MBD assignment 1

ME20B072
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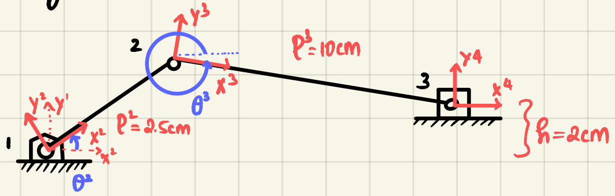


Driving constraint

$\theta^2 = 40\pi t$

$R^1_x = 0 \quad R^1_y = 0 \quad \theta^1 = 0$

at revolute joint 1

$$R^1 = R^2 + A(\bar{o}) \Rightarrow R^2_x = 0 \quad R^2_y = 0$$

at revolute joint 2

$$R^2 + A(\bar{u}) = R^3 \Rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \cos\theta^2 & -\sin\theta^2 \\ \sin\theta^2 & \cos\theta^2 \end{bmatrix} \begin{bmatrix} \rho^2 \\ 0 \end{bmatrix} = R^3$$

$$R^3_x = \rho^2\cos\theta^2 \qquad R^3_y = \rho^2\sin\theta^2$$

at revolute joint 3

$$R^3 + A'(\bar{u}') = R^4 \Rightarrow \begin{bmatrix} \rho^2\cos\theta^2 \\ \rho^2\sin\theta^2 \end{bmatrix} + \begin{bmatrix} \cos\theta^3 & -\sin\theta^3 \\ \sin\theta^3 & \cos\theta^3 \end{bmatrix} \begin{bmatrix} \rho^3 \\ 0 \end{bmatrix} = R^4$$

$$R^4_x = \rho^2\cos\theta^2 + \rho^3\cos\theta^3 \qquad R^4_y = \rho^2\sin\theta^2 + \rho^3\sin\theta^3$$

at prismatic joint

$$R^4_y = h \quad \text{and} \quad \theta^4 = 0$$

Constraints with numerical values :

$R^2_x = 0 \qquad R^2_y = 0 \qquad \theta^2 = 40\pi t$

$R^3_x = 2.5\cos\theta^2 \qquad R^3_y = 2.5\sin\theta^2$

$R^4_x = 2.5\cos\theta^2 + 10\cos\theta^3 \qquad R^4_y = 2.5\sin\theta^2 + 10\sin\theta^3 \qquad \theta^4 = 0$

$R^4_y = 2$

(c) $\quad 2 = 2.5\sin\theta^2 + 10\sin\theta^3$

$$\therefore\ 2.5\cos\theta^2 \underbrace{\dot{\theta}^2}_{40\pi} + 10\cos\theta^3\dot{\theta}^3 = 0 \Rightarrow \dot{\theta}^3 = -\frac{\dot{\theta}^2}{4}\frac{\cos\theta^2}{\cos\theta^3} = -10\pi\frac{\cos\theta^2}{\cos\theta^3}$$

$$\dot{R}^4_x = -(2.5\sin\theta^2\dot{\theta}^2 + 10\sin\theta^3\dot{\theta}^3) \Rightarrow \dot{R}^4_x = -\frac{100\pi}{\cos\theta^3}\left(\sin\theta^2\cos\theta^3 + \cos\theta^2\sin\theta^3\right)$$

$$= -2.5\frac{\sin(\theta^2+\theta^3)}{\cos(\theta^3)}\dot{\theta}^2$$

$$= -100\pi\frac{\sin(\theta^2+\theta^3)}{\cos(\theta^3)}$$

(d) $\ddot{\theta}_3 = -10\pi\left(\frac{-\sin\theta^2(\dot{\theta}^2)}{\cos\theta^3} + \cos\theta^2\sec\theta^3\tan\theta^3(\dot{\theta}^3)\right)$

$$-10\pi\left(\frac{-\sin\theta^2(\dot{\theta}^2)}{\cos\theta^3} + \cos\theta^2\sec\theta^3\tan\theta^3\left(-10\pi\frac{\cos\theta^2}{\cos\theta^3}\right)\right)$$

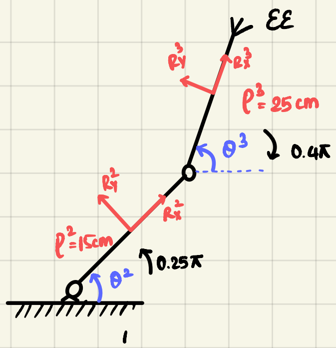
$$100\pi^2\left(4\sin\theta^2\sec\theta^3 + \cos^2\theta^2\sec^2\theta^3\tan\theta^3\right) \Leftarrow \qquad \theta^3 = \sin^{-1}\left(\frac{2-2.5\sin\theta^2}{10}\right)$$

$$\ddot{R}_x^4 = -100\pi\left(\sec\theta^3\cos(\theta^2+\theta^3)(\dot{\theta}^2+\dot{\theta}^3) + \sec\theta^3\tan\theta^3\cos(\theta^2+\theta^3)\dot{\theta}^3\right)$$

$$= -100\pi\left(\sec\theta^3\cos(\theta^2+\theta^3)\left(40\pi - 10\pi\frac{\cos\theta^2}{\cos\theta^3}\right) - 10\pi\sec^2\theta^3\tan\theta^3\cos(\theta^2+\theta^3)\cos\theta^2\right)$$

$$= -1000\pi^2\left(\sec^2\theta^3\cos(\theta^2+\theta^3)(4\cos\theta^3 - \cos\theta^2) - \sec^2\theta^3\tan\theta^3\cos(\theta^2+\theta^3)\cos\theta^2\right)$$

2



assuming that frames 2 & 3 are present at the center of bodies 2 & 3

$R_x^1 = C_1 \qquad R_y^1 = C_2 \qquad \theta^1 = C_3$

$C_1, C_2, C_3 \rightarrow$ constants

for easier computation we take $C_1 = C_2 = C_3 = 0$

$$R^1 = R^2 + A(\bar{u})$$

$$\begin{bmatrix}0\\0\end{bmatrix} = \begin{bmatrix}R_x^2\\R_y^2\end{bmatrix} + \begin{bmatrix}\cos\theta^2 & -\sin\theta^2\\ \sin\theta^2 & \cos\theta^2\end{bmatrix}\begin{bmatrix}-l^2/2\\0\end{bmatrix}$$

$R_x^2 = \frac{l^2}{2}\cos\theta^2 \qquad R_y^2 = \frac{l^2}{2}\sin\theta^2$

$\theta^2 = \frac{\pi}{6} + 0.25\pi t$

driving constraints

$\theta^3 = \frac{\pi}{3} - 0.4\pi t$

$$R^2 + A\bar{u}' = R^3 + A'\bar{u}''$$

$$\begin{bmatrix}l^2/2\cos\theta^2\\ l^2/2\sin\theta^2\end{bmatrix} + \begin{bmatrix}\cos\theta^2 & -\sin\theta^2\\ \sin\theta^2 & \cos\theta^2\end{bmatrix}\begin{bmatrix}l^2/2\\0\end{bmatrix} = \begin{bmatrix}R_x^3\\R_y^3\end{bmatrix} + \begin{bmatrix}\cos\theta^3 & -\sin\theta^3\\ \sin\theta^3 & \cos\theta^3\end{bmatrix}\begin{bmatrix}-l^3/2\\0\end{bmatrix}$$

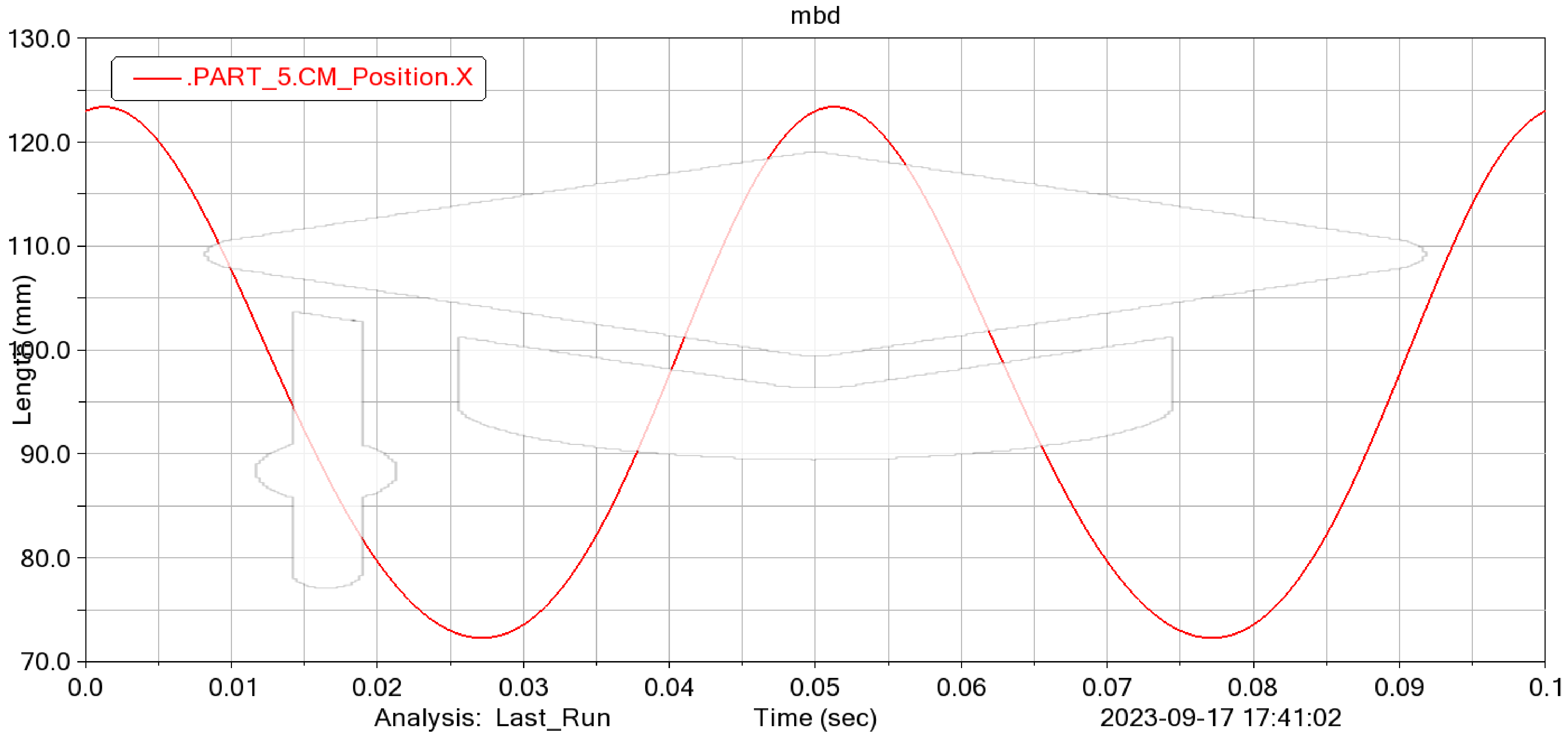
$R_x^3 = l^2\cos\theta^2 + \frac{l^3}{2}\cos\theta^3 \qquad R_y^3 = l^2\sin\theta^2 + \frac{l^3}{2}\sin\theta^3$

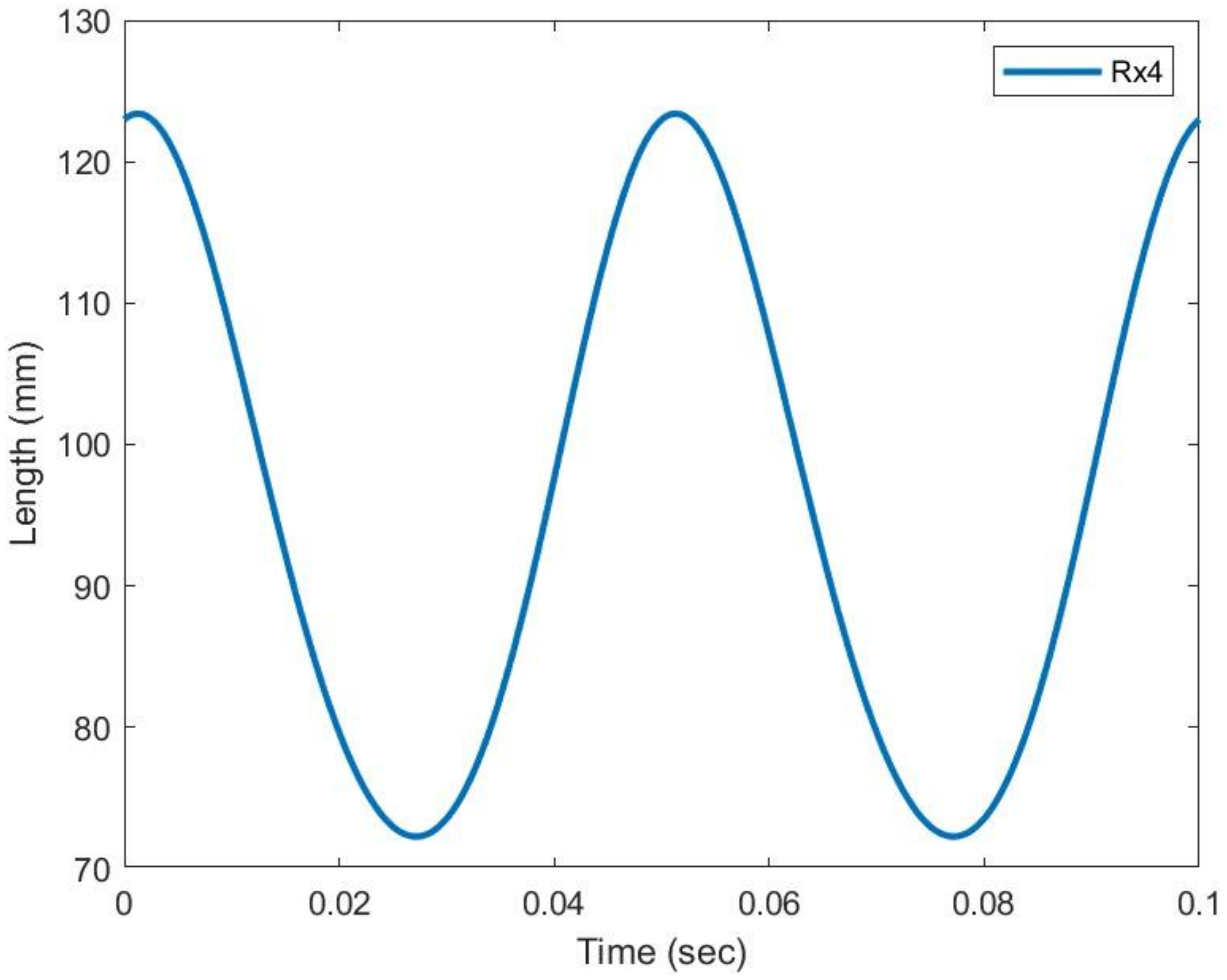
End Effector coordinates

$$E.E = R^3 + A'\bar{m} \Rightarrow R^3 + \begin{bmatrix}\cos\theta^3 & -\sin\theta^3\\ \sin\theta^3 & \cos\theta^3\end{bmatrix}\begin{bmatrix}l^3/2\\0\end{bmatrix}$$

$$\boxed{\begin{aligned}E.E_x &= l^2\cos\theta^2 + l^3\cos\theta^3\\ E.E_y &= l^2\sin\theta^2 + l^3\sin\theta^3\end{aligned}}$$

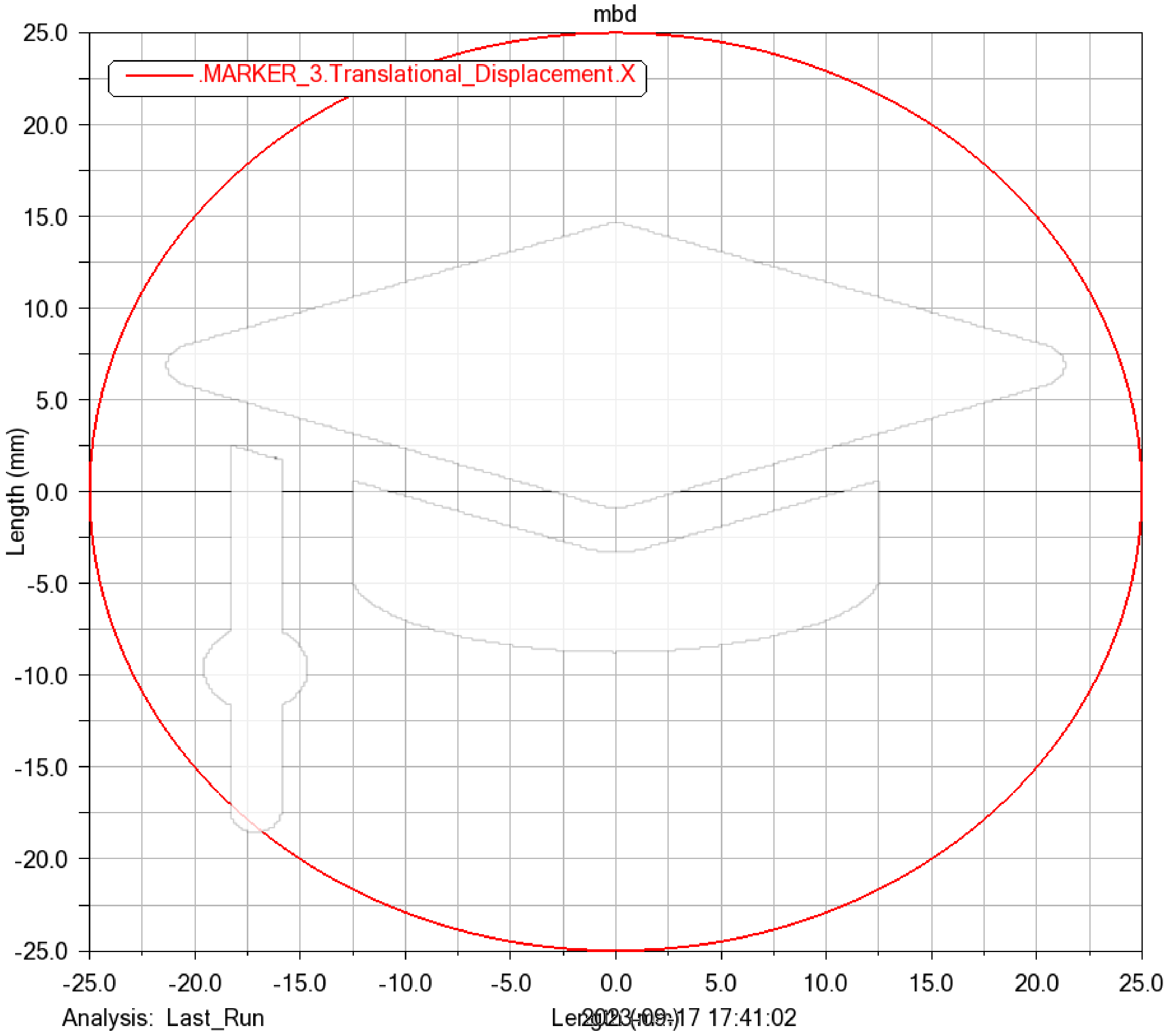
Question 1 : Rx 4 vs time

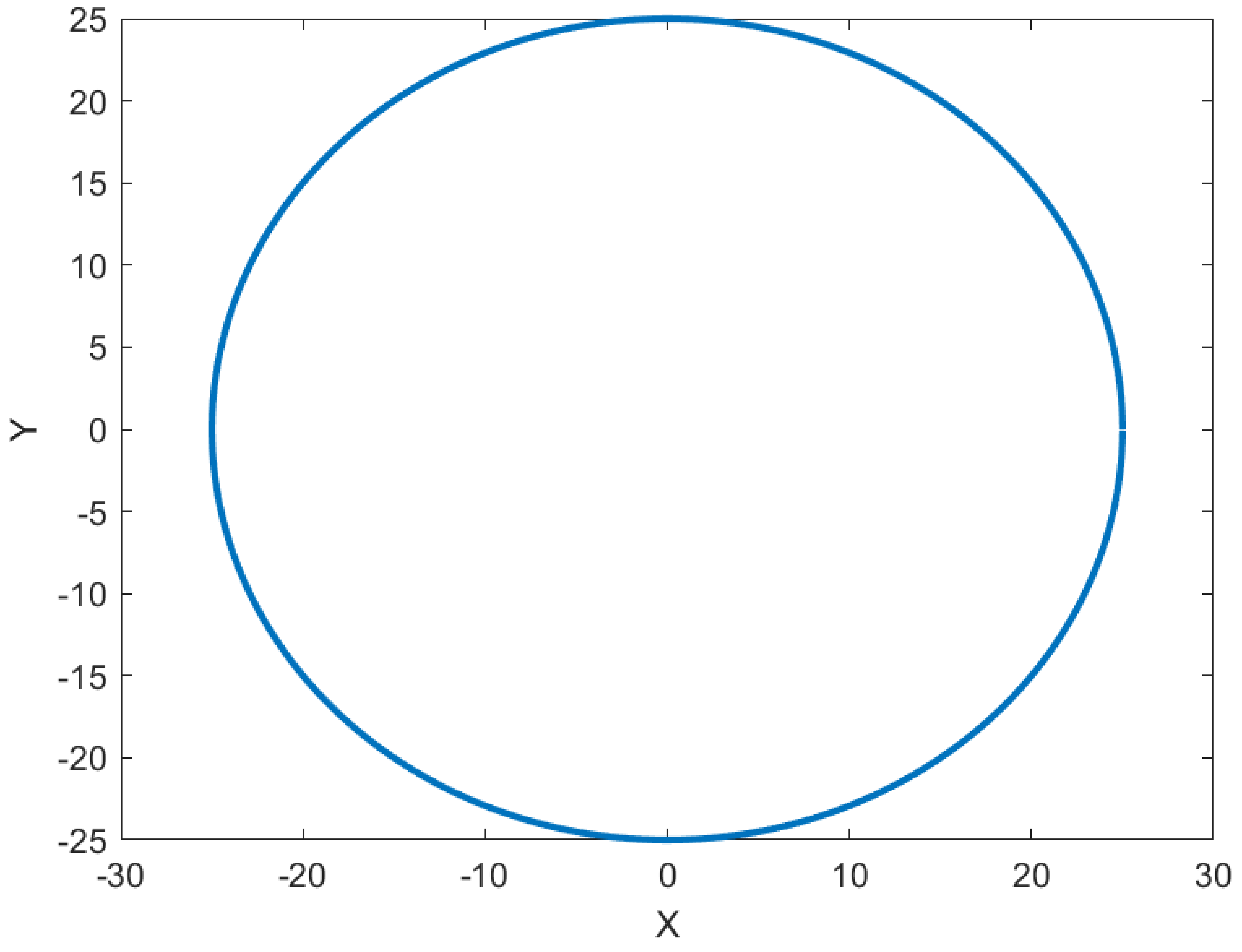




Question 1 : MATLAB generated code for variation of slider's position with time

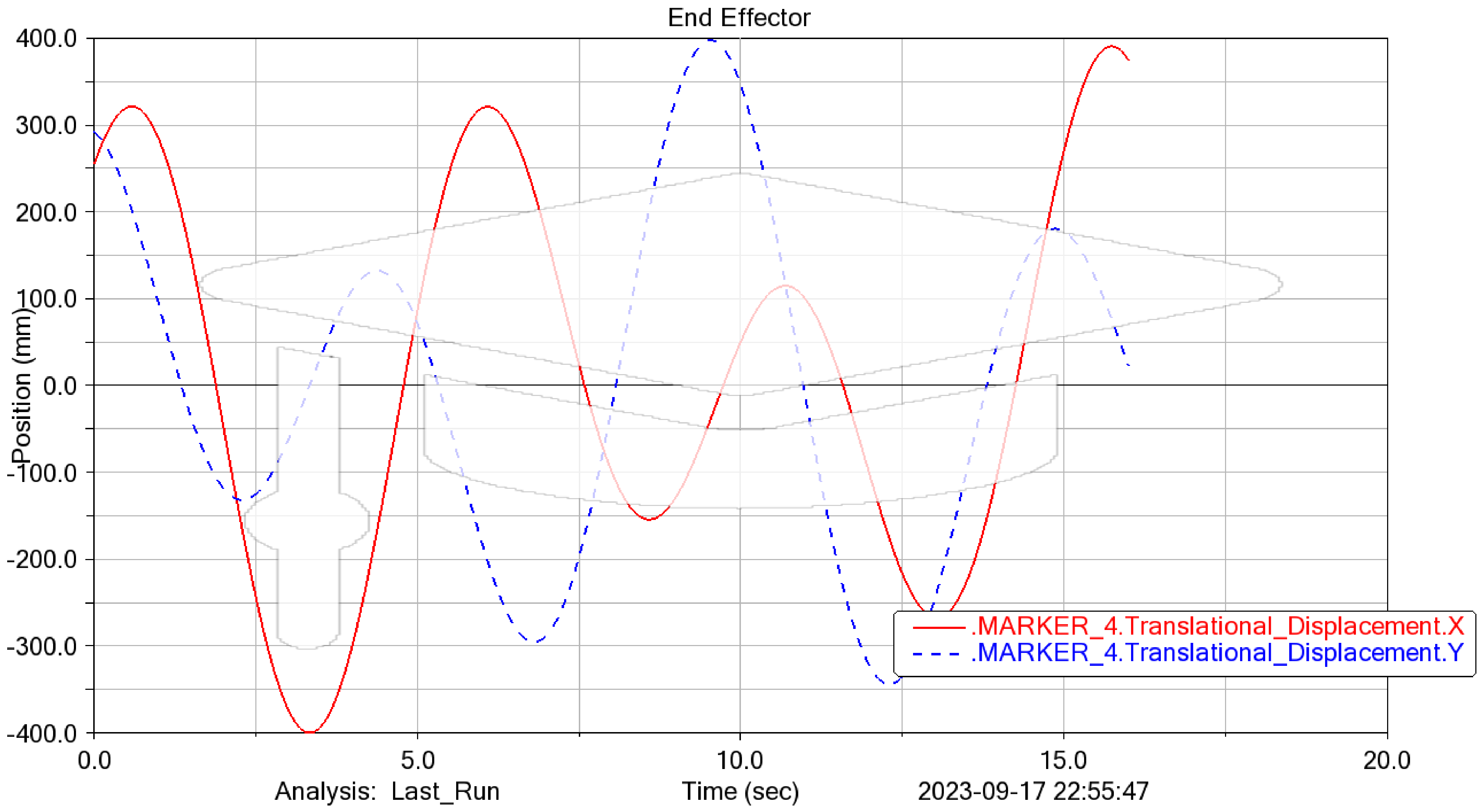
Rx3 (y axis) vs Ry3 (x axis)

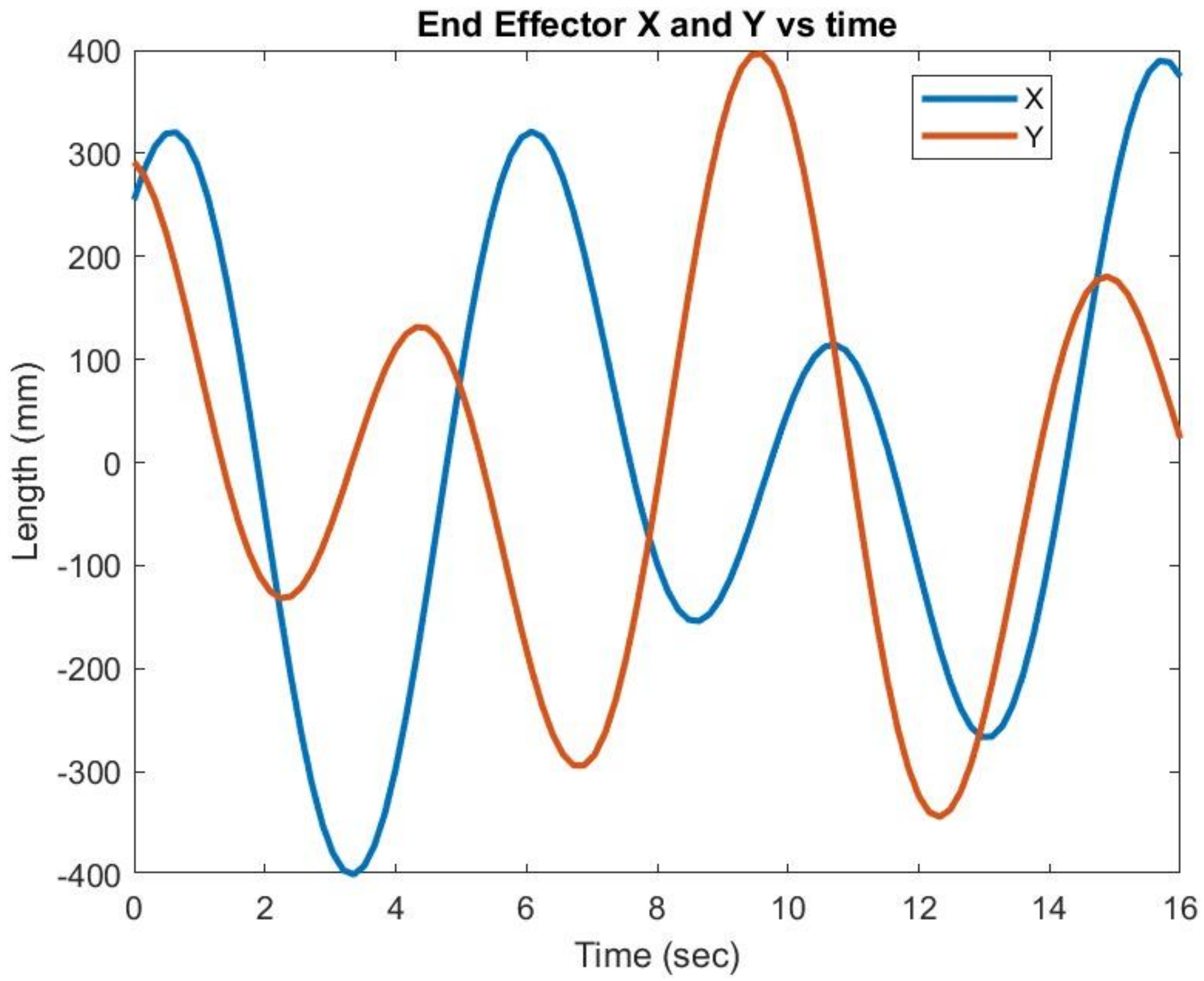




Question 1 : Rx3 vs Ry3 MATLAB Generated Plot

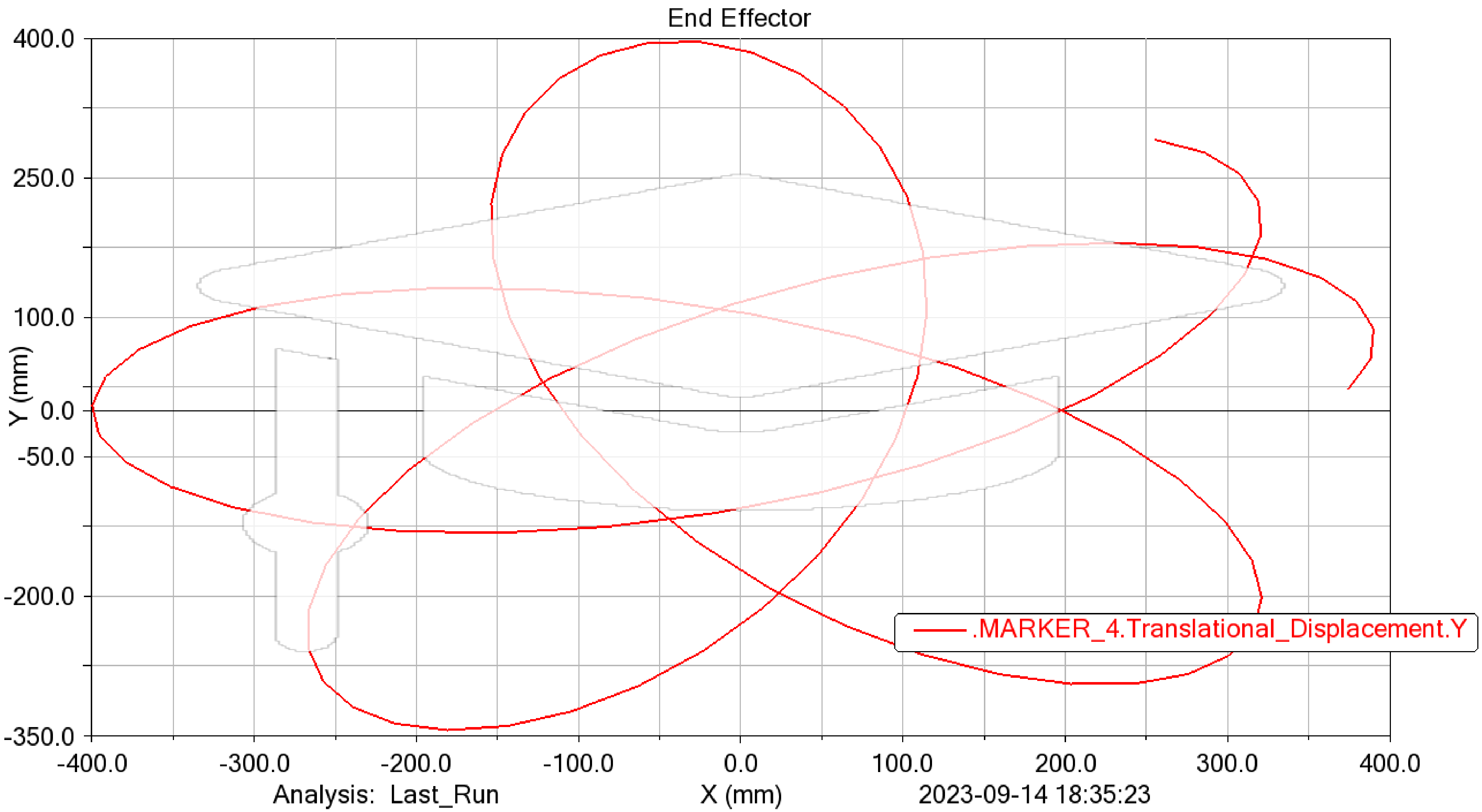
Variation of End Effector Coordinates with time

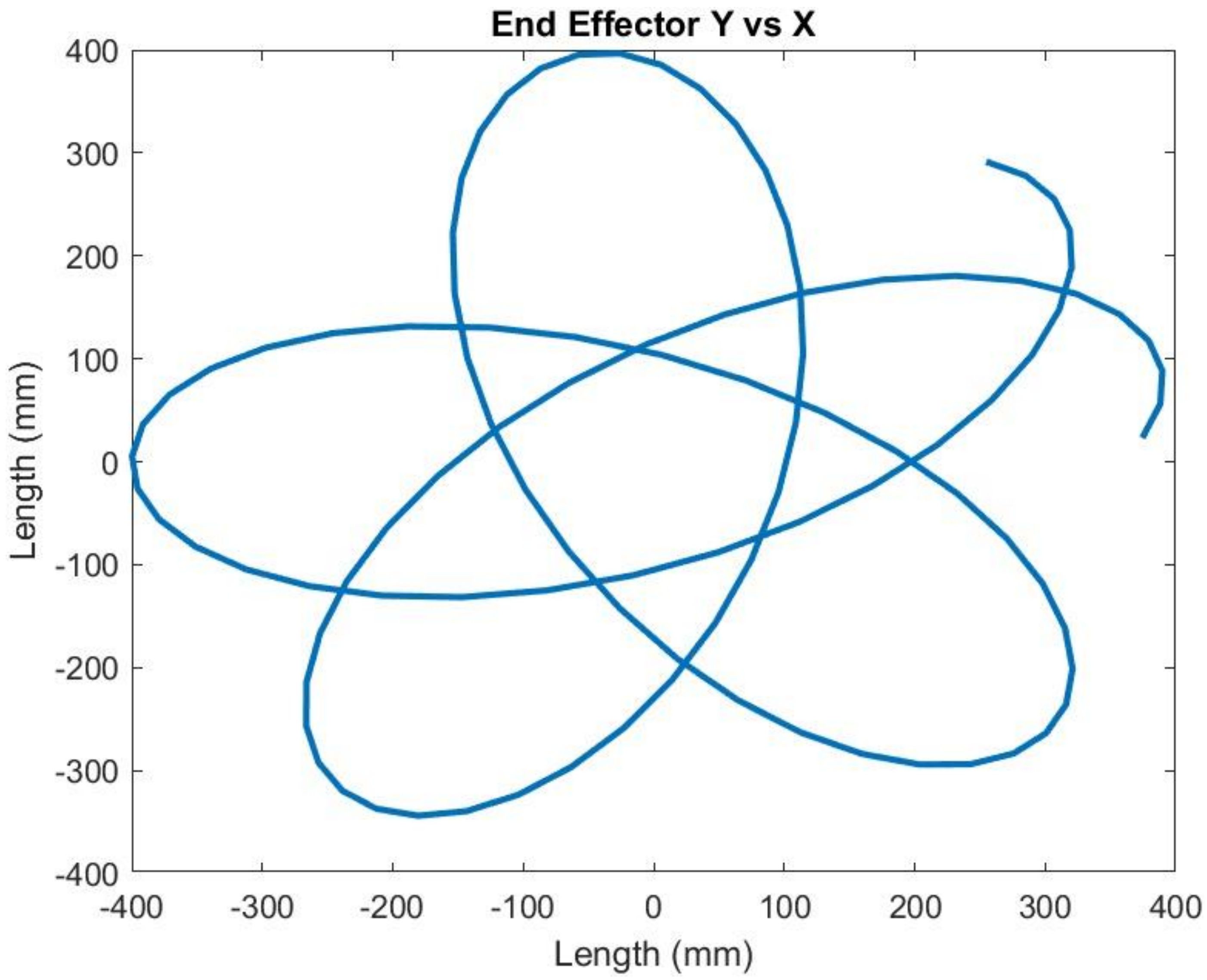




Question 2 : MATLAB Plot for end effector position variation with time

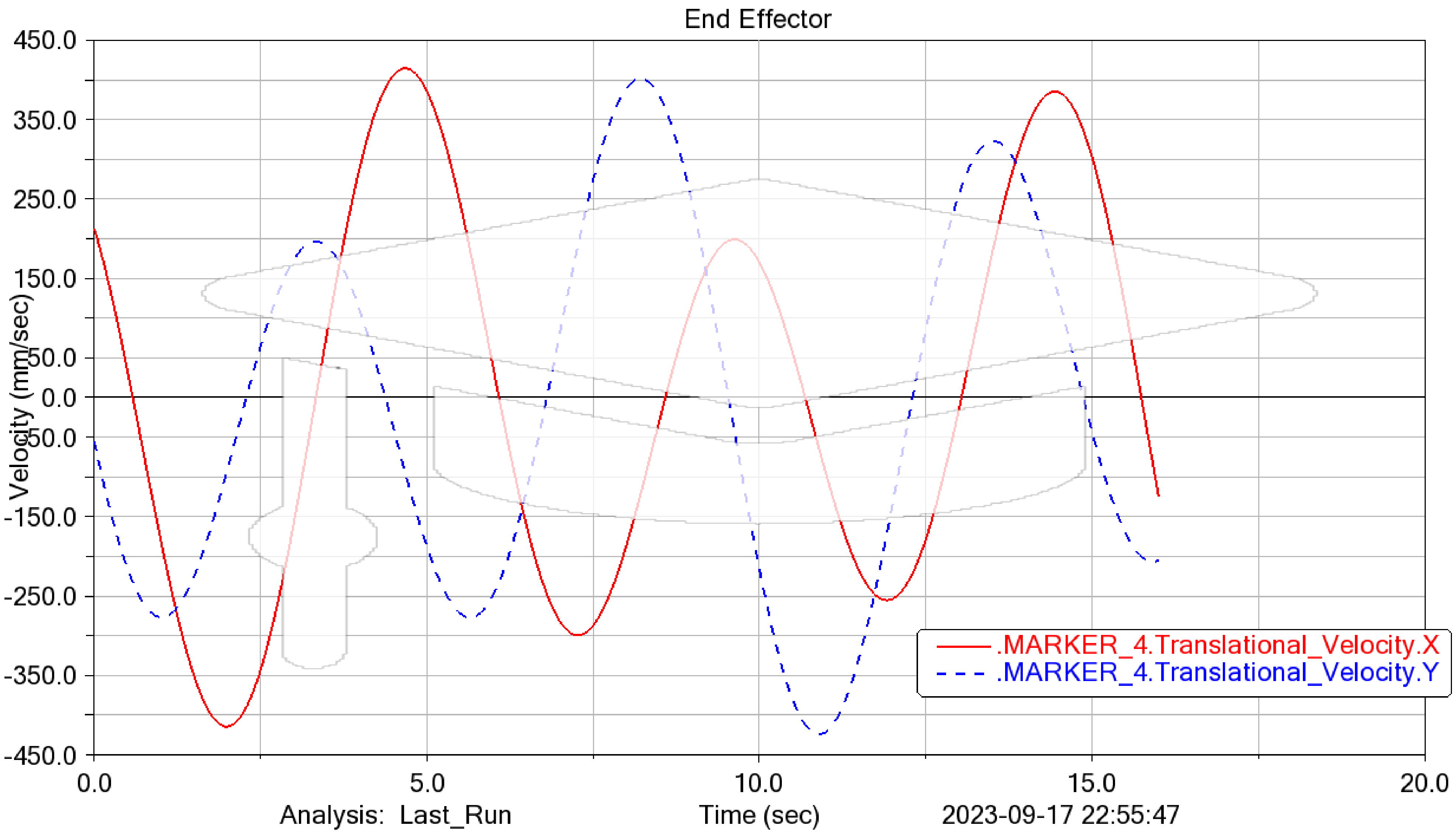
Path traced by end effector

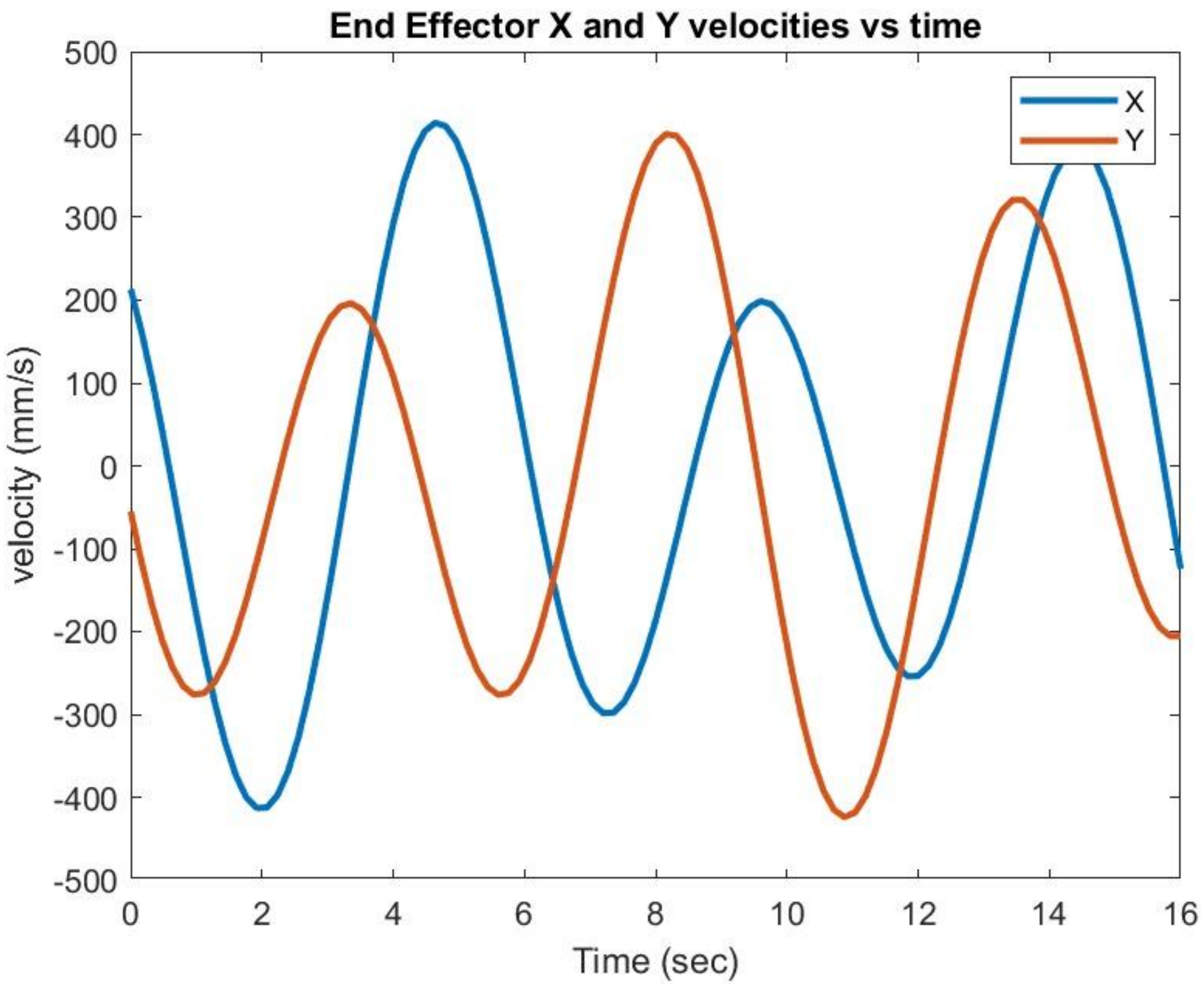




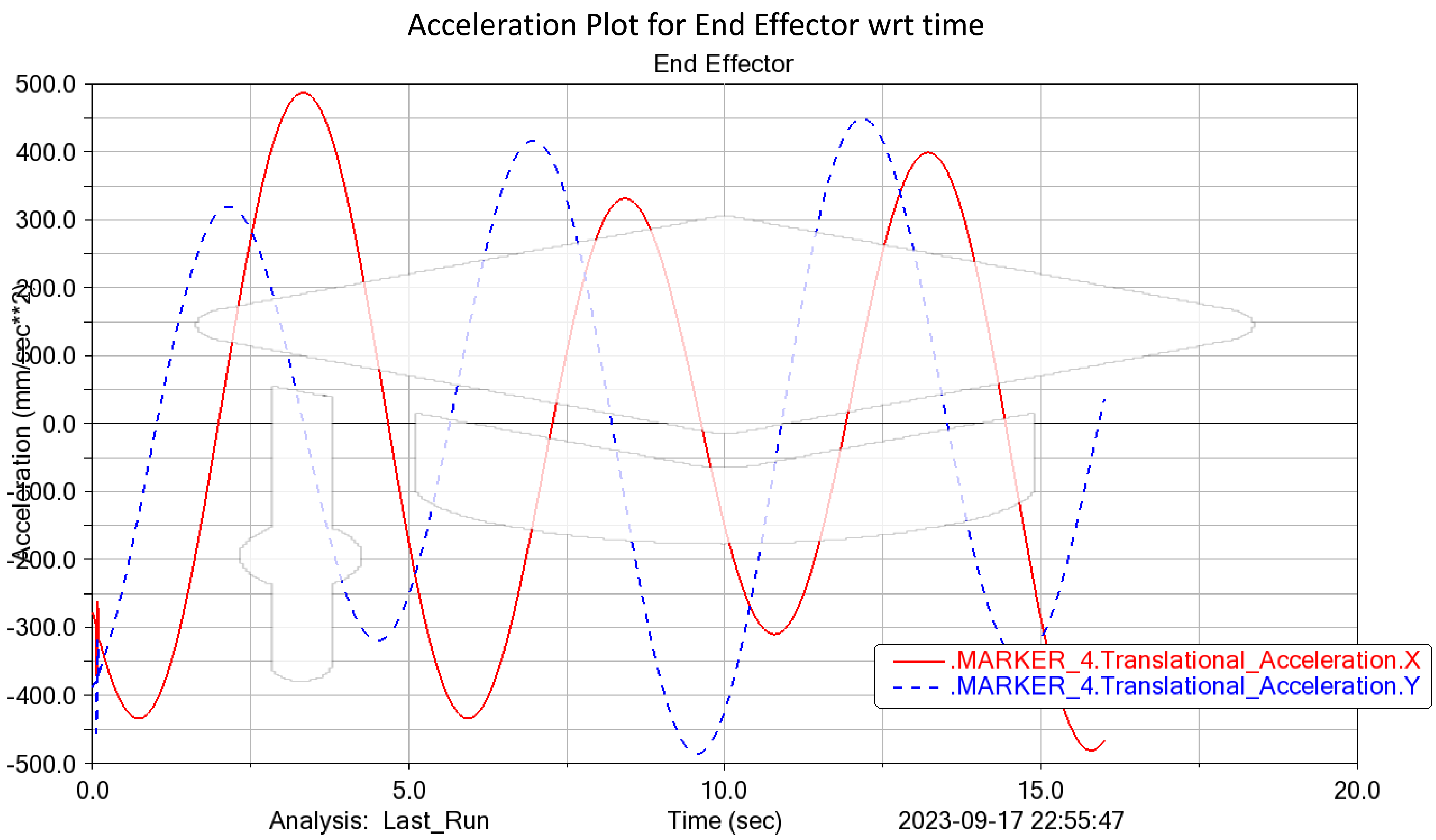
Question 2 : MATLAB Plot for End Effector’s Path

Velocity Plot for end effector w.r.t time

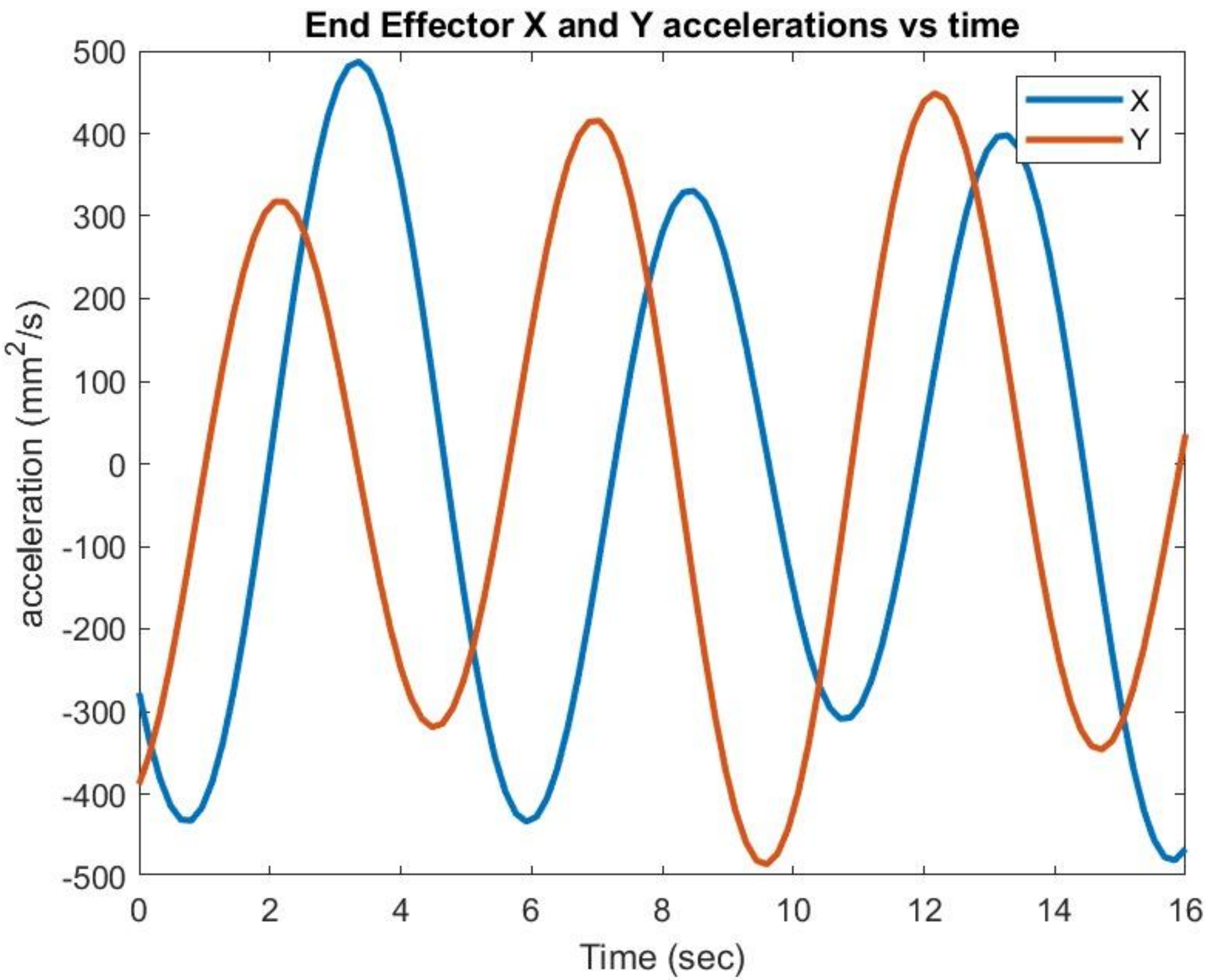




MATLAB generated plot for variation of End Effector Velocity with time



Acceleration Plot for End Effector wrt time
End Effector
500.0
400.0
300.0
200.0
100.0
0.0
-100.0
-200.0
-300.0
-400.0
-500.0
Acceleration (mm/sec**2)
0.0
5.0
10.0
15.0
20.0
Analysis: Last_Run
Time (sec)
2023-09-17 22:55:47
.MARKER_4.Translational_Acceleration.X
.MARKER_4.Translational_Acceleration.Y



MATLAB generated plot for End Effector Acceleration variation with time