Simulating Observations Within the TRAPPIST-1 System

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Abstract

Three of the seven known planets in the TRAPPIST-1 exoplanetary system lie in the Goldilocks zone for liquid surface water. Here we use model emission spectra from Morley et al. (2017) to simulate what one of these planets may be able to observe of another over a range of spectrograph resolutions, telescope collecting areas, exposure times, throughput efficiencies, and observing qualities parameterized by uncorrelated noise.

Introduction

The TRAPPIST-1 system consists of at least seven small roughly Earth-sized planets orbiting an ultracool M-dwarf star. Of these seven, three lie within the star's roughly defined Goldilocks zone for liquid surface water. These planets are prime candidates for further study with transmission spectroscopy with *JWST*, and so Morley et al. (2017) presented model emission spectra for planets in this system for a variety of assumed surface pressures, compositions, and albedos. When the system's discovery was announced, I wondered what it would be like if sapient civilizations emerged on two neighbor planets. How would they see each other and at what point might one definitively recognize the other as alien life? This project answers a much simpler version of those questions: given the emission spectra of TRAPPIST-1 f and g, how would TRAPPIST-1 f observe TRAPPIST-1 g?

Emission Spectra

For TRAPPIST-1 f (the observer planet, where "we" are standing), I use the model that assumes an Earth-like atmospheric composition, has a bond albedo $A_B = 0.3$, and surface pressure $P_{\text{surf}} = 1$ bar. For TRAPPIST-1 g (our target planet), I selected the model with a Titan-like atmospheric composition, $A_B = 0$, and $P_{\text{surf}} = 0.001$ bar. Since the albedo is set to 0, we can neglect any reflectance from the host star and do not need to consider TRAPPIST-1's spectrum for this simulation.

The emission spectrum models show the flux emitted from the top of the atmosphere of their respective planet both of which are shown in Figure 1. An algebraic expression for this flux is given by

$$F_\lambda = \left(B_\lambda(T_{\text{surf}})e^{-\tau_\lambda} + \int_0^{\tau_\lambda} B_\lambda(T_{\tau'_\lambda})e^{-\tau'_\lambda}d\tau'_\lambda\right) \times 2\pi\,(1-\cos\theta) \tag{1}$$

where τ_λ is the optical depth through the whole atmosphere, T_{surf} is the surface temperature of the planet, θ is the angular radius of the planet, and $T(\tau'_\lambda)$ is the temperature of a thin slab of atmosphere with optical depth τ'_λ.

Observing Atmosphere Contribution

None of this matters for the target planet since we know the flux we receive is simply the given model emission spectra. But to account for the atmosphere we are looking through, we must know

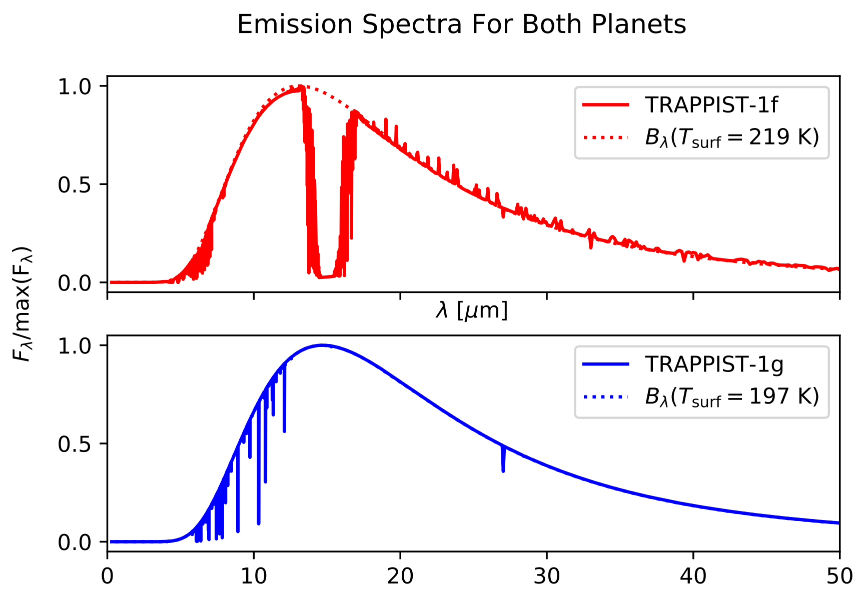

Fig. 1.— The model spectrum for the observer planet, TRAPPIST-1 f, is shown in the top subplot with the solid red line. The model spectrum for the target planet, TRAPPIST-1 g, is shown in the bottom subplot with the solid blue line. For both, we fit a blackbody radiation curve to find the surface temperature and plot the corresponding fit with a dotted line of the same color as the spectrum.

what flux our own atmosphere on TRAPPIST-1 f emits down towards our telescope pointed at TRAPPIST-1 g. Assuming an isotropically emitting atmosphere that does not scatter the radiation emitted by the ground back down to us, along with the more realistic assumption that we are observing at night and the sky is not scattering TRAPPIST-1's starlight, this atmospheric emission is given by the integral term in Equation 1. Unfortunately, Morley et al. (2017) does not provide the temperature-pressure profile or τ_λ for each model, only the final emission spectrum product.

Conveniently, there is a very strong absorption feature at $\sim 15\mu$m that we can safely assume shows the greatest optical depth along the entire spectrum. If we label the optical depth at this absorption feature $\tau_{\lambda_{\rm max}}$ and assume the temperature follows a simple linear trend from $T_{\rm skin}$ at the top of the atmosphere to $T_{\rm surf}$ at the surface, tracking the optical depth from 0 to $\tau_{\lambda_{\rm max}}$, we can express $T(\tau'_\lambda)$ as

$$T(\tau'_\lambda) = T_{\rm skin} + \frac{T_{\rm surf} - T_{\rm skin}}{\tau_{\lambda_{\rm max}}} \times \tau'_\lambda. \tag{2}$$

Then we use a Taylor expansion to make the integral term analytical and use the model spectrum to give us F_λ, turning Equation 1 into a transcendental equation we can solve for τ_λ. Given τ_λ (shown in Figure 2) and our linear temperature profile (Equation 2), we can solve for the integral term

$$\text{Observer Atmosphere Specific Intensity} = \int_0^{\tau_\lambda} B_\lambda(T_{\tau'_\lambda}) e^{-\tau'_\lambda} d\tau'_\lambda. \tag{3}$$

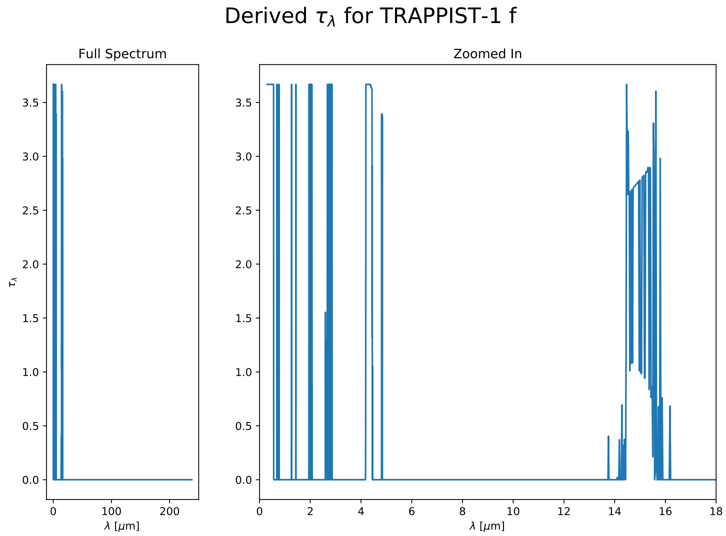

Fig. 2.— The left panel shows the optical depth over the whole spectrum while the right zooms into the absorption feature $\sim 15\mu$m. The fact that the optical depth does not go to $\tau_{\lambda_{\max}}$ in the middle of the feature shows that this is a rough approximation at best.

Signal-to-Noise Ratio Derivation

The model provided must be consistent with the spectrograph we observe with, so we convolve it with a Gaussian kernel to match the resolution we assume, and then multiply by the throughput of the telescope (f, letting through anywhere between 0% and 100% of the light). The flux we observe from our target is this convolved emission spectrum of TRAPPIST-1 g modified by the angular size of the target, the collecting area of our telescope (A), and the exposure time (t):

$$\text{Signal} = (F_\lambda * G(R \times \lambda_{\text{mean}})) \times \frac{\pi R_p^2}{4\pi d^2} \times A \times t \times f \tag{4}$$

where R_p is the radius of TRAPPIST-1 g and d is the distance between TRAPPIST-1 f and g, which we assume is simply the difference between their orbital distances (TRAPPIST-1 g is at perfect opposition to us and both planets are on perfectly circular orbits).

The noise is a combination of the Poisson noise of our signal, the Poisson noise of the flux from our own atmosphere which we circuitously (and only approximately) solved for above, and white noise due to instrumental limitations (σ^2):

$$\text{Noise} = \sqrt{\text{Signal} + \left(\int_0^{\tau_\lambda} B_\lambda(T_{\tau'_\lambda}) e^{-\tau'_\lambda} d\tau'_\lambda \times f \times A \times t \times 2\pi\,(1 - \cos\theta)\right) + \sigma^2} \tag{5}$$

Then the SNR is simply:

$$\text{SNR} = \frac{\text{Signal}}{\text{Noise}} \tag{6}$$

For the purposes of our simulation, the parameters we can vary are $R, f, t, A,$ and σ^2. Of these, A we vary by changing the diameter of the telescope, and use $\log\sigma^2$ and $\log R$ instead of the values directly. For a wavelength range between λ_{min} and λ_{max} we can show the SNR as a function of wavelength and the true SNR is a mean over this function. In the project notebook, we allow these parameters to be varied and show the mean SNR integrated over the range specified using an interactive widget. A few example outputs from the widget are shown in Figure 3 along with an explanation of the slider parameters in Figure 4.

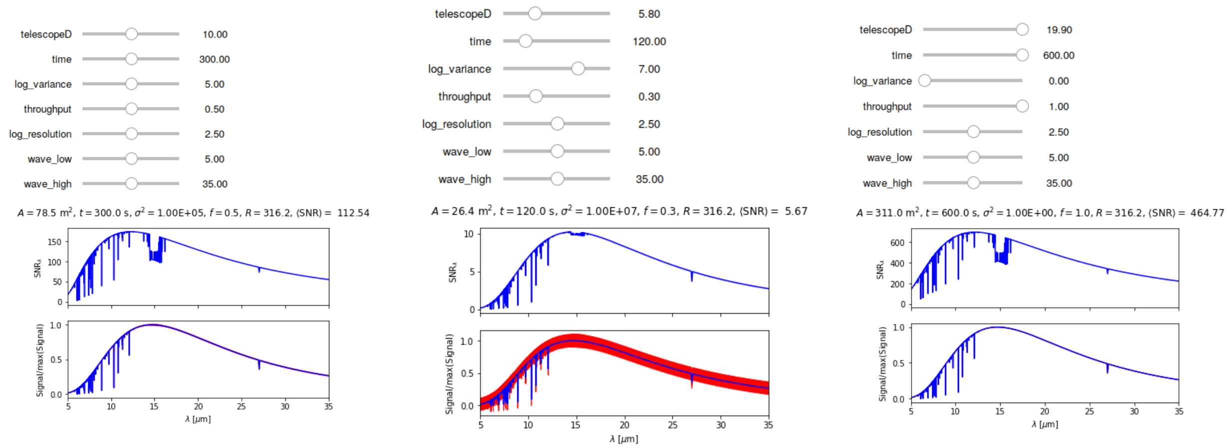

Fig. 3.— For each figure, the top subplot shows SNR at each λ and the bottom shows the signal received in blue with a red band indicating the range of noise (normalized by the maximum signal).

"telescopeD": Telescope diameter in m

"time": t in s

"log_variance": $\log \sigma^2$

"throughput": f

"log_resolution": $\log R$

"wave_low": λ_{min}

"wave_high": λ_{max}

Fig. 4.— This figure expands the names of the slider parameters above the plots in Figure 3.

References

Morley, C. V., Kreidberg, L., Rustamkulov, Z., Robinson, T., & Fortney, J. J. 2017, ApJ, 850, 121