# Lucas Theorem

**Algorithmic Problem Solving 17ECSE309**

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# Why we require Lucas Theorem?...

Consider the case of solving combinatorial problem ,Which can result in hundreds of digits ,Which becomes much difficult to compute .

So most of the competitive coding platforms seek the result by applying modulus operator with the result .

# Lucas Theorem

Lucas theorem states that for non-negative integers n and r, and a prime p,

For non-negative integers n and r and a prime p, the following congruence relation holds:

$$\binom{n}{r} \equiv \prod_{i=0}^{k} \binom{n_i}{r_i} \pmod{p},$$

where

$$n = n_k p^k + n_{k-1} p^{k-1} + \cdots + n_1 p + n_0,$$

and

$$r = r_k p^k + r_{k-1} p^{k-1} + \cdots + r_1 p + r_0$$

# Example:-

Consider (9498 C 6542) % 11

**N=9498 R =6789 P=11**

**Note :- Lucas Theorem applies only when P is a Prime number**

**(9498 C 6789) % 11 =?**

**Represent 9498 in base 11 format ===== 7155**

**Represent 6789 in base 11 format ===== 5112**

**By Lucas Theorem ,The answer reduces to**

**( (7 C 1) * (1 C 1) * (5 C 1) * (5 C 2) ) % 11**

**= ( 7 * 1 * 5 * 10 ) % 11**

**= ( 350 ) % 11**

**= 9**

**By normal method ,the answer would have been 7.915523502*E*+2463 % 11,**
**Which most of the compilers fail to compute such a huge number .**

# Applications:-

1. In reducing Binomial Coefficients

2. To get a remainder when binomial coefficients are divided by prime numbers.

# References:-


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