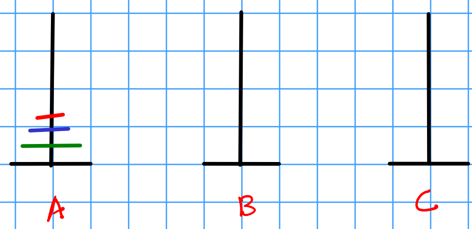


Move all the discs from source to destination (A to C)

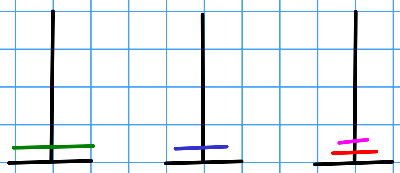
1) Only one disc can be moved at a time

2) At no point should a larger disc be on top of a smaller one

3) Only the disc on the top can be moved

1.) Move Blue from A to B
2.) Move Red from A to C
3) Move Blue from B to C
4) Move Green from A to B
5) Move Blue from C to A
6) Move Red from C to B
7) Move Blue from A to C
8) Move Red from B to A
9) Move Blue from C to A
10) Move Green from B to C
11) Move Blue from A to B
12) Move Red from A to C
13) Move Blue from B to C

1, 3, 7, 14, ?



A (source) B (auxiliary) C (destination)

If $n = 1$

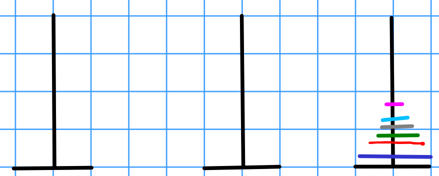
move A to C

Move from A to C using B

Move Pink from A to B — (1)
Move Red from A to C — (2)
Move Pink from B to C — (3)
Move Blue from A to B — (4)
Move Pink from A to C — (5)
Move Red from C to B — (6)
Move Pink from A to B — (7)
Move Green from A to C — (8)
Move Pink from B to C — (9)
Move Red from B to A — (10)
Move Pink from C to A — (11)
Move Blue from B to C — (12)
Move Pink from A to B — (13)
Move Red from A to C — (14)
Move Pink from B to C — (15)

1.) Is there a pattern for number of steps?

2) Are we using minimum steps?



A source | B auxiliary | C destination

$n=1$	1
$n=2$	3
$n=3$	7
$n=4$	?

1) Move $n-1$ discs from source (A) to auxiliary (B) using C a_{n-1}

2) Move last disk from source (A) to destination (C)   1

3) Move $n-1$ discs from B (auxiliary) to C (destination) using A (source) a_{n-1}

Lets assume we need a_{n-1} steps for 1)

we need 1 step for 2)

we need a_{n-1} steps for 3)

For n disks lets say it takes a_n steps

$$a_n = a_{n-1} + 1 + a_{n-1} = 2a_{n-1} + 1$$

$$a_0 = 0$$

$$a_1 = 2a_{1-1} + 1 = 2a_0 + 1 = 2(0) + 1 = 1$$

$$a_2 = 2a_{2-1} + 1 = 2a_1 + 1 = 2(1) + 1 = 3$$

$$a_3 = 2a_{3-1} + 1 = 2a_2 + 1 = 2(3) + 1 = 7$$

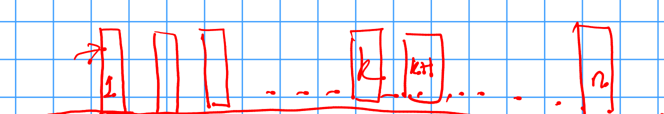
$$a_4 = 2a_{4-1} + 1 = 2a_3 + 1 = 2(7) + 1 = 15$$

$\vdots$

0, 1, 3, 7, 15, 31, 63, 127, 255, 511, 1023 ...   (10^2)

$2^0-1,\ 2^1-1,\ 2^2-1,\ 2^3-1,\ 2^4-1,\ 2^5-1,\ \ldots\ 2^{10}-1$

Induction   Mathematical induction   Recursion is induction



Dominoe effect

$P(n)$: $2^n - 1$ is a number in ToH series

$P(1)$: $2^1 - 1 = 1$.

$P(k)$: $2^k - 1$ is a number in ToH series

Induction

$P(k+1)$: 2^{k+1} is also a number in TOH series