Relations

Let $\rho \subseteq A \times A$ be a relation on a set A.

1. **Reflexive:**
   $\forall a \in A, (a, a) \in \rho$

2. **Symmetric:**
   $(a, b) \in \rho$ implies $(b, a) \in \rho$

3. **Transitive:**
   $(a, b) \in \rho$ and $(b, c) \in \rho$ implies $(a, c) \in \rho$

4. **Antisymmetric:**
   $(a, b) \in \rho$ and $(b, a) \in \rho$ simultaneously hold iff $a = b$

- $\rho \subseteq A \times A$ is an **equivalence relation** if it is reflexive, symmetric and transitive.

- $\rho \subseteq A \times A$ is an **ordering** if it is reflexive, anti-symmetric and transitive.

  e.g. Let $\mathbb{Z}^+$ be the set of positive integers.
  We say $a \sim b$ if $a|b$ (a divides b) $\longrightarrow$ *Ordering*

  e.g. Let X be any set and $P(X)$ be the set of all subsets of X [Power set of X, sometimes denoted as 2^X].
  For $A, B \in P(X)$ define $A \sim B$ if $A \subseteq B \longrightarrow$ *(Ordering)*

- An ordering ρ is called a **linear ordering** if $\forall x, y \in A$ either $(x, y) \in \rho$ or $(y, x) \in \rho$.

Equivalence Class

Let $\rho \subseteq A \times A$

i) For every $a \in A, cl(a)$ is non-empty.

ii) For any $a, b \in A$ either $cl(a) = cl(b)$ or $cl(a) \cap cl(b) = \phi$

Proof: Let $a, b \in A$
Case I: $(a, b) \in \rho$
Let $x \in cl(a)$, then $(x, a) \in \rho$