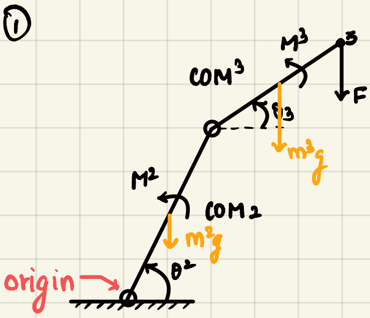
ME20B072 Girish Madhavan V MBD ASSIGNMENT 2

①



$$R_{COM^2} = \begin{bmatrix} \ell^2\cos\theta^2/2 \\ \ell^2\sin\theta^2/2 \end{bmatrix}$$

$$R_{COM^3} = \begin{bmatrix} \ell^2\cos\theta^2 + \ell^3\cos\theta^3/2 \\ \ell^2\sin\theta^2 + \ell^3\sin\theta^3/2 \end{bmatrix}$$

Gravity's virtual work:

$$-m^2g\,\delta R^y_{COM^2} - m^3g\,\delta R^y_{COM^3}$$

$$-m^2g\left(\frac{\ell^2}{2}\cos\theta^2\right)\delta\theta^2 - m^3g\left(\ell^2\cos\theta^2\delta\theta^2 + \frac{\ell^3}{2}\cos\theta^3\delta\theta^3\right)$$

Joint forces virtual work for entire body = 0

F's virtual work $\Rightarrow -F\,\delta R^y_3 = -F\left(\ell^2\cos\theta^2\delta\theta^2 + \ell^3\cos\theta^3\delta\theta^3\right)$

virtual work done by moments $= M^2\delta\theta^2 + M^3\delta\theta^3$

total virtual work $\Rightarrow \delta\theta^2\underbrace{\left[-\frac{m^2g}{2}\ell^2\cos\theta^2 - m^3g\ell^2\cos\theta^2 - F\ell^2\cos\theta^2 + M^2\right]}_{F_1} + \delta\theta^3\underbrace{\left[-\frac{m^3g}{2}\ell^3\cos\theta^3 - F\ell^3\cos\theta^3 + M^3\right]}_{F_2}$

Generalized forces

$$F_1 = -\frac{0.5(10)(0.5)\cos(45)}{2} - (1)(10)(0.5)\cos(45) - 10(0.5)\cos(45) + 3 = \boxed{-4.95}$$

$$F_2 = -\frac{(1)(10)(1)\cos(30)}{2} - 10(1)\cos(30) + 3 = \boxed{-9.99}$$

② Q_e was computed in ① now we compute Q_d and equate them

$$R^2_{COM} = \left[\frac{\ell^2}{2}\cos\theta^2 \quad \frac{\ell^2}{2}\sin\theta^2\right]^T$$

$$\dot{R}^2_{COM} = \frac{\ell^2}{2}\left[-\sin\theta^2 \quad \cos\theta^2\right]^T\dot{\theta}^2$$

$$a^2_{COM} = \frac{\ell^2}{2}\ddot{\theta}^2\left[-\sin\theta^2 \quad \cos\theta^2\right]^T + \frac{\ell^2}{2}(\dot{\theta}^2)^2\left[-\cos\theta^2 \quad -\sin\theta^2\right]^T$$

$$J^2 = \frac{m^2(\ell^2)^2}{12} = \frac{0.5\times0.5^2}{12} = 0.01042$$

$\theta^2 = 45^\circ$ $\quad\dot{\theta}^2 = 70$ rad/s $\quad\ddot{\theta}^2 = 120$ rad/s² $\quad\ell^2 = 0.5$m

$$= 0.25\times120\left[-1/\sqrt{2} \quad 1/\sqrt{2}\right]^T - 0.25(70)^2\left[1/\sqrt{2} \quad 1/\sqrt{2}\right]^T$$

$$= \left[-21.21 \quad 21.21\right]^T - \left[866.2 \quad 866.2\right]^T$$

$$= \left[-887.41 \quad -844.99\right]^T$$

$$\delta R^2_{COM} = \left[-0.177 \quad 0.177\right]^T\delta\theta^2$$

$$R^3_{COM} = \left[\ell^2\cos\theta^2 + \frac{\ell^3}{2}\cos\theta^3 \quad \ell^2\sin\theta^2 + \frac{\ell^3}{2}\sin\theta^3\right]^T$$

$$\dot{R}^3_{COM} = \left[-\ell^2\sin\theta^2\dot\theta^2 - \frac{\ell^3}{2}\sin\theta^3\dot\theta^3 \quad \ell^2\cos\theta^2\dot\theta^2 + \frac{\ell^3}{2}\cos\theta^3\dot\theta^3\right]^T$$

$$J^3 = \frac{(1)(1)^3}{12} = 0.08333$$

$$a^3_{com} = 2a^2_{com} + \frac{\ell^3}{2}\ddot\theta^3\left[-\sin\theta^3 \quad \cos\theta^3\right]^T - \frac{\ell^3(\dot\theta^3)^2}{2}\left[\cos\theta^3 \quad \sin\theta^3\right]^T$$

$\ell^3 = 1\,m \quad \theta^3 = 30^\circ \quad \dot\theta^3 = 40\ rad/s \quad \ddot\theta^3 = 180\ rad/s^2$

$$\delta R^3_{COM} = 2\delta R^2_{COM} + \left[-0.25 \quad 0.433\right]^T\delta\theta^3$$

$$= \left[-0.354 \quad 0.354\right]^T\delta\theta^2 + \left[-0.25 \quad 0.433\right]^T\delta\theta^3$$

$$a^3_{com} = \left[-2512.66 \quad -2012.04\right]^T$$

$$Q_d = m^2 a^{2T}_{com}\,\delta R^2_{COM} + J^2\ddot\theta^2\delta\theta^2 + m^3 a^{3T}_{com}\,\delta R^3_{COM} + J^3\ddot\theta^3\delta\theta^3$$

$$= 0.5(7.5)\delta\theta^2 + 1.25\delta\theta^2 + 177\,\delta\theta^2 - 243.1\,\delta\theta^3 + 15\delta\theta^3$$

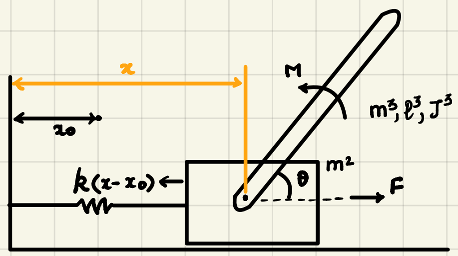
$$= 182\,\delta\theta^2 - 228.1\delta\theta^3$$

for dynamic equ. $\quad Q_e = Q_d$

$$\Rightarrow \quad -7.95 + M^2 = 182$$
$$-12.99 + M^3 = -228.1$$

$$\boxed{\begin{aligned} M^2 &= 189.95\ Nm \\ M^3 &= -215.11\ Nm \end{aligned}}$$

③



Total Kinetic Energy $= KE_{rod} + KE_{block}$

$$KE_{rod} = \frac{m^3}{2}\left((\dot R^3_x)^2 + (\dot R^3_y)^2\right) + \frac{1}{2}J^3(\dot\theta)^2$$

$$R^3_x = x + \frac{\ell^3}{2}\cos\theta \qquad R^3_y = \frac{\ell^3}{2}\sin\theta$$

$$\dot R^3_x = \dot x - \frac{\ell^3}{2}\sin\theta\,\dot\theta \qquad \dot R^3_y = \frac{\ell^3}{2}\cos\theta\,\dot\theta$$

$$KE_{rod} = \frac{m^3}{2}\left(\dot x^2 + \frac{(\ell^3)^2}{4}\sin^2\theta\,\dot\theta^2 + \frac{(\ell^3)^2}{4}\cos^2\theta\,\dot\theta^2 - \ell^3\dot x\sin\theta\dot\theta\right) + \frac{J^3\dot\theta^2}{2}$$

$$= \frac{m^3}{2}\left(\dot x^2 + \left(\frac{\ell^3\dot\theta}{2}\right)^2 - \ell^3\dot x\dot\theta\sin\theta\right) + \frac{J^3\dot\theta^2}{2}$$

$$KE_{block} = \frac{m^2(\dot x)^2}{2}$$

$$\Rightarrow \quad T = \frac{(m^3+m^2)}{2}\dot x^2 + \frac{1}{2}\left(m^3\left(\frac{\ell^3}{2}\right)^2 + J^3\right)\dot\theta^2 - \frac{m^3\ell^3\dot x\dot\theta\sin\theta}{2}$$

Generalized external force computation (Q_e)

$$M\delta\theta + F\delta x - k(x-x_0)\delta x - m^3 g\frac{\ell^3}{2}\cos\theta\,\delta\theta$$

$$③ - \left(M - \frac{m^3 g\ell^3\cos\theta}{2}\right)\delta\theta + \left(F - k(x-x_0)\right)\delta x \rightarrow ④$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot x}\right) - \frac{\partial T}{\partial x} = (m^3+m^2)\ddot x - \frac{m^3\ell^3}{2}(\cos\theta\,\dot\theta^2 + \ddot\theta\sin\theta) - 0 \qquad -①$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot\theta}\right) - \frac{\partial T}{\partial \theta} = \left(J^3 + \frac{m^3(\ell^3)^2}{4}\right)\ddot\theta - \frac{m^3\ell^3}{2}(\ddot x\sin\theta + \dot x\dot\theta\cos\theta) + \frac{m^3\ell^3}{2}\dot x\dot\theta\cos\theta \qquad -②$$

According to Lagrange's formulation

① = ④ and ② = ③

$$\boxed{\begin{aligned} (m^3+m^2)\ddot{x} - \frac{m^3 \ell^3}{2}\left(\cos\theta\,\dot{\theta}^2 + \ddot{\theta}\sin\theta\right) &= F - k(x-x_0) \\ \left(J^3 + \frac{m^3(\ell^3)^2}{4}\right)\ddot{\theta} - \frac{m^3 \ell^3}{2}\left(\ddot{x}\sin\theta\right) &= M - \frac{m^3 g \ell^3}{2}\cos\theta \end{aligned}}$$

$\}$ 2 differential equations of system