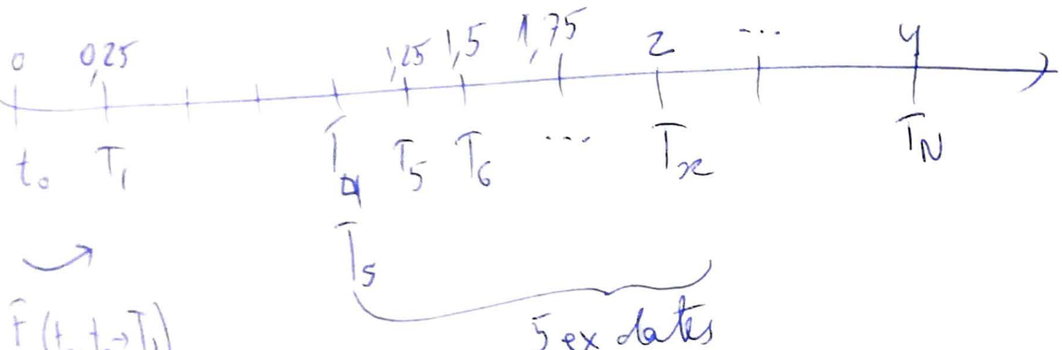
Problem set M

$K = 0,05$

→ Price a Bermudean swap option.
↳ option on a swap.
↳ can exercise at a predetermined set of dates.



$F(t_0, t_0 \to T_1)$

$F(t_0, T_1 \to T_2)$

if exercised at T_e the first payoff occurs at T_{e+1}.

$F_k(t) = F(t, T_{k-1}, T_k)$: forward rate contracted at t for the period T_{k-1} to T_k

Dynamics of how forwards evolve through time

$$F_k(T_j) = F_k(T_{j-1}) \exp\left[\left(\underbrace{\mu_k(T_{j-1})} - \frac{1}{2}\sigma_k(T_{j-1})^2\right)\underset{0,25}{\tau} + \sigma_k(T_{j-1})\Delta W_k(T_j)\right]$$

$$\mu_k(T_{j-1}) = \sigma_k(T_{j-1}) \cdot \sum_{b=\beta(t)}^{k} \frac{\tau \rho_{bk}\sigma_b(T_{j-1})F_b(T_{j-1})}{1+\tau F_b(T_{j-1})}$$

$t = T_j(?) \Rightarrow \beta = j?$

$\rho_{bk} = \cos(\theta_b - \theta_k)$

$\beta(t)$ = index of the first F alive

$$\sigma_k = \begin{cases} 0,2 & \forall k \in [2, 7] \\ 0,22 & \forall k \in [8, 11] \\ 0,24 & \forall k \in [12, 16) \end{cases} \qquad \sigma_k(\cancel{T_{j-1}})$$

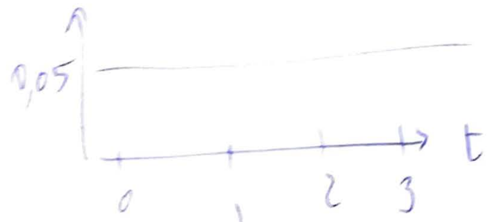
2. $dW_k = \cos(\theta_k)\, dW^{(1)} + \sin(\theta_k)\, dW^{(2)}$

$$\theta_k = \frac{\pi}{2} \cdot \frac{k-2}{14}$$

1) Generate the paths of the forward rates

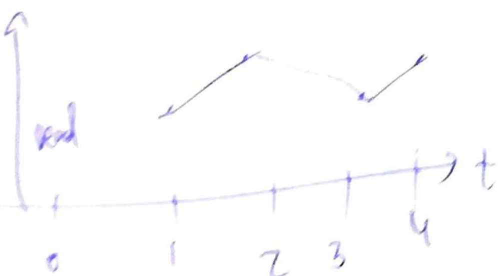
intuition

at $t=0$ flat term structure



$f(t=0, F_{k-1}, F_k) = 0,05 \ \forall k$

at $t=1$ use dynamics to gen F



$F(t=1, T_{k-1}, T_k)$ $\boxed{\text{at } 1 \ \beta = 1}$ $\beta(T_j) = j$?

value of the underlying swap at $T_j = V(T_j)$ [payer]

⚠ swap is not like in the slides.

$$V(T_j) = \tau \sum_{k=j+1}^{N} \underbrace{P(T_j, T_k)}_{\prod_{i=j+1}^{k} \frac{1}{1+\tau F_i(T_j)}} \cdot (F_k(T_j) - K)$$

→ sum of discounted payoffs.

2) Andersen Algorithm: → Backward induction

2.1) Generate all forward paths.

2.2) At T_x (last ex date), exercise swaption if the underlying swap is in the money ↳ compute the $V(T_x)$

2.3) At previous date T_{x-1} ← for now select any value

exercise if $V > H$ ← later test many values.

if not, exercise at T_x if $V_{T_x} > 0$

↳ gives a payoff at T_x or at T_{x-1}

and so discount and get W_H: price of swaption for this particular simulation if only 2 ex dates available and depending on H.

Do 2,3) for many simulated paths and aggregate.

→ gives mean (W_H)

→ do this for many values of H.

4) Select the H which gives the highest mean.
H will be the optimal rule at T_{X-1}

5) Repeat step 3 for previous ex dates
until you get all optimal stopping rules
∀ ex dates

6) Once you have opti H generate new
paths & use H for optimal price