

# **CRACK A HACK**

## **BUILD A STRING**

**Course: ALGORITHMIC PROBLEM SOLVING**

**Course code: 17ECSE309**

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# Problem Statement:

Greg wants to build a string  $S$ , of length  $N$ . Starting with an empty string, he can perform 2 operations:

1. Add a character to the end of  $S$  for  $A$  dollars.
2. Copy any substring of  $S$ , and then add it to the end of  $S$  for  $B$  dollars.

Calculate minimum amount of money Greg needs to build  $S$ .

## Solution:

**The problem is to be solved dynamically and knowledge of suffix array must be used.**

- $dp[x]$  is the answer for prefix size of  $x$ .
- $dp[x-1] \leq dp[x]$ .

Variants:

```
1) we have got prefix(x) from prefix(x-1). that means  $dp[x] = dp[x-1] + A$  and  $A > 0$ 
   so  $dp[x-1] < dp[x]$ 

2) we have got prefix(x) NOT from prefix(x-1). that means  $dp[x] = dp[x-k] + B$  and  $B > 0$  and  $k > 0$ .
   so:
   if  $k=1$  then:  $dp[x] = dp[x-1] + B$  so  $dp[x-1] < dp[x]$ 
   if  $k>1$  then:  $dp[x] = dp[x-k] + B$  and  $dp[x-1] = \min(dp[x-k] + B, \text{other variants})$ 
                  because  $\text{substr}[x-k+1, x-1]$  is in  $\text{substr}[x-k+1, x]$ 
                  so  $dp[x-1] \leq dp[x]$ 
```

So that means if we want to build  $\text{prefix}(x)$ , there are two variants.

- 1) build  $\text{prefix}(x-1)$  and add last symbol.
- 2) build  $\text{prefix}(x-k)$  and copy substring size of  $k$ , where  $k$  is maximum.

So for each  $x$  we need to find maximum  $k$  where  $\text{substr}[x-k+1, x]$  is in  $\text{substr}[1, x-k]$ . because of *Lemma 1*, we can search  $k$  using binary search.

Now we need to check if  $\text{substr}[x-k+1, x]$  is in  $\text{substr}[1, x-k]$  or not.

we will use suffix array, in sorted suffixes we can easily find all substrings which are equal to given substring.

## Python Code:

```
# Enter your code here. Read input from STDIN. Print output to
STDOUT
def sol():
    N, A, B = map(int, raw_input().strip().split())
    S = raw_input().strip()
    res = [0]*N
    res[0] = A
    maxl = 0
    for i in range(1,N):
        minv = res[i-1] + A
        cp, idx, newl = False, i, 0
        for k in range(maxl,-1,-1):
            if S[i-k:i+1] in S[0:i-k]:
                cp, idx, newl = True, i-k, k+1
                break
        if cp: minv = min(minv, res[idx-1]+B)
        maxl = newl
        res[i] = minv
    print res[-1]
T = int(raw_input().strip())
for x in range(T):
    sol()
```

## References:

- 1) <https://www.geeksforgeeks.org/suffix-array-set-1-introduction/>
- 2) [https://www.researchgate.net/publication/47841538\\_Fast\\_Lightweight\\_Suffix\\_Array\\_Construction\\_and\\_Checking](https://www.researchgate.net/publication/47841538_Fast_Lightweight_Suffix_Array_Construction_and_Checking)
- 3) [https://en.wikipedia.org/wiki/Maximum\\_subarray\\_problem](https://en.wikipedia.org/wiki/Maximum_subarray_problem)
- 4) <https://discuss.codechef.com/questions/21385/a-tutorial-on-suffix-arrays>