Multiplication of large numbers using Divide and Conquer

Algorithmic Problem Solving

Course Code:17ECCSE309

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Introduction

Large integers with over 100 decimal digits long are too long to fit in a single word of a modern computer, hence require special algorithms to treat them. One of such algorithms is using Divide and Conquer. The basic step of this algorithm is a formula that allows one to compute the product of two large numbers x and y using three multiplications of smaller numbers, each with about half as many digits as x or y.

For very large numbers, this algorithm can be used recursively.

Algorithm

Let A & B be two n-digits integers where n is a positive even number.

Let

a1 - first half of a's digits

a0 – second half of a's digits

b1 - first half of b's digits

b0 – second half of b's digits

Then,

$c = A * B$

$= (a_1 10^{n/2} + a_0) * (b_1 10^{n/2} + b_0)$

$=(a_1 * b_1)10^n + (a_1 * b_0 + a_0 * b_1)10^{n/2} + (a_0 * b_0)$

Examples

- If A=34 and B=23

  $C=A*B=(3*10^2+4)*(2*10^2+3)$

  $=(3*2)\ 10^2+((3*3)+(4*2))\ 10^1+(4*3)$

Similarly,

- If A = 123456, B = 117933:

Then,

$C = A*B = (123*10^3+456)*(117*10^3+933)$

$=(123 * 117)10^6 + (123 * 933 + 456 * 117)10^3 + (456 * 933)$

References

- http://www.allsyllabus.com/aj/note/Computer_Science/Analysis_and_Design_of_Algorithms/Unit4/Multiplication%20of%20large%20integers.php#.WwWKyEiFNPY
- https://en.wikipedia.org/wiki/Divide_and_conquer_algorithm