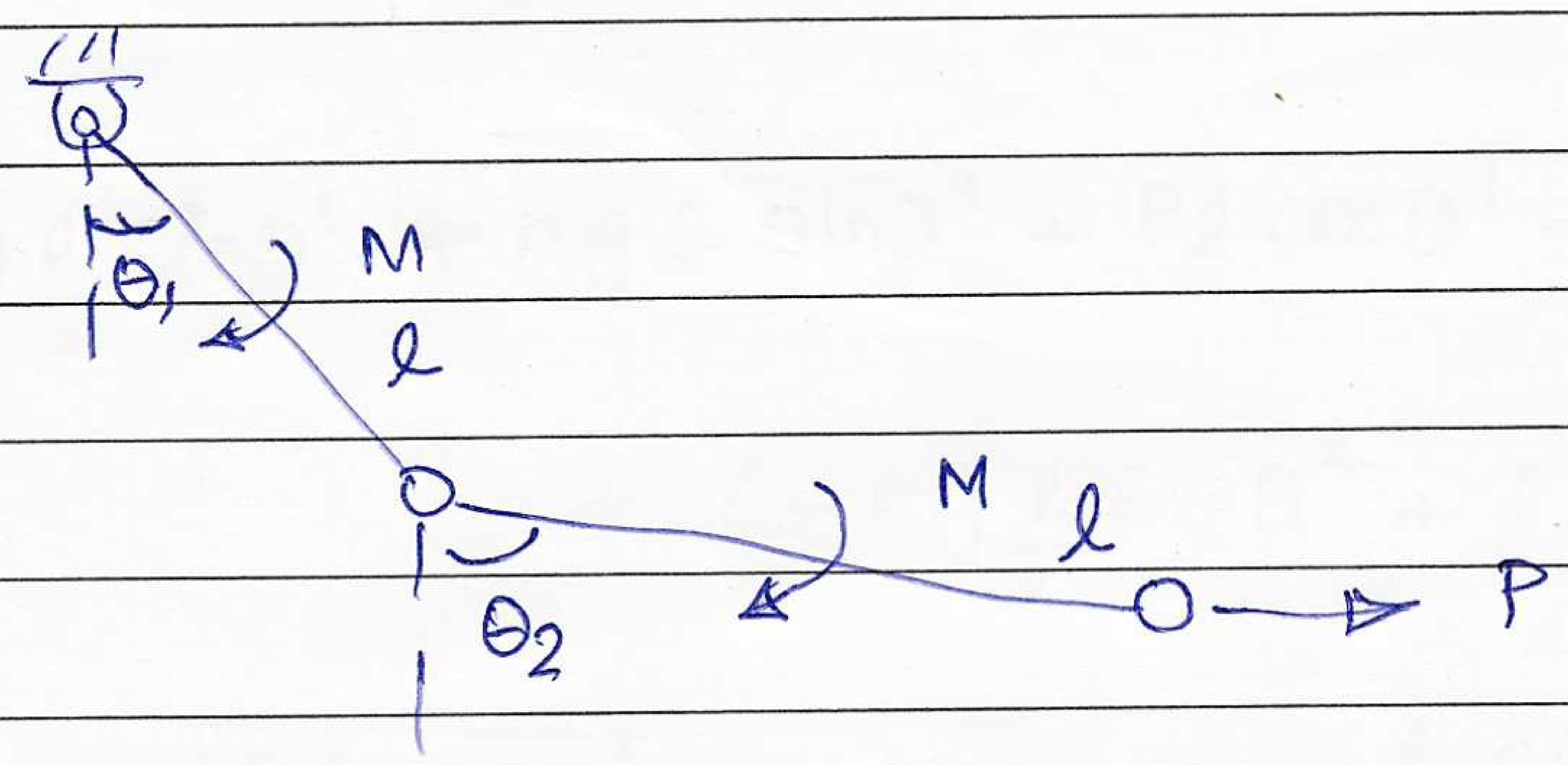
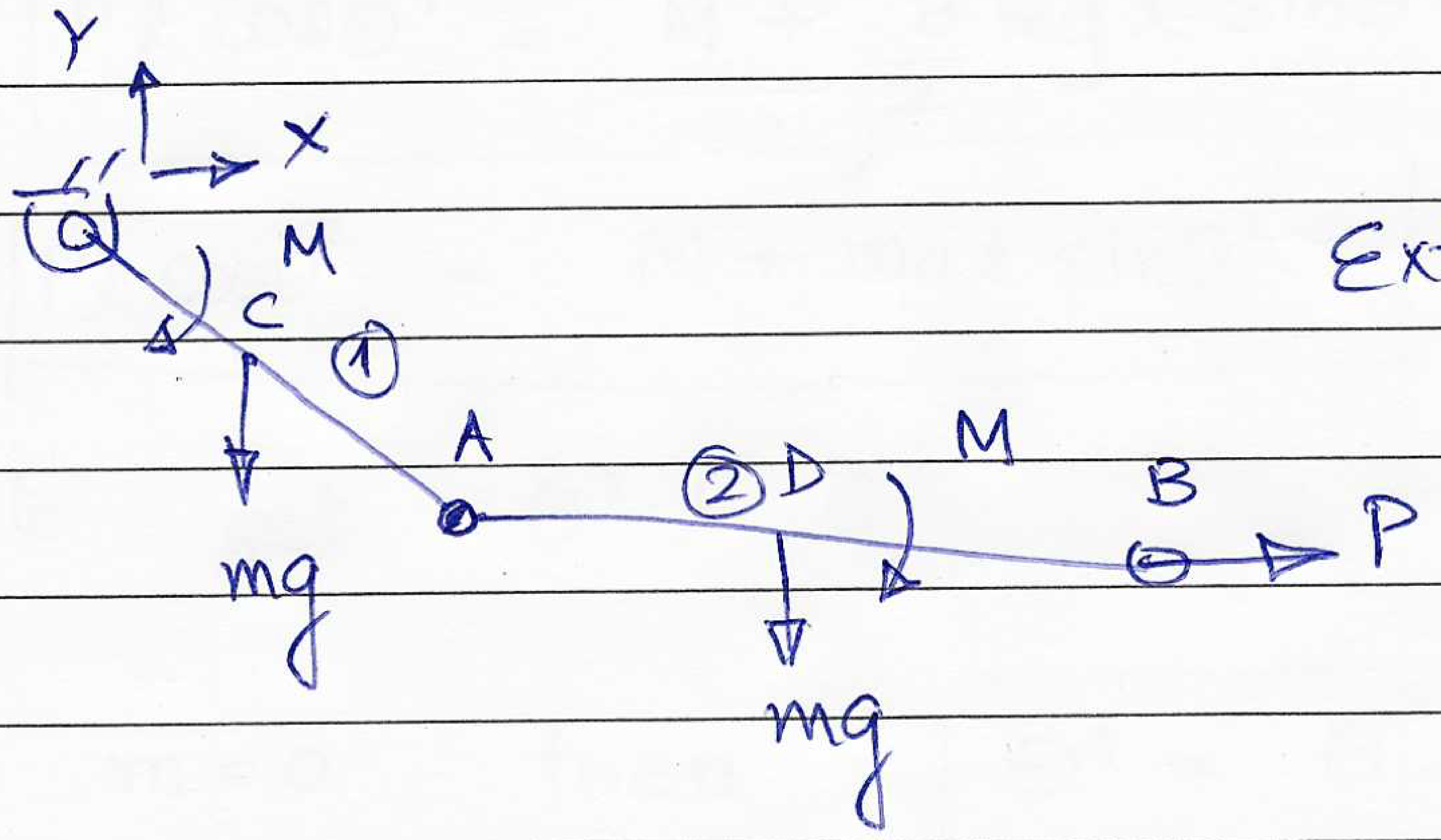
ME 5233 MBD & Applications

Quiz 2 solution

1





External forces

Principle of Virtual work (statics)

$$\boxed{\delta W_e = 0}$$

The co-ordinates of C (CM of link 1)

$$R_X^1 = \frac{l}{2}\sin\theta^1, \quad R_Y^1 = -\frac{l}{2}\cos\theta^1$$

similarly

$$R_X^2 = l\sin\theta^1 + \frac{l}{2}\sin\theta^2$$

$$R_Y^2 = -l\cos\theta^1 - \frac{l}{2}\cos\theta^2$$

Coordinates of B;

$$R_X^B = l\sin\theta^1 + l\sin\theta^2$$

$$R_Y^B = -l\cos\theta^1 - l\cos\theta^2$$

Now

$$-mg\,\delta R_Y^1 - mg\,\delta R_Y^2 + P\,\delta R_X^B - M\delta\theta_1 - M\delta\theta_2 = 0$$

$$\delta R_Y^1 = \frac{l}{2}\sin\theta^1\,\delta\theta^1$$

$$\delta R_Y^2 = l\sin\theta^1\,\delta\theta^1 + \frac{l}{2}\sin\theta^2\,\delta\theta^2$$

$$\delta R_X^B = l\cos\theta^1\,\delta\theta^1 + l\cos\theta^2\,\delta\theta^2$$

Gathering all terms related to $\delta\theta^1$ & $\delta\theta^2$ we have

$$\left(-mg\frac{l}{2}\sin\theta^1 - mgl\sin\theta^1 + Pl\cos\theta^1 - M\right)\delta\theta^1$$

$$+\left(-mg\frac{l}{2}\sin\theta^2 + Pl\cos\theta^2 - M\right)\delta\theta^2 = 0$$

$$\Rightarrow \boxed{Pl\cos\theta^1 = M + \frac{3}{2}mgl\sin\theta^1}$$

$$\boxed{Pl\cos\theta^2 = M + mg\frac{l}{2}\sin\theta^2}$$

Solve to get θ^1 & θ^2

When $m = 0$ then $\boxed{\theta^1 = \theta^2 = \cos^{-1}\left(\frac{M}{Pl}\right)}$

(2) The governing equation is of the form:

$$l\ddot{\theta} + \frac{m}{m+M}\ddot{x}\cos\theta + g\sin\theta = 0$$

Also $\boxed{\dot{x} + l\dot{\theta}\cos\theta = \text{constant}}$

The bar's only undergoing curvilinear translation

Now CM of bar $[l\sin\theta, -l(1-\cos\theta)]$

$\dot{R}_x = +l\cos\theta\,\dot{\theta}$ $\qquad$ $\dot{R}_y = -l\sin\theta\,\dot{\theta}$

Now bar KE $= \frac{1}{2}M(l\cos\theta\,\dot{\theta})^2 + \frac{1}{2}M(l\sin\theta\,\dot{\theta})^2$

$$= \frac{1}{2}Ml^2\dot{\theta}^2$$

The velocity of the ball

$$R_x^b = \dot{x} + l\cos\theta\,\dot{\theta}$$

$$R_y^b = -l\sin\theta\,\dot{\theta}$$

$\Rightarrow$ KE of the ball is $\frac{1}{2}m(\dot{x} + l\cos\theta\,\dot{\theta})^2 + \frac{1}{2}m(-l\sin\theta\,\dot{\theta})^2$

$$= \frac{1}{2}m\dot{x}^2 + m\dot{x}\dot{\theta}\,l\cos\theta + \frac{1}{2}ml^2\dot{\theta}^2$$

The generalized force due to gravity is

$$-\overset{(m+M)}{\cancel{m}}g\,l\sin\theta$$

$\therefore$ $\frac{d}{dt}\left\{\frac{\partial KE}{\partial\dot{\theta}}\right\} = Qe$

$$\Rightarrow (M+m)l^2\ddot{\theta} + ml\ddot{x}\cos\theta + \overset{(m+M)}{\cancel{m}}g\,l\sin\theta = 0$$

$$\Rightarrow \boxed{l\ddot{\theta} + \frac{m}{m+M}\cos\theta\,\ddot{x} + g\sin\theta = 0}$$

For small θ $\sin\theta \simeq \theta$ & $\cos\theta \simeq 1$

$$\Rightarrow \boxed{l\ddot{\theta} + \frac{m}{m+M}\ddot{x} + g\theta = 0}$$

Now $\frac{d}{dt}\left\{\frac{\partial KE}{\partial\dot{x}}\right\} = 0 \Rightarrow \frac{d}{dt}\left\{m\dot{x} + ml\cos\theta\,\dot{\theta}\right\} = 0$

$$\Rightarrow \boxed{\dot{x} + l\cos\theta\,\dot{\theta} = \text{Const}}$$