Multibody dynamics Assignment 0

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① $A = \begin{bmatrix} -1 & 2 & 1 \\ 2 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ $tr(A) = -1-1+1 = -1$

$|A| = -1(-1) - 2(2-1) = -1$

$Adj(A) = \begin{bmatrix} -1 & -2 & -2 \\ -1 & -1 & -1 \\ -1 & -2 & -3 \end{bmatrix}^T$

$A^{-1} = \dfrac{Adj(A)}{|A|} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$

$B = \begin{bmatrix} 0 & -3 & 5 \\ -2 & 2 & -3 \\ 6 & -2 & 0 \end{bmatrix}$

$tr(B) = 0+2+0 = 2$

$Adj(B) = \begin{bmatrix} -6 & -10 & -1 \\ -18 & -30 & -10 \\ -8 & -18 & -6 \end{bmatrix}$ $|B| = 3(18)+5(-8)$
$= 14$

$B^{-1} = \begin{bmatrix} -3/7 & -5/7 & -1/14 \\ -9/7 & -15/7 & -5/7 \\ -4/7 & -9/7 & -3/7 \end{bmatrix}$

② $A = A_s + A_w$ $A \rightarrow$ arbitary square matrix

$A = \dfrac{(A+A^T) + (A-A^T)}{2}$

$A+A^T \rightarrow$ symmetric matrix
$(A+A^T)^T = A^T + A = A + A^T$

$\Rightarrow A_s = A+A^T$
$A_w = A-A^T$

$A-A^T \rightarrow$ skew symmetric matrix
$(A-A^T)^T = A^T - A = -(A-A^T)$

③ $f = \begin{Bmatrix} \sin(q_1) + q_1 q_2^2 + q_3 q_1^2 \\ q_1 q_3^2 + t^2 \\ q_3 q_2^2 t \end{Bmatrix}$

$$f' = \begin{bmatrix} \cos(q_1) + q_2^2 + 2q_1 q_3 & 2q_1 q_2 & q_1^2 & 0 \\ q_3^2 & 0 & 2q_1 q_3 & 2t \\ 0 & 2q_2 q_3 t & t q_2^2 & q_3 q_2^2 \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ 1 \end{Bmatrix}$$

$\dot{q}_i = \frac{dq_i}{dt}$

$i = 1, 2, 3$

④ $a = [-2 \ 5 \ 9]^T$ $\qquad$ $b = [18 \ -3 \ 10]^T$

$$a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 5 & 9 \\ 18 & -3 & 10 \end{vmatrix} = \hat{i}(77) + \hat{j}(182) - 84\hat{k}$$

anti parallel vectors

$$b \times a = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 18 & -3 & 10 \\ -2 & 5 & 9 \end{vmatrix} = \hat{i}(-77) + \hat{j}(-182) + 84\hat{k}$$

along the same line but opposite in direction

matrix method for cross product

$$a \times b = \begin{bmatrix} 0 & -9 & 5 \\ 9 & 0 & 2 \\ -5 & -2 & 0 \end{bmatrix} \begin{bmatrix} 18 \\ -3 \\ 10 \end{bmatrix} = \begin{bmatrix} 77 \\ 182 \\ -84 \end{bmatrix}$$

$$b \times a = \begin{bmatrix} 0 & -10 & -3 \\ 10 & 0 & -18 \\ 3 & 18 & 0 \end{bmatrix} \begin{bmatrix} -2 \\ 5 \\ 9 \end{bmatrix} = \begin{bmatrix} -77 \\ -182 \\ 84 \end{bmatrix}$$

⑤ $a^T a$ = constant prove $\dot{a}^T a = 0$

on differentiating this product wrt time, RHS = 0

$\dot{a}^T a + a^T \dot{a} = 0$ $\dot{a}^T a = \sum_{i=1}^{d} \dot{a}_i a_i$ $a^T \dot{a} = \sum_{i=1}^{d} a_i \dot{a}_i$

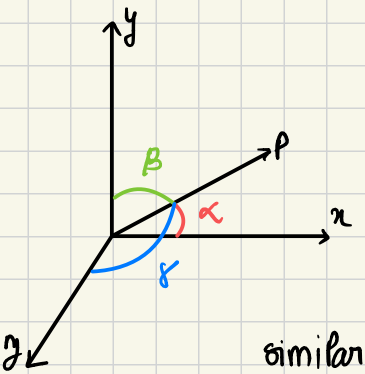
therefore $\dot{a}^T a = a^T \dot{a}$

$\Rightarrow 2\dot{a}^T a = 0$

$\Rightarrow \underline{\underline{\dot{a}^T a = 0}}$

⑥ $P = [1\ 1\ 1]^T$ direction cosines?

$\bar{P}$ = unit vector along $P = \frac{1}{\sqrt{3}}[1\ 1\ 1]^T = \frac{P}{L_2(P)}$



direction cosine along x axis = cosine of angle b/w P and x axis = $\cos\alpha$

$\therefore \cos\alpha = \bar{P}^T \hat{x} = \frac{1}{\sqrt{3}}[1\ 1\ 1]\begin{bmatrix}1\\0\\0\end{bmatrix} = \frac{1}{\sqrt{3}}$

similarly $\cos\beta = \bar{P}^T \hat{y} = \frac{1}{\sqrt{3}}$

$\cos\gamma = \bar{P}^T \hat{z} = \frac{1}{\sqrt{3}}$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \Rightarrow (1/\sqrt{3})^2 + (1/\sqrt{3})^2 + (1/\sqrt{3})^2 = 1$ //

⑦ $$\begin{bmatrix} x_1^j \\ x_2^j \\ x_3^j \end{bmatrix} = \begin{bmatrix} \cos(54.74) & \cos(54.74) & \cos(54.74) \\ \cos(65.90) & \cos(65.90) & \cos(144.74) \\ \cos(135) & \cos(45) & \cos(90) \end{bmatrix} \begin{bmatrix} x_1^i \\ x_2^i \\ x_3^i \end{bmatrix}$$

$$= \begin{bmatrix} 0.577 & 0.577 & 0.577 \\ 0.408 & 0.408 & -0.817 \\ -0.707 & 0.707 & 0 \end{bmatrix} \begin{bmatrix} x_1^i \\ x_2^i \\ x_3^i \end{bmatrix}$$

unit vector in i's coordinate frame

now $P = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}^T$

now components of P in j's coordinate frame are the dot products of P with respective coordinate axes in j

projection of P along x_1^j

$$\Rightarrow \quad P^T x_1^j = \frac{1}{\sqrt{3}} [1 \; 1 \; 1] \begin{bmatrix} 0.577 \\ 0.577 \\ 0.577 \end{bmatrix} = 0.577\sqrt{3}$$

lly for P along x_2^j

$$= \frac{1}{\sqrt{3}} [1 \; 1 \; 1] \begin{bmatrix} 0.408 \\ 0.408 \\ -0.817 \end{bmatrix} \approx 0$$

for P along $x_3^j = \frac{1}{\sqrt{3}} [1 \; 1 \; 1] \begin{bmatrix} -0.707 \\ 0.707 \\ 0 \end{bmatrix} = 0$

therefore coordinates in j's coordinate system

$$\Rightarrow \quad \underline{\underline{\begin{array}{ccc} & \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T & \\ x_1^j & x_2^j & x_3^j \end{array}}}$$