CATALAN NUMBERS AND APPLICATIONS.

The Catalan numbers are a sequence of positive integers that appear in many counting problems in combinatorics. They count certain types of lattice paths, permutations, binary trees, and many other combinatorial objects.

- They satisfy the following equation:

$$C(n)=(2n)!/(n+1)!*(n)!$$

- The first person to discover catalan numbers was Leonhard Euler. In 1751,he discussed number of ways to cut a polygon into triangles with lines without any of the lines intersecting each other.

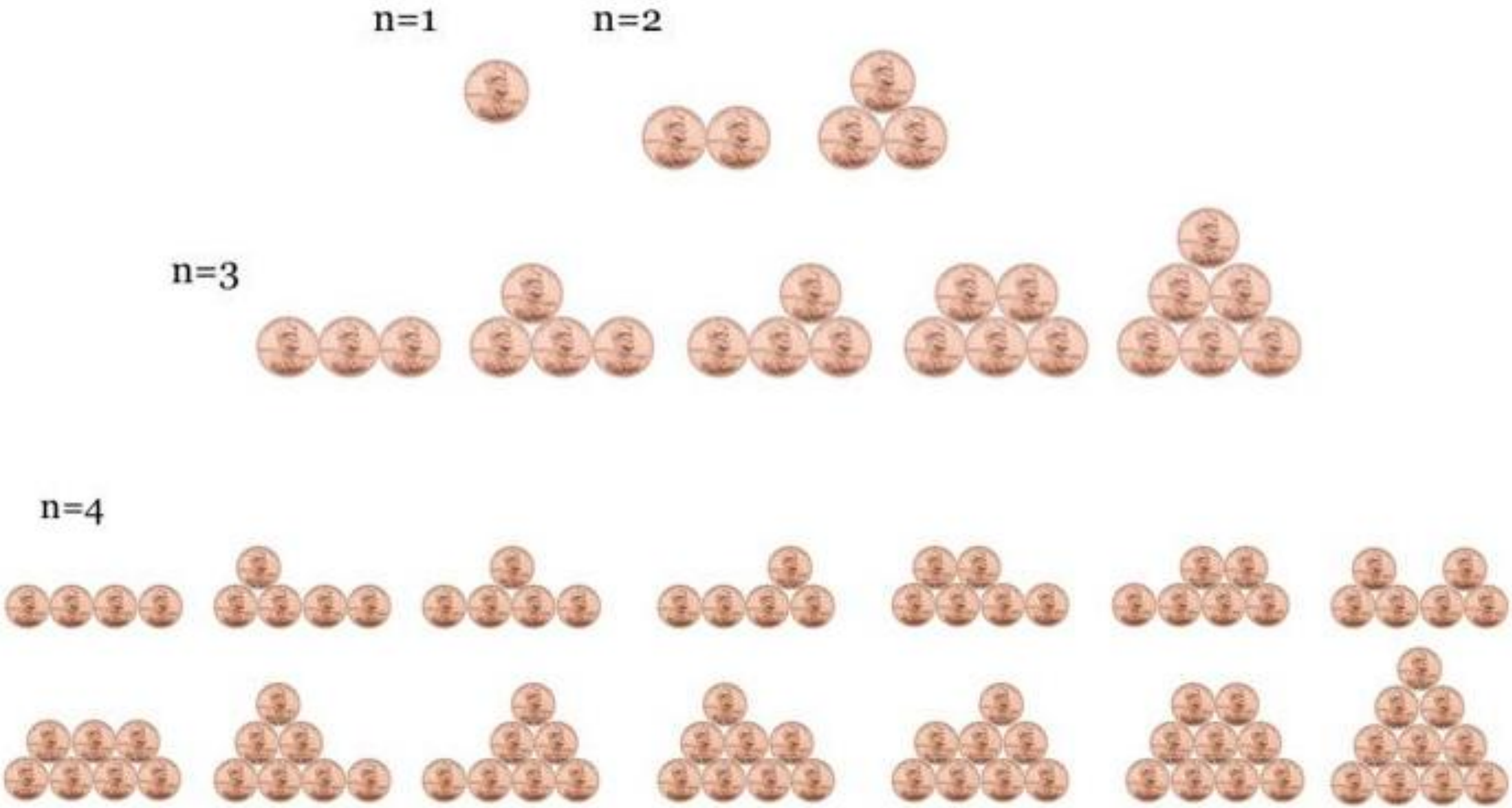
- The first few catalan numbers are given below:

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
C_n	1	1	2	5	14	42	132	429	1430	4862	16796	58786	208012	742900	2674440	9694845	35357670

APPLICATIONS.

1)Stacking coins:

We are going to stack coins on a bottom row that consists of n consecutive coins. It is not allowed to put the coins on the two sides of the bottom coins. How many ways there are to stack coins on the n coins?


n=1
n=2
n=3
n=4

2) Balanced parenthesis:We want to group a string of parentheses. Each open parenthesis must have a matching closed parenthesis. Therefore, "(( )( ))" is valid, but ")( )) ((" and "( ))( ) (" are not. How many groupings are there to group n pairs of parentheses?

$n = 0$	Do Nothing!					1 solution
$n = 1$	( )					1 solution
$n = 2$	(( ))		( )( )			2 solutions
$n = 3$	((( )))	(( )( ))	(( )) ( )	( ) (( ))	( )( )( )	5 solutions

<u>3)Mountain Ranges</u>: We want to form mountain ranges on a line with n upstrokes and n downstrokes. Same as the matching rule of the parentheses grouping problem, each upstroke must have a matching downstroke. How many mountain ranges are there for each value of n?

<table>
<tr><td>$n = 0$</td><td colspan="5">Do Nothing!</td><td>1 solution</td></tr>
<tr><td>$n = 1$</td><td colspan="5">/\</td><td>1 solution</td></tr>
<tr><td>$n = 2$</td><td colspan="2">/\/\</td><td colspan="3"> /\
/ \</td><td>2 solutions</td></tr>
<tr><td>$n = 3$</td><td>/\/\/\</td><td>   /\
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/   \</td><td>5 solutions</td></tr>
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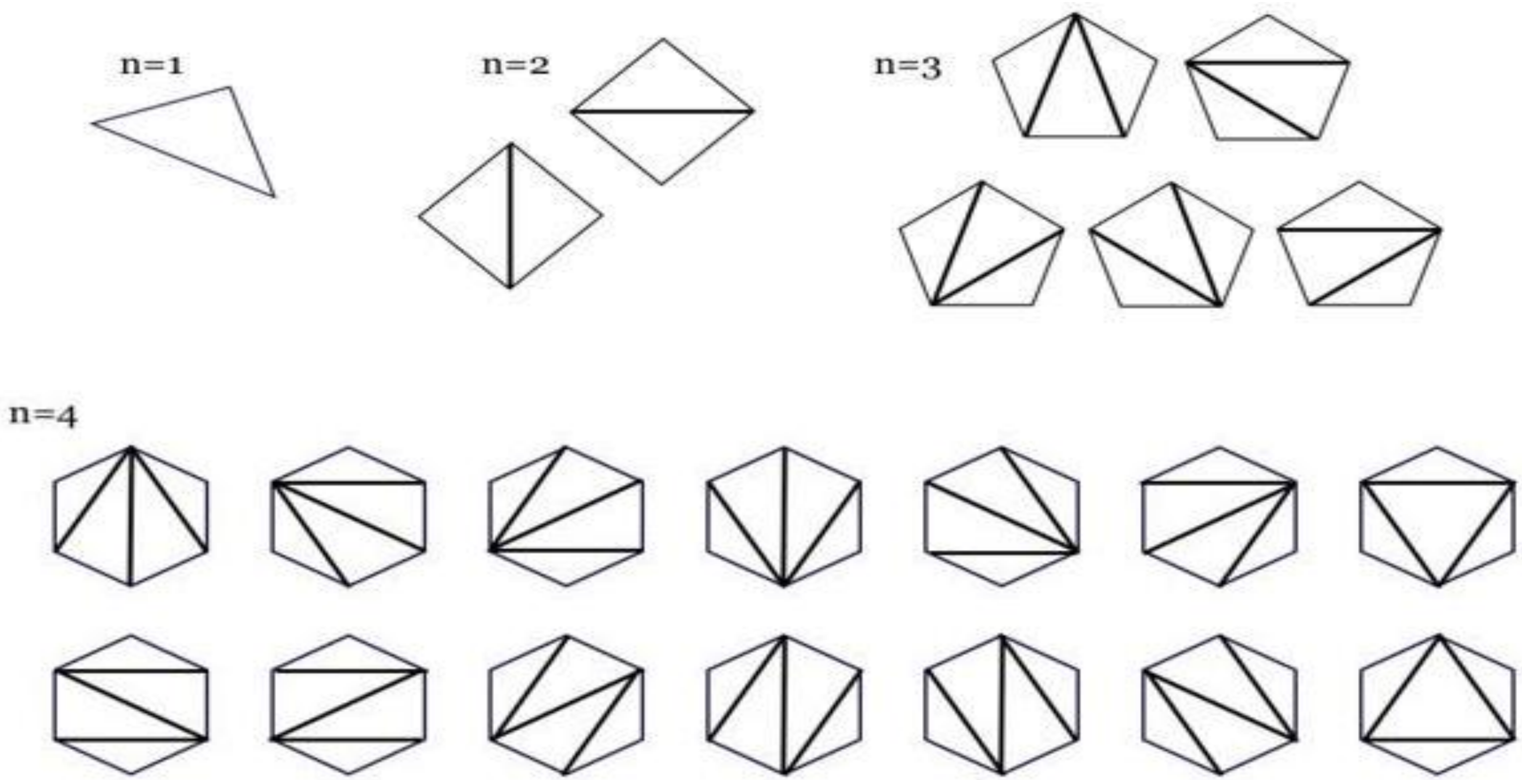
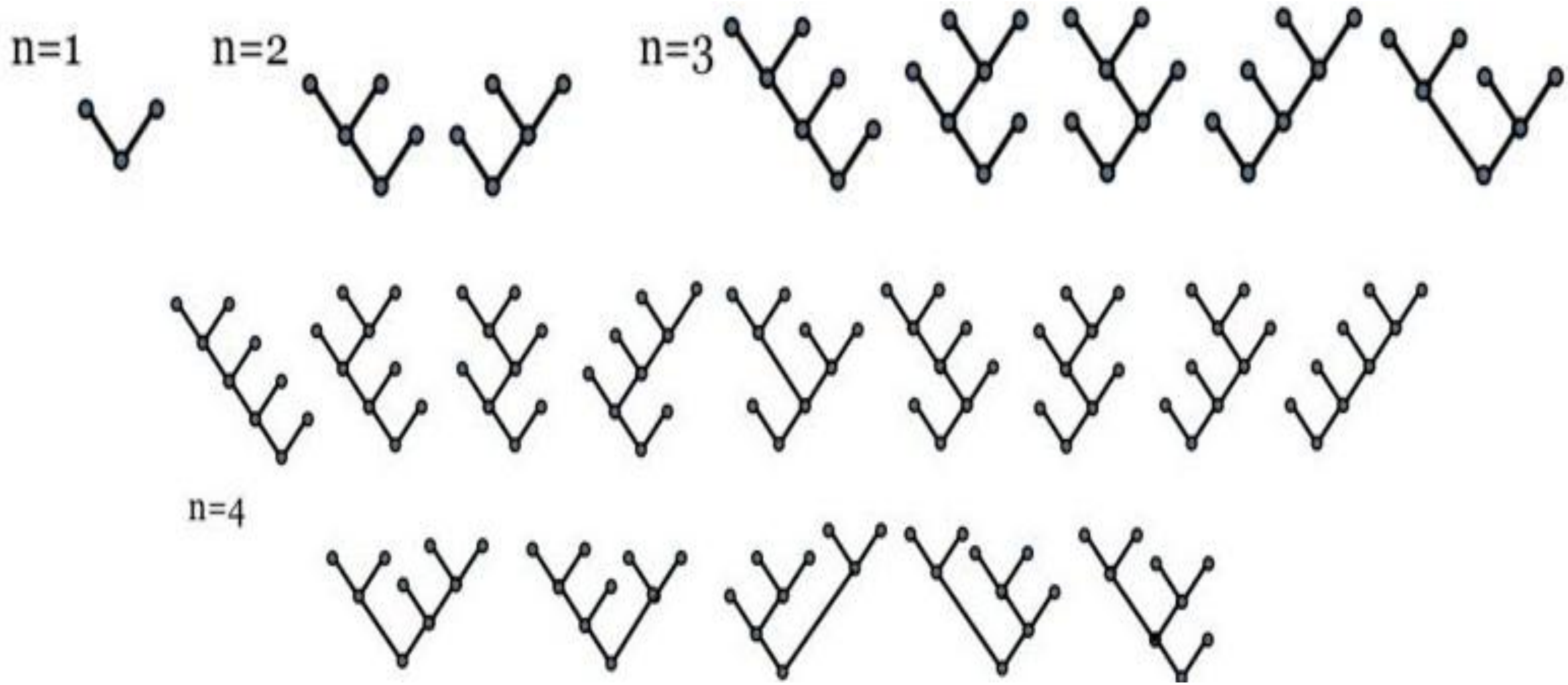
4) Polygon Triangulation:

We want to cut convex polygons into triangles by connecting the vertices with straight, non-intersecting lines. How many different ways are there for a polygon with n+2 sides?

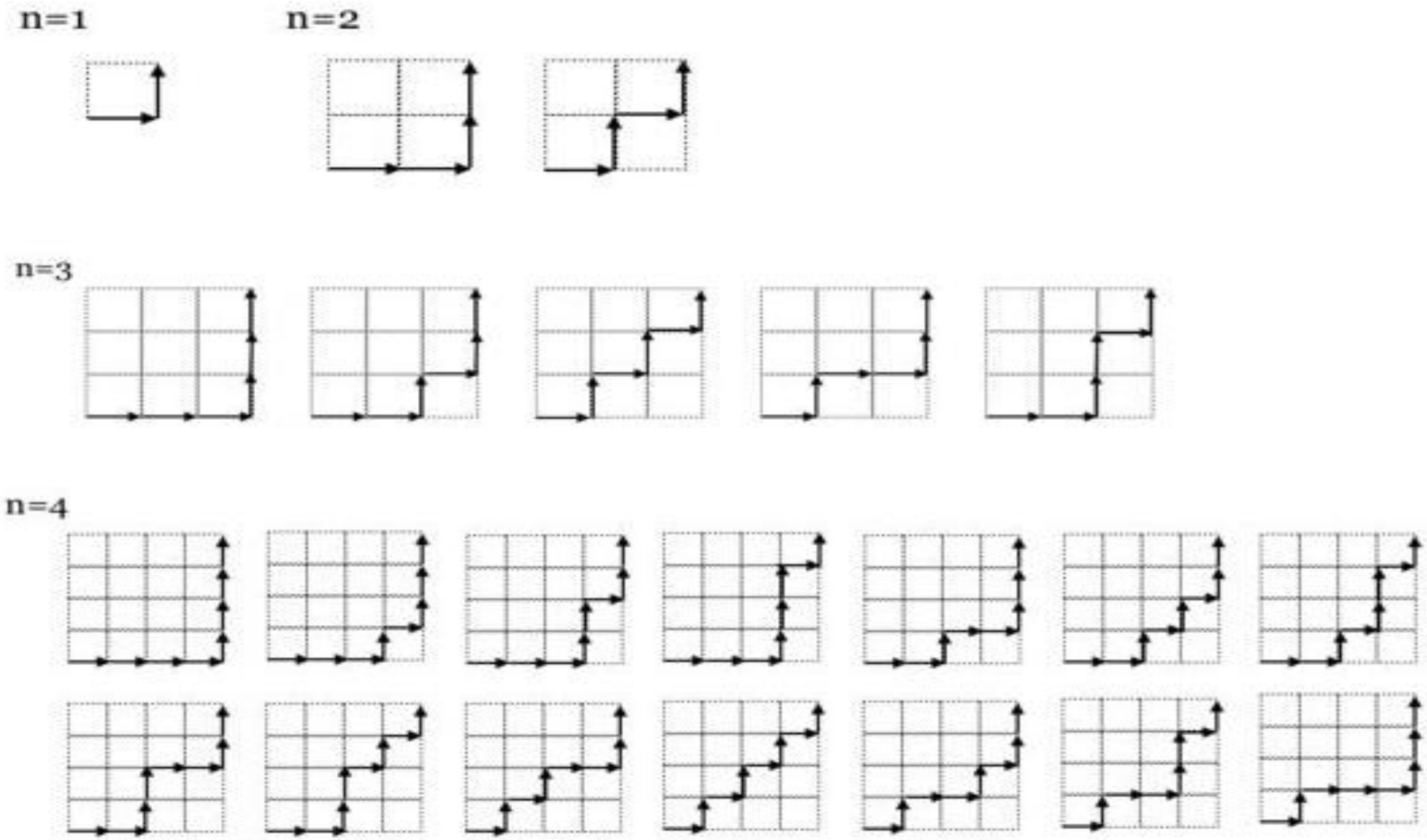


5)Binary Trees: How many full binary trees there are in order to have n internal nodes?

n: The number of internal nodes on full binary trees.



6) Binary Paths:In a n × n grid, we are going to joint the lower left point A and the upper right point B by a path. We are only allowed to go to the right or upwards for each unit, and cannot pass above the diagonal connecting A and B.



REFERENCES.

- http://mathforum.org/mathimages/index.php/Catalan_Numbers
- https://en.wikipedia.org/wiki/Catalan_number

THANK YOU