Department of Mechanical Engineering
Indian Institute of Technology Madras

ME 5233 Multi-body Dynamics & Its Applications

Assignment 0

Due on: **August 23, 2023**

1. Find the trace and inverse of the following matrices

$$\mathbf{A} = \begin{bmatrix} -1 & 2 & -1 \\ 2 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 0 & -3 & 5 \\ -2 & 2 & -3 \\ 6 & -2 & 0 \end{bmatrix}$$

2. Show that any arbitrary square matrix $\mathbf{A}$ can be expressed as $\mathbf{A} = \mathbf{A}_s + \mathbf{A}_w$ where $\mathbf{A}_s$ is a symmetric square matrix and $\mathbf{A}_w$ is a skew-symmetric square matrix.

3. Find the total derivative of the vector function given below with respect to parameter t; q_1, q_2 and q_3 are implicit functions of t.

$$\mathbf{f} = \begin{Bmatrix} \sin(q_1) + q_1 q_2^2 + q_3 q_1^2 \\ q_1 q_3^2 + t^2 \\ q_3 q_2^2 t \end{Bmatrix}$$

   Also express the derivative of $\mathbf{f}$ with respect to $\mathbf{q} = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}^{\mathrm{T}}$

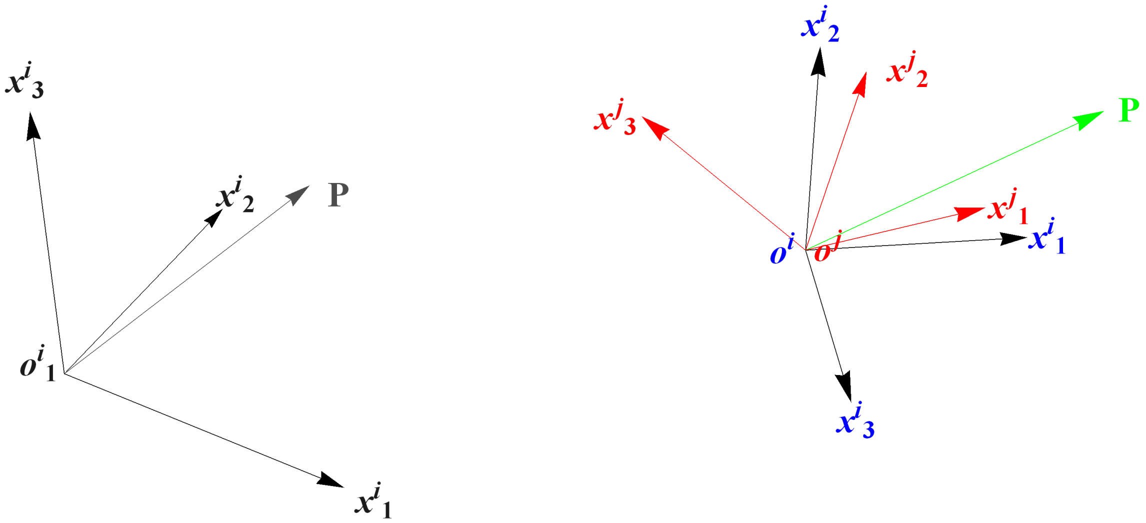
4. Find the cross product $\mathbf{a}\times\mathbf{b}$ and $\mathbf{b}\times\mathbf{a}$ where $\mathbf{a} = \begin{bmatrix} -2 & 5 & 9 \end{bmatrix}^{\mathrm{T}}$ and $\mathbf{b} = \begin{bmatrix} 18 & -3 & 10 \end{bmatrix}^{\mathrm{T}}$. What is the relation between $\mathbf{a} \times \mathbf{b}$ and $\mathbf{b} \times \mathbf{a}$. Now use the skew-symmetric form $\tilde{\mathbf{a}}$ and $\tilde{\mathbf{b}}$ to arrive at the cross-products.

5. If $\mathbf{a}$ is a time-dependent vector of constant length, show that $\dot{\mathbf{a}}^{\mathrm{T}}\mathbf{a} = 0$.

6. Find the direction cosines of the vector $\mathbf{P} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^{\mathrm{T}}$; see Figure 1a.

7. The orientation angles (**in deg**) between two Cartesian co-ordinate systems are given in the table below (see Figure 1b). Find the direction cosine based transformation matrix. If a vector $\mathbf{P} = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}^{\mathrm{T}}$ in the $X_1^i X_2^i X_3^i$ frame, find its co-ordinates in the $X_1^j X_2^j X_3^j$ frame.

	X_1^i	X_2^i	X_3^i
X_1^j	54.74	54.74	54.74
X_2^j	65.90	65.90	144.74
X_3^j	135	45	90



(a) (b)

Figure 1: Figures for Problems 6 and 7.