Pigeon and Pigeon hole Principle

Algorithmic Problem Solving
17ECCSE309

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Introduction

In mathematics, the pigeonhole principle states that if n items are put into m containers, with n > m, then at least one container must contain more than one item.[1] This theorem is exemplified in real life by truisms like "in any group of three gloves there must be at least two left gloves or two right gloves". It is an example of a counting argument.

Theorem

If “A” is the average number of pigeons per hole, where A is not an integer then:

- At least one pigeon hole contains ceil[A] (smallest integer greater than or equal to A) pigeons
- Remaining pigeon holes contains at most floor[A] (largest integer less than or equal to A) pigeons

Examples-1 : Pigeons

If (Kn+1) pigeons are kept in n pigeon holes where K is a positive integer, what is the average no. of pigeons per pigeon hole?

Solution: average number of pigeons per hole = (Kn+1)/n = K + 1/n

Therefore at least a pigeonholes contains (K+1) pigeons i.e., ceil[K +1/n] and remaining contain at most K i.e., floor[k+1/n] pigeons.

i.e., the minimum number of pigeons required to ensure that at least one pigeon hole contains (K+1) pigeons is (Kn+1).

Example-2 : Marbles

A bag contains 10 red marbles, 10 white marbles, and 10 blue marbles. What is the minimum no. of marbles you have to choose randomly from the bag to ensure that we get 4 marbles of same color?

Solution: Apply pigeonhole principle.

No. of colors (pigeonholes) n = 3

No. of marbles (pigeons) K+1 = 4

Therefore the minimum no. of marbles required = Kn+1

By simplifying we get Kn+1 = 10.

Verification: ceil[Average] is [Kn+1/n] = 4

[Kn+1/3] = 4

Kn+1 = 10

i.e., 3 red + 3 white + 3 blue + 1(red or white or blue) = 10

References

- https://www.geeksforgeeks.org/discrete-mathematics-the-pigeonhole-principle/
- https://en.wikipedia.org/wiki/Pigeonhole_principle
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