APS Topic Presentation

TRAVELING SALESPERSON PROBLEM

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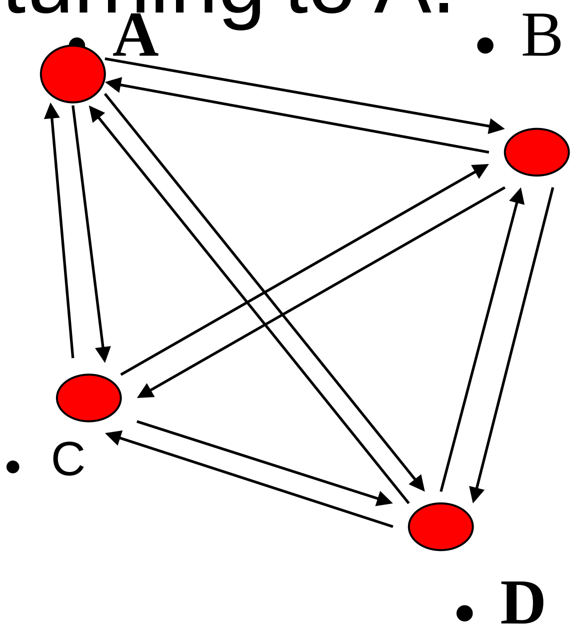
Problem Statement

Find the least cost tour starting at A, traveling through the other three cities exactly once and returning to A.



	A	B	C	D
A	-	3	4	6
B	2	-	5	2
C	7	4	-	3
D	5	6	7	-

Applications

- School Bus routing.

- Robotic welding in the car industry.

- Printed circuit board drilling and laser cutting of integrated circuits.
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- Job processing: Chemical plants – cost of setup to produce chemicals.

Given: Set of cities $\{c_1, c_2, \ldots, c_N\}$.

For each pair of cities $\{c_i, c_j\}$, a distance $d(c_i, c_j)$.

Find: Permutation that minimizes the overall cost of the travel(brute force approach).

Number of possible tours by generating all permutations:

$N! = 1\times2\times3\times\cdots\times(N\text{-}1)\times N = \Theta(2^{N\log N})$

$10! = 3{,}628{,}200$

$20! \sim 2.43\times10^{18}$ (2.43 quadrillion)

Dynamic Programming Solution:

$O(N^2 2^N) = o(2^{N\log N})$

Dynamic Programming Algorithm

For each subset C' of the cities containing c_1, and each city $c \in C'$, let $f(C',c)$ = Length of shortest path that is a permutation of C', starting at c_1 and ending at c.

$f(\{c_1\}, c_1) = 0$

For $x \notin C'$, $f(C' \cup \{x\},x) = \mathrm{Min}_{c \in C'} f(C',c) + d(c,x)$.

Optimal tour length $= \mathrm{Min}_{c \in C} f(C,c) + d(c, c_1)$.

Running time: $\sim(N-1)2^{N-1 \text{ items}}$ to be computed, at time N for each = $O(N^2 2^N)$

How hard is the problem

Number of possible tours generating all permutations:

$N! = 1\times2\times3\times\cdots\times(N\text{-}1)\times N = \Theta(2^{N\log N})$

$10! = 3{,}628{,}200$

$20! \sim 2.43\times10^{18}$ (2.43 quadrillion)

Dynamic Programming Solution:

$O(N^2 2^N)$

$10^2 2^{10} = 102{,}400$

$20^2 2^{20} = 419{,}430{,}400$

References

- https://www.geeksforgeeks.org/travelling-salesman-problem-set-1/
- https://en.wikipedia.org/wiki/Travelling_salesman_problem
- https://www.tutorialspoint.com/design_and_analysis_of_algorithms/design_and_analysis_of_algorithms_travelling_salesman_problem.htm

Thank You