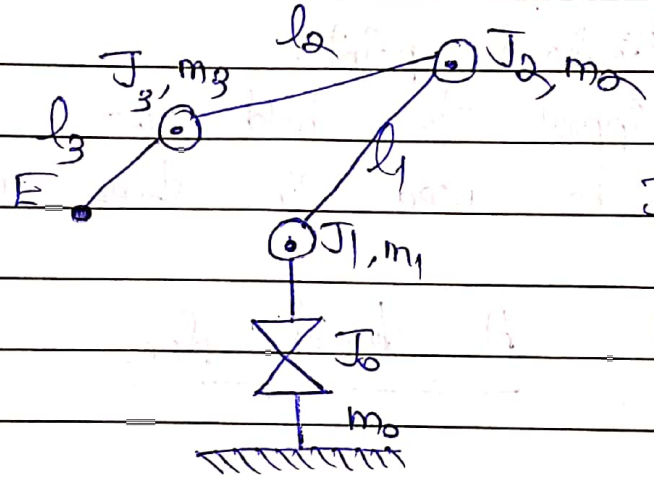
→ D-H Convention



J_0, J_1 : shoulder

J_2 : elbow

J_3 : wrist

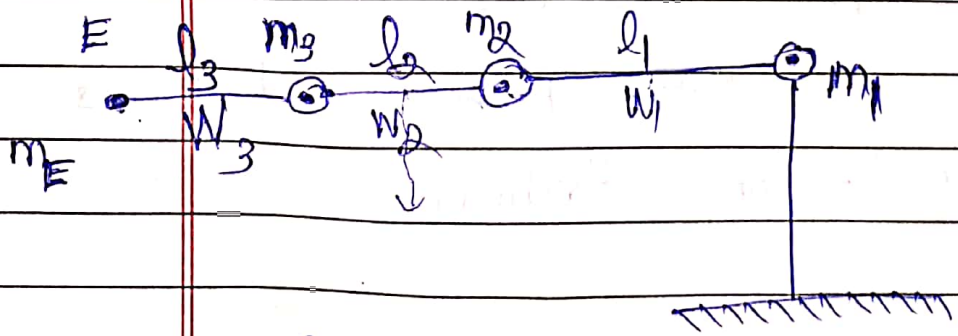
E : end effector

J : joints

m : motors

Analysis 1 : Ignore J_0 and m_0.

Keep a stretched out position for max-torque calculations.



(Worst case scenario)

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notations

Joints/motors	mass	links	weight
m_0	m_0		
m_1	m_1	l_1	W_1
m_2	m_2	l_2	W_2
m_3	m_3	l_3	W_3
E (end-effector including the object to be picked up)	m_E		

Assumption: uniform mass distribution

Torque about m_1 (maximum):

$$\tau_{m_1} = \left(\frac{W_1 l_1}{2}\right) + (m_2 g l_1) + W_2\left(l_1 + \frac{l_2}{2}\right) + m_3 g (l_1 + l_2) + W_3\left(l_1 + l_2 + \frac{l_3}{2}\right) + m_E (l_1 + l_2 + l_3)$$

Torque about m_2:

$$\left(\frac{W_2 l_2}{2}\right) + (m_3 g l_2) + W_3\left(l_2 + \frac{l_3}{2}\right) + m_E (l_2 + l_3)$$

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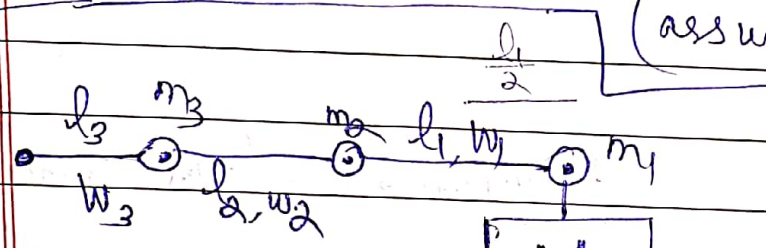
Torque about m_3 :

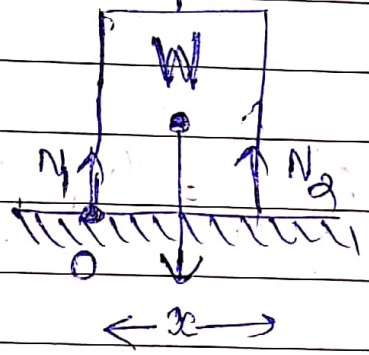
$$\left(\frac{W_3 l_3}{2}\right) + \left(m_E l_3\right)$$

If τ_{m_1} is consistent with servo motor, then τ_{m_2} & τ_{m_3} will become consistent automatically. (assuming same servos are used)



TOPPLING

Assumption : m_1 is kept at centre of base.



No sliding

$$W = N_1 + N_2$$

Torque about O :

$$\left(m_1 g + W\right)\left(\frac{x}{2}\right) = (N_2 x) + W_1\left(\frac{l_1}{2} - \frac{x}{2}\right)$$

$$+ m_2 g\left(l_1 - \frac{x}{2}\right) + W_2\left(\frac{l_2}{2} + l_1 - \frac{x}{2}\right)$$

$$+ m_3 g\left(l_2 + l_1 - \frac{x}{2}\right) + W_3\left(\frac{l_3}{2} + l_2 + l_1 - \frac{x}{2}\right)$$

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$$+ m_E\left(l_3 + l_2 + l_1 - \frac{x}{2}\right)$$

$$N_2 = \left(\frac{m_1 g + W}{2}\right) - \left(\frac{T}{x}\right)$$

Arm will topple if $N_2 \leq 0$.

For $N_2 \geq 0 \Rightarrow$ arm is stable.

$$\Rightarrow \frac{m_1 g + W}{2} - \frac{T}{x} \geq 0$$

$$\frac{m_1 g + W}{2} \geq \frac{T}{x}$$

$$m_1 g + W \geq \frac{2T}{x}$$

$$W \geq \frac{2T}{x} - m_1 g$$

(Condition for robot to not topple)

W should also include the weight of J_0/m_0 - base motor.

Base should be quite heavy and broader. (x value is high)

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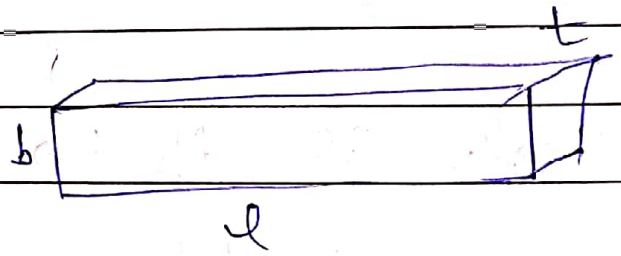
Assume that links are cuboidal.



All links will have same breadth and thickness, only length will differ.

Assume that density of the material is ρ.

W_1 = weight of link 1 = $\rho l_1 b t g$

W_2 = " " 2 = $\rho l_2 b t g$

W_3 = " " 3 = $\rho l_3 b t g$

Use these in the torque equations.

The worst-case scenario is not when the arm is static, but when it is

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accelerated from '0' to some angular velocity. In this case the moment of inertia should also be included. For approximation a factor of safety value '2' is used.

$$I_{m_1}' = 2 I_{m_1}$$

$$I_{m_2}' = 2 I_{m_2}$$

$$I_{m_3}' = 2 I_{m_3}$$

$\Rightarrow$ worst case scenario.

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