# Introduction to Machine Learning Course

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# Contents

# 课程教学大纲

- 机器学习基础（3）
- Linear Regression and Gradient Descent （3）
  线性回归与梯度下降
- Linear Classification and Stochastic Gradient Descent （3）
  线性分类、支持向量机、随机梯度算法
- Logistic Regression and Ensemble Methods (Decision Tree, Adaboost) （3）
  逻辑回归与集成学习算法
- Overfitting, Underfitting, Regularization and Cross-Validation (3)
  过拟合、欠拟合、正则化与交叉验证
- Multiclass Classification and Cross-entropy Loss （3）
  多类分类和交叉熵损失函数

# 课程教学大纲

- ~~Clustering and~~ Dimension Reduction (PCA, Feature Selection) （3）
  ~~聚类算法与~~维度约简

- ~~**Recommendation Systems （3）推荐系统**~~

- Neural Networks and Deep Learning (Basics) （3）
  神经网络与深度学习

- Image Processing Basics and Convolutional Neural Networks （3）
  神经网络与深度学习

- 序列模型(RNN)、Transformer、Bert（3）

- Markov Decision Process, Reinforcement Learning and AlphoGO（3）
  马尔可夫决策过程、强化学习及AlphoGo

# 实验教学大纲

## 随堂实验

- Linear Regression and Gradient Descent (2)
  线性回归与梯度下降

- Linear Classification with Stochastic Gradient Descent (2)
  线性分类、支持向量机、随机梯度算法

## 课程实验

- Classification with AdaBoost (4)
  科技论文阅读、写作；
  逻辑回归与集成学习算法

- Face Detection and Recognition (4)
  人脸检测与识别基础

- 基于Transformer的中英文翻译 (4)

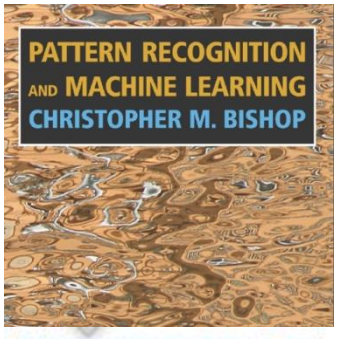
# 考核标准+参考书

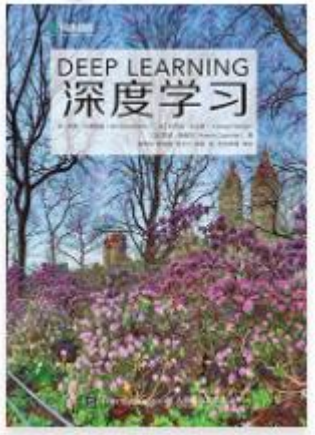
- **考核标准**

  考试（50%）+ **平时成绩（25%）+ 技术报告（25%）**

- **参考书**

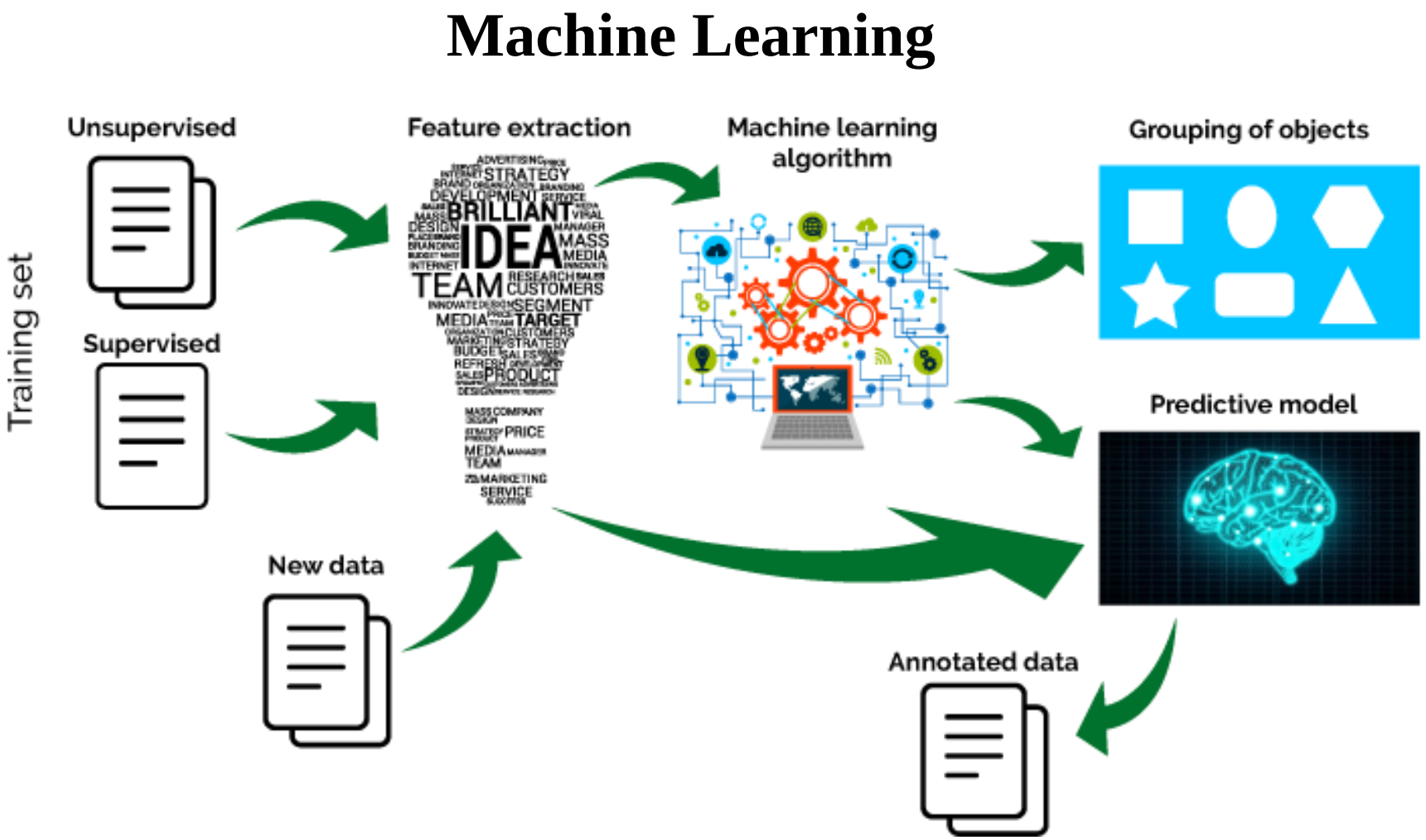
  - Pattern Recognition and Machine Learning **By Bishop**
  - Understanding Machine Learning: From Theory to Algorithms By Shai Shalev-Shwartz and Shai Ben-David
  - 深度学习 by Ian Goodfellow（伊恩 古德费洛）
  - 《机器学习》By 周志华



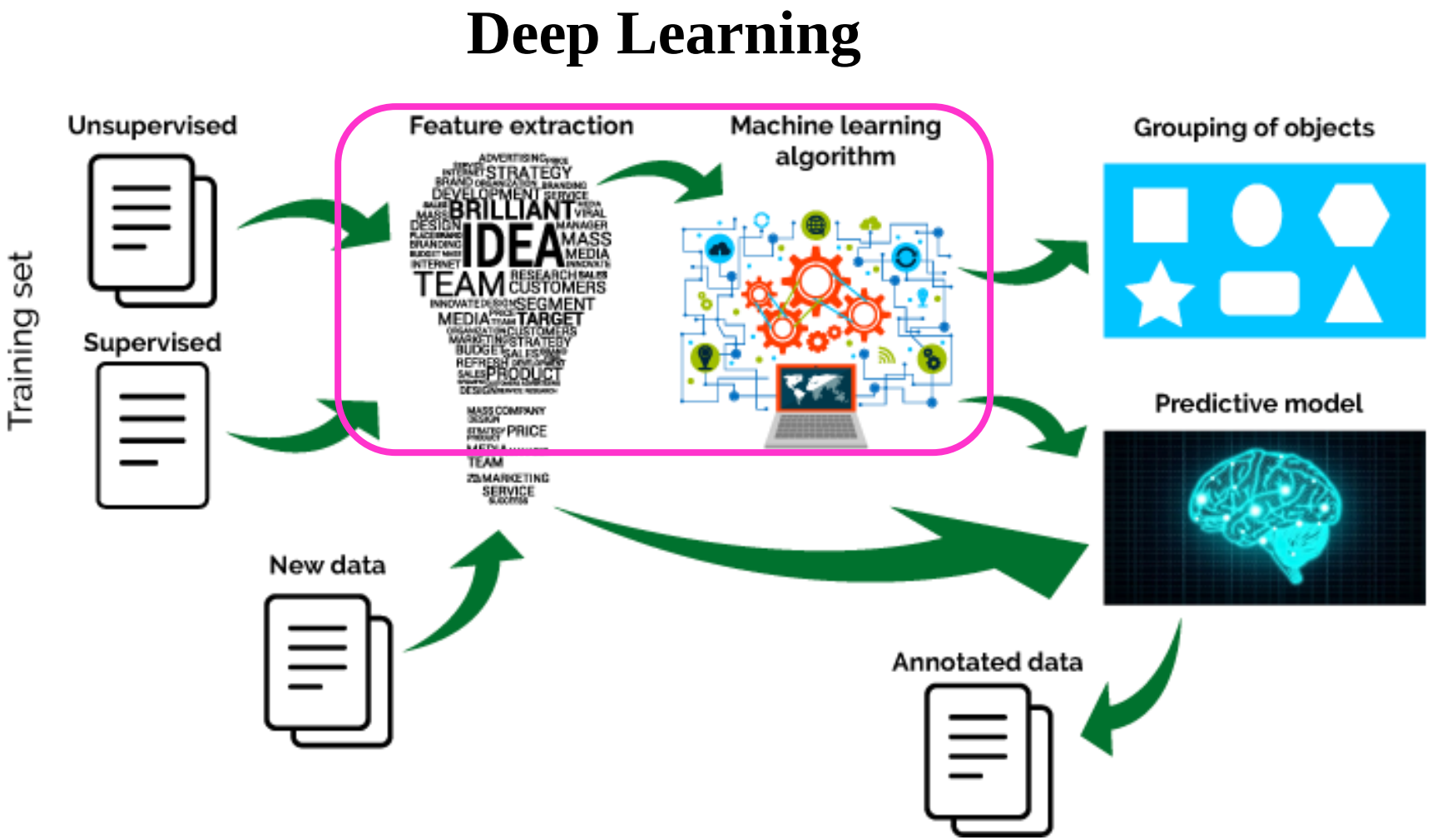



# Contents

# Big Picture: Machine Learning

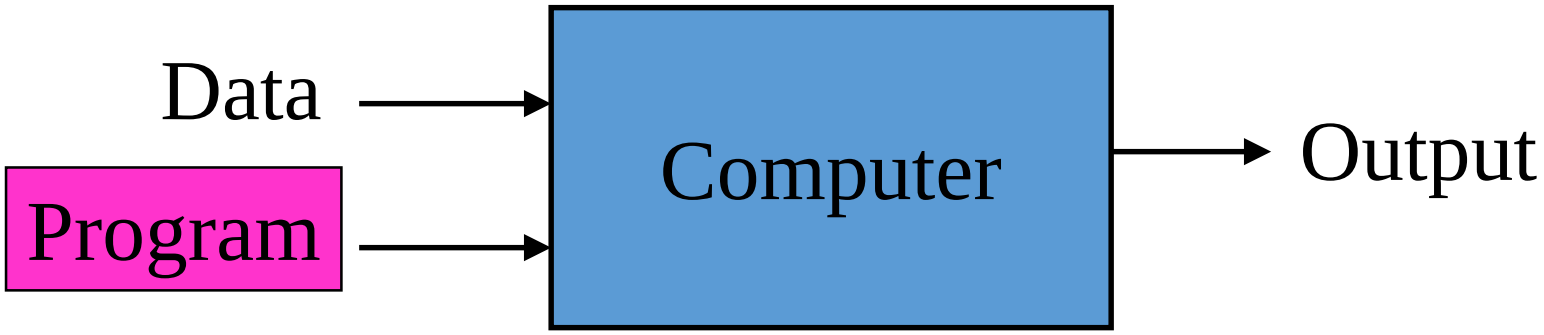


# Big Picture: Machine Learning

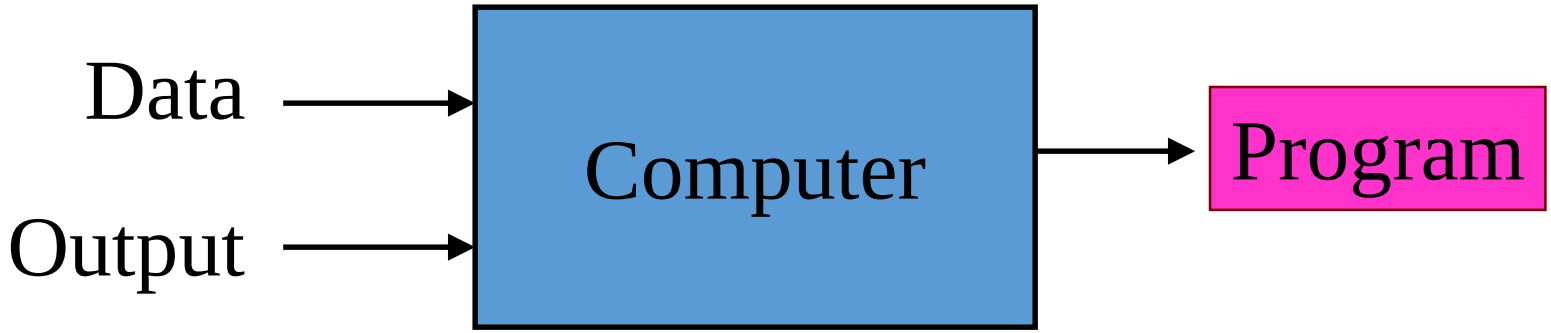


# Traditional Programming and Machine Learning

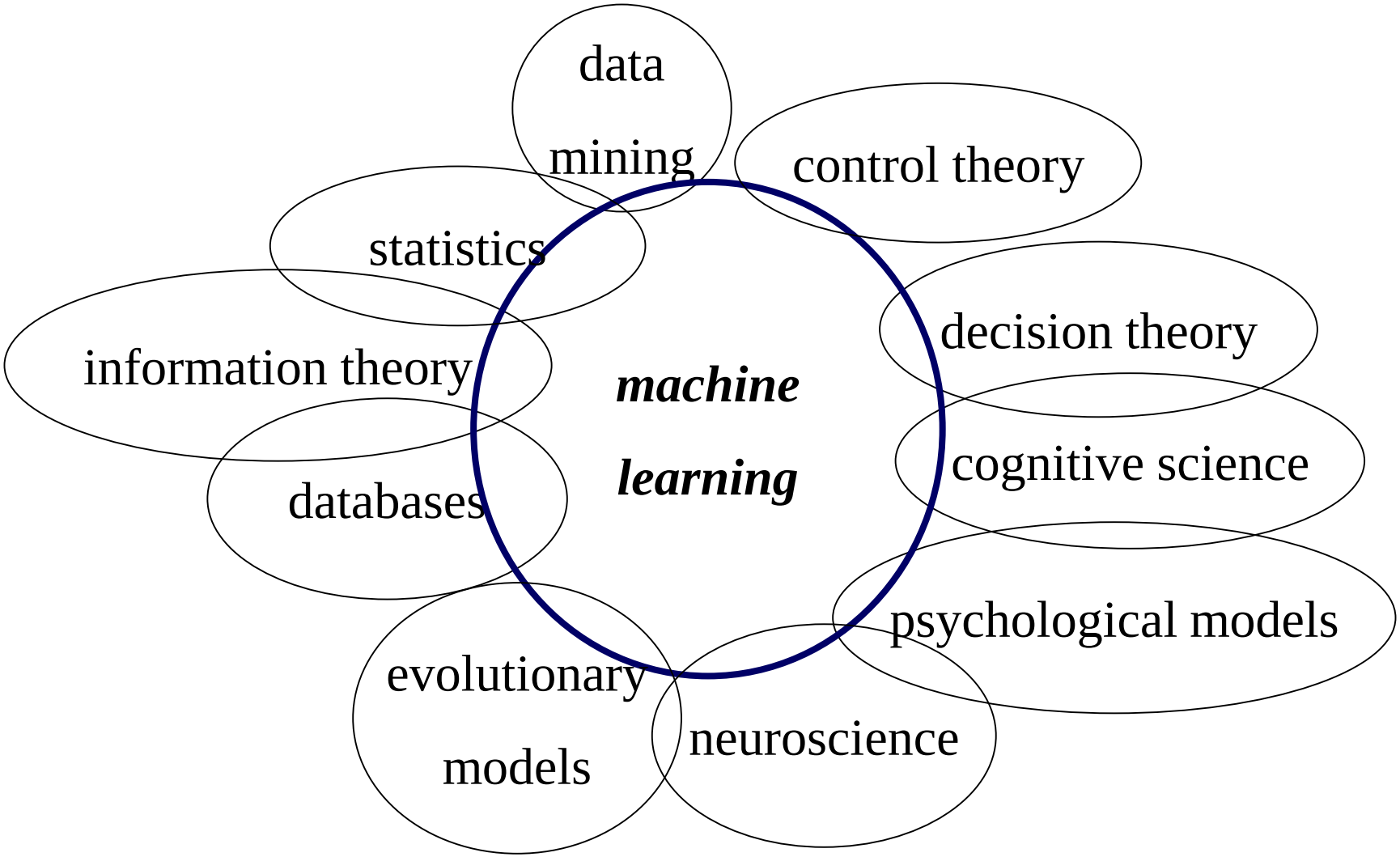
- **Traditional Programming**




- **Machine Learning**



# Related Fields



# Contents

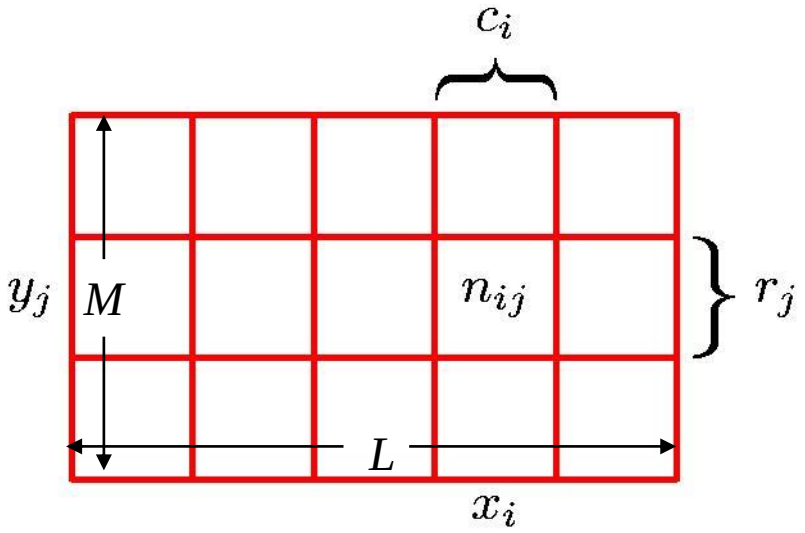
# Probability Theory

- **Random Variables**

$$P(A) = \frac{1}{6}, A = 1, 2, \ldots, 6$$

- Random variables describe the outcome of a random experiment in terms of a (real) number
- A random experiment is an experiment that can (in principle) be repeated several times under the same conditions
- Discrete or continuous random variables
- Independent and identically distributed (iid) experiment vs non-iid experiment

# Probability Theory




- **Marginal Probability**

$$P(X = x_i) = \frac{c_i}{L}$$

- **Joint Probability**

$$P(X = x_i, Y = y_i) = \frac{n_{ij}}{L \times M} = \frac{c_i \times r_j}{L \times M}$$

- **Conditional Probability**

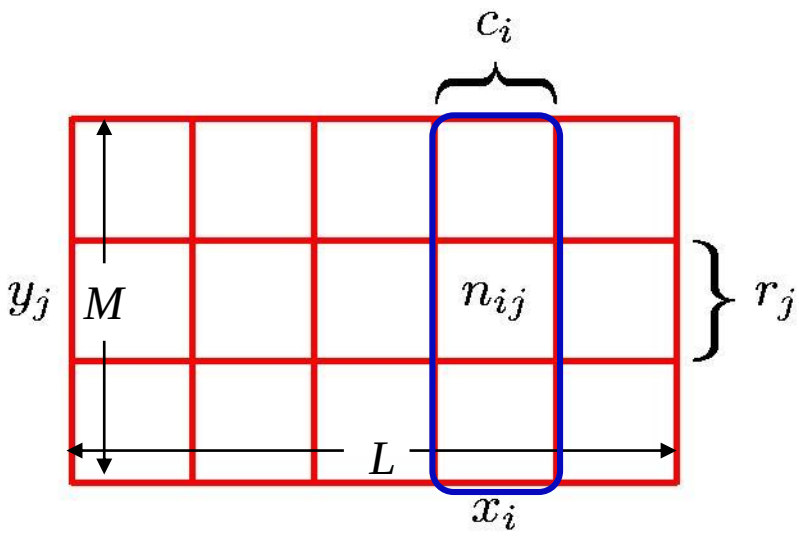
$$P(Y = y_j \mid X = x_i) = \frac{r_j}{M}$$

# Probability Theory

## Sum Rule

$$P(X = x_i) = \frac{c_i}{L} = \frac{1}{L \times M} \sum_j n_{ij}$$

$$= \sum_j P(X = x_i, Y = y_j)$$




## Product Rule

$$P(X = x_i, Y = y_i) = \frac{n_{ij}}{L \times M} = \frac{r_j}{M} \cdot \frac{c_i}{L}$$

$$= P\left(Y = y_j \mid X = x_i\right) P(X = x_i)$$

# Marginalization

Marginal Probability — Joint Probability

$$P(X = x_i) = \sum_j P(X = x_i, Y = y_j)$$

$$= \sum_j P(X = x_i | Y = y_j) P(Y = y_j)$$

Conditional Probability — Marginal Probability

| Y \ X | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $p_Y(Y)\downarrow$ |
|---|---|---|---|---|---|
| $y_1$ | $4/32$ | $2/32$ | $1/32$ | $1/32$ | $8/32$ |
| $y_2$ | $2/32$ | $4/32$ | $1/32$ | $1/32$ | $8/32$ |
| $y_3$ | $2/32$ | $2/32$ | $2/32$ | $2/32$ | $8/32$ |
| $y_4$ | $8/32$ | 0 | 0 | 0 | $8/32$ |
| $p_X(X) \rightarrow$ | $16/32$ | $8/32$ | $4/32$ | $4/32$ | $32/32$ |

Margin

This concept is called "marginal" because it can be found by summing values in a table along rows or columns, and writing the sum in the **margins** of the table

# Contents

# Bayes' Theorem

## The Rules of Probability

$$\text{Sum Rule: } P(X) = \sum_{Y} P(X, Y)$$

$$\text{Product Rule: } P(X, Y) = P(Y|X)P(X)$$

## Bayes' Theorem

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)} \qquad P(X) = \sum_{Y} P(X|Y)P(Y)$$

# Bayes' Theorem

$$\text{posterior} \propto \text{likelihood} \times \text{prior}$$

**Posterior probability $P(Y|X)$:** the likelihood of event $Y$ occurring given that $X$ is true, $P(Y|X)$ is a conditional probability

**Posterior probability $P(X|Y)$:** the likelihood of event $X$ occurring given that $Y$ is true, $P(X|Y)$ is a conditional probability

**Prior probability $P(X)$ and $P(Y)$:** the probabilities of observing $X$ and $Y$ independently of each other (the marginal probability)

# Bayes' Theorem

$$P(\text{"taking a shower"}|\text{"wet"}) = P(\text{"wet"}|\text{"taking a shower"})\frac{P(\text{"taking a shower"})}{P(\text{"wet"})}$$

$$P(\text{reason}|\text{observation}) = P(\text{observation}|\text{reason})\frac{P(\text{reason})}{P(\text{observation})}$$

- Often useful in diagnosis situations,
  since $P(\text{observation}|\text{reason})$ might be easily determined
- Useful for reasoning
- Often delivers surprising results

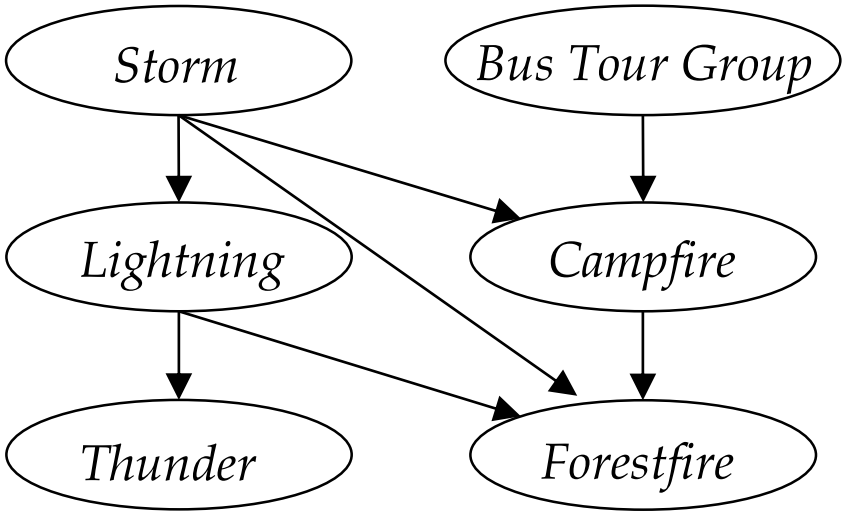
# Bayes' Theorem in Bayesian Learning

$$P(h|D) = \frac{P(D|h)P(h)}{P(D)}$$

- $P(h)$: prior probability of hypothesis $h$
- $P(D)$: prior probability of training data $D$
- $P(h|D)$: posterior probability of $h$ given $D$
- $P(D|h)$: posterior probability of $D$ given $h$

# Bayesian Net

- Network represents conditional independence assertions
- Each node conditionally independent of its non-descendants, given its immediate predecessors (e.g. Campfire and Lightning are independence conditioned on Storm)




conditional probability tables (CPT)

| | $S \wedge B$ | $S \wedge \neg B$ | $\neg S \wedge B$ | $\neg S \wedge \neg B$ |
|---|---|---|---|---|
| $C$ | 0.4 | 0.1 | 0.8 | 0.2 |
| $\neg C$ | 0.6 | 0.9 | 0.2 | 0.8 |

*C: Campfire*

*S: Storm*

*B: Bus Tour Group*

# Example

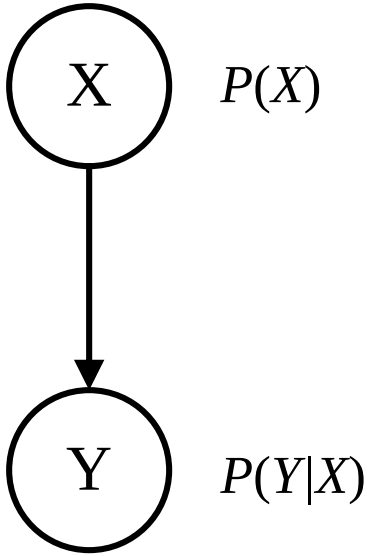
- **Random variables *X* and *Y***

  *X*: It is raining
  *Y*: The grass is wet

- ***X* affects *Y***

  Or, *Y* is a symptom of *X*

- **Draw two nodes and link them**




  - Define the CPT(conditional probability tables) for each node
    - $P(\mathrm{X})$ and $P(Y|X)$
  - Typical use: we observe Y and we want to query $P(X|Y)$
  - - Y is an evidence variable
    - X is a query variable

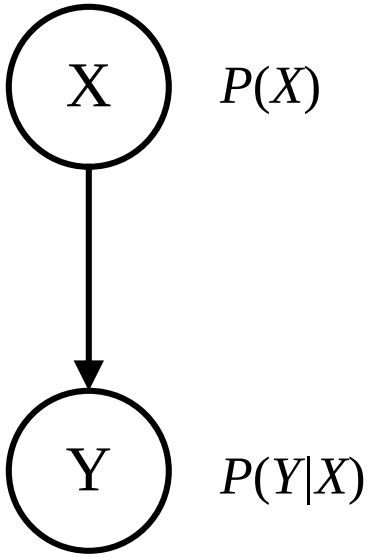
# Example

- **What is *P*(*X*|*Y*)?**
  - Given that we know the CPTs of each node in the graph

$$P(X \mid Y) = \frac{P(Y \mid X)P(X)}{P(Y)}$$

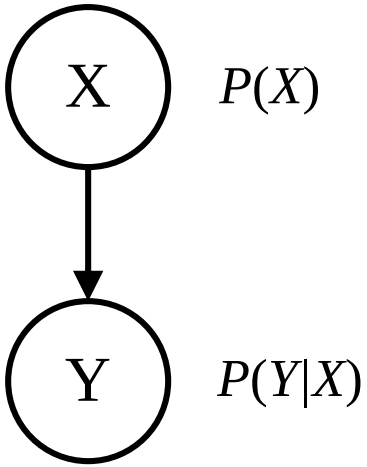
$$= \frac{P(Y \mid X)P(X)}{\sum_X P(X, Y)}$$

$$= \frac{P(Y \mid X)P(X)}{\sum_X P(Y \mid X)P(X)}$$

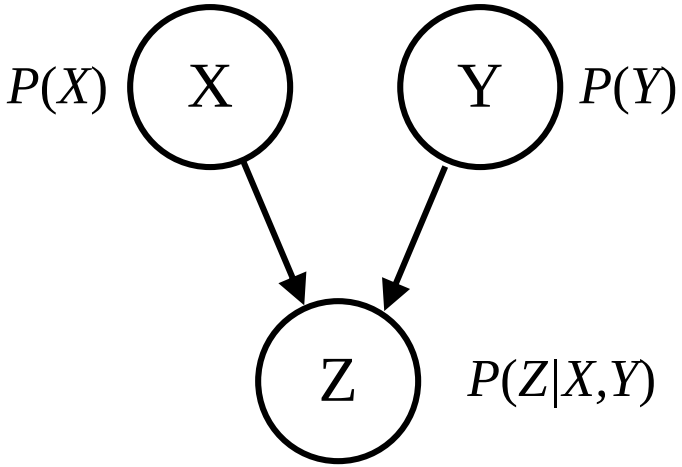


# Belief Nets Represent Joint Probability

- **The joint probability function can be calculated directly from the network**
- **It is the product of the CPTs of all the nodes**
- $P(var_1, \ldots, var_n) = \prod_i P(var_i|\text{Parents}(var_i))$




$P(X,Y) = P(X)P(Y|X)$




$P(X,Y,Z) = P(X)\ P(Y)\ P(Z|X,Y)$

# Probability Densities

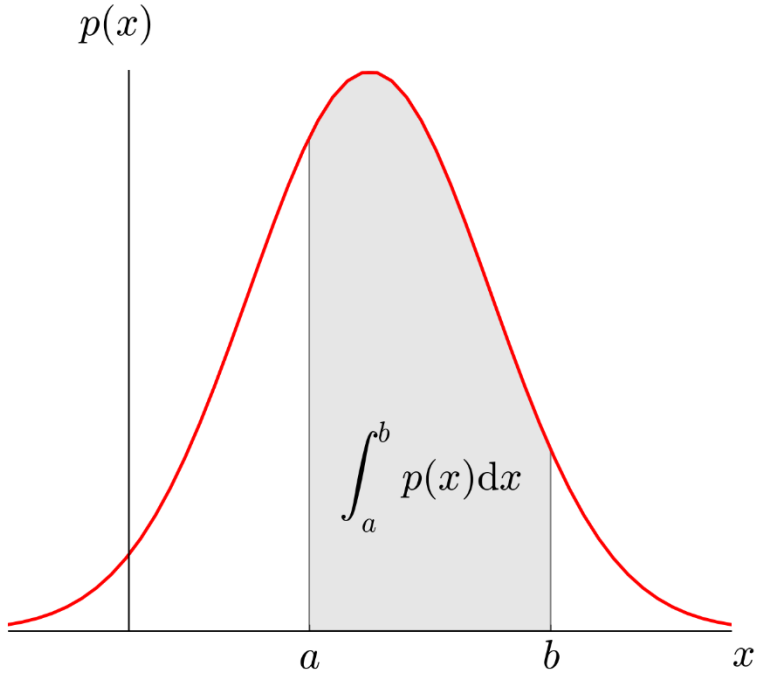
- **The probability density function p(x) has the following properties**

$$p(x) \geqslant 0$$

$$p(x \in (a,b)) = \int_a^b p(x)\,\mathrm{d}x$$

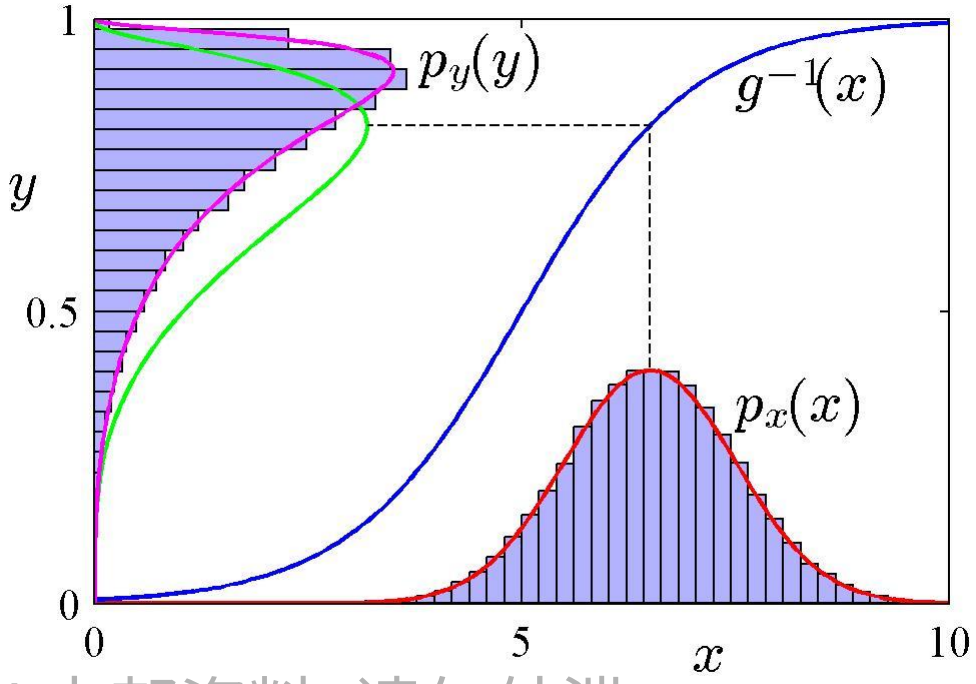
$$P(z) = \int_{-\infty}^{z} p(x)\,\mathrm{d}x$$

$$\int_{-\infty}^{\infty} p(x)\,\mathrm{d}x = 1$$



# Transformed Densities

- $x$ has a probability density $p_x(x)$
- $y = h(x)$ is some strictly monotonic continuous function
- Probability density $p_y(y)$ can be transformed from $p_x(x)$




$$y = h(x) = g^{-1}(x)$$

$$\begin{aligned} p_y(y) &= p_x(x)\left|\frac{\mathrm{d}x}{\mathrm{d}y}\right| \\ &= p_x(g(y))\,|g'(y)| \end{aligned}$$

# Maximum Likelihood Estimation

- **A density $f$ usually contains parameters $\theta \in \Omega$: $f(x|\theta)$**
  **Parameters $\theta$: a scalar or a vector**

$$f(x|\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- **Question: How to estimate $\theta$ given data $\mathcal{D} = \{x_i\}$ ?**
- **Likelihood function of $\theta$ given $x$:**

$$L(\theta|x) = P(X = x|\theta)$$

- **Likelihood function of $\theta$ given $\mathcal{D} = \{x_i\}$:**

$$L_{\mathcal{D}}(\theta) = P(\mathcal{D}|\theta) = \prod_m P(x_i|\theta)$$

# Maximum Likelihood Estimation

- **Likelihood function of $\theta$ given $\mathcal{D} = \{x_i\}$ (iid $x_i$)**

$$L_{\mathcal{D}}(\theta) = P(\mathcal{D}|\theta) = \prod_i P(x_i|\theta)$$

- **Estimate $\theta$ by**

$$\theta_* = \underset{\theta}{\operatorname{argmax}} \left(\prod_i P(x_i|\theta)\right)$$

- **In practice, often use log likelihood function**

$$\theta_* = \underset{\theta}{\operatorname{argmax}} \log\left(\prod_i P(x_i|\theta)\right)$$

- **Then, we have**

$$\theta_* = \underset{\theta}{\operatorname{argmax}} \left(\sum \log\big(P(x_i|\theta)\big)\right)$$

# Maximum a Posteriori Estimation

- **Replace the likelihood in the MLE formula with the posterior, and we get:**

$$\theta_{MAP} = \underset{\theta}{\operatorname{argmax}}\, P(X|\theta)P(\theta)$$

$$= \underset{\theta}{\operatorname{argmax}}\, \log P(X|\theta) + \log P(\theta)$$

$$= \underset{\theta}{\operatorname{argmax}}\, \log \prod_i P(x_i|\theta) + \log P(\theta)$$

$$= \underset{\theta}{\operatorname{argmax}} \sum_i \log P(x_i|\theta) + \log P(\theta)$$

# MLE vs MAP

- **If we use uniform prior in MAP estimation, $P(\theta)$ is a const, so we have:**

$$\theta_{MAP} = \text{argmax} \sum_i \log P(x_i|\theta) + \log P(\theta)$$

$$= \text{argmax} \sum_i \log P(x_i|\theta) + const$$

$$= \text{argmax} \sum_i \log P(x_i|\theta) = \theta_{MLE}$$

- **MLE is a special case of MAP, where the prior is uniform**

# Contents

# Probability and Information Theory

- **Information measure of an event *A***

$$I(A) = -\log_b P(A)$$

***I*(*A*):** self-information or information content, random variable

***P*(*A*):** probability of the event happening

***b*:** base, usually $b$=2

base 2 = bits  base 3 = trits
base 10 = Hartleys  base e = nats

# Information and Probability

## Examples

The Chinese football team lost:

$P(A)=1$ $\qquad\qquad$ $I(A) = -\log_2 P(A) = 0$

The Chinese table tennis team lost:

$P(A)=0$ $\qquad\qquad$ $I(A) = -\log_2 P(A) = +\infty$

Probability $P(A)$: The degree of uncertainty of an event

Self-information $I(A)$: The elimination of uncertainty

# Entropy

- **Entropy is simply the average (expected) amount of the information from the event**

$$H(A) = -E[\log_2 P(A)] = -\sum_A P(A)\log_2 P(A)$$

**$H(A)$ is maximized when $P(A) = \frac{1}{n}$ for all $A$**

- **Joint Entropy**

$$H(A,B) = -E[\log_2 P(A,B)] = -\sum_{A,B} P(A,B)\log_2 P(A,B)$$

- **Conditional entropy of $A$ given $B$**

$$H(A|B) = -E[\log_2 P(A|B)] = -\sum_{A,B} P(A,B)\log_2 P(A|B)$$

# Thank You