Financial Econometrics
HT Week 1 Assignment Answers

February 2021

Exercise 4.5

Write the AR(1) $Y_t = \phi_0 + \phi_1 Y_{t-1} + \varepsilon_t$ as an MA(∞) assuming $|\phi_1| < 1$.

$$
\begin{aligned}
Y_t &= \phi_0 + \phi_1 Y_{t-1} + \varepsilon_t \\
\textbf{using } Y_{t-1} &= \phi_0 + \phi_1 Y_{t-2} + \varepsilon_{t-1} \\
Y_t &= \phi_0 + \phi_1 \left(\phi_0 + \phi_1 Y_{t-2} + \varepsilon_{t-1}\right) + \varepsilon_t \\
&= \phi_0 + \phi_1 \phi_0 + \phi_1^2 Y_{t-2} + \phi_1 \varepsilon_{t-1} + \varepsilon_t \\
\textbf{using } Y_{t-2} &= \phi_0 + \phi_1 Y_{t-3} + \varepsilon_{t-2} \\
&= \phi_0 + \phi_1 \phi_0 + \phi_1^2 \left(\phi_0 + \phi_1 Y_{t-3} + \varepsilon_{t-2}\right) + \phi_1 \varepsilon_{t-1} + \varepsilon_t \\
&= \phi_0 + \phi_1 \phi_0 + \phi_1^2 \phi_0 + \phi_1^3 Y_{t-3} + \phi_1^2 \varepsilon_{t-2} + \phi_1 \varepsilon_{t-1} + \varepsilon_t \\
&= \phi_1^0 \phi_0 + \phi_1^1 \phi_0 + \phi_1^2 \phi_0 + \phi_1^3 Y_{t-3} + \phi_1^2 \varepsilon_{t-2} + \phi_1^1 \varepsilon_{t-1} + \phi_1^0 \varepsilon_{t-0} \\
&= \sum_{i=0}^{\infty} \phi_1^i \phi_0 + \sum_{j=0}^{\infty} \phi_1^j \varepsilon_{t-j} \\
&= \frac{\phi_0}{1-\phi_1} + \sum_{j=0}^{\infty} \phi_1^j \varepsilon_{t-j}
\end{aligned}
$$

Exercise 4.12

Answer the following questions:

1. **Under what conditions on the parameters and errors are the following processes covariance stationary?**

   (a) $Y_t = \phi_0 + \varepsilon_t$
   If ε_t is a white noise.

   (b) $Y_t = \phi_0 + \phi_1 Y_{t-1} + \varepsilon_t$
   If ε_t is a white noise and $|\phi_1| < 1$.

   (c) $Y_t = \phi_0 + \theta_1 \varepsilon_{t-1} + \varepsilon_t$
   If ε_t is a white noise.

   (d) $Y_t = \phi_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \varepsilon_t$
   If ε_t is a white noise and ϕ_1 and ϕ_2 lie within a triangular region bound by (-2,-1),(0,1) and (2,-1) where the coordinates are (ϕ_1, ϕ_2). This can be easily observed using the quadratic formula.

   (e) $Y_t = \phi_0 + \phi_2 Y_{t-2} + \varepsilon_t$
   If ε_t is a white noise and $|\phi_2| < 1$ is finite. Note that technicall we need to check the roots of $z^2 - \phi_2$ but these are just $c = \pm\sqrt{\phi_2}$ so that these are less than 1 in absolute value if $|\phi_2| < 1$.

   (f) $Y_t = \phi_0 + \phi_1 Y_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t$
   If ε_t is a white noise and $|\phi_1| < 1$.

Exercise 4.19

Suppose you observe the three sets of ACF/PACF in the figure below. What ARMA specification would you expect in each case. Note: Dashed line indicates the 95% confidence interval for a test that the autocorrelation or partial autocorrelation is 0.

1. The first appears consistent with an MA(2). The key features are that the ACF cuts off sharply after 2 lags while the PACF appears to be more complex.

2. The second matches the shape that one would expect if the data was generated by an AR(1) with a positive AR coefficient. The ACF decays exponentially while the PACF cuts off sharply after one lag.

3. The third appers are be consistent with a White Noise process. None of the ACF or PACF values are outside of the CI, and so there appears to be no dependence in the data.

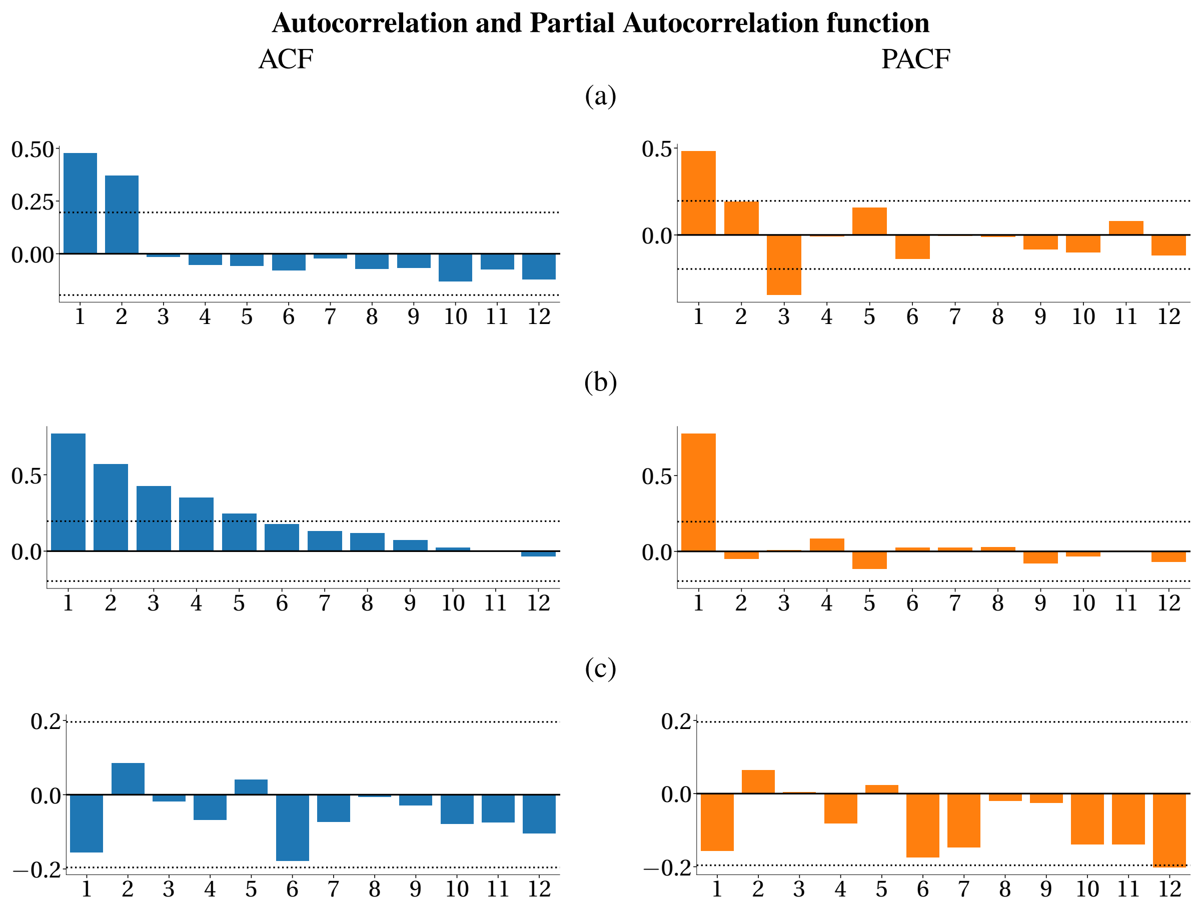

Figure 1: The ACF and PACF of three stochastic processes. Use these to answer the question .