The angle between the sun and a fixed location on earth depends on the particular location (the longitude of the location), the time of year and the time of day. In addition, the time at which the sun rises and sets depends on the longitude of the location. Therefore, the complete modelling of the sun's angle to a fixed position on Earth requires the latitude, longitude, day of the year, and time of day.

Example 2.1. *Determine the sunset hour angle and day-length at a location latitude of 32° on March 30.*

Solution. *Given*: Latitude, $\phi = 32°$; Day the year = March 30

Sunset hour angle, ω_s:

Sunset hour angle is calculated by using the relation:

$$\omega_s = \cos^{-1}(-\tan\phi\tan\delta) \qquad \text{...[Eqn. (2.16)]}$$

Let us first calculate the value of δ by using the relation:

$$\delta = 23.45\sin\left[\frac{360}{365}(284+n)\right] \qquad \text{...[Eqn. (2.3)]}$$

Here, n =number of days = 89 for Mach 30.

$$\left[n = \underset{Jan.}{31} + \underset{Feb.}{28} + \underset{Mar.}{30} = 89\right]$$

$$\therefore \qquad \delta = 23.45\sin\left[\frac{360}{365}(284+89)\right] = 3.22°$$

Hence, $\omega_s = \cos^{-1}(-\tan 32°.\tan 3.22°) = \mathbf{88.2°}$ **(Ans.)**

Day-length, t_{day} (hours):

Day-length is calculated by using the relation:

$$t_{day} = \frac{2}{15}\left[\cos^{-1}(-\tan\phi\tan\delta)\right]\text{hours}$$

$$= \frac{2}{15}\times 88.2° \text{ (already calculated)} = \mathbf{12.78\ hours\ (Ans.)}$$

Example 2.2. *Determine the day-length in hours at Delhi (latitude = 28.6°) on June 28 in a leap year.*

Solution. *Given:* $\phi = 28.6°$; Day of the leap year = June 28.

Day-length, t_{day} (hours):

Day length is given as:

$$t_{day} = \frac{2}{15}[\cos^{-1}(-\tan\phi\tan\delta)]\text{ hours} \qquad \text{...[Eqn. (2.17)]}$$

Declination (δ) is found by using Cooper's equation:

$$\delta = 23.45\sin\left[\frac{360}{365}(284+n)\right] \qquad \text{...[Eqn. (2.3)]}$$

Here, n = number of days = 180 for June 28.

$$\left[n = \underset{Jan.}{31} + \underset{Feb.}{29} + \underset{Mar.}{31} + \underset{Apr.}{30} + \underset{May}{31} + \underset{June}{28} = 180\right]$$

$$\therefore \qquad \delta = 23.45\sin\left[\frac{360}{265}(284+180)\right] = 23.24°$$

Substituting the various values in the above equation, we get :

$$t_{day} = \frac{2}{15}\left[\cos^{-1}(-\tan 28.6°.\tan 23.24°)\right]$$

$$= \frac{2}{15}\cos^{-1}(-0.234) = \mathbf{13.8\ hours}\ \ \textbf{(Ans.)}$$

Example 2.3. *Calculate Local Apparent Time (LAT) and declination at a location latitude 24° 20′ N, longitude 77°30′E at 12.30 IST on July 24. Equation of time correction = – (1′ 06″).*

Solution. *Given:* Latitude = 24° 21′ N; Longitude = 77°30′E′;

IST = 12.30; Day of the year = July 24.

Local apparent time (LAT or LST):

We know that;

LAT = IST – 4 (Standard time longitude – longitude of location) + (Equation of time correction) ...[Eqn. (2.20)]

Inserting the values in the above eqn., we get:

$$\text{LAT} = 12^h\ 30' - 4\ (82°30' - 77°30') - (1'06'')$$

$$= 12^h\ 30' - 4 \times 5 - 1'06'' = \mathbf{12^h\ 8'\ 54''}\ \textbf{(Ans.)}$$

(IST is the local civil time corresponding to **82.5° E longitude.**

Declination) δ:

Using Cooper's equation, we obtain:

$$\delta = 23.45 \sin\left[\frac{360}{365}(284 + n)\right] \qquad \text{...[Eqn. (2.3)]}$$

Here, n = number of days = 205 for July 24

$$\left[n = \underset{Jan.}{31} + \underset{Feb.}{28} + \underset{Mar.}{31} + \underset{Apr.}{30} + \underset{May}{31} + \underset{June}{30} + \underset{July}{24} = 205\right]$$

$$\therefore \quad \delta = 23.45 \sin\left[\frac{360}{365}(284 + 205)\right] = \mathbf{19.82°}\ \textbf{(Ans.)}$$

Example 2.4. *Calculate the angle made by beam radiation with the normal to a flat plate collector, pointing the south location in New Delhi (27° 30′N, 76° 42′E) at 10.00 hour solar time on October 29. The collector is tilted at an angle of 35° with the horizontal. Also calculate the day-length.*

Solution. *Given:* φ = 27°30′ = 27.5°; solar time on October 29 = 10.00 hour (LST). Angle of tilt, β = 35°.

Incident angle, θ :

Since the surface is facing south, γ = *0*, therefore, the using following relation, we get:

$$\cos\theta = \sin\delta \sin(\phi - \beta) + \cos\delta \cos\omega \cos(\phi - \beta) \qquad \text{...[Eqn. (2.11)]}$$

Let us first calculate the declination (δ) by using the relation:

$$\delta = 23.45 \sin\left[\frac{360}{365}(284 + n)\right] \qquad \text{...[Eqn. (2.3)]}$$

Here, n = number of days = 302 for October 29

[n = 31+ 28 + 31 + 30 + 31 + 30 + 31 + 31 + 30 + 29 = 302]

$$\therefore \quad \delta = 23.45 \sin\left[\frac{360}{365}(284 + 302)\right] = -14.43°$$

Also, hour angle, $\omega = 15\ (12 - LST) = 15\ (12 - 10) = 30°$

Inserting various values in the above eqn., we have:

$$\cos\theta = \sin(-14.43°)\sin(27.5° - 35°) + \cos(-14.43)\cos 30°\cos(27.5° - 35°)$$
$$= 0.0325 + 0.8315 = 0.864$$

$\therefore$ $\theta =$ **30.23° (Ans.)**

Day-length, t_{day}:

$$t_{day} = \frac{2}{15}\left[\cos^{-1}(-\tan\phi\tan\delta)\right] \text{ hour} \qquad \text{...Eqn.(2.17)}$$

$$= \frac{2}{15}\left[\cos^{-1}\{-\tan 27.5°.\tan(-14.43°)\}\right]$$

$=$ **10.97 hours (Ans.)**

Example 2.5. *Calculate the angle made by beam radiation with normal to a flat plate collector on November 30, at 9.00 A.M. solar time for a location at 27°30′ N. The collector is tilted at an angle of latitude plus 12°, with the horizontal and is pointing due south.*

Solution. *Given:* $\phi = 27°30° = 27.5°$; Day of the year = November 30;

Local Solar Time (LST) = 9.00 AM; Tilt angle, $\beta = 27.5 + 12 = 39.5°$.

Angle made by the beam radiation, θ:

Since collector is pointing due south, therefore, $\gamma = 0$.

In such a case, we shall use the following equation:

$$\cos\theta = \sin\delta\sin(\phi - \beta) + \cos\delta\cos\omega\cos(\phi - \beta) \qquad \text{...[eq. (2.11)]}$$

Declination, δ:

In order to calculate declination (δ), using Cooper's equation, we get:

$$\delta = 23.45\sin\left[\frac{360}{365}(284 + n)\right]$$

(Here, n = number of days = 334 for November 30).

$$\therefore \quad \delta = 23.45\sin\left[\frac{360}{365}(284 + 334)\right] = -21.97°$$

Also, $\omega = 15\ (12 - LST) = 15\ (12 - 9) = 45°$

Inserting the various values in the above eqn., we obtain:

$$\cos\theta = \sin(-21.97°)\sin(27.5° - 39.5°) + \cos(-21.97°)\cos 45°\cos(27.5° - 39.5°)$$
$$= 0.0778 + 0.6414 = 0.7192$$

or, $\theta = \cos^{-1}(0.7192) =$ **44.01° (Ans.)**

Example 2.6. *Calculate the sun's attitude zenith and solar azimuth angles, at 9.00 A.M. solar time on August 30 at latitude 25°N.*

Solution. *Given*: Latitude, $\phi = 25°$; Local Solar Time (LST) on August 30 = 9.00 A.M.

Zenith angle, θ_z:

We know that: Hour angle, $\omega = 15\ (12 - LST)$

$$= 15\ (12 - 9) = 45°$$

δ (declination) can be found by using the relation:

$$\delta = 23.45 \sin\left[\frac{360}{365}(284+n)\right]$$

where, n = number of days = 242 for August 30

$$\left[n = \underset{Jan.}{31} + \underset{Feb.}{28} + \underset{Mar.}{31} + \underset{Apr.}{30} + \underset{May}{31} + \underset{June}{30} + \underset{July}{31} + \underset{Aug.}{30} = 242\right]$$

$$\therefore \quad \delta = 23.45 \sin\left[\frac{360}{365}(284+242)\right] = 8.48°.$$

In order to calculate θ_z, use the relation:

$$\cos\theta_z = \sin\phi \sin\delta + \cos\phi \cos\delta \cos\omega \qquad \text{...[Eqn. (2.10)]}$$
$$= \sin 25°.\ \sin 8.48° + \cos 25°.\ \cos 8.48°.\ \cos 45°$$
$$= 0.0623 + 0.6338 = 0.6961$$

$\therefore$ $\theta_z = \cos^{-1}(0.6961) = \mathbf{45.88°}$ **(Ans.)**

Solar azimuth angle, γ_s:

Using the relation:

$$\cos\gamma_s = \frac{\cos\theta_z \sin\phi - \sin\delta}{\sin\theta_z . \cos\phi} \qquad \text{...[Eqn. (2.14)]}$$
$$= \frac{\cos 45.88° . \sin 25° - \sin 8.48°}{\sin 45.88° . \cos 25°}$$
$$= \frac{(0.2942 - 0.1475)}{0.6506} = 0.2254$$

$\therefore$ $\gamma_s = \cos^{-1}(0.2254) = \mathbf{76.97°}$ **(Ans.)**

Example 2.7. *Calculate the sun's altitude and azimuth angle at 8.30 A.M. solar time on March 18 for a location at 35° N latitude.*

Solution: *Given* : Latitude, ϕ= 35° N; Day of the year = March 18; Local solar time (LST) = 8.30 A.M.

Sun's altitude angle, α:

ω (hour angle) is calculated from the relation:

$$\omega = 15\,(12 - \text{LST})$$
$$= 15\,(12 - 8.5) = 52.5°.$$

δ (declination) can be found by using the relation:

$$\delta = 23.45 \sin\left[\frac{360}{365}(284+n)\right]$$

where n = number of days = 77 for March 18.

$$\left[n = \underset{Jan.}{31} + \underset{Feb.}{28} + \underset{Mar.}{18} = 77\right]$$

$$\therefore \quad \delta = 23.45 \sin\left[\frac{360}{365}(284+77)\right] = -1.613°$$

α (altitude angle) can be calculated from the relation:

$$\sin\alpha = \sin\phi \sin\delta + \cos\phi \cos\delta \cos\omega \qquad \text{...[Eqn. (2.10)]}$$
$$= \sin 35° \sin(-1.613°) + \cos 35° \cos(-1.613°) \cos 52.5°$$

$$= -0.0161 + 0.4984 = 0.5145$$

$$\therefore \quad \alpha = \sin^{-1}(0.5145) = \mathbf{30.96° \ (Ans.)}$$

Solar azimuth angle, γ_s:

Using the relation:

$$\sin \gamma_s = \sec \alpha . \cos \delta . \sin \omega \qquad \text{...[Eqn. (2.73)]}$$

$$= \sec 30.96°. \cos(-1.613°) \sin 52.5°$$

$$= 0.9248$$

or, $$\gamma_s = \sin^{-1}(0.9248) = \mathbf{67.64° \ (Ans.)}$$

2.3. MEASUREMENT OF SOLAR RADIATION

It is important to measure solar radiation, owing to the increasing number of solar heating and cooling applications, and the necessity for *accurate solar radiation data to predict performance.*

The following *three* devices are used for measuring the solar radiations.

1. Pyranometer;
2. Pyrheliometers;
3. Sunshine recorders.

2.3.1. Pyranometer

A **pyranometer** is *a device used to measure the "total hemispherical solar radiation"*. The total solar radiation arriving at the outer edge of the atmosphere is called the '*solar constant*'.

The *working principle* of this instrument is that sensitive surface is exposed to total (beam, diffuse and reflected from the earth and surrounding) radiations.

The description of a pyranometer is given below:

Refer to Fig. 2.9.

Construction. It consists of a "black surface" which receives the beam as well diffuse radiations which rises heat. A "*glass dome*" prevents the loss of radiation received by the black surface. A "*thermopile*" is a temperature sensor, and consists of a number of thermocouples connected in series to increase the sensitivity. The "*supporting stand*" keeps the black surface in a proper position.

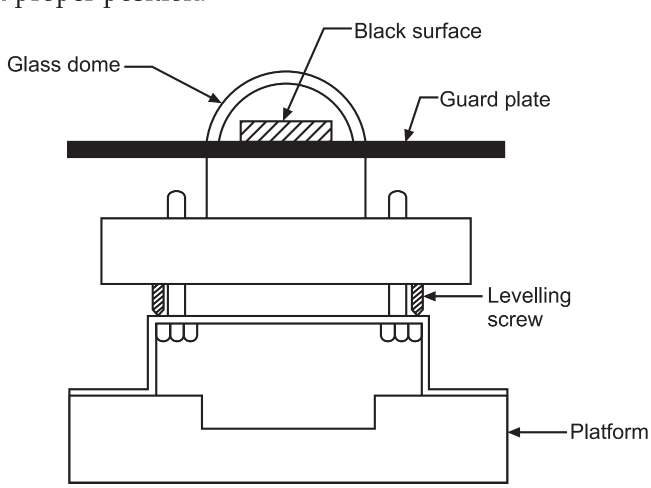

Fig. 2.9. Pyranometer.

Working: When the pyranometer is exposed to sun, it starts receiving the radiations. As a result, the surface temperature starts rising due to absorption of the radiation. The increase in the temperature of the absorbing surface is detected by the *thermopile*. The thermopile generates a *thermo emf* which is *proportional to the radiations absorbed;* this thermo emf is *calibrated in terms of the received radiations.* This will measure the global solar radiations.

2.3.2. Pyrheliometer

A **pyrheliometer** *is a device used to measure "beam or direct radiations".* It collimates the radiation to determine the beam intensity as a function of incident angle.

This instrument uses a collimated detector for measuring solar radiation from the sun and from a small portion of the sky around the sun at normal incidence.

The description of a thermoelectric type pyrheliometer is given below:

Refer to Fig. 2.10.

Construction: In this instrument, two identical blackened manganin strips A and B are arranged in such a way that either can be exposed to radiation at the base of *collimating tubes* by moving a *reversible shutter.*

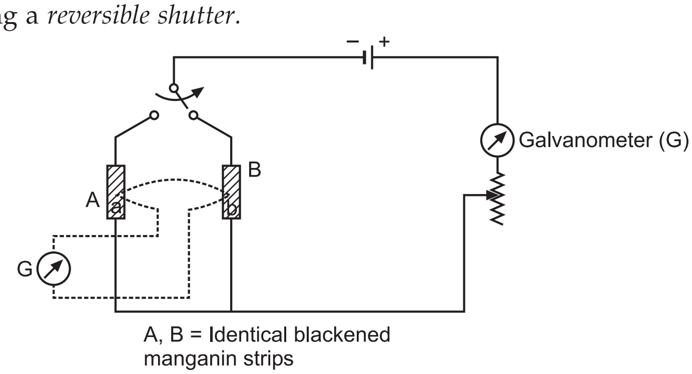

Fig. 2.10. Circuit diagram for the thermoelectric type pyrheliometer.

Working: One strip is placed in radiation and a current is passed through the *shaded strip* to heat it to the same temperature as the *exposed strip*. When there is *no difference in temperature*, the electrical energy supplied to shaded strip *must equal the solar radiation absorbed by the exposed strip*. Solar radiation is then determined by *equating the electrical energy to the product of incident solar radiation, strip area and absorptance.*

2.3.3. Sunshine Recorder

A **sunshine recorder** *is a device used to measure the "hours of bright sunshine in a day".*

The description of a sunshine recorder is given below:

Refer to Fig. 2.11.

Construction: It consist of a "*glass-sphere*" installed in a section of "*spherical metal bowl*" having grooves for holding a *recorder card strip"* and the glass sphere.

Working. The glass-sphere, which acts as a convex lens, focusses the sun's rays/beams to

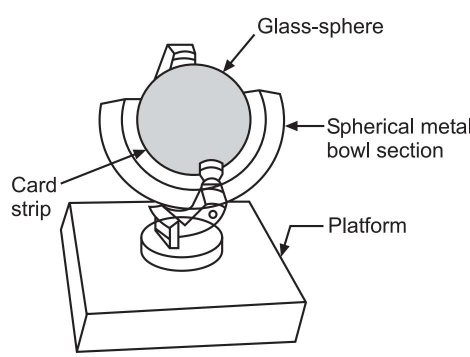

Fig. 2.11. Sunshine recorder.

a point on the card strip held in a groove in the spherical bowl mounted concentrically with the sphere.

Whenever there is a *bright sunshine, the image formed is intense enough to burn a spot on the card strip*. Through the day, the sun moves across the sky, the image moves along the strip. Thus a burnt space whose *length is proportional to the duration of sunshine* is obtained on the strip.

2.4. SOLAR RADIATION DATA

2.4.1. General Aspects

Solar radiation data should include the following informations:

(*i*) Whether they are *instantaneous* measurements or values *integrated* over some period of time (usually hour or days,) (*ii*) The *time* or *time period* of the measurements; (*iii*) Whether the measurements are of *beam*, *diffuse* or *total radiation* and the *instrument used*; (*iv*) The receiving surface *orientation* (usually horizontal, it may be inclined at a fixed slope or normal); (*v*) If averaged, the *period* over which they are averaged.

An instrument called **Solarimeter** is used to measure most of the data on solar radiation received on the surface of the earth. It gives *readings for instantaneous measurements at rate throughout the day for total radiation on a horizontal surface*. Integrating the plot of rate of energy received per unit area per unit time over a whole day gives the "*langleys*" of radiation received on a horizontal surface (the unit 'langley' has been adopted in honour of Samuel Langley who made the first measurement of the spectral distribution of the sun; 1 *langley = 1 cal/cm*^{2}).

- When data are *not* available, '**Maps**' can be used as a source of average radiation. **Charts** are also available for *clear day horizontal radiation for any period for any latitude.* **Tables** are also available for *hours of sunshine for various locations*.
- The solar radiation data is collected for various locations in the world on the basis of:

1. Solar power calculations with reference to the movement of the sun, latitude of the location etc.
2. Hourly measurements of solar radiation at the location and calculation of:
   (*i*) "*Daily average*" global radiation (H_{dg}) at the location for the month ($kJ/m^2.day$); (*ii*) "*Monthly average*" global radiation (H_{mg}) at the location for various months ($kJ/m^2.\ month$); (*iii*) "*Yearly average*" global radiation (H_{yg}) at the location for a few years ($kJ/m^2.\ year$).

The data in terms of kJ/m^2. day or kWh/m^2. day for various days/months/an year can be readily used for calculating:

(*i*) Available solar energy at the location;
(*ii*) Determining the surface area of the solar collectors;
(*iii*) Determining rating of solar plant.

2.4.2. Solar Radiation Data for India

- India is in the "*northern hemisphere*" within latitudes of 7° and 37.5° N.
- The average solar radiation values for India are between 12.5 and 22.7 MJ/m.^{2} day.
- The *peak solar radiation in India* occurs in some parts of Rajasthan and Gujrat and is equal to *25 MJ/m*^{2}.
- The solar radiation *reduces to about 60 percent* during monsoon months.

2.4.3. Solar Insolation

The **solar insolation** *is the solar radiation received on a flat horizontal surface at a particular location on earth at a particular instant of time*.

The *unit of solar radiation is* **W/m²**.

The *parameters of the solar insolation for a given flat horizontal surface are:*

(*i*) Daily variation (hour angle); (*ii*) Seasonal variation and geographical location of the particular surface; (*iii*) Atmospheric clarity; (*iv*) Shadows of trees, tall structures, adjacent solar panels etc.; (*v*) Degree of latitude for location; (*vi*) Surface area m^2; (*vii*) Tilt angle.

2.5. ESTIMATION OF AVERAGE SOLAR RADIATION

"Angstrom's equation" for average daily global radiation:

The *expression* for the average radiation on a horizontal surface in terms of constants *a* and *b* and observed values of average length of solar days, as suggested by **Angstrom** (1924) is given by:

$$\frac{H_g}{H_c} = a + b\left(\frac{L_a}{L_m}\right) \quad \text{...(2.21)}$$

where, H_g = Monthly average of the daily global radiation on a horizontal surface at the location (kJ/m² day),

H_c = Monthly average of the daily global radiation on the same horizontal surface at the same location but on *clear* sky day (kJ/m². day),

a, b = Constants determined from various cities in the world by measurements,

L_a = Average length of *solar day* for a particular month calculated/observed (hours), and

L_m = Length of the *longest solar day* in the month (hours).

Modified *Angstrom's equation:*

The modified Angstrom's equation is written by replacing H_c by H_0 in Eqn. (2.21) as:

$$\frac{H_g}{H_o} = a + b\left(\frac{L_a}{L_m}\right) \quad \text{...(2.22)}$$

where, H_o = Monthly average of the daily *extraterrestrial radiation*, which would fall on a horizontal surface at the location under consideration (kJ/m². day).

The expression for H_o is given by:

$$H_o = I_{sc}\left[1 + 0.033\cos\left(\frac{360n}{365}\right)\right]\int[(\sin\phi\sin\delta) + (\cos\phi\cos\delta\sin\omega)]\,dt \quad \text{...(2.23)}$$

where, n = Number of days in the year,

I_{sc} = Solar constant = 4870.8 kJ/m² hour,

ϕ = Angle of latitude for the location,

δ = Angle of declination of equatorial plane, and

ω = Hour angle for local apparent time.

The integral $\int dt$ in Eqn. (2.23) is in terms of hours. It can be changed to angle $\int d\omega$ in radians.

Now, $\quad 1 \text{ hour} = 15° = \frac{15\pi}{180}$ rad.

$\therefore \quad dt = \frac{180}{15\,\pi}d\omega = \frac{12}{\pi}d\omega$

Inserting the value of dt in Eqn. (2.23), and integrating between *sunrise* ($-\omega_s$) to sunset ($+\omega_s$) for total day light period, we get:

$$H_o = \frac{12}{\pi} I_{sc}\left[1 + 0.003\cos\left(\frac{360}{365}n\right)\right]\int_{-\omega_s}^{+\omega_s}(\sin\phi\sin\delta + \cos\phi\cos\delta\cos\omega)d\omega \quad ...(2.24)$$

By solving the integral, we get

$$H_o = \frac{24}{\pi} I_{sc}\left[\left\{1 + 0.033\cos\left(\frac{360}{365}n\right)\right\}(\omega_s\sin\phi\sin\delta + \cos\phi\cos\delta\sin\omega_s)\right]\text{kJ/m}^2\text{. day}$$

(where, ω_s is in radian) ...(2.25)

From Eqn. (2.25), the value of H_o is calculated in terms of kJ/m^2. day for a *clear sky day*, for a *flat surface location*.

Hence, the value of H_g (Daily global radiation-monthly average) for a horizontal surface at the location is then calculated by using the values of H_o, a, b in Eqn. 2.22.

- *H_o can be obtained from charts or can be calculated* by the *following* **Empirical relation:**

$$H_o = \frac{24}{\pi} I_{sc}\left[\left\{1 + 0.033\cos\left(\frac{360}{365}n\right)\right\}\left(\cos\phi\cos\delta\sin\omega_s + \frac{2\pi\omega_s}{360}\sin\phi\sin\delta\right)\right] \quad ...(2.26)$$

where, I_{sc} = Solar constant per hour,

n = Day of the year, and

ω_s = Sunrise hour angle (*degrees*).

The '*declination δ' can be obtained from 'Cooper equation*' and the sunrise hour angle (*i.e.*, ω_s) from the following equation:

$$\omega_s = -\tan\phi\tan\delta$$

Example 2.8. *The following observations were made in Bhopal during the mouth of March:*

Average length of the day = 8.4 hours; Longest day during the month = 9 hours; Angstrom's constants for Bhopal: a = 0.27, b = 0.50; Solar radiation per day for a clear day = 2100 J/m^2. day.

Calculate the average daily global radiation.

Solution. *Given:* L_a = 8.4 hours, L_m = 9 hours; a = 0.27; b = 0.50;

H_c = 2100 J/m^2. day.

Average daily global radiation, H_g:

Angstrom's equation is written as:

$$\frac{H_g}{H_c} = a + b\left(\frac{L_a}{L_m}\right) \quad ...[\text{Eqn. (2.21)}]$$

or,

$$H_g = H_c\left[a + b\left(\frac{L_a}{L_m}\right)\right]$$

Inserting the various values in the above eqn., we get:

$$H_g = 2100\left[0.27 + 0.50\left(\frac{8.4}{9.0}\right)\right] = \textbf{1547 kJ/m}^2\textbf{.day (Ans.)}$$

Example 2.9. *Calculate the average value of solar radiation on a horizontal surface located in Ahmedabad (22°00'N, 73° 10'E), for May 28. Average solar day hours are 10.5 hours. Angstrom's constants are: a = 0.28, b = 0.48.*

Solution: *Given*: ϕ = 22°, Average solar day hours L_a = 10.5 hours; Angstrom's constants: a = 0.28, b = 0.48.

Monthly average of daily global radiation, H_g :

We know that, $$H_g = H_o\left[a + b\left(\frac{L_a}{L_m}\right)\right]$$ ...[Eqn. (2.22)]

where, $$H_o = \frac{24}{\pi} I_{sc}\left[\left\{1 + 0.033\cos\left(\frac{360}{365}n\right)\right\}(\omega_s \sin\phi\sin\delta + \cos\phi\cos\delta\sin\omega_s\right] \text{ kJ/m}^2\text{. day}$$ ...[Eqn. (2.25)]

Calculations for δ, ω_s and L_m:

$$\delta = 23.45\sin\left[\frac{360}{365}(284 + n)\right]$$ ... [Eqn. (2.3)]

Here, n = number of days = 148 for May 28.

[n = 31 + 28 + 31 + 30 + 28 = 148]

$$\therefore \quad \delta = 23.45\sin\left[\frac{360}{365}(284 + 148)\right] = 24.44°$$

Now, sunshine hour angle, $\omega_s = \cos^{-1}(-\tan\phi \cdot \tan\delta)$ ... [Eqn. (2.16)]

Hence, $\omega_s = \cos^{-1}(-\tan 22° \cdot \tan 24.44°) = 100.58°$ (= 1.755 rad.)

Now, $$L_m = \frac{2}{15}\omega_s = \frac{2}{15}\times 100.58 = 13.41 \text{ hours}$$

Substituting the various values in the eqn. of H_0, we obtain:

$$H_o = \frac{24}{\pi}\times 4870.8\left[\left\{1 + 0.033\cos\left(\frac{360}{365}\times 148\right)\right\}\right.$$

$$\left.(1.755\sin 22°\sin 24.44° + \cos 22°\cos 24.44°\sin 100.58°\right]$$

$$= 37210\ (0.9726)\ (0.2720 + 0.8297) = 39871 \text{ kJ/m}^2\text{.day.}$$

Substituting the values in eqn. of H_g (above), we get:

$$H_g = 39871\left[0.28 + 0.48\left(\frac{10.5}{13.41}\right)\right]$$

$$= 26149 \text{ kJ/m}^2\text{.day}\left(26149\times\frac{1353}{4870.8}\right)$$

$$= \mathbf{7263.6\ W/m^2.\ day\ (Ans.)}$$

Example 2.10. *Calculate the average value of global solar radiation on a horizontal surface for March 21, at the latitude of 12°N if the constants are given as equal to 0.28 and 0.50 respectively. The ratio of average length of solar day and length of the longest solar day is 0.68.*

Solution. *Given:* ϕ = 12°; a = 0.28; b = 0.50; $\frac{L_a}{L_m} = 0.68$; *Day of year = March 21.*

Average value of global solar radiation; H_g:

$$H_g = H_o\left[a + b\left(\frac{L_a}{L_m}\right)\right]$$ ...[Eqn. (2.22)]

Declination, $$\delta = 23.45\sin\left[\frac{360}{365}(284 + n)\right]$$ ...[Eqn. (2.3)]

Here, n = 31 + 28 + 21 = 80 for March 21.

$$\therefore \quad \delta = 23.45\sin\left[\frac{360}{365}(284 + 80)\right] = -0.404°$$

Sunshine hour angle, $\omega_s = \cos^{-1}(-\tan\phi\tan\delta)$ ... [Eqn. (2.16)]

$= \cos^{-1}[-\tan 12° \tan(-0.404°)] = 89.9°$

The empirical relation for average insolation at *the top of the atmosphere* is given by:

Now, $$H_o = \frac{24}{\pi} I_{sc}\left[\left\{1 + 0.033\cos\left(\frac{360}{365}n\right)\right\}\left(\cos\phi\cos\delta\sin\omega_s + \frac{2\pi\omega_s}{360}\sin\phi\sin\delta\right)\right]$$ ...[Eq. 2.26]

$$= \frac{24}{\pi}\times 4870.8\left[\left\{1 + 0.033\cos\left(\frac{360}{365}\times 80\right)\right\}\{\cos 12°\cos(-0.404)\sin 89.9° + \frac{2\pi\times 89.9}{360}\times\sin 12°\times\sin(-0.404)\}\right]$$

$= 37210\ (1.006)\ (0.9781 - 0.0023) = 36527$ kJ/m^2. day

Hence, $H_g = 36527\ (0.28 + 0.50 \times 0.68) =$ **22647 kJ/m^2.day (Ans.)**

2.6. SOLAR RADIATION ON AN INCLINED SURFACE

The following three types of solar radiation constitute the total solar radiation on a surface:

(*i*) Beams solar radiation (I_b);

(*ii*) Diffuse solar radiation (I_d);

(*iii*) Solar radiation reflected from the ground and the surroundings.

Usually, I_b and I_d on a horizontal surface are recorded. In case of *non-availability of data* for beam and diffuse radiation, the following *expression* for beam and diffuse radiation on the horizontal surface may be used:

$$I_b = I_N \cos\theta_z \quad ...(2.27)$$

and, $$I_d = \frac{1}{3}(I_{ext.} - I_N)\cos\theta_z \quad ...(2.28)$$

Li and Jordon (1962) suggested the following *formula* to evaluate *total radiation on a surface of "arbitrary orientation":*

$$I_T = I_b R_b + I_d R_d + \rho R_r (I_b + I_d) \quad ...(2.29)$$

where, R_b, R_d *and* R_r = "*Conversion factors*" for *beam, diffuse* and *reflected components respectively;*

ρ = The reflection coefficient of the ground

= 0.2 and 0.7 for ordinary and snow covered ground respectively.

Expressions for Conversion factors:

Refer to Fig. 2.12.

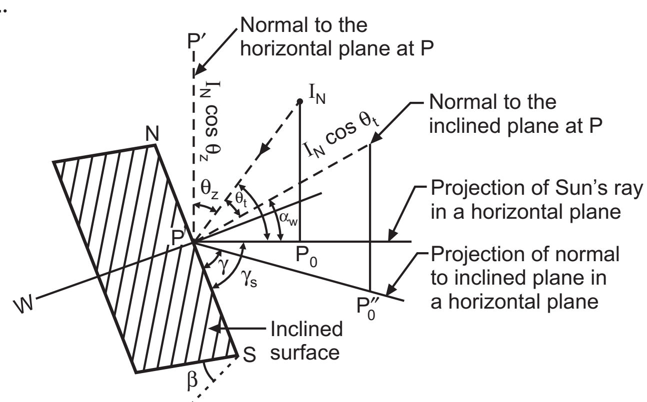

Fig. 2.12

(*i*) **R_b**: It *is defined as the ratio of flux of beam radiation (I_b) incident on an inclined surface (I_{bt})to that on a horizontal surface*. It is called *tilt factor* for beam radiation.

$$I_b = I_N \cos \phi_z \quad \text{...on } \textit{horizontal surface}$$

and,
$$I_{bt} = I_N \cos \phi_t \quad \text{...on tilted/inclined surface}$$

Mathematically,
$$R_b = \frac{I_b}{I_{bt}} = \frac{I_N \cos\theta_t}{I_N \cos z} = \frac{\cos\theta_t}{\cos z} \quad \text{...(2.30)}$$

where, I_N = Intensity of beam radiation,

θ_t = Angle of incidence on the inclined surface (it depends on several variables associated with the location and orientation of the surface and the direction of sun rays), and

θ_z = Angle of incidence on the horizontal surface.

For beam radiation, in most cases, the *tilted surface* faces due south *i.e.*,

$$\gamma = 0, \text{ for this case,}$$

$$\cos\theta = \cos\theta_t = \sin\delta \sin(\phi - \beta) + \cos\delta \cos\omega \cos(\phi - \beta)$$

For *horizontal force* ($\theta = \theta_z$),

$$\cos\theta = \cos\theta_z = \sin\phi \sin\delta + \cos\phi \cos\delta \cos\omega$$

Hence,
$$R_b = \frac{\cos\theta_t}{\cos\theta_z} = \frac{\sin\delta\sin(\phi-\beta) + \cos\delta\cos\omega\cos(\phi-\beta)}{\sin\phi\sin\delta + \cos\phi\cos\delta\cos\omega} \quad \text{...[2.30(a)]}$$

(*ii*) **R_d**: *It is the ratio of the flux of diffuse radiation falling on the tilted surface to that on the horizontal surface*.

If one "*radiation shape factor*" for the tilted surface with respect to sky, is $\frac{1+\cos\beta}{2}$, then

$$R_d = \frac{1+\cos\beta}{2} \quad \text{...(2.31)}$$

(*iii*) **R_r**: The reflected component comes *mainly from the ground and other surrounding objects*. If the considered reflected radiation is *diffuse and isotropic*, then the situation is opposite to that in the above case, and

$$R_r = (1 - \cos\beta)\frac{\rho}{2}$$

where ρ = Reflection coefficient of the ground (0.2 for ordinary and 0.7 for snow covered ground).

Hence combining all the three terms, we get:

$$I_T = I_b R_b + I_d\left(\frac{1+\cos\beta}{2}\right) + (I_b + I_d)\left(\frac{1-\cos\beta}{2}\right)\rho \quad \text{...(2.32)}$$

HIGHLIGHTS

1. '*Solar radiation*' is the energy emitted by sun. The *total radiation* is the sum of beam and diffuse radiations intercepted at the earth's surface per unit area of location.
2. The ratio of radiation received on earth's horizontal surface over a given period to radiation on equal surface area beyond earth's atmosphere in direction prependicular to the beam is called '*Clarity index*'.

3. The declination (δ) in degrees for any day may be calculated from the approximate equation of Cooper:

$$\delta = 234.5 \sin\left[\frac{360}{365}(284 + n)\right]$$

where, n is the day of the year.

4. Hour angle (ω) may be calculated from the follow relation:

$$\omega = 15\,(12 - \text{LST})$$

where, LST means local solar time.

5. *General equation* for $\cos\theta$ is given

$$\cos\theta = [\sin\phi\,(\sin\delta\cos\beta + \cos\delta\cos\gamma\cos\omega\sin\beta) + \cos\phi\,(\cos\delta\cos\omega\cos\delta - \sin\delta\cos\gamma\sin\beta) + \cos\delta\sin\gamma\sin\omega\sin\beta]$$

(*i*) *For surface facing south,* $\gamma = 0$:

$$\cos\theta = \sin\phi\,(\sin\delta\cos\beta + \cos\delta\cos\omega\sin\beta) + \cos\phi\,(\cos\delta\cos\omega\cos\beta - \sin\delta\sin\beta)$$

or,

$$\cos\theta = \sin\delta\sin(\phi - \beta) + \cos\delta\cos\omega\cos(\phi - \beta)$$

(*ii*) *For horizontal surface,* $\beta = 0°;\ \theta = \theta_z$:

$$\cos\theta\ (= \cos z) = \sin\phi\sin\delta + \cos\phi\cos\delta\cos\omega$$

Also,

$$\cos\theta = \cos\theta_z = \sin\alpha \qquad \left(\because \theta_z = \frac{\pi}{2} - \alpha\right)$$

(*iii*) *For vertical surface; $B = 90°$:*

$$\cos\theta = \sin\phi\cos\delta\cos\gamma\cos\omega - \cos\phi\sin\delta\cos\gamma + \cos\delta\sin\gamma\sin\omega$$

For vertical surface facing south, $B = 90°$, $\gamma = 0$:

$$\cos\theta = \sin\phi\cos\delta\cos\omega - \cos\phi\sin\delta$$

Also,

$$\cos\gamma_s = \frac{\cos\theta_z\sin\phi - \sin\delta}{\sin\theta_z\cos\phi}$$

where: θ = Incident angle; α = Altitude angle; ϕ = Latitude angle;
δ = Declination; ω = Hour angle; θ_z = Zenith angle;
γ_s = Solar azimuth angle;
γ = Surface azimuth angle; β = slope or tilt angle.

6. The value of ω_s (hour angle for sunrise and sunset) can be calculated from the following relation:

$$\omega_s = \cos^{-1}(-\tan\phi\tan\delta)$$

7. Total day-length (t_{day}) is given as:

$$t_{day} = \frac{2}{15}\left[\cos^{-1}(-\tan\phi\tan\delta)\right]\text{hours}$$

8. In India standard time is based on *"82.5° E longitude"*.

9. Solar radiation may be measured by following devices:
   (*i*) Pyranometer;
   (*ii*) Pyrheliometers;
   (*iii*) Sunshine recorder.

10. Angstrom's equation for the estimation of average solar radiation is given by:

$$\frac{H_g}{H_o} = a + b\left(\frac{L_a}{L_m}\right)$$

Here,

$$H_o = \frac{24}{\pi} I_{sc}\left[\left\{1 + 0.033\cos\left(\frac{360}{365}n\right)\right\}\left(\cos\phi\cos\delta\sin\omega_s + \frac{2\pi\omega_s}{360}\sin\phi\sin\delta\right)\right]$$

where, H_g = Monthly average of daily global radiation on a horizontal surface at the location (kJ/m^2. day)

H_o = Monthly average of the daily extraterrestrial radiation, which would fall on a horizontal surface at the location under consideration,

I_{sc} = Solar constant, 4870.8 kJ/m^2 hour,

a, b = Constants determined from various cities in the world by measurements,

L_a = *Average length of solar day* for a particular month calculated/observed (hours), and

L_m = Length of the *longest solar day* in the month (hours).

11. Liu and Jorden (1962) suggested the following formula to calculate total radiation on a surface of "arbitrary orientation":

$$I_T = I_b R_b + I_d\left(\frac{1+\cos\beta}{2}\right) + (I_b + I_d)\left(\frac{1-\cos\beta}{2}\right)\rho$$

where, I_T = Total radiation; I_b = Beam radiation; R_b = Tilt factor for beam rodiation; I_d = Diffuse radiation; β = Tilt angle; ρ = Reflection coefficient of the ground (0.2 for ordinary and 0.7 for snow covered ground).

THEORETICAL QUESTIONS

1. List the renewable energy sources which find their origin in sun.
2. State the advantages, disadvantages and applications of solar energy.
3. Define the following terms as applied to solar energy:
   (*i*) Solar radian; (*ii*) Extraterrestrial radiation;
   (*iii*) Beam radiation; (*iv*) Diffuse radiation.
4. State the reasons for variation in solar radiation reaching the earth than received at the outside of the atmosphere.
5. What is solar constant? Explain briefly.
6. What is air mass?
7. Define the terms: (*i*) Clarity index; (*ii*) Concentration ratio.
8. Define the following angles:
   (*i*) Latitude angle; (*ii*) Hour angle;
   (*iii*) Zenith angle; (*iv*) Surface azimuth angle.

9. Give the solar altitude angle expression for angle between incident beam and normal to a plane surface.
10. Explain briefly the following:
    (*i*) Day-length; (*ii*) Local apparent time (LAT)
11. Explain briefly the following devices used for the measurement of solar radiation:
    (*i*) Pyranometer; (*ii*) Pyrheliometer
12. Write a short note on 'Sun recorder'.
13. Define the term 'solar insolation'. State the parameters of solar insolation for a given flat horizontal surface.
14. Discuss briefly the "Angstrom's equation" used for the estimation of average solar radiation.

UNSOLVED EXAMPLES

1. Calculate the number of day-light hours at Delhi on June 21 in a leap year.
   (**Ans. 13.82 hours**)
2. Calculate the sunset hour angle and day-length at a location latitude of 35°N, on Feb. 14. (**Ans.** 80.23°; 10.7 hours)
3. Calculate the local apparent time (LAT) and declination at Ahmedabad (longitude 72°41′; E, latitude 23°00′N) corresponding to 1430, LST on December 15. (Given E = 5′13″). (**Ans.** 13 45′ 27″; –23° 20′ 6.79″)
4. Calculate the zenith and solar azimuth angles at 9 A.M. solar time on September 1 at latitude 23°N. (**Ans.** 45.87°; 78.01°)
5. Calculate the angle made by beam radiation with the normal to a flat plate collector, pointing due south location in New Delhi (28° 38′N, 77° 17′E) at 9:00 hour solar time on December 1. The collector is tilted at an angle of 36° with the horizontal. Also calculate the day-length. (**Ans.** 45.7°; 10.28 hours)
6. Calculate the daily global radiation on a horizontal surface at Baroda (22° 13′ 13N, 73°13′E) during the month of March if constants *a* and *b* are given equal to 0.28 and 0.48 respectively and average sunshine hours for day are 9.5.
   (**Ans.** 22485 kJ/m^2. day)
7. Calculate monthly average of daily global solar radiation on a horizontal surface located in Ahmedabad, (22° 00′N, 73° 10′E) for the month of April. Average solar day hours are 10 hours. Angstrom's constant for Ahmedabad are a = 0.28, b = 0.48. (**Ans.** 28270 kJ/m^2. day)
8. Estimate the average value of solar radiation on a horizontal surface for June 22, at the latitude of 10° N, if constants *a* and *b* are given as equal to 0.31 and 0.51 respectively. The ratio of average length of solar day and length of longest solar day is 0.55. (**Ans.** 21182. 4 kJ/m^2. day)