A Comparative Study of Image Compression Techniques based on SVD, DWT-SVD and DWT-DCT

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A Comparative Study Of Image Compression Techniques Based On Svd, Dwt-Svd And Dwt-Dct

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Abstract

Image compression using DCT (Discrete Cosine Transform) in JPEG standard had been a popular technique before JPEG2000 standard came up in which DWT (Discrete Wavelet Transform) is used. For saving bandwidth of transmission, the storage requirement and in turn the cost, other more efficient compression and coding techniques are reported. Singular Value Decomposition (SVD) was initially explored for image compression and later on tried with DWT, and DCT to get DWT-SVD and DCT-SVD based different techniques for image processing. These techniques are mainly being used for watermarking in copyright protection and security in the industries. But these techniques have never been compared in terms of their compression performances and resultant image quality. Here we undertake a comparative study of SVD, DWT-SVD and DWT-DCT based image compression techniques. We study the PSNR (Peak Signal to Noise Ratio) and percentage of compression variations when these techniques are applied on a standard image data type. We find that DWT-DCT outperforms SVD and to some extent DWT-SVD, providing significant compression due to the use of wavelet transform with optimum PSNR for optimal image quality (above PSNR=20dB).

Keywords

DCT (Discrete Cosine Transform), DWT (Discrete Wavelet Transform), SVD (Singular Valued Decomposition), Image compression, Watermarking, PSNR (Peak Signal to Noise Ratio).

1. Introduction

For saving transmission bandwidth, power, storage space, as is mostly required in wireless multimedia communication, efficient data compression algorithms have become a necessity. Thus it has turned out to be a present day craze and source of competition in the race for technology and research with so much manpower, time and money involved for its development [1] [2].Two widely used image compression standards are JPEG and JPEG2000. The former is based on the Discrete Cosine Transform (DCT), and the latter the Discrete Wavelet Transform (DWT). A few years ago, a third algorithm called the Singular Value Decomposition (SVD) was explored and found to be a most useful tool in various image processing fields. Hybrid or mixed techniques are also reported based on SVD, DCT, and DWT such as DWT-SVD, DWT-DCT, and DCT-SVD.Out of the image compression techniques available, transform coding is the most preferred method [1] [2] [3]. Since energy distribution after transform coding varies with each image, compression in the spatial domain is not an easy task [4].Images do however tend to compact their energy in the frequency domain making compression in the frequency domain much more effective. Transform coding is simply the compression of the images in the frequency domain [5]. Transform coefficients are used to maximize compression. For lossless compression, the coefficients must not allow for loss of any information. We have seen that DCT and DWT have been among the most popular transform coding used. Initially it was DCT which was popularized by the JPEG then came up the JPEG2000 which used the DWT [6] [1] [2]. Based on a comparative study [7] to evaluate the performance difference of the DCT and the Daubechies-6 wavelet based image coding and reconstructions, it was reported that for still images DWT outperforms DCT.

For DWT of a standard image data, we use the Daubechies-6 wavelet and proceed for low & high pass filtering with decimations for both row-wise and column-wise as shown in the parallel cascaded structure of Figure 1. Using the Daubechies-6 wavelet we do not get exactly 75% compressions. The size of the approximated image after the single level DWT varies from 33% to 27% of the original image.

The DCT is a well known algorithm. The 2 D-DCT, $P(i,j)$ of a discrete image block of size 8x8 is represented as $F[i,j]$ as shown in Equation (1). As the transform matrix after DCT has a DC value of significance and most of the AC values of the matrix either zero or very small, we apply masks [7][18] on the DCTed output image with zeros per 8x8 blocks as shown in Fig. 2. This type of masking reduces the number of entries in the image matrix by a good amount.

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$$F[i,j]=\frac{1}{4}C(i)C(j)\sum_{x=0}^{7}\sum_{y=0}^{7}P(x,y)\cos\left(\frac{(2x+1)i\pi}{16}\right)\cos\left(\frac{(2y+1)j\pi}{16}\right)\ldots(1)$$

where x, y, i and j all vary from 0 to 7

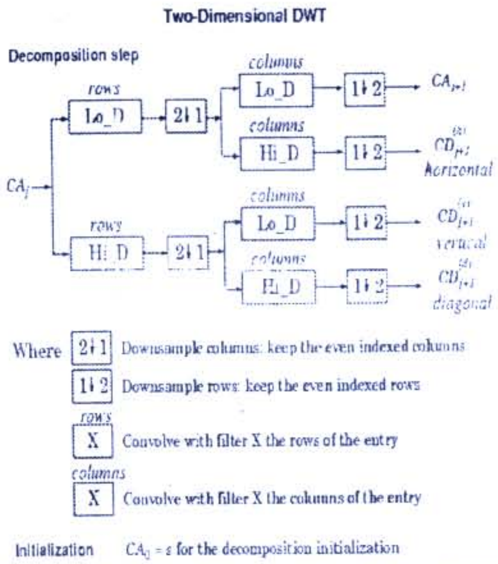

Figure 1: Single level Discrete Wavelet Transform steps to extract approximate image

$$C(i)\text{ and }C(j)=\begin{cases}\frac{1}{\sqrt{2}} & \text{for } i \text{ and } j=0\\ 1 & \text{for all other values of } i \text{ and } j\end{cases}$$

Figure 2: Example Masks to be applied after DCT

The mathematical tool of SVD, which is a generalization of the eigen-value decomposition, is used to analyze rectangular matrices (the eigen-value decomposition is defined only for squared matrices) and is also explored for image processing. The main idea of the SVD is to decompose a rectangular matrix into three simple matrices (two orthogonal matrices and one diagonal matrix) [8]. It has been widely studied [9] [10] [11] [12] and used for data compression (mainly image). If the image-matrix, has low rank, or can be approximated sufficiently well by a matrix of low rank, then SVD can be used to find this approximation. Further this low rank approximation can be represented much more compactly than the original image. Given an image I(x,y) of size N x M, a real matrix with rank K, then it can be factored into its SVD representation $I(x,y)=USV^T$, where S is a diagonal matrix of order NxM having only K nonzero entries along the diagonal ordered in a non-increasing order, and U (NxN) and V (MxM) are orthogonal matrices [13]. Then a rank r approximation to I(x,y) is the matrix $I_r(x,y)=U_rS_rV_r^T$, where S_r is the top-left r x r sub-matrix of S, U_r consists of the first r columns of U, and V_r^T the first r rows of V^T. The SVD decomposition is interesting because U_r, S_r, V_r^T provide the best rank r approximation to X in the sense of packing the maximum energy from X. Thus we achieve compression as X has NxM entries while U_r, S_r, V_r^T together there are only r(N+M+1) entries. For an optimal choice of r which may be much smaller than K, the approximation X_r gets most of the energy of X, and is still visually adequate. And as in this process we multiply U_r and S_r to get US_r so we have total r(N+M) for US_r and V_r^T.

$$I(x,y)=USV^T$$

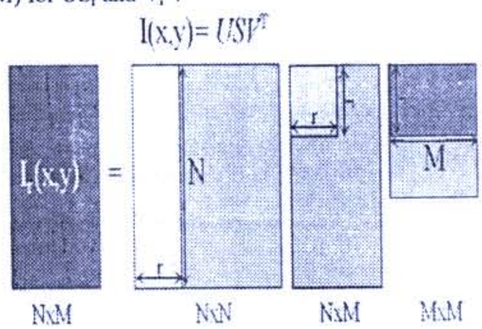

Figure 3: SVD operation on image I

Till now the SVD with DCT hybrid techniques were used for watermarking and copyright protection [14] based on LPSNR (Low peak signal to noise ratio) and image encoding [16]. Hybrid DWT and SVD (HDWTSVD) technique has also been used for these purposes [15] [17].

In this study, DWT is the main tool used to firstly reduce the image size to 1:4 ratio(approx) of the original image (67-73% reduction in the number of entries). Then on the approximate image we apply either DCT or SVD. In this paper, we make a comparative study on the performances of SVD, DWT-SVD and DWT-DCT techniques in terms of the PSNR of the decompressed image and the percentage of compression achieved, compared to the original image. As far as our knowledge goes this is the first comparison between the three techniques.

2. Implementation

The implementation steps are illustrated in the Fig. 4 where we show that the same input image under goes the steps at three different paths providing SVD, DWT-SVD and DWT-DCT, and then percentage of compression is calculated on the compressed data. Thus Image I(x,y) under goes three different processes separately to provide three sets of image matrices SVD(x,y), DWT-SVD(x,y) and DWT-DCT(x,y).

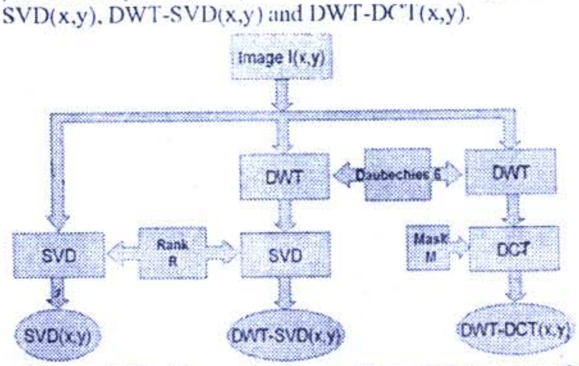

Figure 4: The three sets of operations undergone on the original image for compression

Out of these three, the first that have undergone SVD only is represented as a set of two matrices i.e $US_{SVD}(x,y)$ and $V^T_{SVD}(x,y)$ of rank r where $US_{SVD}(x,y)$ is the product of $U_{SVD}(x,y)$ and $S_{SVD}(x,y)$. The decompressed image then becomes $I_r(x,y)=US_{SVD}(x,y).V^T_{SVD}(x,y)$. The image in the second method where the image has under gone SVD after DWT, is also represented in terms of two matrices as above i.e. $US_{DWT\text{-}SVD}(x,y)$ and $V^T_{DWT\text{-}SVD}(x,y)$. So to decompress it we have to first take multiplication to get the approximate image and then take the inverse DWT (IDWT) with blank horizontal, vertical and diagonal components using the daubechies-6 wavelet to finally get $I_{DWT\text{-}SVD}(x,y)$. In the last case, i.e., in DWT-DCT, the DWTed image is DCTed and after applying masks the reconstructed image i.e. $I_{DWT\text{-}DCT}(x,y)$ is obtained through inverse transforms and zero padding [7].The process is illustrated in Figure 5.

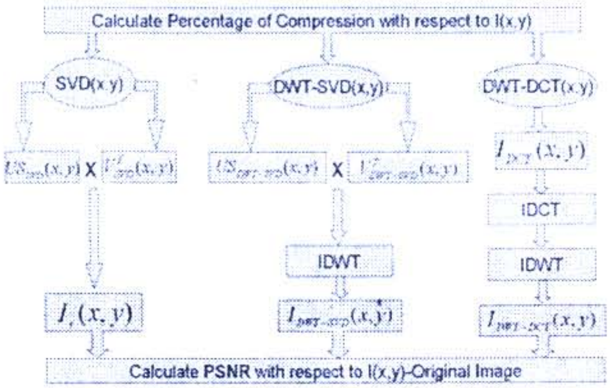

Figure 5: The three sets of operations undergone on compressed image for reconstruction

Based on the final output of the steps as shown in Fig. 4, we calculate the percentage of compression based on the number of entries at the final stage (without any coding) with respect to the original image I(x,y). Then at the end of the reconstruction phase of Fig. 5, based on the reconstructed images $I_r(x,y)$, $I_{DWT\text{-}SVD}(x,y)$, and $I_{DWT\text{-}DCT}(x,y)$ we calculate their respective PSNRs with reference to the original image I(x,y).

To compare the reconstructed image with the original image we apply PSNR criterion. The ratio between the maximum possible power of a signal and the power of corrupting noise that affects the fidelity of its representation is called PSNR (peak signal-to-noise ratio). It is commonly used as a measure of quality of reconstruction in image compression etc. Because many signals have a very wide dynamic range, PSNR is usually expressed in terms of the logarithmic decibel scale. The original image $X(i,j)$ and the reconstructed image $\hat{X}(i,j)$ may be compared for evaluation of the effectiveness of the compression technique by calculating the mean squared error (MSE) as given by Eq. (2).

$$MSE = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}\left\|X(i,j)-\hat{X}(i,j)\right\|^2 \qquad (2)$$

The X(i,j) is considered as a noisy approximation of the $X(i,j)$. PSNR is expressed by Eq. (3)

$$PSNR = 20\log_{10}\left(\frac{MAX_I}{\sqrt{MSE}}\right) \qquad (3)$$

Here, MAX_I is the maximum pixel value of the image. When the pixels are represented using 8 bits per sample, MAX_I is 255.

3. Results

Though we have used a number of commonly available images, we present the results here based on the Babra image of size 512x512. Fig.6 shows the PSNR and percentage of compression variation with r which is the reduced rank, for the SVD operation on the original image. Fig. 7 shows the PSNR and the percentage of compression variation with rank r in the case of DWT-SVD. Fig.8 shows variation of PSNR and percentage of compression with number of mask bits in case of DWT-DCT.

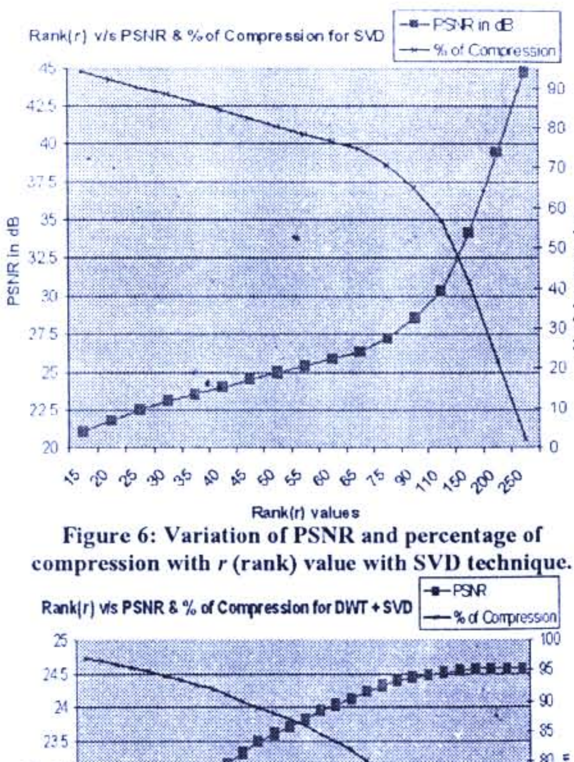

Figure 6: Variation of PSNR and percentage of compression with r (rank) value with SVD technique.

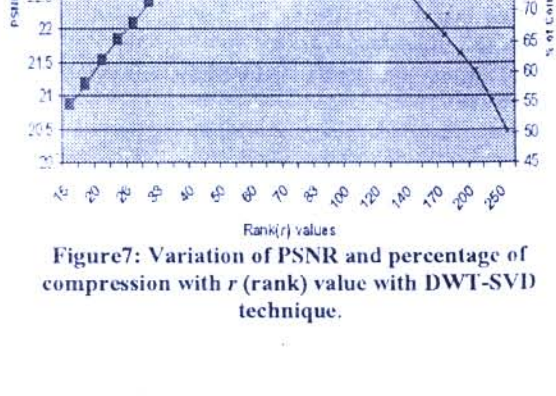

Figure7: Variation of PSNR and percentage of compression with r (rank) value with DWT-SVD technique.

We finally have comparative graphs, based on the above results for all three techniques showing variation of PSNR with %compression, using predictive curve fitting of 6^{th} order polynomial trend lines (solid lines) on the actual results as illustrated in Fig. 9. Here we kept the percentage of compression for the graphical plots within the range of 70% to 100% as due to initial DWT application in DWT-SVD and DWT-DCT techniques, we already have around 67%-73% compression. Moreover the range of the PSNR of these techniques is limited as with DWT application alone we have a PSNR of 24.5 dB. So for DWT-SVD and DWT-DCT the maximum PSNR is 24.5 dB. We also calculated the bits per pixel verses PSNR variation and shown in Fig 10. For this also we have used curve fitting (solid lines) with the 6^{th} order polynomial trend line.

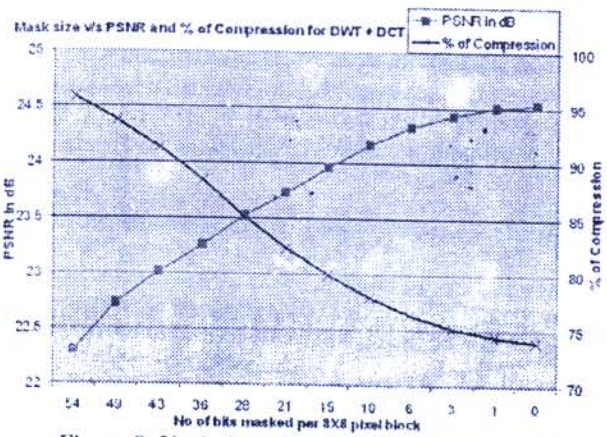

Figure 8: Variation of PSNR and percentage of compression with number of masked bits per 8x8 block value with DWT-DCT technique

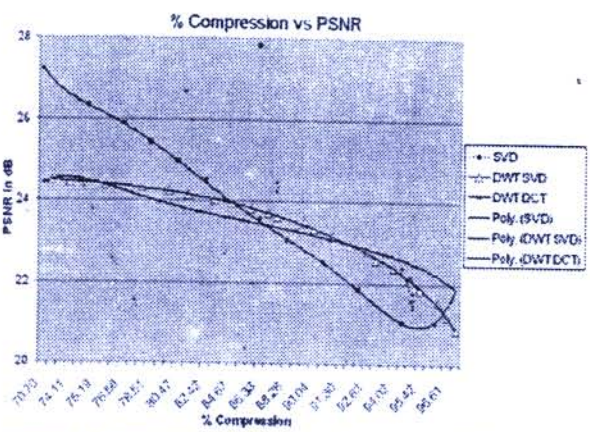

Figure 9: Percentage of compression verses PSNR for all the three techniques with their predicted trend lines.

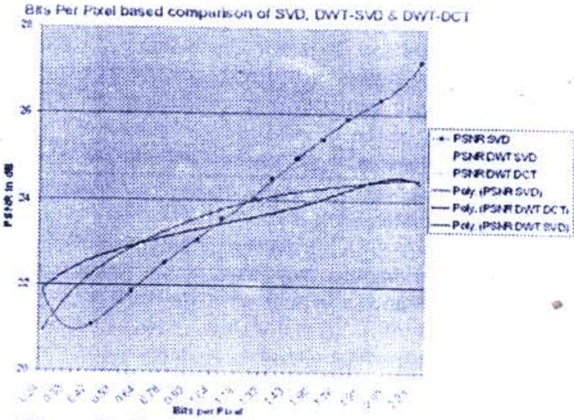

Figure 10: Bits per pixels verses PSNR for all the three techniques with their predicted trend lines.

4. Conclusion

From the results as explained above we can conclude that if we use only SVD then we have a higher PSNR compared to the other two techniques for percentages of compression less than 85%. But for higher percentage of compression we need to apply the hybrid techniques (DWT-DCT, DWT SVD). Due to application of DWT alone, the image has a PSNR value around 24.5dB. For both DWT-SVD and DWT-DCT technique we find that, for percentage of compression in between 75% to 92%, the PSNR of the reconstructed image lies in between 24.5dB to 23dB. So as there is not much significant difference in PSNR value, i.e. image quality, so we may go for a minimum of 92% compression of the original image using these mixed or hybrid techniques. Moreover it is evident from the graphs of Fig. 9 and Fig. 10 that, if we compress even upto 96-97% then also PSNR degrades only by 3.5 dB from 24.5dB of the DWTed image. Now if we compare performances between DWT-SVD and DWT-DCT techniques, we find that they do not have much difference in characteristics due to initial application of DWT. But in case of higher percentages of compression i.e. 94% to 97%, DWT-DCT gives around 1dB more PSNR compared to DWT-SVD. For lower percentages of compression in the range of 75%-92%, the DWT-SVD image has higher PSNR than DWT-DCT image.

We may finally conclude that DWT-DCT and DWT-SVD techniques both give better PSNR values than SVD alone for compression of the image above 85%. For compressions below 85% the PSNR of SVD is better. Moreover these percentages of compressions are without entropy coding, and we know that DWT-DCT image data would reduce further on entropy coding compared to DWT-SVD or SVD alone. So to get better percentage of compression with reasonable PSNR value on the reconstructed image it may be useful to apply DWT-DCT technique with entropy coding like RLE (Run-Length Encoding) or Huffman coding [7].

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