Fake Coin Problem

Algorithm Problem Solving
17ECSE309

By,
Yogesh Hiremath
343
01fe15bcs235

What is the problem?

Problem Statement:

Given a two pan fair balance and N identically looking coins, out of which only one coin is **lighter (or heavier)**. To figure out the odd coin, how many minimum number of weighing are required in the worst case?

Challenge:

Given a two pan fair balance and N identically looking coins out of which only one coin is defective. How can we trace which coin, if any, is odd one and also determine whether it is lighter or heavier in minimum number of trials in the worst case?

Example of N=8 coins

1. Compare the sum of weights 1-3, s_1 , and weights 4-6, s_2. There are three possibilities: $s_1 = s_2$,$s_1 < s_2$, and $s_1 > s_2$. If $s_1 = s_2$, then the counterfeit coin is isolated as the pair of coins 7 and 8, and the other six are all of standard weight. When coins 7 and 8 are compared with each other, one of them will be heavier. The other two cases use the second comparison to isolate a similar pair of coins and provide relative weight information. At the third weighing, one of the isolated pair is compared to a standard coin, and the answer becomes apparent.

1.1 $s_1 = s_2$,: The bad coin is isolated in the pair $w[7]$ and $w[8]$, and so they are compared to each other.

1.1.1 $w[7] > w[8]$: One of them, say $w[7]$, is compared with a standard coin, say $w[1]$.

1.1.1.1 $w[7] >$ *standard:* Then $w[7]$ is the bad coin, and it is heavy. Otherwise, $w[8]$ is the bad coin, and it is light.

1.1.2 $w[7] < w[8]$: This is symmetric to 1.1.1 above.

1.2 $s_1 < s_2$: Now $w[7]$ and $w[8]$ are known to be standard. Two of the coins in the triplets forming s_1 and s_2 need to be isolated, and two are switched. For example, the comparison might be between $\mathrm{s}_3 = w[1] + w[4]$ and $\mathrm{s}_4 = w[2] + w[5]$, isolating $w[3]$ and $w[6]$ from the other four.

1.2.1 s_3: $< \mathrm{s}_4$: Clearly coins 3 and 6 were standard, and either coin 1 is light or coin 5 is heavy because switching coins 2 and 4 left the balance tipped in the same direction. Either coin 1 or coin 5 is to be tested against a standard, because these two are now isolated as the bad pair.

1.2.2 $\mathrm{s}_3 = \mathrm{s}_4$: Then coin 3 or coin 6 is counterfeit, and one should be tested against one of the six known standards.

1.2.3 $s_1 > \mathrm{s}_4$: The switching of coins 2 and 4 caused the balance to shift, and so one of them is counterfeit.

1.3 $s_1 > s_2$: This is entirely symmetric to 1.2 above.

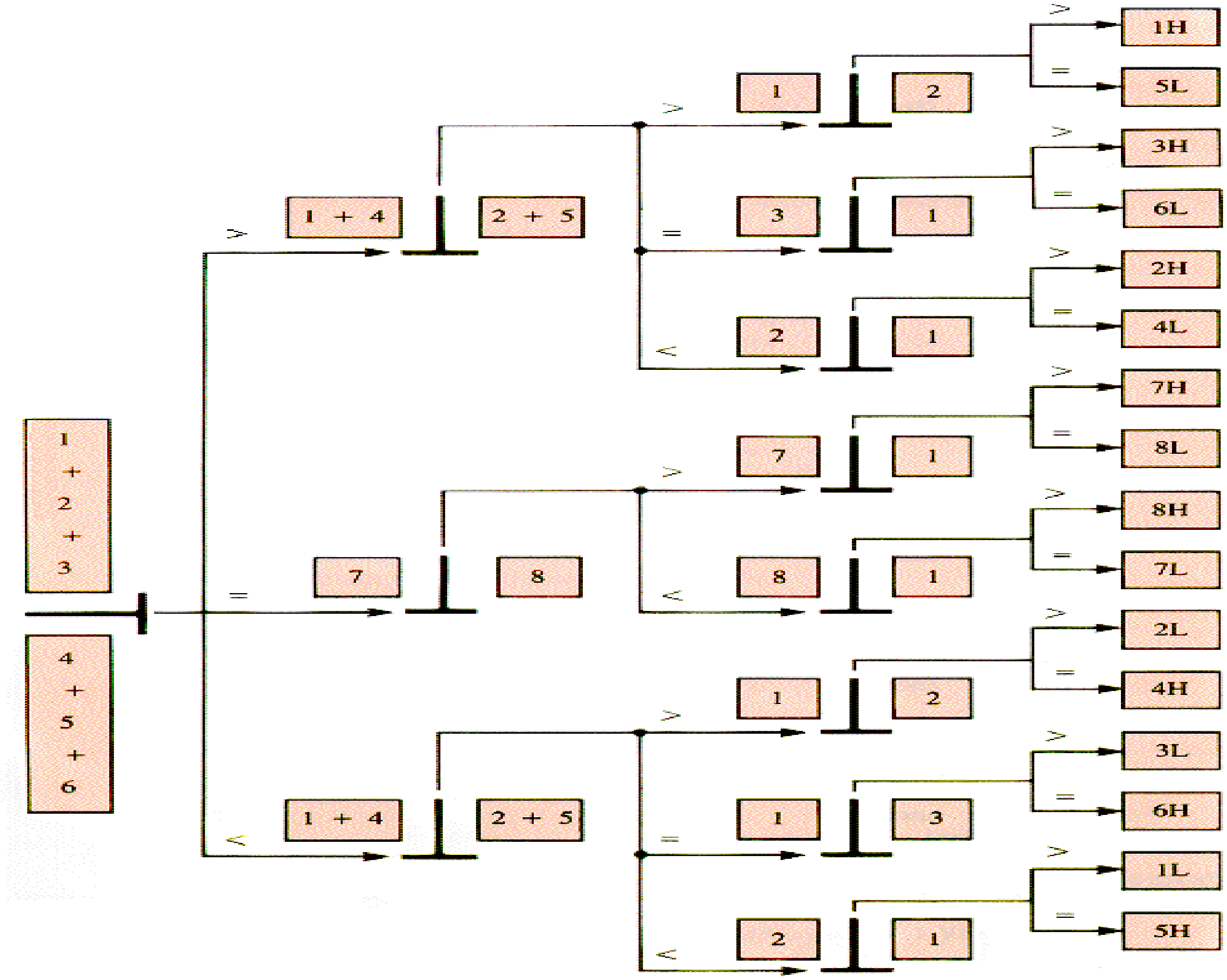

Figure 1: Illustrates the Comparison to be made for 8 fake coin problem

Applications

- Niobe, the protagonist of Piers Anthony's novel *With a Tangled Skein*, must solve the twelve-coin variation of this puzzle to find her son in Hell: Satan has disguised the son to look identical to eleven other demons, and he is heavier or lighter depending on whether he is cursed to lie or able to speak truthfully.
- Beremiz, the main character from Júlio César de Mello e Souza's book *The Man Who Counted*, encounters an Indian merchant that challenges him with the standard balance puzzle with eight identical-shaped pearls (one pearl slightly lighter than the rest).
- In the episode "The Wedding Scammer" of *Cyberchase*, the group of protagonists must find a lighter key out of eight keys (the other seven weigh the same), and they solve it suboptimally, with three weightings, when two suffices.

References

- https://en.wikipedia.org/wiki/Balance_puzzle
- https://www.cut-the-knot.org/blue/EightCoins.shtml
- http://www.icodeguru.com/vc/10book/books/book4/secv.htm