

Consider initially the two drunkards start from  $x=0$  (origin). Each drunkard can move left or right with equal probability  $= 1/2$ .

Consider at any time ~~time~~  $t$  the position or the displacement from origin of the drunkards.

Now, the position of the drunkard is given by displacement  $P$

$$P = \text{No. of steps towards right} \\ - \text{No. of steps towards left.}$$

Let the drunkard take  $r$  steps right.

$$\begin{aligned} \Rightarrow P &= r - (N - r) \\ &= r - N + r \\ &= 2r - N \end{aligned}$$

Probability of taking  $r$  steps towards right

$$\begin{aligned} P(x=r) &= {}^N C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{N-r} \\ &= {}^N C_r \left(\frac{1}{2}\right)^N \end{aligned}$$

$$P = 2r - N \Rightarrow r = \frac{P+N}{2}$$

Now

$$\begin{aligned} \Pr[X = (P+N/2)] &= \frac{N!}{\left(\frac{P+N}{2}\right)! \left(N - \left(\frac{P+N}{2}\right)\right)!} \left(\frac{1}{2}\right)^N \\ &= \frac{N!}{\left(\frac{P+N}{2}\right)! \left(\frac{N-1}{2}\right)! \times 2^N} \end{aligned}$$

If the two drunkards meet after  $N$  steps relative displacement  $P=0$ .

$$\Rightarrow \Pr[X = N/2] = \frac{N!}{(N/2)! (N/2)! \cdot 2^N}$$

For two drunkards total steps taken is  $2N$

Probability that they meet each other at same position =  $\frac{(2N)!}{2^{2N} (N!)^2}$