Reaction Wheel Pendulum Filter Design Project

1 Overview

In this project you will design and test a Kalman filter and an extended Kalman filter for estimating the states of a reaction wheel pendulum control system. You will be provided with a mathematical model of the system. You will then implement the system in Simulink and develop a linearized model of the system. From the linearized model you will develop a Kalman filter to estimate the states from some sensor measurements. You will also develop an extended Kalman filter and compare the performances. You will compare the filter estimates to the true states to validate performance.

2 Specifics

The formal written report should be typed and should contain the following sections.

1. Introduction. Describe the overall objective of the project.
2. Reaction Wheel Model. Describe the reaction wheel mathematical model and the Simulink model. Simulate the system and verify performance. (Demonstrate that the system responses to initial conditions and external voltages are physically reasonable.)
3. Sensors. Describe the sensors you are going to use to help in estimating the system state. These sensors should include, at a minimum, an encoder to measure motor angle and an encoder or a gyro to measure pendulum angle. Provide model numbers for these components and provide specifications.
4. Kalman filter. Explain the development of your Kalman filter. Describe how you determined the appropriate sampling time. Provide linearized discrete-time models, the Kalman filter equations, and the Kalman gain matrix. The Kalman filter performance should be demonstrated in Simulink. You should demonstrate the performance to several initial conditions and for a variety of input voltages. (You can assume that the angles are generally in the neighborhood of zero.)
5. Extended Kalman filter. Describe the design of the extended Kalman filter. Demonstrate the performance through Simulink simulations.

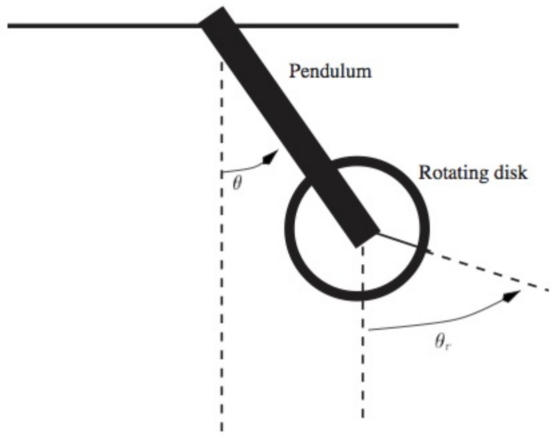

Figure 3.1: Reaction Wheel Pendulum Schematic (from Block, Astrom, Spong, 2007)

6. Conclusions. Discuss the advantages and disadvantages of both of the filters that you tested. This is an important section of your report. Discuss what your learned.

7. Include any MATLAB programs you use for the project and instructions for running your models in an appendix. Also include printouts of the Simulink block diagrams, if you have not included them in the body of the report. Put a zipped file containing your final Simulink models for the each of your filters and any data files required to simulate your systems in the D2L Dropbox. Provide instructions for running your software. All pages of the report (including appendices) should be on 8 1/2" by 11" paper.

3 Reaction Wheel Model

Figure 3.1 is a schematic diagram of the reaction wheel system. A motor is attached to the rotating disk (wheel). The pendulum is pivoted at the other end, so that it is free to rotate. As the motor turns the wheel, a reaction torque is applied to the pendulum, which causes the pendulum to move.

The parameters of the system are defined as follows:

- m_p = mass of the pendulum
- m_r = mass of the wheel
- $m = m_p + m_r$ = combined mass of rotor and pendulum
- J_p = moment of inertia of the pendulum about its center of mass
- J_r = moment of inertia of the rotor about its center of mass
- l_p = distance from pivot to the center of mass of the pendulum

- l_r = distance from pivot to the center of mass of the rotor
- l = distance from pivot to the center of mass of pendulum and rotor
- θ = angle of pendulum
- θ_r = angle of wheel
- $\theta_m = \theta_r - \theta$ = angle of motor

We can also define some combination parameters that will be used in the mathematical model of the reaction wheel system:

- $ml = m_p l_p + m_r l_r$
- $J = J_p + m_p l_p^2 + m_r l_r^2$

Assume that the values of the parameters are given by

- $m_p = 0.2164kg$
- $J_p = 2.233 \times 10^{-4} kgm^2$
- $m_r = 0.0850kg$
- $Jr = 2.495 \times 10^{-5} kgm^2$
- $m = 0.3014kg$
- $l = 0.1200m$
- $l_p = 0.1173m$
- $l_r = 0.1270m$
- $J = 4.572 \times 10^{-3} kgm^2$

The equations of motion for the reaction wheel pendulum are

$$J\ddot{\theta} + mglsin(\theta) = -\tau \tag{3.1}$$

$$J_r \theta_r = \tau \tag{3.2}$$

where τ is the torque produced by the dc motor. The equations for the motor operation are

$$Ri = e - K_b \dot{\theta} \tag{3.3}$$

$$\tau = K_t i \tag{3.4}$$

where K_b is the back emf constant, R is the armature resistance, i is the armature current, K_t is the motor torque constant and e is the voltage applied to the armature (the input u to the reaction wheel pendulum system). When SI units are used, it can be shown that the torque constant and the back emf constant are always equal. Assume the following values for the motor

- $K_t = 27.4 \times 10^{-3} Nm/A$
- $R = 12.1 ohms$
- $K_b = 27.4 \times 10^{-3} V/(rad/s)$

4 CONCLUSION

The written report will be due on May 6 before 5:00pm in my mailbox (Room 202 ES). If you can turn in your report before that time it will be appreciated. This project should be your own work; it should be done individually, not in teams.