

## Girls Talk Math - Money Math

The word **finance** describes all activities associated with the management of money including banking, investments, credit, capital markets, and debt. The finance field includes three major subcategories: personal finance, public/government finance, and corporate finance.

- **Personal finance** is the management of an individual's money. This includes planning for retirement, purchasing financial products like credit cards, insurance and mortgages, banking and investments.
- **Public/government finance** is the management of government funds. This includes taxing constituents, spending funds, budgeting funds, and policy making that affects the government's capacity to administer public services.
- **Corporate finance** is specific to the money management related to running a corporation. This includes raising funds through stock offerings or bond issues, budgeting a corporation's capital, and deciding on financial opportunities. Most corporations have specific departments, divisions or offices dedicated to supervise these activities.

We will study various financial concepts including taxes, entrepreneurship, building an investment portfolio with stocks and bonds, and strategies for comparing investment opportunities.

### What is income tax?

The Federal government, most states, and many cities collects taxes from the income earned by citizens and use the money to pay for the government's expenses. In the USA, the government agency in charge of collecting taxes is called the Internal Revenue Service (IRS).

Each person and company must calculate the amount of taxes they owe each year based on how much money they earned and a variety of other variables. For individuals, those variables include their family situation and if they donate to charities. Calculating taxes owed and making any required payments is called "filing taxes" and has a deadline of April 15th every year.

The fact that individuals and companies have to pay taxes each year means there is an immense demand for professional, certified accountants who take care of the calculations and complex problem solving. Accounting can be a lucrative and rewarding career full of complex problem solving.

Income is the total amount of money earned working in a year. Income is recorded in a document called a Form W-2. Each January, employers must provide their employees with a W-2 form that describes the employee's income in the previous year. The employee then uses the information from a W-2 to file their taxes or pays an accountant to figure it out for them.

A Form W-2 from 2014 looks like this:

a Employee's social security number <b>123-45-6789</b>	OMB No. 1545-0008	Safe, accurate, FAST! Use  Visit the IRS website at <a href="http://www.irs.gov/efile">www.irs.gov/efile</a>	
b Employer identification number (EIN) <b>11-2233445</b>	1 Wages, tips, other compensation <b>48,500.00</b> 2 Federal income tax withheld <b>6,835.00</b>		
c Employer's name, address, and ZIP code  <b>The Big Company</b> <b>123 Main Street</b> <b>Anywhere, PA 12345</b>	3 Social security wages <b>50,000.00</b> 4 Social security tax withheld <b>3,100.00</b> 5 Medicare wages and tips <b>50,000.00</b> 6 Medicare tax withheld <b>725.00</b> 7 Social security tips      8 Allocated tips		
d Control number <b>A1B2</b>	9      10 Dependent care benefits		
e Employee's first name and initial      Last name  <b>Jane A      DOE</b> <b>123 Elm Street</b> <b>Anywhere Else, PA 23456</b>	Suff.	11 Nonqualified plans  12a See instructions for box 12 <b>D</b> <b>1,500.00</b> 12b <b>DD</b> <b>1,000.00</b> 12c <b>P</b> <b>4,800.00</b> 12d	
f Employee's address and ZIP code  15 State      Employer's state ID number <b>PA</b> <b>1235</b>	16 State wages, tips, etc. <b>50,000</b>	17 State income tax <b>1,535</b>	18 Local wages, tips, etc. <b>50,000</b> 19 Local income tax <b>750</b> 20 Locality name <b>MU</b>
<b>W-2 Wage and Tax Statement</b> Form <b>2014</b>		Department of the Treasury—Internal Revenue Service	
Copy B—To Be Filed With Employee's FEDERAL Tax Return. This information is being furnished to the Internal Revenue Service.			

The W-2 includes information identifying the employer, the employee, and the employee's earnings:

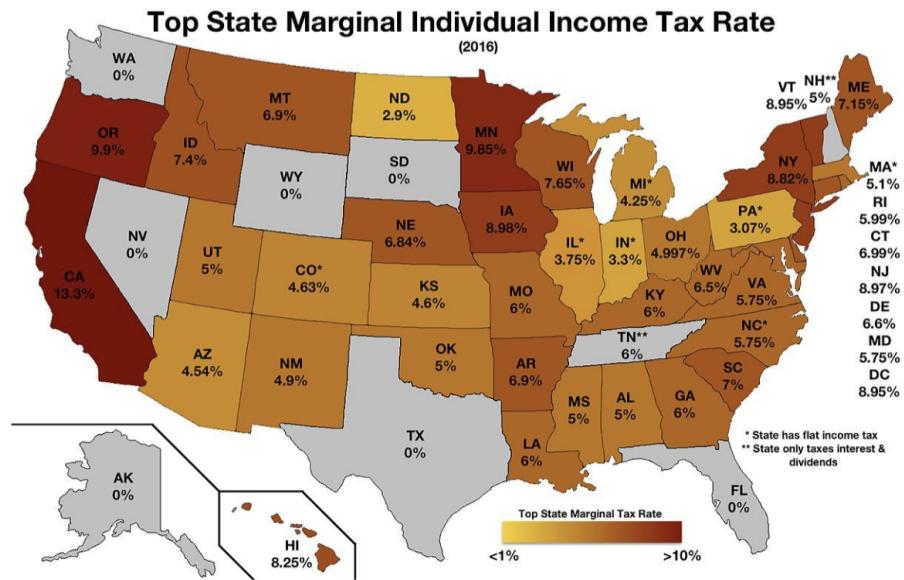
- Box 1 shows the total amount of money earned by the worker, \$48,500.00 in the above example.
- Box 2 represents the amount of income tax that was withheld by the employer throughout the year, \$6,835 in the above example.
- Boxes 3-8 are used to calculate taxes for Social Security and Medicare benefits. The Social Security and Medicare programs provide money to people in need and is collected from all taxpaying citizens.

Withholding (Box 2) means the employer sets aside money from an employee's paycheck and sends the money directly to the IRS in the employee's name. The IRS requires that tax payments be made throughout the year as income is earned. Withholding is a convenient way to pay taxes throughout the year, reducing the burden of making a large payment each April and avoiding penalty payments associated with paying all at once.

For example, if you owe \$6,000 in taxes for the year and your employer withheld \$4,000 from your paychecks that year, it would be up to you to pay the remaining \$2,000 in taxes when it's time to file in April. New employees

provide their company with a Form W-4 that shows how much withholding they want from their income.

### State Taxes

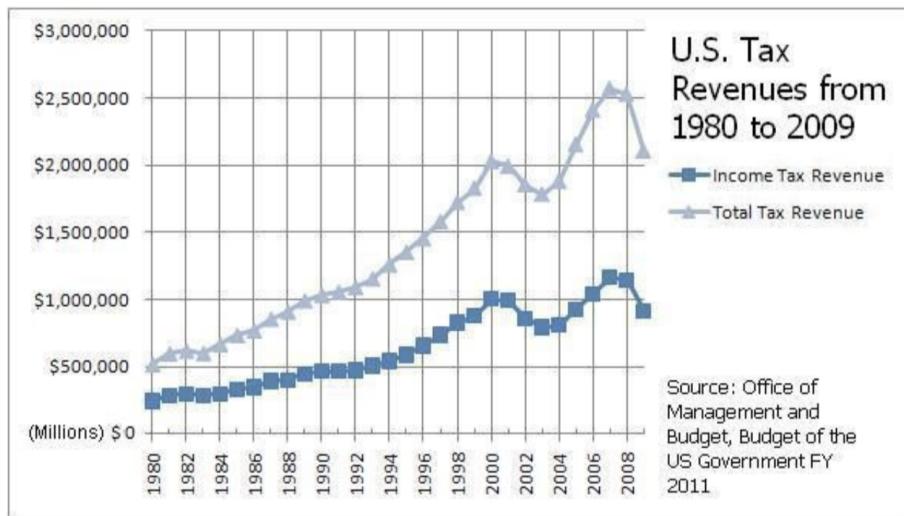


The W-2 also shows information for filing taxes in your state of residence. Every state in the US charges varying levels of income tax in addition to the tax owed to the federal government except for Alaska, Florida, Nevada, South Dakota, Texas, Washington, and Wyoming. Tennessee and New Hampshire only tax income from investments and not income earned from working.

**Question:** Which state has the highest income tax rate in the above map?

### Tax Revenues

The total money collected in taxes by the government is used to pay interest on the national debt, build roads, fund the military, pay for medical and social safety net programs, and generally keep the government in operation. Taxes collected from individuals and corporations generate huge sums of revenue each year:



**Question:** What were the approximate total U.S. tax revenues in 2000? Note the scale of the Y-axis.

#### How are taxes calculated?

The more money you earn in a year, the higher the portion of your earnings must be paid in taxes. The amount of taxes owed is calculated as a percentage of the money that was earned in that year. These percentages are referred to as “tax brackets”. The below table summarizes the Federal tax brackets from 2019:

**Tax Brackets and Rates, 2019**

Rate	For Unmarried Individuals, Taxable Income Over	For Married Individuals Filing Joint Returns, Taxable Income Over	For Heads of Households, Taxable Income Over
10%	\$0	\$0	\$0
12%	\$9,700	\$19,400	\$13,850
22%	\$39,475	\$78,950	\$52,850
24%	\$84,200	\$168,400	\$84,200
32%	\$160,725	\$321,450	\$160,700
35%	\$204,100	\$408,200	\$204,100
37%	\$510,300	\$612,350	\$510,300

The amount of income earned corresponds to a percentage referred to as the rate. The amount of income needed to reach the next tax rate on the table is different if a person is single, married, or the head of a household.

For example, in 2019 an unmarried individual that earned \$300,000 was in the 35% tax bracket. Any income above \$204,100 and below \$510,300 would fall into the 35% tax bracket.

Thankfully, this doesn't mean you owe 35% of \$300,000 to the government.

#### Graduated tax

Income tax is calculated on a graduated scale. This means that the amount of tax owed is calculated in each tax bracket as income increases. For example, the graduated tax means an individual is taxed 10% on the first \$9,700 of income, 12% on the amount of income between \$9,700 and \$39,475, and so on down the table of rates until all of their income is counted.

**Question:** *A married couple each receive a Form W-2 from their employer. Box 1 shows \$100,000 and \$140,000 on the respective forms. Box 2 shows \$15,000 on both of their forms. Based on the graduated tax calculation, how much would they owe in Federal taxes? When do they need to send the check?*

What if their employers withheld \$30,000 for each of them? When taxes have been overpaid through withholding, a refund of the difference between the amount paid through withholding and amount of taxes due is issued. In this case, the government writes a check to you.

**Question:** *What is the couple's refund in this scenario?*

#### How does Debt Work? The Power of Compounding Interest

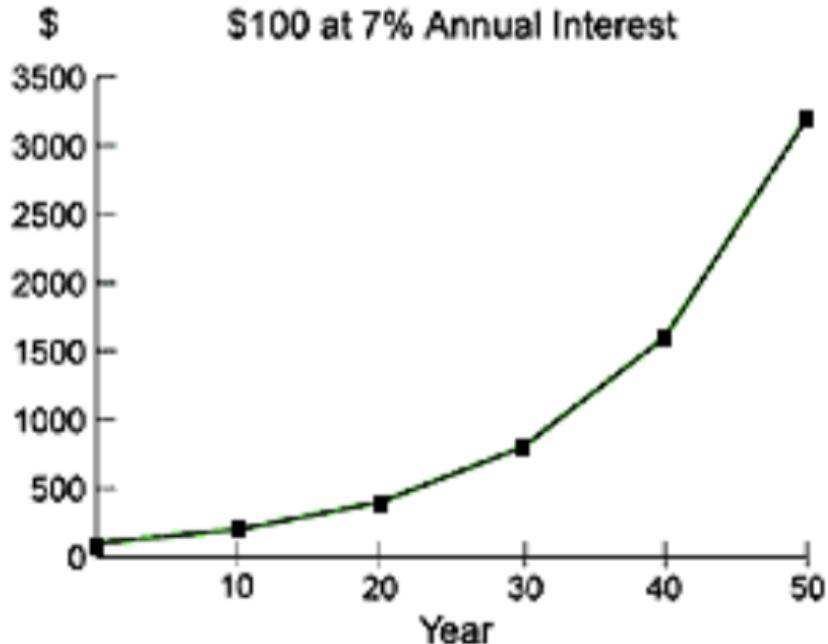
There are many reasons to get a loan. Starting a business, buying a house, paying for college, or simply paying the bills all require money that is sometimes easier to borrow and pay back later. There are many types of loans. A mortgage is a loan used to buy a house, a credit card is a loan to an individual, a bond is a loan to a government or company.

No matter the type of loan, whoever is lending the money will charge interest payments to the borrower in exchange for handing over the money. Interest is expressed as a percentage of the amount of money borrowed, also called the "principal". For example, if you borrow \$5,000 from the bank, the principal of the loan is \$5,000.

The interest percentage of the loan varies based on the type of loan. Credit card interest rates are high, currently about 18% on average.

**Question:** *Can you think of a reason credit cards interest rates might be higher than those for other loans?*

Interest is calculated and charged on a regular schedule, which could be weekly, monthly, or annually. Credit cards and other personal loans are usually charged monthly.



If someone spent \$100 on a credit card and did not pay the money back within one month, interest will be charged on the outstanding balance of \$100. If the interest rate of the credit card is 20%, they would now owe 20% of \$100 plus the original \$100, or \$120. If another month passes the interest will be calculated on the new balance of \$120 and suddenly they owe \$144.

This kind of growth over time is referred to as “compounding” interest. Compounding is the growth of the total amount owed based on accumulating interest.

**Question:** If your credit card charges 15% interest, how many months until you owe more in interest charges than the original balance if you spent \$100?

#### Compounding Growth Rate Equation

The Compounding Growth Rate (CGR) equation can be used to solve for the ending value, beginning value, rate of growth, or number of compounding periods.

To solve for the growth rate the equation is expressed as:

$$R = \left(\frac{E}{B}\right)^{\frac{1}{n}} - 1$$

where:

- $R$  = compounding growth rate per period (annual, monthly, etc.)

- $E$  = Ending Value
- $B$  = Beginning Value
- $n$  = Number compounding of periods (months, years, etc.)

The equation can also be written to solve for the future value of a loan or investment as:

$$E = B(1 + R)^n$$

**Question:** *What is the monthly the growth rate for an investment worth \$25,000 if you paid \$10,000 for it 5 years ago?*

**Question:** *Suppose a company had \$3 million in revenue 2010 that grew to \$30 million by 2020. In this case, what was the compounding annual growth rate?*

#### How are investments taxed?

The money earned from investing is taxed unless you're investing to save for your retirement in a retirement account. Interest payments from bonds are taxed at the same rates as income using the federal tax brackets on page 4. The interest earned from bonds is added to other income for the purpose of calculating taxes.

Money made in the stock market is only taxed if you sell your stock. Stock sale proceeds are taxed as "capital gains". The rate of capital gains tax depends on how long the investor owned the stock before selling it. Stocks held for more than one year before selling are taxed as long-term capital gains at a rate of 0%, 15%, or 20%, depending on the total income that was earned that year.

#### **Long Term Capital Gains Tax Brackets**

Tax Bracket/ Rate	Single	Married filing jointly	Head of household
0%	\$0 - \$39,375	\$0 - \$78,750	\$0 - \$52,750
15%	\$39,376 - \$434,550	\$78,751 - \$488,850	\$52,751 - \$461,700
20%	\$434,551+	\$488,851+	\$461,701+

Short-term capital gains, or the money earned from selling stock that the investor held for less than one year, are taxed at the same rate as ordinary income using the federal tax brackets. Just like interest payments from bonds, short term capital gains are added to the total amount of income. Taxes are higher on short-term capital gains

to discourage investors from buying and selling investments over short time spans, creating volatility. Volatility is rapidly changing prices caused by people quickly buying and selling.

Retirement savings accounts, like an IRA or 401(k), allow people to invest their earnings tax-free. Many employers match their employees 401(k) contributions, so it is best to first maximize contributions to tax-free accounts before buying taxable investments.

**Question:** *A married couple each receive a W-2. One W-2 shows \$40,000 in Box 1 and \$10,000 in Box 2. The other shows \$65,000 in Box 1 and \$3,000 in Box 2. During the year, the couple made \$5,000 selling stock they held for less than a year and \$15,000 selling stock they held for more than a year. The bonds the couple owned paid them \$500 in interest throughout the year. How much in taxes does the couple owe this year? What advice might you give them to pay less in taxes next year?*

## Investing Basics - What is stock?

**Stock** is a piece of ownership in a company that can be bought and sold by investors. Investors buying and selling stock in companies around the world is referred to as the “stock market”.

Owning stock in a company entitles the investor to a portion of the company’s earnings as well as the ability to vote on some company decisions. Companies sell pieces of ownership called “shares” of stock and use the money raised from investors to sustain their operations, expand, or pay for improvements in their business. Shares of stock can be represented by pieces of paper, like printed money, or exist as records in computers.



An example of a stock certificate, representing 100 shares of Allied Paper Corporation

What is a company worth? Who decides how much stock in different companies is worth? The value of a company includes its assets as well as its ability to generate money in the future. Assets can include buildings like offices and factories, equipment, or products that hasn’t been sold. When people buy stock from a company they consider the value of the company’s assets as well as how much money they think the company can make.

People around the world buy and sell stock at prices based on how high the demand is for ownership in that company. Hopefully over time a company will earn more money and become successful. As a company becomes

successful, a share of ownership in the company becomes more valuable, so the price of the stock goes up. An investor could sell their shares of the company and profit now that the shares are worth more.

Market capitalization or “cap” is the total dollar amount of stock shares of a company. The market cap of Microsoft, the largest in the world, is roughly \$95 billion dollars. The market cap is the total number of shares that exist for that particular company times the current price per share of stock.

**Question:** If the price of Microsoft is approximately \$125 per share, how many shares of Microsoft exist in the world?

Stocks are represented by a series of letters called a ticker. For example, the ticker for Apple stock is AAPL. Nike is NKE and Tesla is represented by TSLA. You may have seen a series of these tickers together with numbers and arrows on TV:



At the bottom of the screen we see the semiconductor chip company Nvidia (NVDA) is trading for a price of \$267.46 per share, and that the price has gone up \$1.76 that day.

The biggest stock market in the world, the New York Stock Exchange (NYSE), allows trading between 9:30 A.M. and 4:00 P.M. almost every weekday of the year.

### Making money investing in stock

If you could go back in time to 1980, you could purchase one share of Apple Computers for \$22. Over time, Apple has become hugely successful and iPhones are everywhere. Today that same share of Apple purchased for \$22

in 1980 could be sold for about \$200. Imagine if you bought 10,000 shares instead of just 1.

**Question:** *Just how much would you profit if you had bought 10,000 shares in 1980 and sold them today?*

**Question:** *What is the average annual percentage growth of Apple since 1980 from \$22 to \$200?*

**Question:** *Imagine today you invest \$100 in Apple stock and expect the same average return for the next 20 years. What's your stock worth in 20 years? (Hint: Recall what we learned about compounding interest for this question and the next.)*

**Question:** *Imagine instead of buying a drink at Starbucks you buy \$5 worth of Starbucks stock. Over the next 45 years your investment grows at an average rate of 6%. What's your investment worth when you retire in 45 years? How many drinks could you buy in retirement?*

It's clear that investing can be a profitable endeavor. Working as an investment professional is a rewarding and exciting career and there is a high demand for smart and creative individuals in the field.

The downside of investing in stock is that it can be risky. The value of a company could also go down and the investor who owns the stock would lose money as the value of the stock decreased. The value of a stock can go down to \$0 if the company goes out of business and the investor would lose all of the money they used to buy the stock. Usually, the value of stock fluctuates over time.

As risky as investing as stocks may be, stocks can be a great way to increase your net worth outside of your career and prepare for retirement. We will return to stocks later and look at some investing strategies.

#### What is a bond?

A **bond** is simply a loan. Like other loans it consists of a principal payment provided by the lender to the borrower with the expectation of interest payments. Issuing bonds is like giving out IOUs to the investors who buy them. All kinds of institutions raise money by selling bonds, including all levels of government and companies. Bonds are considered safer than stocks because they tend to retain their value over time and are guaranteed legal protections if the bond issuer can't pay back the loan. Unlike a stock which could drop in value to \$0, bonds tend to retain their value. Because the investor is taking on less risk when buying the bond as compared to stock, the expected return on the investment is lower for a bond.

The safest bonds are issued by the federal government. The government issues bonds to raise money and provide regular interest payments to investors. There is very low risk the government won't pay make payments on the loans they take out from Americans.

#### What is an index?

The performance of the stock market is tracked using indexes. An index is the average performance (growth

in value) of a group of companies. There are many different types of indexes. You could imagine an index that represents the average performance of all of the companies in the USA, all the companies in the world, in a specific region, or in a particular industry like energy or biotech. A commonly used index to represent the US stock market is called the S&P 500. The S&P 500 tracks the performance of 500 of the biggest US companies. You can see that over time the stock market, as represented by the S&P 500, fluctuates and has volatility in the short term. Over longer periods of time values generally go up.



Consider the growth of \$1 invested in the S&P 500:

Visit the following site: <https://www.icmarc.org/x3333.html?RFID=W1470>

You can see there are periods where the stock market retracts in value quickly, such as the 51% decline in the value of the S&P 500 during the financial crisis of 2008. There are many reasons the stock market might lose value including fear of global conflict or a perception that the economy isn't producing as much as it should. The stock market is prone to volatility ultimately because of the emotions of investors and what they think will happen with a company or the global economy in the near term.

Less risky investments, like bonds, tend to earn lower returns than stocks but are less prone to fluctuating in value. This is the tradeoff between risk and reward that exists in all forms of investing.

#### Diversification

An important term in investing is diversification. That means you spread your money around to different types of investments but also to different regions of the world and sizes of businesses. The reason for diversification is that

it lowers the risk of investing. You may have heard the phrase “don’t put all of your eggs in one basket” because if you drop the basket, you lose all of your eggs. This concept holds true in investing, too. In any given year no one country or company will be the best performer. By spreading your money around to different investments, you get the best average return and reduce your risk.

**Question:** *Imagine you invest your first paycheck of \$1,000 (after taxes) in a portfolio consisting of 80% stocks with an expected annual return of 6% and 20% bonds with an expected annual return of 2%. What is your portfolio worth in 40 years if you don't add any money to it and return expectations are accurate?*

**Question:** *Imagine you invest instead in 80% bonds and 20% stock with the same return expectations. What is your portfolio worth?*

**Question:** *Why might someone invest in more bonds than stocks? Why might you invest more in stocks than bonds?*

**Question:** *What if after investing in the 80% stock and 20% bond portfolio, stocks decreased in value on an average of 4% and the bonds maintained their growth rate of 2%. How would you have done if you instead invested in the 20% stock and 80% bond portfolio with stocks declining 4% and bonds growing at 2%?*

In our examples above we assume an initial investment of \$1,000. In reality, it pays to save early and often. The more time your money has to grow and the more you add to the portfolio, the more money you'll have available when you're ready to stop working. When you're young and saving for retirement, it makes sense to invest your savings mostly in stocks rather than bonds. Over longer time periods, the higher risk of investing in stock is rewarded with higher average returns. As you get older, your investments could be moved into bonds if you are spending from the portfolio and need certainty that the money will be available. Anyone can begin investing starting at age 18.

## Investment Strategies

There are many strategies you can use to choose how to diversify your portfolio. Once you choose how much of each type of investment option you want your portfolio to consist of, you must choose exactly what stocks and bonds to buy. While buying bonds from the federal government is safest, the same is not true for buying any stock that's traded in a reputable stock market, like the NYSE. There are risks associated with purchasing all stocks which must be considered. While there is no way to eliminate all risk, you can make choices you feel more confident in by using strategies to invest. The following are some strategies for choosing what companies and industries to invest in.

## Summations

Before we get start looking at investment strategies, it is important to understand summation notation. The word summation involves the word “sum” and literally means “the process of adding things together” or “the process of summing things.” In mathematics, many formulas require adding many variables together. **Summation notion**, also called sigma notation, is a concise way to express the sum of a set of variable values.

Assume we have a set of  $n$  numbers,  $x_1, x_2, x_3, \dots, x_n$ . The first number in this set is  $x_1$ . The  $i$ th number in this set is  $x_i$ . To express the sum of these variables using summation notation, we use the Greek letter sigma,  $\Sigma$ , as the **summation sign**. This tells us to sum the elements of some sequence. The **variable of summation** is the variable that is being summed. The variable of summation is written with a subscript, often  $i$ , called an **index**. The index of the summation is written below the summation sign. It is usually written in an expression like  $i = 1$ . The value on the right hand side, in this case 1, is the starting point or **lower limit of summation**. The stopping point or **upper limit of summation** is written above the summation sign. The index will assume all values starting with the lower limit of summation and ending with the upper limit of summation. Therefore, the sum of the  $n$  numbers,  $x_1, x_2, x_3, \dots, x_n$ , can be written

$$\sum_{i=1}^n x_i = x_1 + x_2 + x_3 + \dots + x_n.$$

If we wanted to sum the second through fifth elements of this sequence, we could instead write

$$\sum_{i=2}^5 x_i = x_2 + x_3 + x_4 + x_5.$$

## Concentration in the Market

An **industry** is a group of businesses/firms that provide a particular product or service. **Industry competition** is competition between businesses that sell similar products or provide similar services. Different industries will have different levels of competition in the stock market. For example, the energy industry includes companies that are involved in the development, production and sale of energy. This industry includes firms you may know like BP (BP) and Chevron Corporation (CVX). On the other hand, the pharmaceutical industry includes firms that are

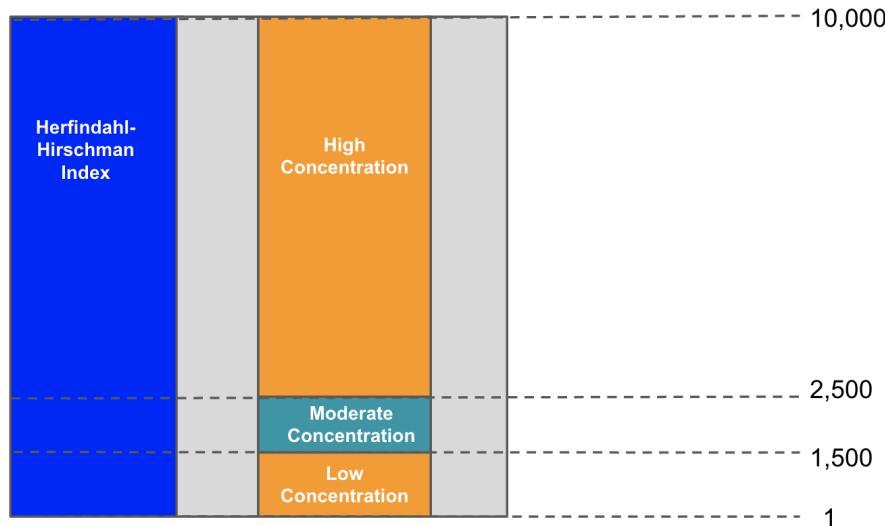
involved in developing, discovering, producing, and selling pharmaceutical drugs. This industry includes companies like Pfizer (NYSE:PFE) and Johnson & Johnson (NYSE:JNJ). Companies are always worried about their competition, however, they are mainly worried about competition within their own industry.

Low market competition can lead to several problems. A **monopoly** is a market structure where a single firm has the exclusive possession or control of the supply of or trade in a commodity or service. A **duopoly** is when the market is controlled by two companies. An **oligopoly** is when the market is controlled by two or more companies, but still a small number of companies, in such a way that no single company can hinder the others from having significant influence. These type of markets can be very problematic because they can block new newcomers to the market, decrease inventiveness and increase prices. All of these things are detrimental to consumers. When a few companies are in charge and are able to set the prices, profit margins end up being higher than what would be the case in a more competitive market.

One way to measure market competitiveness is with the **Herfindahl-Hirschman Index (HHI)**. The HHI is the sum of the squared market shares of all of the firms in a market. The formula is

$$HHI = \sum_i s_i^2$$

where  $s_i$  is the market share of firm  $i$ , expressed as a percentage (so a firm with 25% of all market shares would have  $s_i = 25$ ). The sum of the  $s_i$  values should equal 100 ( $\sum_i s_i = 100$ ). The highest possible HHI is 10,000. In this case we have a monopoly. The lowest possible HHI is 0. A low HHI is an indicator that there is a fair amount of competition within market, where no one firm controls all the customers. A high HHI shows a small number of firms have control of the entire market and may indicate an oligopoly.



The U.S. Department of Justice and the Federal Trade Commission have the responsibility of regulating market competition. The U.S. Department of Justice considers markets based on their HHI in the following manner.

- $HHI < 1,500$ : competitive marketplace
- $HHI$  between 1,500 and 2,500: moderately concentrated marketplace
- $HHI > 2,500$ : highly concentrated marketplace

A highly concentrated marketplace implies minimal competition. One important time the HHI is calculated is before and after mergers and acquisitions. This helps the U.S. Department of Justice and the Federal Trade Commission to ensure the merging of certain firms doesn't lead to problems associated with an oligopoly. Generally, mergers that lead to an increase of the HHI by 200 points or more raise concerns in concentrated markets. Data for all firms in every market is collected every 5 years.

The HHI has advantages and disadvantages. The HHI is advantageous because it is simple to compute and does not require much data. However, the simplicity of this measure completely ignores the complexities that exist within specific markets so is not an extremely accurate gauge of market competitiveness.

Consider an industry with 4 firms:

- Firm A market share: 20%
- Firm B market share: 30%
- Firm C market share: 25%
- Firm D market share: 25%

The HHI is calculated in the following manner:

$$\begin{aligned}
 HHI &= \sum_i s_i^2 \\
 &= s_1^2 + s_2^2 + s_3^2 + s_4^2 \\
 &= (20)^2 + (30)^2 + (25)^2 + (25)^2 \\
 &= 2550
 \end{aligned}$$

As expected, we have a highly concentrated marketplace because we have only four firms. However, the number of firms is not always enough to get an accurate picture of market concentration. This is why we need the HHI.

**Question:** Suppose 7 new firms enter the market. The market shares are now such that Firm A has 90% and all other firms have an equal share of the remainder. What is the value of the HHI in this case? Has the increase

*in firms led to an increase in market competitiveness? Do increases in the HHI lead to increases or decreases in competition?*

### Buying and Holding Stocks

Once you choose a competitive market and the companies you wish to invest in, you will buy stocks that will fluctuate in value over time. In order to build wealth, it is important to buy and hold stocks. This avoids the accrual of capital gains tax. These taxes are not accrued when stocks are held, they are only accrued when the stocks are sold. Holding a stock allows you to build equity that will be tax-free until the stock is sold. We will look more at taxes and taxing investments later on in this packet.

### Price-to-Book Ratio

One way to find undervalued companies and avoid overvalued companies is to examine the **price-to-book ratio (P/B ratio)**. The P/B ratio is the ratio of the market value of a company's shares to its book value of equity.

$$\text{price-to-book ratio} = \frac{\text{market price per share}}{\text{book value per share}}$$

A company's **book value of equity** is the difference between the book value of assets and the book value of liabilities when considering the company's assets on a balance sheet.

$$\text{book value} = \text{assets} - \text{liabilities}$$

$$\text{book value per share} = \frac{\text{assets} - \text{liabilities}}{\text{number of shares outstanding}}$$

If a P/B ratio is equal to one, a stock's trading price is aligned with the company's book value. This means the stock has a fair price, with respect to P/B ratios. Lower P/B ratios, especially those below 1, may mean a stock is undervalued because the stock's trading price is low with respect to the value of the company's assets. Higher P/B ratio values, or those greater than 1, may mean a stock is overvalued relative to the company's assets. A company's P/B ratio is best compared to the ratios of other companies within that industry.

We need the following things in order to calculate the P/B ratio:

- market price per share of the stock
- total amount of assets
- total amount of liabilities
- total number of outstanding equity shares

While the P/B ratio is helpful in determining overvalued and undervalued company's, it is not without flaws.

It is considered most helpful when looking at “capital-intensive” industries. These include energy, automobile manufacturing, telecommunications, and transportation sectors. The P/B ratio also has shortcomings because the book value does not consider intangible assets like intellectual property and branding and also does not account for debt possibly boosting asset values. The ratio can also be misleading because while a low P/B ratio can mean a company is undervalued, it can also mean the company has a poor or negative return on assets.

**Question:** *Jan is looking for company's to invest in. After looking at a Company A's balance sheet, she finds the following:*

*Assets: \$200 million*

*Liabilities: \$ 50 million*

*Outstanding shares: 10 million*

*Stock price: \$10 per share*

1. *What is the company's book value? What is the company's book value per share?*
2. *What is the company's P/B ratio? Based strictly on this value, would you say the stock is undervalued, overvalued, or fairly priced?*
3. *An inside source revealed to Jan that Company A has 30 million dollars in debt that has not been accounted for? How should this effect Jan's consideration of the P/B ratio? Based on this, would you now say the stock is undervalued, overvalued, or fairly priced? Why?*

Price to Earnings Ratio The **price-to-earnings ratio (P/E ratio)** is a statistic that compares a company's current share price to its earnings per-share (EPS). The P/E ratio is also called the price multiple or earnings multiple.

$$\text{P/E ratio} = \frac{\text{market value per share}}{\text{earnings per share}}$$

. Therefore, we can easily calculate this value by dividing a company's current stock price by the EPS. The P/E ratio tells us the dollar amount an investor should expect to invest in a company in order to receive a dollar in earnings. This is how much an investor should expect to pay per dollar of earnings, which is why it is also called the price or earnings multiple.

There are two types of P/E ratios you may see given for a company. One of these is the “P/E (TTM),” also called a trailing P/E, which is the P/E ratio available on many financial sites. The acronym “TTM” means “trailing twelve months” and shows the company's performance over the last year. This is the most popular P/E ratio because, assuming honest and accurate reporting on behalf of the company, it is fairly objective. This is not an estimate based

on a particular investor's viewpoint, but rather strictly based on a company's performance. However, using this to make choices in investments can be risky because a company's past performance will not always indicate how it will behave in the future.

The **earnings per share (EPS)** is a company's profit divided by the companies outstanding shares of common stock.

$$\text{Earnings per Share} = \frac{\text{net income-preferred dividends}}{\text{end-of-period common shares outstanding}}$$

This value tells us how much a company makes from each share of stock. In order to calculate this, a company's balance sheet and income statement are required for finding net income/earnings, period-end number of common shares, and dividends paid on preferred stock. Higher EPS values typically indicate higher profitability when evaluating companies, however, comparing P/E ratios rather than EPS values is generally regarded as a better practice among investors.

Company's will also release a "forward P/E" that is a company's expectation of earnings in the future based on performance to date. This is also called an "estimated price to earnings." Investors should be careful when considering a company's released forward P/E ratio because this is not a perfect metric. Different investors might produce different estimates that are not very close to the company's estimate which can be confusing.

The P/E ratio helps investors compare a stock's market value to the company's earnings. Investors are generally expecting higher earnings from companies with higher P/E ratios than those with lower P/E ratios. A low P/E ratio might show that a company's current stock price is low relative to earnings and, therefore, undervalued. It might also mean a company is doing extremely well with respect to how it behaved historically. On the other hand, a high P/E ratio might tell us that a stock's price is high relative to company earnings and that the stock is overvalued. If a company has a P/E ratio listed as "N/A" it may mean the company has no earnings or that it is posting losses because negative P/E ratios are not commonly calculated.

Question: *Company A's stock price closed at \$85.20 on November 12, 2018. The number of shares outstanding was 2.8 billion. The company's profit for the fiscal year that ended January 31, 2018 was US\$12.2 billion. The company paid \$0 in preferred dividends. Calculate the EPS and P/E ratio.*

#### Distributions of Discrete Variables

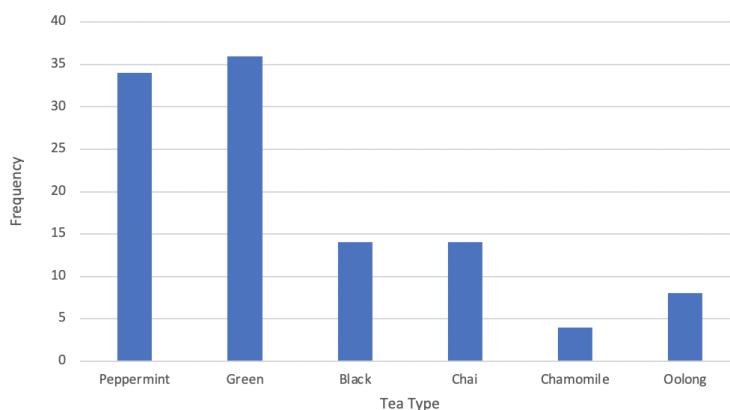
We can express data in different ways. Consider the following set of data. Kenya bought a bag of assorted teas. There were six types of tea. After counting the tea bags, there were 110 tea bags: 34 peppermint tea bags, 36 green tea bags, 14 black tea bags, 14 chai tea bags, 4 chamomile tea bags and 8 oolong tea bags.

First, Carla decided to use the data to make a frequency table. A **frequency table** is a table that contains the number of occurrences in each class of data; for example, the number of each type of tea in an assorted bag of teas. This table describes the distribution of tea type frequencies. This type of distribution is called a **frequency**

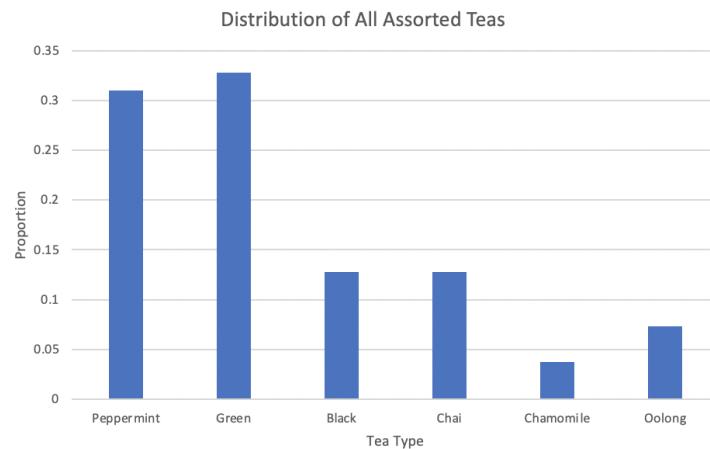
**distribution.** Carla decided to express the frequency distribution graphically as well.

Tea Type	Frequency
Peppermint	34
Green	36
Black	14
Chai	14
Chamomile	4
Oolong	8

Distribution in a Bag of Assorted Teas



The distribution from Carla's data is specific to her single bag of assorted teas. However, what about the distribution of all the teas? Carla contacted the company and was able to gather some information about this. Based on all of the tea bags ever produced, the company reported the proportions of each of the six teas. From the information given, Carla was unsure of how many tea bags were produced in general and the frequency of each tea type produced. Carla represented this data graphically.



The company only makes six types of tea so every tea bag ever made is one of the six flavors from Carla's frequency table. Because of this, the six proportions sum to one. The figure above is called a **probability distribution**. This is because the probability of getting a certain type of tea bag when choosing a tea bag at random is equal to the proportion of tea bags that are that kind. For example, the probability of choosing a peppermint tea bag at random is equal to the proportion of peppermint tea bags, which is approximately .31 or 31%.

Analyzing the graphs, we can see that they are different. The first graph shows the distribution in a sample of 110 tea bags. The second shows the proportions for all tea bags. The machines used by the company to produce the tea bags and package the bags of assorted tea bags introduce random variation into the different bags produced. When you purchase a random bag of assorted teas, you may have one bag that is very close to the second graph while another may be further away.

### Transitioning to Continuous Variables

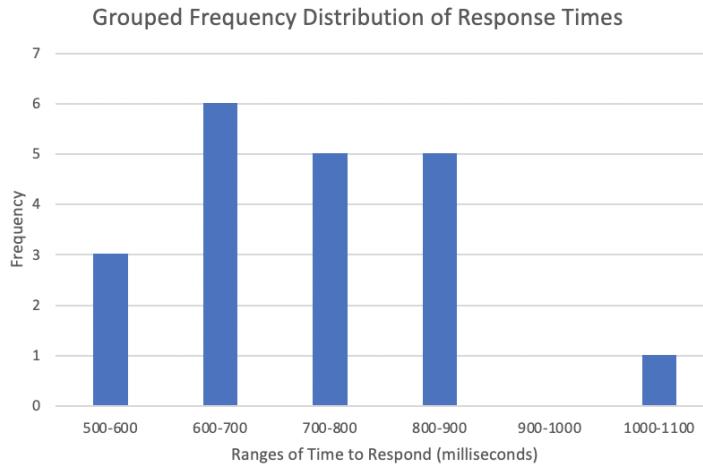
In the example we just considered with Carla and the tea bags, “type of tea” is a variable that is considered a discrete variable. The associated distribution os also considered discrete. A **discrete variable** is a variable that can only take on a distinct list of (usually finite) possible values that you can count. A **continuous variable** is a variable that can take on any value from a spectrum of possibilities. We can extend our understanding of distributions to continuous variables.

Consider a company wants to keep track of call response times for their receptionists. They kept track of the time needed (in milliseconds) to answer a call in a series of 20 trials. The variable “time to respond” is a continuous variable. If time is measured accurately (for example, in microseconds or nanoseconds) then no two delivery times would be expected to be the same. In psychology, measuring time in milliseconds is usually accurate enough to approximate a continuous variable. As we see, measuring time in this way produced data for call response times such that no two response times were the same.

Response Times			
571	643	720	824
577	657	728	832
581	668	729	865
628	696	777	875
641	703	808	1007

Consider the frequency distribution for this table. This table would consist of 20 times in the experiment, each having a frequency of 1. The table would be extremely uninformative. However, we can instead use a grouped frequency distribution. Using a **grouped frequency distribution**, response times falling within carious ranges are counted instead. A grouped frequency distribution for the 20 times is given below.

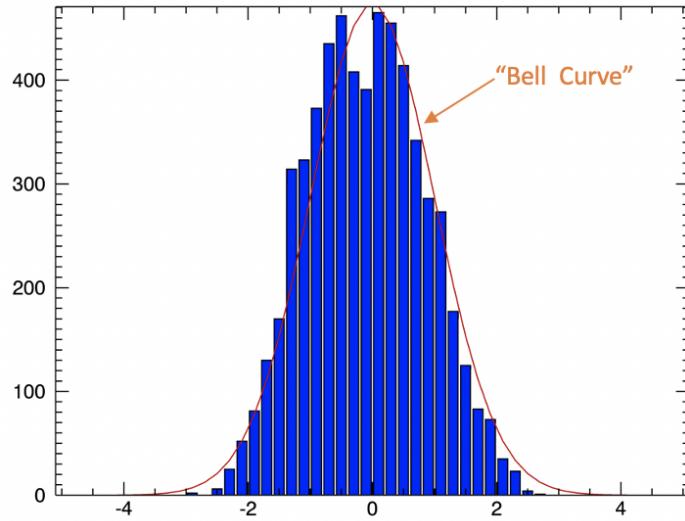
Range	Frequency
500 – 600	3
600 – 700	6
700 – 800	5
800 – 900	5
900 – 1000	0
1000 – 1100	1



The graph above shows the times from the 20 trials in a single experiment done by the company. It is possible for a response time to be any positive amount of time. We can represent the probability of an arbitrary response time by representing all potential times at once. To do this, we need to plot the distribution for the continuous, rather than discrete, variable. A **continuous distribution** expresses the probabilities of possible values of a continuous random variable. These are also referred to as probability density.

### The Normal Distribution

Data can be distributed in different ways, for example, it can be all jumbled up, spread out more toward the left or right or be centered around some value. There are many cases where we find collected data has no bias towards the left or right and tends to surround a central value. Data like this gets close to a “Normal Distribution.” A Normal Distribution is also called the “Bell Curve” because the shape of the curve resembles a bell. The data below is an example of a case where a collected group frequency table of a group of data resembles a normal distribution pretty closely, but not perfectly (which is typically the case).



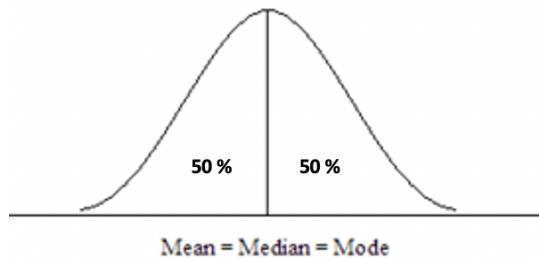
Examples of data that behave like a Normal Distribution:

- test grades
- heights of people
- baby birth weights
- shoe size
- rolling fair dice
- IQ
- income distribution

Why do we say the data is distributed “normally”? A **Normal Distribution** has

- symmetry about the center,
- 50% of values greater than the mean
- 50% of values less than the mean
- mean, median and mode are all equal.

## Symmetric Distribution



The **mean** of a set of data is the sum of all the data values, divided by how many values there are. In other words,

$$\text{mean} = \frac{\sum_{i=1}^N d_i}{N}$$

where  $N$  is the number of data values and each  $d_i$  is a data value. The mean is often denoted with the Greek letter mu,  $\mu$ . For example, the mean of the set of weights of young adult patients

Patient A: 125 lbs Patient B: 180 lbs Patient C: 145 lbs Patient D: 155 lbs Patient E: 145 lbs

is calculated in the following manner

$$\begin{aligned}\text{mean} = \mu &= \frac{\sum_{i=1}^N d_i}{N} = \frac{\sum_{i=1}^5 d_i}{5} \\ &= \frac{125 + 180 + 145 + 155 + 145}{5} = 150\end{aligned}$$

which means the mean of the set of data is 150 lbs. The mean is also referred to as the average of a set of data.

The **median** of a set of data is the “middle” of the sorted list of values. To calculate the median of the set of data above, we first sort the data from least to greatest and then choose the middle value.

125 lbs, 145 lbs, 145 lbs, 155 lbs, 180 lbs

If there are an even number of data values, we take the median to be the average of (or the halfway point between) the two middle values. To do this, we simply sum the two numbers and divide by two, or take the mean.

The **mode** of a set of data is the value that appears most frequently. In the set of data we have been looking

at of patient weights, there is only one weight that appears more than once,

125 lbs, 145 lbs, 145 lbs, 155 lbs, 180 lbs

which is 145 lbs. This is the mode. There are times when a set of data has more than one mode. Two values may appear in a set of data an equal number of times, which is more times than any other value appears in the data. A set of data with two modes is called **bimodal**. A set of data with more than two modes is called **multimodal**.

So far, we have discussed the mean, median and mode of a set of data. We can also calculate the standard deviation. The **standard deviation** for a set of data measures how spread out the values in our set of data are. The standard deviation is typically denoted with the Greek letter sigma,  $\sigma$ . The standard deviation is the square root of the variance. The **variance** of a set of data is the average of the squared differences from the mean. The variance is denoted  $\sigma^2$ . We calculate the variance by

1. Find the mean of the set of data.
2. For each value in the data set, subtract the mean and square the resulting value. This is the squared difference.
3. Find the average (or the mean) of those squared differences.

We can write this

$$\text{variance} = \sigma^2 = \frac{\sum_{i=1}^N (d_i - \mu)^2}{N}$$

where  $N$  is the number of data values, each  $d_i$  is a data value, and  $\mu$  is the mean of the set of data.

Let's return to our data. We will calculate the variance and the standard deviation. Recall, we have already calculated the mean,  $\mu = 150$  lbs.

$$\begin{aligned}\text{variance} = \sigma^2 &= \frac{\sum_{i=1}^N (d_i - \mu)^2}{N} = \frac{\sum_{i=1}^5 (d_i - 150)^2}{5} \\ &= \frac{(125 - 150)^2 + (145 - 150)^2 + (145 - 150)^2 + (155 - 150)^2 + (180 - 150)^2}{5} \\ &= \frac{(-25)^2 + (-5)^2 + (-5)^2 + (5)^2 + (30)^2}{5} \\ &= \frac{625 + 25 + 25 + 25 + 900}{5} = \frac{1600}{5} = 320\end{aligned}$$

$$\text{standard deviation} = \sigma = \sqrt{\sigma^2} = \sqrt{320} = 17.89$$

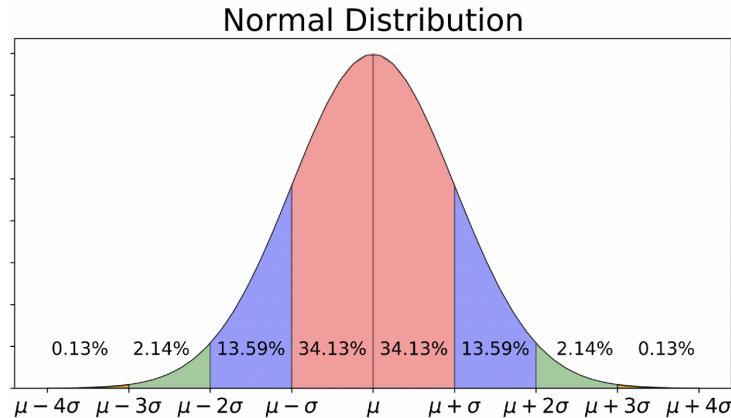
Therefore, the standard deviation is 17.89 lbs, which is 18 lbs if we round to the nearest lb. Which weights are

within one standard deviation (or 18 lbs) of the mean? We can see that three of the weights, 145 lbs, 145 lbs, and 155 lbs are within one standard deviation of the mean.

For the normal distribution that we were discussing previously, we find that generally

- 68% of values fall within **1 standard deviation** of the mean
- 95% of values fall within **2 standard deviations** of the mean
- 99.7% of values fall within **3 standard deviations** of the mean

This means that any value is “likely” to be within one standard deviation of the mean, “highly likely” to be within two standard deviations of the mean and is “almost certainly” within three standard deviations of the mean.



**Question:** The *closing price* of a stock (the last price at which a stock trades during a regular trading session) is recorded for a given company’s stock for a year. The company finds that 95% of the closing prices recorded were between \$1.10 and \$1.70. Assuming the data is normally distributed, calculate the mean and standard deviation.

#### Using the Normal Distribution to Help Make Investments

When investor’s look for stocks they consider the risk and the return of their investment. Ideally, an investor wants to find low-risk, high-reward situations, stocks that offer the lowest risk possible with the highest return possible. Stock prices are assumed to be normally distributed. The normal distribution is able to help evaluate risk and return through evaluating aspects of the mean and standard deviation of these stock prices.

We can consider the mean of a stock’s price over any given day. Company A’s mean stock price is given over 5 days.

Day 1: \$0.90 Day 2: \$1.17 Day 3: \$1.76 Day 4: \$2.28 Day 5: \$3.42

We can also look at the way in which this changes from day to day. From Day 1 to Day 2, we know the mean stock price changed from \$0.90 to \$1.17. This is a

$$.90p = \$1.17 \rightarrow p = 1.3$$

which is a growth factor of 1.3 or a 30% increase. Looking at the rest of the days,

% Change Day 1-2: 30% % Change Day 2-3: 50% % Change Day 3-4: 30% % Change Day 4-5: 50%

Taking the average, we see

$$\text{Average \% Change} = \frac{30\% + 50\% + 30\% + 50\%}{4} = 40\%$$

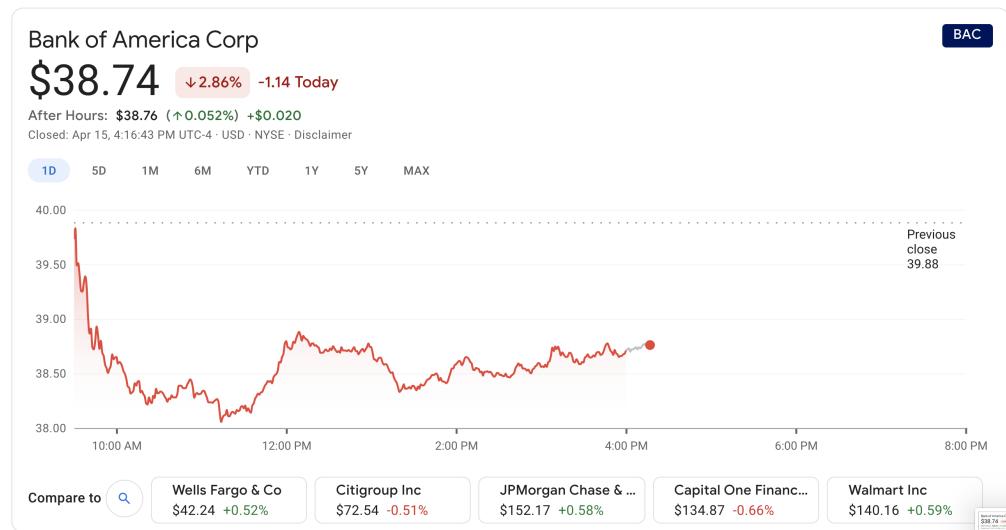
For this company, the mean change of the company's stock price was about 40% daily. This means, the price of the stock increased by 40% daily, on average. This mean value can help us to make predictions on what to expect in the future. With a large enough data set of historical daily price changes, investor's can make judgements on how much they can expect their investments to grow in the future. The higher the mean change is for a given company, the better investment it is likely to be.

Question: *Company B looks at their stock prices over 5 years and found that the the mean change of the company's stock price was about 40% increase monthly. An investor wants to know what they can expect the company's stock price to be next month if it is currently \$23.45 and trends remain the same. What price can the investor expect?*

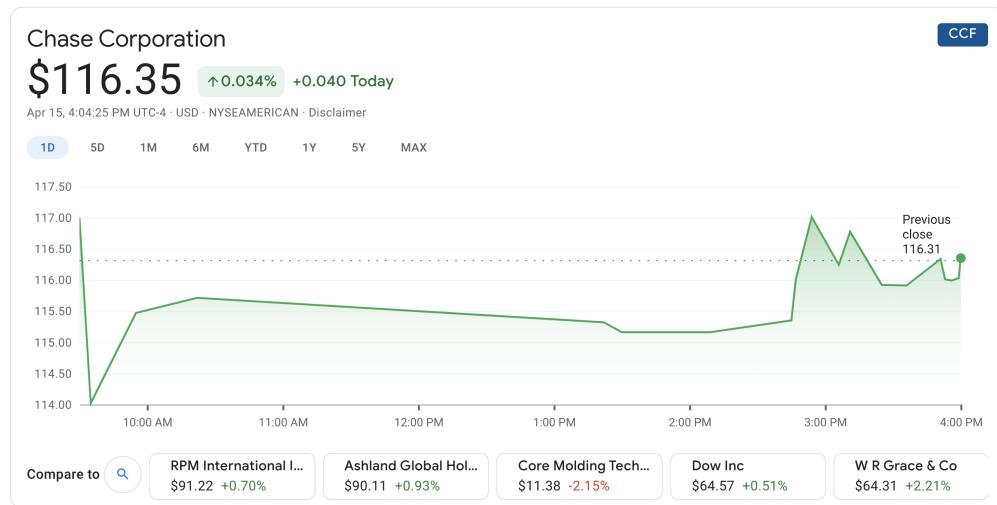
While the mean is useful in evaluating the possible reward of an investment, the standard deviation can be used to evaluate the possible risk. The standard deviation tells us exactly how much data values deviate from the mean, on average. Investors usually associate higher standard deviations with riskier investments. This is because the higher the standard deviation, the more uncertainty there is.

Question: *Company A and Company B record their mean stock price daily over a year and calculate the mean and standard deviation of this data. The mean stock price for Company A is \$20.50 and the standard deviation is 0.50. The mean stock price for Company A is \$25.00 and the standard deviation is 2.50. First, find the range for which 99% of all values fall for both Company A and Company B. Which is the riskier investment? In your opinion, which is the better investment?*

Putting Investment Strategies Into Practice Look at the following key statistics for Bank of America and Chase. This data is from Google Finance and was accessed after market close on April 15, 2021.



Key stats	
PREVIOUS CLOSE	\$39.88
DAY RANGE	\$38.05 - \$39.84
YEAR RANGE	\$20.10 - \$40.38
MARKET CAP	333.66B USD
VOLUME	57.63M
P/E RATIO	20.66
DIVIDEND YIELD	1.86%
CDP CLIMATE CHANGE SCORE	A
PRIMARY EXCHANGE	NYSE



### Key stats

PREVIOUS CLOSE	\$116.31
DAY RANGE	\$114.02 - \$117.01
YEAR RANGE	\$77.00 - \$123.13
MARKET CAP	1.10B USD
VOLUME	21.41K
P/E RATIO	28.41
DIVIDEND YIELD	0.69%
PRIMARY EXCHANGE	NYSEAMERICAN

We can see that the same statistics are given for both companies so that we can easily compare. The stock price for a single stock is listed below the company name. For Bank of America, the price of one stock was \$38.74 and for Chase, the price of one stock was \$116.35. Next to these values are the percentage increase of the stock price and the dollar increase for the current day. Bank of America's stock price was down 2.86% and down \$1.14 from the start of the day. Chase's stock was up 0.034% and up \$0.04 from the start of the day. **After-hours trading** is buying and selling of securities that occurs after the market closes and is done without the use of a traditional stock exchange. Not all stocks are traded during after-hours trading. For Bank of America, we can see that the price of the stock had gone up during after-hours trading to \$38.76. There is no after-hours trading information available for Chase because the information was collected too close to the end of the market day (4:00 pm EST).

The charts below show the price action of the respective stocks throughout the day. There are choices to see the price action over the last five days, one month, six months, year, five years, and over the maximum possible time. When a stock price has increased from the previous day's close, the chart will be green. When a stock price has decreased from the previous day's close, the chart will be red.

There are various key statistics given for these stocks. **Previous close** shows the price of the stock at market close the previous day. **Day range** shows the maximum and minimum value the stock reached during the given day. **Year range** shows the maximum and minimum value the stock reached during the last year. **Market cap** is short for market capitalization and shows the total value of all a companies shares of stocks. **Volume** is the total number of shares that are traded during the trading day. **P/E ratio** shows the price-to-earnings ratio discussed earlier. **Dividend yield** shows the percentage of a company's share price that it pays out in dividends each year. For certain people like retirees, much of their income depends on dividends so higher dividend yields are preferred. Younger investors who are willing to invest money and watch companies grow over time may be less interested in dividend yield. **Primary exchange** is the largest and most significant stock exchange in a given country. In the case of the United States of America, this is the New York Stock Exchange (NYSE or NYSEAMERICAN).

For Bank of America, an extra key statistic is listed. **CDP climate change score** is a carbon disclosure rating that measures of the environmental sustainability of a corporation. The CDP is a non-profit organization that helps companies and cities disclose their environmental impact. Bank of America has a CDP climate change score of an A.

### The Stock Market Game

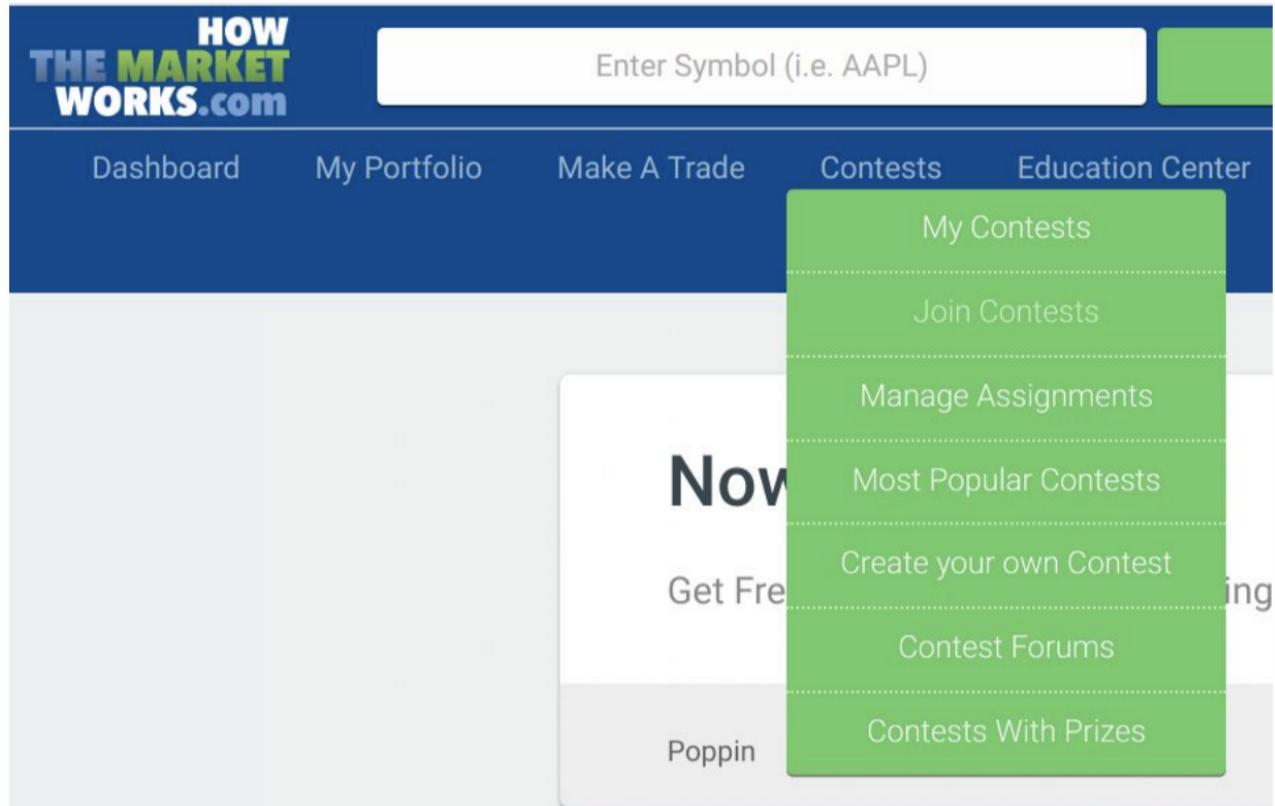
At this point you've learned the basics of stocks and bonds. Let's put your skills to the test with the stock market game. You will start out with a cash balance of \$100,000 to spend on whatever stocks or bonds you desire. Our simulated stock market will close on the day before the last day of camp and whoever has the most money when the market closes on that day will be the winner!

First, register your account here:

<https://www.howthemarketworks.com/register/154626>

Create your account by selecting a username, password, age range, country, name, select 'no' for "Are you an instructor?" and 'no' for "Do you currently own a brokerage account?". You can enter an email address if you want but if you don't you can't reset your password if you forget it!

After logging in, select 'Contests' near the top of the screen, then select 'Join Contest' on the top of the screen:



Search for "GTM 2021" and select 'Join', use the password 'talkinmath'.

CONTEST NAME	ADMIN	DATES	TOTAL PLAYERS
GTM 2019 This competition is for participants in the University of Maryland's 2019 Girls Talk Math Program.	samclayt	06/18/2019 - 06/27/2019	2

Next, head to your ‘Dashboard’ using the menu near the top of the screen. Our simulation website is set up just like a real stock trading website. You should see your portfolio value of \$100,000 on the Dashboard:

MY DASHBOARD | CONTEST SELECT < GTM 2019 >

< PORTFOLIO SUMMARY: PORTFOLIO VALUE: \$100,000.00 CASH BALANCE: \$100,000.00 BUYING POWER: \$1

Now you’re free to start trading. Select ‘Made a Trade’ from the main menu at the top of the screen.

To start, select “Buy” and enter the ticker of the stock you want to buy. Think of your favorite companies that you’ve considered in terms of the investment strategies you’ve learned and Google search ‘“company name” stock’ to find the company’s stock ticker. Enter the quantity you wish to purchase and keep the order type as “Market”. A market order means your purchase will happen as soon as possible. For example, the below order will purchase 20 shares of Google:

CHOOSE: UNITED STATES 🇺🇸 TORONTO 🇨🇦

ACTION Buy	SYMBOL THROW A DART! GOOG	QUANTITY 20	TYPE Market	LIMIT/STOP PRICE 0	ORDER TERM Good-till-Day
DATE mm/dd/y					

Scroll down, select “Preview Order” and then select “Confirm Order” if it looks good to you.

Build out your portfolio in any way you choose, buying and selling stocks and bonds over the course of the program using your \$100,000.

If you are interested in buying bonds to reduce your risk, purchase the ticker “SHV”.

Use the investment strategies you’ve learned along with whatever research you want to determine what stocks

to buy. You could even Google search “What stocks to buy” and see how you do with those. Whoever has the most money when the markets close at the end of the program is the winner!

GOOD LUCK!

**Question:** *While playing the stock market game, use Google Finance to look up your investments. Which statistics did you care most about as you chose your stocks? Which statistics did you care least about? Are you interested in more long term investments or short term investments? How would your portfolio compare and differ as a student graduating high school and an adult preparing for retirement? Which investments would you recommend for a person in each of these positions and why?*

## Starting a Business

Imagine you want to start a business on Etsy selling monogrammed hats. You think you can make some money reselling the hats but you want to run the numbers before jumping into a new business. To determine how many hats you'll have to sell to make a profit for yourself, you can express your business as a series of functions. Your business profits can be expressed as:

$$\text{Profit} = \text{Revenue} - \text{Cost}.$$

**Revenue** is the money you earn through your sales and **costs** are the bills you pay to keep your business running. The difference between those amounts is your **profit**.

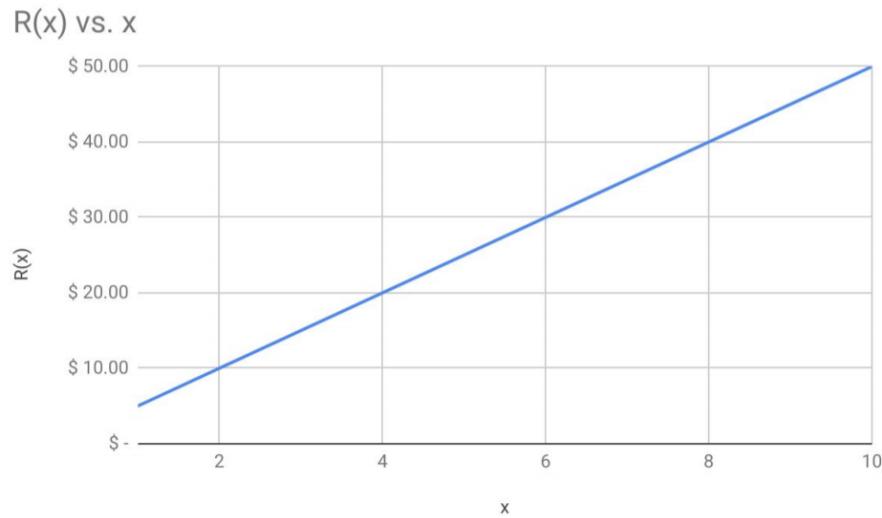
Imagine you represent the number of hats you make as the variable  $x$ . The above profit equation can be written as the function:

$$P(x) = R(x) - C(x).$$

Each of the components of your business: the profits, revenue, and costs are all a function of how many hats are sold,  $x$ . For example, if a company sells pens for \$5 each, the revenue function of the pen business could be expressed as:

$$R(x) = 5x$$

We can plot this equation as a simple linear regression:



For example, if the company sells 8 pens, their revenue is \$40.

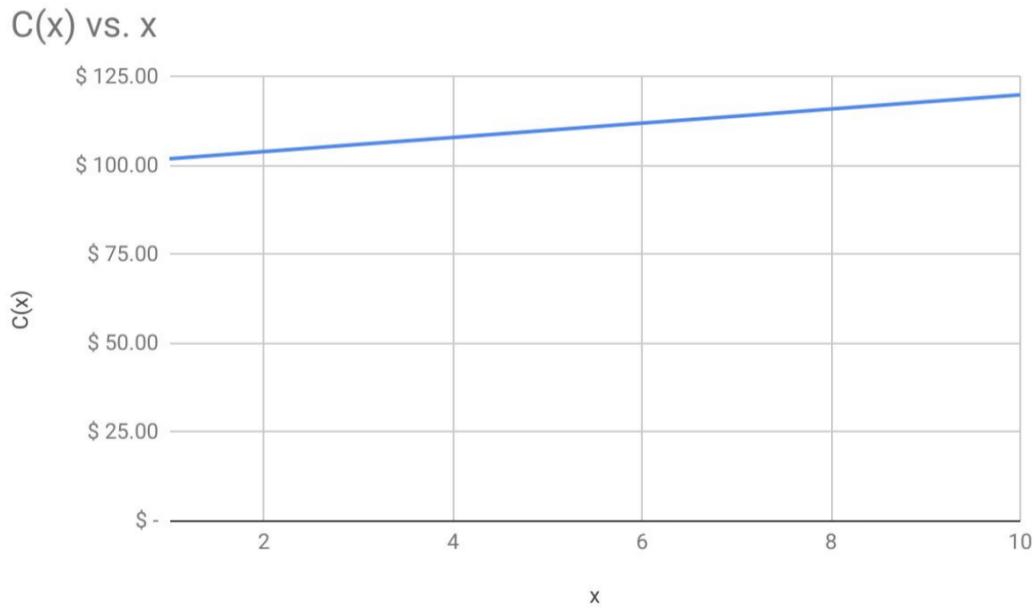
Let's consider the costs of starting a business. Costs include machinery, materials, time, and a wide variety of other costs. Costs like machinery or monthly office rent are considered fixed costs because the costs are the same no

matter how much is produced. Costs such as materials or worker salaries are considered variable costs because the costs vary depending on how much is produced. Total costs are the sum of fixed and variable costs.

For example, if the machinery (a fixed cost) in a pen factory cost \$100 and each pen cost \$2 in materials (a variable cost) to produce, the cost function could be expressed as:

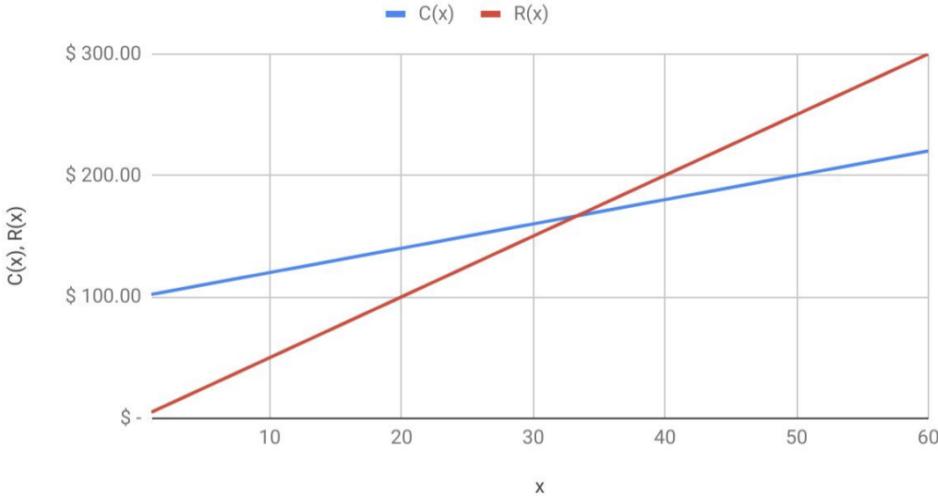
$$C(x) = 100 + 2x$$

We can plot the cost function as follows:



As you can tell, the higher the fixed costs of a business the more revenue must be generated to make a profit.  
Let's look at the cost function and revenue function on the same graph:

## Cost and revenue functions



**Question:** What does the point where the cost and revenue functions meet represent? What does it mean when your costs are the same as your revenue?

**Question:** Using the cost and revenue functions from the pen example, solve for  $x$  when the costs are equal to revenue. How many pens must be sold to break even?

Let's return to the Etsy business. To start your new business you buy an embroidery machine for \$400. Next, you find a supplier that will sell you blank hats for \$5 per hat. It costs you 50 cents in materials to embroider each hat.

**Question:** How would you express these costs as a function of the number of hats produced?

You do some research on Etsy and determine \$12 to be the going rate of embroidered hats.

**Question:** How do you express the revenue of your hats as a function,  $R(x)$ ?

**Question:** Express the profit function of your hats,  $P(x)$ .

**Question:** What are your profits if you sell 300 hats?

**Question:** How many hats do you have to sell to make back your \$400 investment in the embroidery machine?