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RM 294 Optimization Project 3

Non-Linear Programming

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Executive Summary

This report examines two extensions to the standard newsvendor model (NVM) to improve production and pricing decisions for our publishing company. While the standard NVM is a foundational tool for determining optimal print quantities under uncertain demand, it does not account for important factors like operational costs from overproduction and underproduction or the effect of price on demand. Extension 1 improves upon the NVM by including rushed printing and disposal costs. In contrast, Extension 2 takes it further by incorporating price-dependent demand, allowing both price and production quantity to be optimized simultaneously.

Our analysis found that Extension 2 is the better model, achieving a maximum profit of $234.42 compared to $231.48 for Extension 1. While the profit difference is slight, Extension 2's ability to adjust pricing dynamically provides more flexibility and responsiveness to market conditions. Additionally, we observed a clear logarithmic-like relationship between price and quantity, where reducing price leads to diminishing increases in production, emphasizing the need for careful pricing strategies. Extension 2's sensitivity to price adjustments and consistent results across various scenarios make it a more robust and practical tool. For these reasons, we recommend adopting Extension 2 to replace the standard NVM, offering more accurate and effective decision-making for our publishing operations.

Introduction & Business Problem

One of the most significant operational challenges in the publishing industry is deciding how many copies of a publication to print. Publications must balance the risk of printing too many, leading to unsold copies and waste, against the risk of printing too few, resulting in unmet demand and lost revenue. This decision is particularly difficult due to the inherent uncertainty in predicting customer demand.

Currently, our publishing company utilizes the standard NVM to guide printing decisions. The NVM is a well-established operations management tool designed to maximize expected profit under uncertain demand. The model determines the optimal quantity of a product to produce by balancing the cost of overproduction (e.g., printing excess copies) against the cost of underproduction (e.g., missed sales opportunities). The objective function for this model is:

$$\max_q E[p\, \min(q, D) - qc]$$

where:

- q: quantity to print,
- p: price per unit sold,
- D: random demand,
- c: cost to produce each unit.

If demand (D) is greater than the quantity printed (q), all printed copies are sold. If demand is lower than q, unsold copies incur a loss. To estimate demand, the model relies on historical data to calculate the expected demand distribution.

While the newsvendor model provides a straightforward framework, it has limitations. Notably, it does not account for:

1. **Cost Variations for Rushed Printing and Disposal**: The current model does not incorporate additional costs incurred if demand exceeds supply (e.g., rushed printing) or costs for managing surplus copies (e.g., disposal or recycling).
2. **Demand Sensitivity to Price**: The NVM assumes demand is static and independent of price, whereas real-world demand often varies with changes in pricing strategies.

To address these limitations, our team is investigating two extensions of the standard newsvendor model:

1. **Extension 1**: Introduces rushed printing costs for unmet demand and disposal costs for surplus production.
2. **Extension 2**: Incorporates a price-dependent demand function, allowing for simultaneous optimization of pricing and production decisions.

Both extensions aim to refine our approach by accounting for additional real-world complexities. Through nonlinear programming techniques and optimization tools like Gurobi, we evaluate whether these extensions can deliver superior profit outcomes compared to the current model.

Method

To address the limitations of the standard newsvendor model (NVM), we tested two extensions aimed at improving its accuracy and applicability in the publishing field. The first extension incorporates additional costs for rushed printing and disposal, while the second integrates demand as a function of price. These modifications refine the decision-making framework by accounting for real-world complexities that the standard model overlooks. Below, we describe the methodologies for each extension, step by step, and how they improve upon the standard NVM.

Extension 1: Incorporating Rushed Printing and Disposal Costs

The standard NVM assumes a static cost structure and does not account for additional costs associated with overproduction or underproduction. However, in reality, unsold copies of a publication incur disposal costs, and unanticipated demand exceeding the initial print run necessitates rushed printing at a higher cost. Extension 1 introduces these cost components, making the model more realistic and reflective of actual business operations. By integrating rushed printing and disposal costs into the objective function, this extension ensures that the production quantity optimizes profit while minimizing potential waste and additional expenses.

We began by generating demand scenarios (D_i) to capture the stochastic nature of customer demand. Using historical price-demand data, we fit a linear regression model to establish the deterministic relationship between price and demand, extracting coefficients β_0 and β_1. The residuals from this regression were used to simulate variability around the predicted demand. This step was implemented as follows:

```
# Fit linear regression
coefficients = np.polyfit(prices, demands, 1)
beta_1, beta_0 = coefficients
```

```
# Calculate predicted demands
predicted_demands = beta_0 + beta_1 * prices

# Calculate residuals
residuals = demands - predicted_demands
```

```
# Fixed price
p_fixed = 1

# Compute base demand at p = 1
base_demand = beta_0 + beta_1 * p_fixed

# Generate demand data
D_i = base_demand + residuals

# Ensure non-negative demand
D_i = np.maximum(D_i, 0)
```

Next, we set the parameters for production cost ($c = 0.5$), rushed printing cost ($g = 0.75$), and disposal cost ($t = 0.15$) and formulated the optimization problem as a linear program (LP). The objective function maximized average profit by balancing production costs, rushed printing costs, disposal costs, and sales revenue. We used Gurobi to define the decision variable (q) for production quantity and auxiliary variables (s_i^+ and s_i^-) to capture overproduction and underproduction.

```
# Create a Gurobi model
model = gp.Model("Newsvendor_LP")
model.setParam('OutputFlag', 0)  # Suppress output

# Decision variable: q
q = model.addVar(lb=0, vtype=GRB.CONTINUOUS, name="q")

# Auxiliary variables: s_i^+, s_i^-
s_plus = model.addVars(n, lb=0, vtype=GRB.CONTINUOUS, name="s_plus")
s_minus = model.addVars(n, lb=0, vtype=GRB.CONTINUOUS, name="s_minus")

# Objective function
profit = gp.quicksum((p * D_i[i] - c * q - g * s_plus[i] - t * s_minus[i]) for i in range(n)) / n
model.setObjective(profit, GRB.MAXIMIZE)

# Constraints
for i in range(n):
    model.addConstr(s_plus[i] >= D_i[i] - q, name=f"s_plus_constr_{i}")
    model.addConstr(s_minus[i] >= q - D_i[i], name=f"s_minus_constr_{i}")
```

Finally, we solved the optimization problem using Gurobi, which provided the optimal production quantity while accounting for rushed printing and disposal costs.

Extension 2: Incorporating Price-Dependent Demand

The standard NVM assumes that demand is independent of price, which limits its applicability in real-world scenarios where price strongly influences customer behavior. Extension 2 improves upon this by incorporating a price-demand relationship into the model, allowing us to optimize both production quantity (q) and price (p) simultaneously. By introducing this flexibility, the extension enables dynamic pricing strategies that align production and pricing decisions to maximize profit.

To model price-dependent demand, we extended the linear regression framework to include price as a variable. Demand (D_i) was expressed as a function of price (p), with the equation

$$D_i = \beta_0 + \beta_1 p + \epsilon_i,$$

where ϵ_i represents the residual variability. The parameters β_0 and β_1 were obtained from the regression analysis in the previous step, while the residuals were used to simulate variability.

We then formulated the optimization problem as a Quadratically Constrained Program (QCP), as the objective function now included the product $p \cdot D_i$, which is nonlinear in p. Using Gurobi, we defined the decision variables for price (p) and production quantity (q), as well as auxiliary variables (s_i^+ and s_i^-) for overproduction and underproduction. The setup is detailed below:

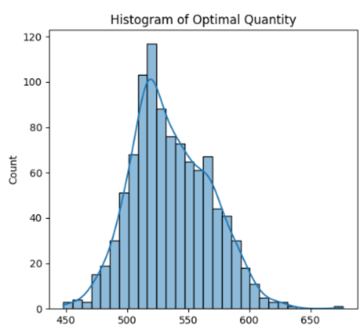
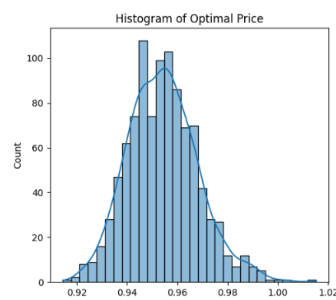
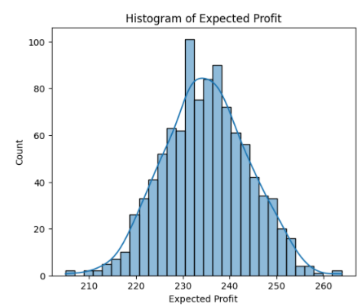
```
# Create a Gurobi model
model = gp.Model("Newsvendor_QCP")
model.setParam('OutputFlag', 0)

# Decision variables: p and q
p = model.addVar(lb=0, vtype=GRB.CONTINUOUS, name="p")
q = model.addVar(lb=0, vtype=GRB.CONTINUOUS, name="q")

# Auxiliary variables: s_i^+, s_i^-
s_plus = model.addVars(n, lb=0, vtype=GRB.CONTINUOUS, name="s_plus")
s_minus = model.addVars(n, lb=0, vtype=GRB.CONTINUOUS, name="s_minus")

# Precompute constants
epsilon_i = data['residuals'].values

# Objective function
profit_terms = []
for i in range(n):
    D_i = beta_0 + beta_1 * p + epsilon_i[i]
    profit_term = (p * D_i - c * q - g * s_plus[i] - t * s_minus[i]) / n
    profit_terms.append(profit_term)

# Set objective
model.setObjective(gp.quicksum(profit_terms), GRB.MAXIMIZE)

# Constraints
for i in range(n):
    D_i = beta_0 + beta_1 * p + epsilon_i[i]
    model.addConstr(s_plus[i] >= D_i - q, name=f"s_plus_constr_{i}")
```

Finally, we solved the QCP using Gurobi to determine the optimal price and production quantity simultaneously.

By addressing the limitations of the standard NVM, these extensions provide a more robust and adaptable framework for production and pricing decisions, aligning better with the realities of the publishing industry. Extension 1 improves cost realism, while Extension 2 introduces the ability to leverage dynamic pricing for profit maximization.

Results & Analysis

Building on the methodologies described in the previous section, we evaluated two extensions to the standard newsvendor model: one that accounts for rushed printing and disposal costs (Extension 1), and another that incorporates price-dependent demand (Extension 2). Below, we present the results for Extensions 1 and 2.

	Optimal Production Quantity (*q*)	**Optimal Price (*p*)**	**Maximum Profit**
Extension 1: Incorporating Rushed Printing and Disposal Costs	471.87	1.00 (fixed)	$231.48
Extension 2: Incorporating Price-Dependent Demand			
	535.29	0.95	$234.42

Extension 1: Incorporating Rushed Printing and Disposal Costs

This extension improves upon the standard NVM by explicitly modeling additional costs that arise in real-world scenarios. By introducing parameters for rushed printing (g) and disposal (t), Extension 1 ensures that the optimal production quantity reflects actual cost trade-offs. This results in higher profitability compared to the standard model, as it avoids the financial penalties of overproduction and underproduction. Additionally, the linear programming approach makes the model computationally efficient.

However, Extension 1 does not allow for dynamic pricing. The price is assumed to be fixed, which limits the ability to respond to market conditions. This constraint makes the model less flexible than Extension 2, especially in scenarios where pricing plays a critical role in influencing demand.

Extension 2: Incorporating Price-Dependent Demand

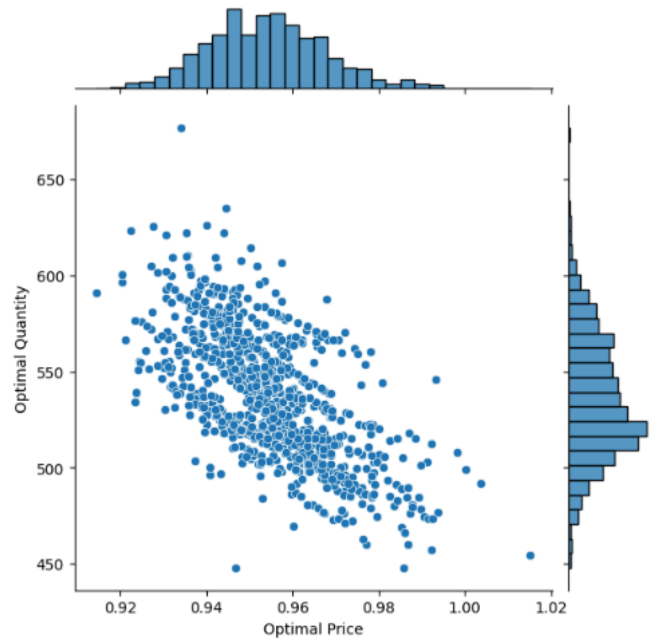
Extension 2 further refines the decision-making process by introducing price sensitivity into the demand model. This allows for simultaneous optimization of production quantity (q) and price (p), enabling dynamic pricing strategies that adapt to market conditions. The inclusion of price-dependent demand increases profitability compared to Extension 1, as shown by the higher maximum profit of $234.42.

However, this increased flexibility comes at the cost of greater complexity. The model requires fitting a regression to the price-demand relationship and solving a quadratically constrained programming problem, which is computationally more demanding. Additionally, it assumes an accurate estimation of demand variability, which might require more extensive data collection.

Conclusion & Recommendation

In this project, we tested two extensions of the standard newsvendor model (NVM) to determine which would better improve our production and pricing decisions. Extension 1 modified the NVM by including rushed printing and disposal costs, allowing it to account for the extra expenses associated with overproduction and underproduction. Extension 2 went further by incorporating price-dependent demand, making it possible to optimize both price and production quantity. While both extensions provide improvements, Extension 2 stands out as the more effective option for addressing real-world challenges in our operations.

Between the two models, Extension 2 achieved a slightly higher maximum profit of $234.42 compared to Extension 1’s $231.48. Although the profit difference is small, the benefits of Extension 2 outweigh those of Extension 1. By optimizing price alongside production quantity, Extension 2 provides more flexibility and the ability to respond to changing market conditions. This added adaptability makes Extension 2 a better fit for maximizing profitability in the publishing industry, where demand can be highly price-sensitive.



An important observation from Extension 2 is the logarithmic-like relationship between optimal price and production quantity. As the price decreases, the quantity increases, but the rate of increase slows over time. This means that while lowering the price boosts demand, the additional gains in production eventually become smaller. This insight shows the importance of carefully balancing price and quantity to avoid unnecessary production costs. The clustering of solutions around a price of $0.95 and a quantity of 535 units also highlights the model's robustness, as it consistently finds optimal solutions even with demand variability.

Another advantage of Extension 2 is its higher sensitivity to price adjustments compared to production quantity. Minor price changes have a more significant effect on profit, showing that price is a critical factor in optimizing overall revenue. This sensitivity allows the model to fine-tune pricing decisions precisely, making it an excellent tool for boosting profitability. Combined with its ability to adjust to market conditions, Extension 2 clearly offers more practical value for real-world decision-making.

In summary, while Extension 1 provides a more straightforward and improved alternative to the standard NVM, the benefits of Extension 2 make it the better choice. Its ability to optimize both price and quantity while remaining robust and sensitive to changes ensures it can effectively handle the challenges of demand uncertainty. For these reasons, Extension 2 is the recommended model to replace the standard NVM, offering a stronger foundation for increasing profitability and improving decision-making in our publishing operations.

Appendix

We used ChatGPT to help with the coding portion of this project.