EULERIAN PATH AND CIRCUIT

ALGORITHMIC PROBLEM SOLVING 17ECSE309

Pavankumar Hittalamani
01FE15BCS129

What is Eulerian Path?

- An Euler path is a path that uses every edge of a graph exactly once.

What is Eulerian Circuit?

- An Euler circuit is a circuit that uses every edge of a graph exactly once.

- An Euler path starts and ends at different vertices
- An Euler circuit starts and ends at the same vertex.

The Criterion for Euler Paths

- Suppose that a graph has an Euler path P.
- For every vertex v other than the starting and ending vertices, the path P enters v the same number of times that it leaves v (say s times).
- Therefore, there are 2s edges having v as an endpoint. Therefore, all vertices other than the two endpoints of P must be even vertices.

- Suppose the Euler path P starts at vertex x and ends at y.
- Then P leaves x one more time than it enters, and leaves y one fewer time than it enters.
- Therefore, the two endpoints of P must be odd vertices.
- If a graph G has an Euler path, then it must have exactly two odd vertices.

The Criterion for Euler Circuits

- Suppose that a graph G has an Euler circuit C.
- For every vertex v in G, each edge having v as an endpoint shows up exactly once in C.
- The circuit C enters v the same number of times that it leaves v (say s times), so v has degree 2s.
- That is, v must be an even vertex

- If a graph G has an Euler circuit, then all of its vertices must be even vertices.

The Handshaking Theorem

The Handshaking Theorem says that

In every graph, the sum of the degrees of all vertices equals twice the number of edges.

If there are n vertices V1, . . . , Vn, with degrees d1, . . . , dn, and there are e edges, then

$$d1 + d2 + \cdots + dn-1 + dn = 2e$$

Or, equivalently,

$$e = d1 + d2 + \cdots + dn-1 + dn / 2$$

Fleury's Algorithm

To find an Euler path or an Euler circuit:

- Make sure the graph has either 0 or 2 odd vertices.
- If there are 0 odd vertices, start anywhere. If there are 2 odd vertices, start at one of them.
- Follow edges one at a time. If you have a choice between a bridge and a non-bridge, always choose the non-bridge.
- Stop when you run out of edges.

References

1. https://en.wikipedia.org/wiki/Eulerian_path
2. https://www.geeksforgeeks.org/eulerian-path-and-circuit /