# Algorithmic Problem Solving

## 17ECSE309

# **Keith Numbers**

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# Keith Number

A Keith number is an n-digit integer N>9 such that if a Fibonacci-like sequence (in which each term in the sequence is the sum of the previous terms) is formed with the first n terms taken as the decimal digits of the number N, then N itself occurs as a term in the sequence. For example, 197 is a Keith number since it generates the sequence 1, 9, 7, 1+9+7=17, 9+7+17=33, 7+17+33=57, 17+33+57=107, 33+57+107=197, …(Keith). Keith numbers are also called repfigit (*rep*etitive *fi*bonacci-like di*git*) numbers.

There is no known general technique for finding Keith numbers except by exhaustive search. Keith numbers are much rarer than the primes, with only 84 Keith numbers with <26 digits. The first few are 14, 19, 28, 47, 61, 75, 197, 742, 1104, 1537, 2208, 2580, 3684, 4788, 7385, 7647, 7909, ... (OEIS A007629). As of Mar. 31, 2006, there are 95 known Keith numbers (Keith). The number of Keith numbers having d=1,2, ... digits are 0, 6, 2, 9, 7, 10, 2, 3, 2, 0, 2, 4, 2, 3, 3, 3, 5, 3, 5, 3, 1, 1, 3, 1, 1, 3, 7, 1, 2, 5, 2, 4, 6, 3, ... (OEIS A050235), as summarized in the following table given in next slide.

| $d$ | $d$-digit Keith numbers |
|---|---|
| 2 | 14, 19, 28, 47, 61, 75 |
| 3 | 197, 742 |
| 4 | 1104, 1537, 2208, 2580, 3684, 4788, 7385, 7647, 7909 |
| 5 | 31331, 34285, 34348, 55604, 62662, 86935, 93993 |
| 6 | 120284, 129106, 147640, 156146, 174680, 183186, 298320, 355419, 694280, 925993 |
| 7 | 1084051, 7913837 |
| 8 | 11436171, 33445755, 44121607 |
| 9 | 129572008, 251133297 |
| 10 | (none) |
| 11 | 24769286411 96189170155 |
| 12 | 171570159070, 202366307758, 239143607789, 296658839738 |
| 13 | 1934197506555, 8756963649152 |
| 14 | 43520999798747, 74596893730427, 97295849958669 |
| 15 | 120984833091531, 270585509032586, 754788753590897 |
| 16 | 3621344088074041, 3756915124022254, 4362827422508274 |
| 17 | 11812665388886672, 14508137312404344, 16402582054271374, 69953250322018194, 73583709853303061 |
| 18 | 119115440241433462, 166308721919462318, 301273478581322148 |
| 19 | 1362353777290081176, 3389041747878384662, 5710594497265802190, 5776750370944624064, 6195637556095764016 |
| 20 | 12763314479461384279, 27847652577905793413, 45419266414495601903 |
| 21 | 855191324330802397989 |
| 22 | 7657230882259548723593 |
| 23 | 26842994422637112523337, 368992775938526099974 03, 6133385360212981918966 8 |
| 24 | 229146413136585558461227 |
| 25 | 9838678687915198599200604 |
| 26 | 18354972585225358067718266, 19876234926457288511947945, 98938191214220718050301312 |
| 27 | 153669354455482560987178342, 154677881401007799974564336, 133118411174059688391045955, 154140275428339949899922650, 29576823736129170864522 7474, 95663372046411451589031 8410, 98824231039386039006691 1414 |

# Algorithm

- Store the ‘n’ digits of given number “x” in an array “terms”.
- Loop for generating next terms of sequence and adding the previous ‘n’ terms.
- Keep storing the next_terms from step 2 in array “terms”.
- If the next term becomes equal to x, then x is a Keith number. If next term becomes more than x, then x is not a Keith Number.

# References


- http://mathworld.wolfram.com/KeithNumber.html
- https://www.geeksforgeeks.org/keith-number/