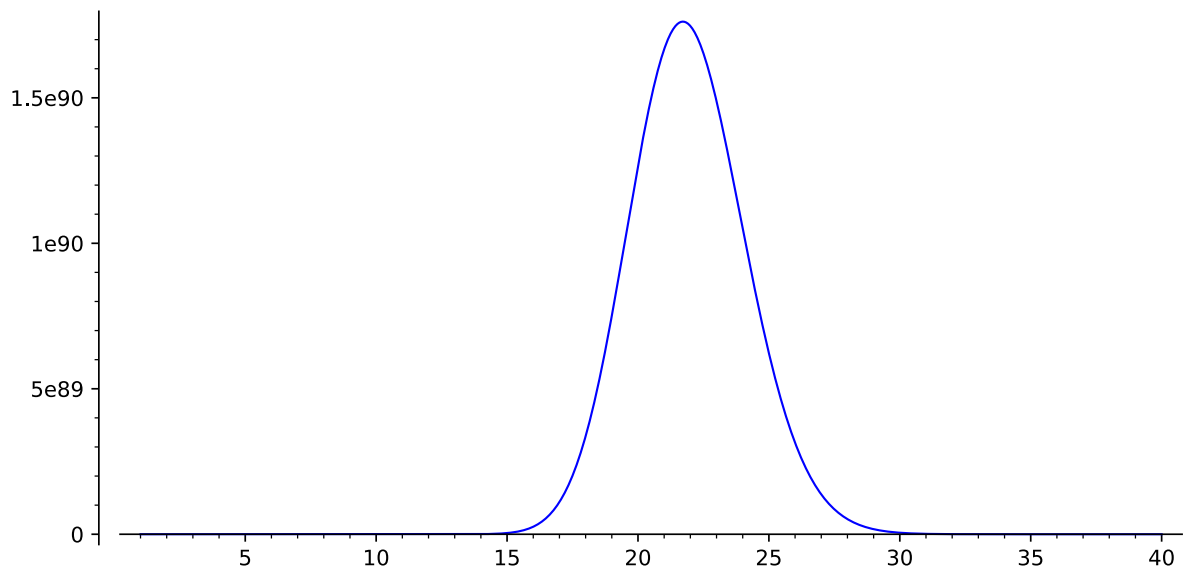
```
#Sequences
```

```
u(n) = n^100 / 100^n
u(1.);u(2.);u(3.);u(4.);u(5.);u(6.);u(7.);u(8.);u(9.);u(10.)
0.010000000000000000
1.26765060022823e26
5.15377520732011e41
1.60693804425899e52
7.88860905221012e59
6.53318623500071e65
3.23447650962476e70
2.03703597633449e74
2.65613988875875e77
1.00000000000000e80
```

```
#To get an idea of the variation of the sequence, we can draw the \
    graph of
```

```
#the function
```

```
plot(u(x), x, 1, 40)
```



```
#Derivative diff(f(x), x)
#n-th derivative diff(f(x), x, n)
#Antiderivative integrate(f(x), x)
#Numerical integration integral_numerical(f(x), a, b)
#Symbolic summation sum(f(i), i, imin, imax)
#Limit limit(f(x), x=a)
#Taylor expansion taylor(f(x), x, a, n)
#Power series expansion f.series(x==a, n)
#Graph of a function plot(f(x), x, a, b)
```

```
diff(1 + x + x^2, x)
2*x + 1
```

```
numerical_integral(1 + x + x^2, 0, 3)[0] # [1] gives error bound
16.500000000000004
```

```
diff(sin(x^2), x)
2*x*cos(x^2)
```

```
sin(x).integral(x, 0, pi/2)
1
```

```
integrate(1/(1+x^2), x, -infinity, infinity)
pi
```