Pure Mathematics
Advanced Level

"Once your soul has been enlarged by a truth, it can never return to its original size."
-Blaise Pascal

Notes By

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Introduction

Let an event be denoted by A. The probability that event A occurs is denoted by $\mathrm{P}(A)$ and is given by

$$\Pr(A) = \frac{\text{no. of ways in which } A \text{ can occur}}{\text{total no. of outcomes}}$$

Also note that the probability of event A not happening is defined by $1 - \Pr(A)$ and is denoted by $\Pr(A^c)$.

Ex. 1. A letter is chosen at random from the world 'CALCULUS'. Find the probability that it is:

a) a 'C'.
b) a vowel.

Ex. 2. A card is chosen at random from a pack of 52 playing cards. Find the probability that the card is:

a) an ace.
b) black.
c) a heart.
d) a royal card.

Using Permutations and Combinations

In the following problem, the number of successful outcomes and the total number of outcomes are calculated using the counting techniques in the previous chapter.

Ex. 1. A team of 6 children is chosen at random from a class of 10 girls and 9 boys. Find the probability thattge selected team contains:

a) girls only.
b) boys only.
c) more girls than boys.
d) the oldest 5 children in the class.

Bernoulli Trials

A Binomial Experiment is an experiment which satisfies the following 4 conditions:

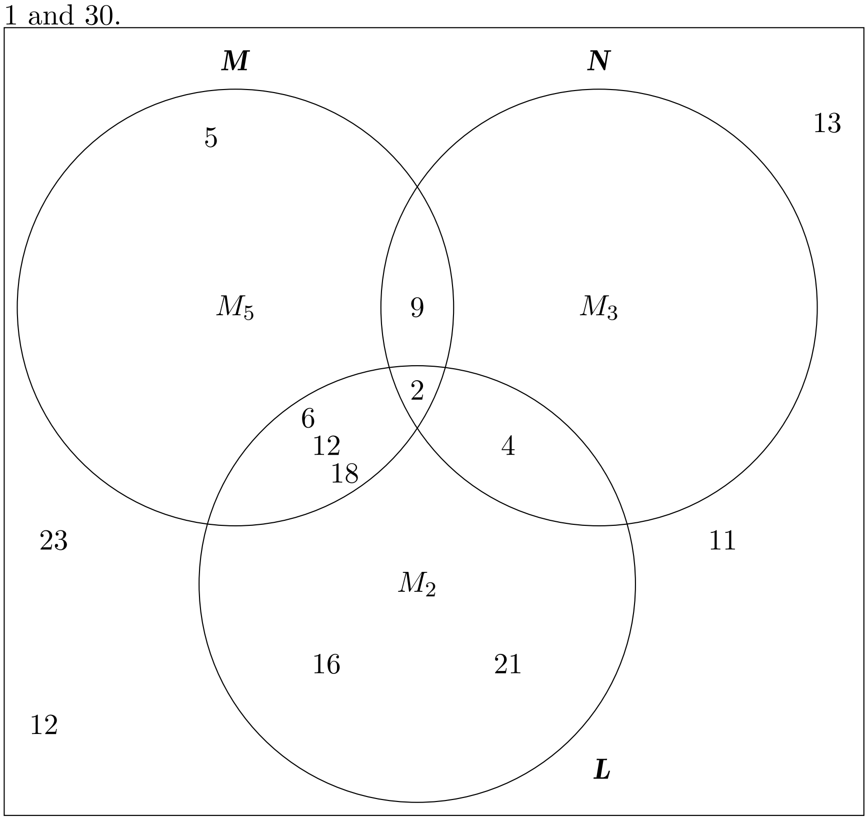
- A fixed number of trials.
- Each trial is independent of the others.
- There are only **two** outcomes.
- The probability of each outcome remains constant from trial to trial.

Probability Space Diagrams and Tree Diagrams.

Tree diagrams and possibility spaces are simple graphical tools which help us calculate probabilities. Possibility space diagrams are particularly useful when the problem involves independent events.

Venn Diagrams

Venn diagrams ilustrate sets of objects or items and set operations. Consider for instance the sets M_2, M_3 and M_5 which stand for multiples of 2, 3 and 5 respectively. The **universal** set U is the set of all integers between 1 and 30. For convenience, let us consider the set P to be the set of all primes between 1 and 30.



- The intersection between sets M_2 and M_3 is denoted by $M_2 \cap M_3$ and hence $M_2 \cap M_3 = \{6, 12, 18, 25, 30\}$ and $M_2 \cap M_3 \cap M_5 = \{30\}$.
- The union between the sets M_3 and M_5 is denoted by $M_3 \cup M_5$ and hence $M_3 \cup M_5 = \{3, 5, 6, 7, 10, 12, 15, 18, 20, 21,$
- We already know that the set that is formed by the members of the universal set which are not in the set M_2 is called the complement of the set M_2 denoted by M_2^c. Hence, $M_2^c = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 2$

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