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1 // Linux	1
1 for i in {A..Z}; do cp tem.cpp \$i.cpp; done	1
1 // Windows	1
2 'A'..'Z' % { cp tem.cpp "\$_.cpp" }	2
1.2 run	2
2 g++ -std=c++20 -DPEPPA -Wall -Wextra -Wshadow -O2 -fsanitize=	2
2 undefined \$1.cpp -o \$1 && ./ \$1	2
1.3 tem	2
2 #include <bits/stdc++.h>	2
3	3
3 using namespace std;	3
3 using i64 = long long;	3
3	3
3 #define int i64	3
3 #define all(a) a.begin(), a.end()	3
4 #define rep(a, b, c) for (int a = b; a < c; a++)	4
5	5
5 #ifdef PEPPA	5
5 template <typename R>	5
5 concept I = ranges::range<R> && !std::same_as<ranges::	5
5 range_value_t<R>, char>;	5
5 template <typename A, typename B>	5
6 std::ostream &operator<<(std::ostream &o, const std::pair<A, B>	6
6 &p) {	6
6 return o << "(" << p.first << ", " << p.second << " ";	6
6 }	6
6 template <I T>	6
6 std::ostream &operator<<(std::ostream &o, const T &v) {	6
6 o << "{";	6
6 int f = 0;	6
6 for (auto &i : v) o << (f++ ? " " : "") << i;	6
7 return o << "}";	7
7 }	7
7 void debug__(int c, auto &&...a) {	7
7 std::cerr << "\e[1;" << c << "m";	7
7 (... , (std::cerr << a << " "));	7
8 std::cerr << "\e[0m" << std::endl;	8
8 }	8
8 #define debug_(c, x...) debug__(c, __LINE__, "[" + std::string	8
8 (#x) + "]", x)	8
8 #define debug(x...) debug_(93, x)	8
8 #else	8
8 #define debug(x...) void(0)	8
9 #endif	9
9	9
10 bool chmin(auto& a, auto b) { return (b < a and (a = b, true));	10
10 }	10
10 bool chmax(auto& a, auto b) { return (a < b and (a = b, true));	10
11 }	11
11	11
11 void solve() {	11
11 //	11
11 }	11
12	12
12 int32_t main() {	12
12 std::ios::sync_with_stdio(false);	12
12 std::cin.tie(nullptr);	12
12	12
12 int t = 1;	12
12 std::cin >> t;	12
13	13
13 while (t--) {	13
13 solve();	13
14 }	14
14	14
14 return 0;	14
17 }	17
1.4 debug	18
18 #ifdef PEPPA	18
18 template <typename R>	18
18 concept I = ranges::range<R> && !std::same_as<ranges::	18
18 range_value_t<R>, char>;	18
18 template <typename A, typename B>	18
19 std::ostream& operator<<(std::ostream& o, const std::pair<A, B	19
19 &p) {	19
19 return o << "(" << p.first << ", " << p.second << " ";	19
19 }	19
19 }	19
19 template <I T>	19
20 std::ostream& operator<<(std::ostream& o, const T& v) {	20

```

o << "{";
int f = 0;
for (auto &i : v) o << (f++ ? " " : "") << i;
return o << "}";
}

void debug__(int c, auto&&... a) {
    std::cerr << "\e[1;" << c << "m";
    (... , (std::cerr << a << " "));
    std::cerr << "\e[0m" << std::endl;
}

#define debug_(c, x...) debug__(c, __LINE__, "[" + std::string
    (#x) + "]", x)
#define debug(x...) debug_(93, x)
#else
#define debug(x...) void(0)
#endif

```

1.5 run.bat

```

@echo off
g++ -std=c++23 -DPEPPA -Wall -Wextra -Wshadow -O2 %1.cpp -o %1.
exe
if "%2" == "" ("%.exe") else ("%1.exe" < "%2")

```

1.6 random

```

std::mt19937_64 rng(std::chrono::steady_clock::now().
    time_since_epoch().count());
inline i64 rand(i64 l, i64 r) { return std::
    uniform_int_distribution<i64>(l, r)(rng); }

```

1.7 TempleHash

```

cat file.cpp | cpp -dD -P -fpreprocessed | tr -d "[:space:]" |
md5sum | cut -c-6

```

2 Misc

2.1 FastIO

```

#include <unistd.h>
int OP;
char OB[65536];
inline char RC() {
    static char buf[65536], *p = buf, *q = buf;
    return p == q && (q = (p = buf) + read(0, buf, 65536)) == buf
        ? -1 : *p++;
}

inline int R() {
    static char c;
    while ((c = RC()) < '0');
    int a = c ^ '0';
    while ((c = RC()) >= '0') a *= 10, a += c ^ '0';
    return a;
}

inline void W(int n) {
    static char buf[12], p;
    if (n == 0) OB[OP++] = '0';
    p = 0;
    while (n) buf[p++] = '0' + (n % 10), n /= 10;
    for (--p; p >= 0; --p) OB[OP++] = buf[p];
    if (OP > 65520) write(1, OB, OP), OP = 0;
}

// another FastIO
char buf[1 << 21], *p1 = buf, *p2 = buf;
inline char getc() {
    return p1 == p2 && (p2 = (p1 = buf) + fread(buf, 1, 1 << 21,
        stdin), p1 == p2) ? 0 : *p1++;
}

template<typename T> void Cin(T &a) {
    T res = 0; int f = 1;
    char c = getc();
    for (; c < '0' || c > '9'; c = getc()) {
        if (c == '-') f = -1;
    }
    for (; c >= '0' && c <= '9'; c = getc()) {
        res = res * 10 + c - '0';
    }
    a = f * res;
}

template<typename T, typename... Args> void Cin(T &a, Args &...
    args) {
    Cin(a), Cin(args...);
}

template<typename T> void Cout(T x) { // there's no '\n' in
    output

```

```

if (x < 0) putchar('-'), x = -x;
if (x > 9) Cout(x / 10);
putchar(x % 10 + '0');
}

```

2.2 stress.sh

```

#!/usr/bin/env bash
g++ $1.cpp -o $1
g++ $2.cpp -o $2
g++ $3.cpp -o $3
for i in {1..100}; do
    ./$3 > input.txt
    # st=$(date +%s%N)
    ./$1 < input.txt > output1.txt
    # echo "$(((date +%s%N) - $st)/1000000))ms"
    ./$2 < input.txt > output2.txt
    if cmp --silent -- "output1.txt" "output2.txt" ; then
        continue
    fi
    echo Input:
    cat input.txt
    echo Your Output:
    cat output1.txt
    echo Correct Output:
    cat output2.txt
    exit 1
done
echo OK!
./stress.sh main good gen

```

2.3 Timer

```

struct Timer {
    int t;
    bool enable = false;

    void start() {
        enable = true;
        t = std::clock();
    }

    int msec() {
        assert(enable);
        return (std::clock() - t) * 1000 / CLOCKS_PER_SEC;
    }
};

```

3 Data Structure

3.1 Fenwick Tree

```

template<class T>
struct Fenwick {
    int n;
    vector<T> a;
    Fenwick(int _n) : n(_n), a(_n) {}
    void add(int p, T x) {
        for (int i = p; i < n; i = i | (i + 1)) {
            a[i] = a[i] + x;
        }
    }
    T qry(int p) { // sum [0, p]
        T s{};
        for (int i = p; i >= 0; i = (i & (i + 1)) - 1) {
            s = s + a[i];
        }
        return s;
    }
    T qry(int l, int r) { // sum [l, r]
        return qry(r - 1) - qry(l - 1);
    }
    pair<int, T> select(T k) { // first position >= k
        T s{};
        int p = 0;
        for (int i = 1 << __lg(n); i; i >= 1) {
            if (p + i <= n and s + a[p + i - 1] < k) {
                p += i;
                s = s + a[p - 1];
            }
        }
        return {p, s};
    }
};

```

3.2 Li Chao

```
struct Line {
    // y = ax + b
    i64 a{0}, b{-inf<i64>};
    i64 operator()(i64 x) {
        return a * x + b;
    }
};

// max LiChao
struct Seg {
    int l, r;
    Seg *ls{}, *rs{};
    Line f{};
    Seg(int l, int r) : l(l), r(r) {}
    void add(Line g) {
        int m = (l + r) / 2;
        if (g(m) > f(m)) {
            swap(g, f);
        }
        if (g.b == -inf<i64> or r - l == 1) {
            return;
        }
        if (g.a < f.a) {
            if (!ls) {
                ls = new Seg(l, m);
            }
            ls->add(g);
        } else {
            if (!rs) {
                rs = new Seg(m, r);
            }
            rs->add(g);
        }
    }
    i64 qry(i64 x) {
        if (f.b == -inf<i64>) {
            return -inf<i64>;
        }
        int m = (l + r) / 2;
        i64 y = f(x);
        if (x < m and ls) {
            chmax(y, ls->qry(x));
        } else if (x >= m and rs) {
            chmax(y, rs->qry(x));
        }
        return y;
    }
};
```

3.3 PBDS

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template<typename T> using RBT = tree<T, null_type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;

/*
.find_by_order(k) 回傳第 k 小的值 (based-0)
.order_of_key(k) 回傳有多少元素比 k 小
*/
struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }

    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM = chrono::steady_clock::
            now().time_since_epoch().count();
        return splitmix64(x + FIXED_RANDOM);
    }
};

// gp_hash_table<int, int, custom_hash> ss;
```

3.4 Sparse Table

```
template<class T>
struct SparseTable {
    function<T(T, T)> F;
    vector<vector<T>>> sp;
    SparseTable(vector<T> &a, const auto &f) {
        F = f;
        int n = a.size();
```

```
        sp.resize(n, vector<T>(__lg(n) + 1));
        for (int i = n - 1; i >= 0; i--) {
            sp[i][0] = a[i];
            for (int j = 1; i + (1 << j) <= n; j++) {
                sp[i][j] = F(sp[i][j - 1], sp[i + (1 << j - 1)][j - 1]);
            }
        }
    }
    T query(int l, int r) { // [l, r)
        int k = __lg(r - l);
        return F(sp[l][k], sp[r - (1 << k)][k]);
    }
};
```

3.5 Treap

```
struct Treap {
    Treap *l, *r;
    int key, size;
    Treap(int k) : l(nullptr), r(nullptr), key(k), size(1) {}
    void pull();
    void push() {};
};

inline int SZ(Treap *p) {
    return p == nullptr ? 0 : p->size;
}

void Treap::pull() {
    size = 1 + SZ(l) + SZ(r);
}

Treap *merge(Treap *a, Treap *b) {
    if (!a || !b) return a ? a : b;
    if (rand() % (SZ(a) + SZ(b)) < SZ(a)) {
        return a->push(), a->r = merge(a->r, b), a->pull(), a;
    }
    return b->push(), b->l = merge(a, b->l), b->pull(), b;
}

// <= k, > k
void split(Treap *p, Treap *a, Treap *b, int k) { // by key
    if (!p) return a = b = nullptr, void();
    p->push();
    if (p->key <= k) {
        a = p, split(p->r, a->r, b, k), a->pull();
    } else {
        b = p, split(p->l, a, b->l, k), b->pull();
    }
}

// k, n - k
void split2(Treap *p, Treap *a, Treap *b, int k) { // by size
    if (!p) return a = b = nullptr, void();
    p->push();
    if (SZ(p->l) + 1 <= k) {
        a = p, split2(p->r, a->r, b, k - SZ(p->l) - 1);
    } else {
        b = p, split2(p->l, a, b->l, k);
    }
    p->pull();
}

void insert(Treap *p, int k) {
    Treap *l, *r;
    p->push(), split(p, l, r, k);
    p = merge(merge(l, new Treap(k)), r);
    p->pull();
}

bool erase(Treap *p, int k) {
    if (!p) return false;
    if (p->key == k) {
        Treap *t = p;
        p->push(), p = merge(p->l, p->r);
        delete t;
        return true;
    }
    Treap *t = k < p->key ? p->l : p->r;
    return erase(t, k) ? p->pull(), true : false;
}

int Rank(Treap *p, int k) { // # of key < k
    if (!p) return 0;
    if (p->key < k) return SZ(p->l) + 1 + Rank(p->r, k);
    return Rank(p->l, k);
}

Treap *kth(Treap *p, int k) { // 1-base
    if (k <= SZ(p->l)) return kth(p->l, k);
    if (k == SZ(p->l) + 1) return p;
    return kth(p->r, k - SZ(p->l) - 1);
}
```

```

}
// pref: kth(Rank(x)), succ: kth(Rank(x+1)+1)
tuple<Treap*, Treap*, Treap*> interval(Treap *s0, int l, int r)
{ // 1-based
  Treap *a, *b, *c; // b: [l, r]
  split2(o, a, b, l - 1), split2(b, b, c, r - l + 1);
  return make_tuple(a, b, c);
}

```

4 Matching and Flow

4.1 Dinic

```

template<typename T>
struct Dinic {
  const T INF = numeric_limits<T>::max() / 2;
  struct edge {
    int v, r; T rc;
  };
  vector<vector<edge>> adj;
  vector<T> dis, it;
  Dinic(int n) : adj(n), dis(n), it(n) {}
  void add_edge(int u, int v, T c) {
    adj[u].pb({v, adj[v].size(), c});
    adj[v].pb({u, adj[u].size() - 1, 0});
  }
  bool bfs(int s, int t) {
    fill(all(dis), INF);
    queue<int> q;
    q.push(s);
    dis[s] = 0;
    while (!q.empty()) {
      int u = q.front();
      q.pop();
      for (const auto& [v, r, rc] : adj[u]) {
        if (dis[v] < INF || rc == 0) continue;
        dis[v] = dis[u] + 1;
        q.push(v);
      }
    }
    return dis[t] < INF;
  }
  T dfs(int u, int t, T cap) {
    if (u == t || cap == 0) return cap;
    for (int &i = it[u]; i < (int)adj[u].size(); ++i) {
      auto &[v, r, rc] = adj[u][i];
      if (dis[v] != dis[u] + 1) continue;
      T tmp = dfs(v, t, min(cap, rc));
      if (tmp > 0) {
        rc -= tmp;
        adj[v][r].rc += tmp;
        return tmp;
      }
    }
    return 0;
  }
  T flow(int s, int t) {
    T ans = 0, tmp;
    while (bfs(s, t)) {
      fill(all(it), 0);
      while ((tmp = dfs(s, t, INF)) > 0) {
        ans += tmp;
      }
    }
    return ans;
  }
  bool inScut(int u) { return dis[u] < INF; }
};

```

4.2 General Matching

```

struct GeneralMatching { // n <= 500
  const int BLOCK = 10;
  int n;
  vector<vector<int>> g;
  vector<int> hit, mat;
  priority_queue<pair<int, int>, vector<pair<int, int>>,
    greater<pair<int, int>>> unmat;
  GeneralMatching(int _n) : n(_n), g(_n), mat(n, -1), hit(n) {}
  void add_edge(int a, int b) { // 0 <= a != b < n
    g[a].push_back(b);
    g[b].push_back(a);
  }
};

```

```

int get_match() {
  for (int i = 0; i < n; i++) if (!g[i].empty()) {
    unmat.emplace(0, i);
  }
  // If WA, increase this
  // there are some cases that need >= 1.3*n^2 steps for BLOCK
  // no idea what the actual bound needed here is.
  const int MAX_STEPS = 10 + 2 * n + n * n / BLOCK / 2;
  mt19937 rng(random_device{}());
  for (int i = 0; i < MAX_STEPS; ++i) {
    if (unmat.empty()) break;
    int u = unmat.top().second;
    unmat.pop();
    if (mat[u] != -1) continue;
    for (int j = 0; j < BLOCK; j++) {
      ++hit[u];
      auto &e = g[u];
      const int v = e[rng() % e.size()];
      mat[u] = v;
      swap(u, mat[v]);
      if (u == -1) break;
    }
    if (u != -1) {
      mat[u] = -1;
      unmat.emplace(hit[u] * 100ULL / (g[u].size() + 1), u);
    }
  }
  int siz = 0;
  for (auto e : mat) siz += (e != -1);
  return siz / 2;
};

```

4.3 KM

```

template<class T>
T KM(const vector<vector<T>> &w) {
  const T INF = numeric_limits<T>::max() / 2;
  const int n = w.size();
  vector<T> lx(n), ly(n);
  vector<int> mx(n, -1), my(n, -1), pa(n);
  auto augment = [&](int y) {
    for (int x, z; y != -1; y = z) {
      x = pa[y];
      z = mx[x];
      my[y] = x;
      mx[x] = y;
    }
  };
  auto bfs = [&](int s) {
    vector<T> sy(n, INF);
    vector<bool> vx(n), vy(n);
    queue<int> q;
    q.push(s);
    while (true) {
      while (q.size()) {
        int x = q.front();
        q.pop();
        vx[x] = 1;
        for (int y = 0; y < n; y++) {
          if (vy[y]) continue;
          T d = lx[x] + ly[y] - w[x][y];
          if (d == 0) {
            pa[y] = x;
            if (my[y] == -1) {
              augment(y);
              return;
            }
            vy[y] = 1;
            q.push(my[y]);
          } else if (chmin(sy[y], d)) {
            pa[y] = x;
          }
        }
      }
    }
  };
  T cut = INF;
  for (int y = 0; y < n; y++)
    if (!vy[y])
      chmin(cut, sy[y]);
  for (int j = 0; j < n; j++) {
    if (vx[j]) lx[j] -= cut;
    if (vy[j]) ly[j] += cut;
  }
};

```

```

    else sy[j] -= cut;
}
for (int y = 0; y < n; y++)
    if (!vy[y] and sy[y] == 0) {
        if (my[y] == -1) {
            augment(y);
            return;
        }
        vy[y] = 1;
        q.push(my[y]);
    }
}
for (int x = 0; x < n; x++)
    lx[x] = ranges::max(w[x]);
for (int x = 0; x < n; x++)
    bfs(x);
T ans = 0;
for (int x = 0; x < n; x++)
    ans += w[x][mx[x]];
return ans;
}

```

4.4 MCMF

```

template<class T>
struct MCMF {
    const T INF = numeric_limits<T>::max() / 2;
    struct edge { int v, r; T f, w; };
    vector<vector<edge>> adj;
    const int n;
    MCMF(int n) : n(n), adj(n) {}
    void addEdge(int u, int v, T f, T c) {
        adj[u].push_back({v, ssize(adj[v]), f, c});
        adj[v].push_back({u, ssize(adj[u]) - 1, 0, -c});
    }
    vector<T> dis;
    vector<bool> vis;
    bool spfa(int s, int t) {
        queue<int> que;
        dis.assign(n, INF);
        vis.assign(n, false);
        que.push(s);
        vis[s] = 1;
        dis[s] = 0;
        while (!que.empty()) {
            int u = que.front(); que.pop();
            vis[u] = 0;
            for (auto [v, r, f, w] : adj[u])
                if (f && chmin(dis[v], dis[u] + w))
                    if (!vis[v]) {
                        que.push(v);
                        vis[v] = 1;
                    }
        }
        return dis[t] != INF;
    }
    T dfs(int u, T in, int t) {
        if (u == t) return in;
        vis[u] = 1;
        T out = 0;
        for (auto [v, rev, f, w] : adj[u])
            if (f && !vis[v] && dis[v] == dis[u] + w) {
                T x = dfs(v, min(in, f), t);
                in -= x;
                out += x;
                f -= x;
                adj[v][rev].f += x;
                if (!in) break;
            }
        if (in) dis[u] = INF;
        vis[u] = 0;
        return out;
    }
    pair<T, T> flow(int s, int t) { // {flow, cost}
        T a = 0, b = 0;
        while (spfa(s, INF)) {
            T x = dfs(s, INF, t);
            a += x;
            b += x * dis[t];
        }
        return {a, b};
    }
};

```

4.5 Model

- Maximum/Minimum flow with lower bound / Circulation problem
 - Construct super source S and sink T .
 - For each edge (x, y, l, u) , connect $x \rightarrow y$ with capacity $u - l$.
 - For each vertex v , denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
 - If $in(v) > 0$, connect $S \rightarrow v$ with capacity $in(v)$, otherwise, connect $v \rightarrow T$ with capacity $-in(v)$.
 - To maximize, connect $t \rightarrow s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T . If $f \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
 - To minimize, let f be the maximum flow from S to T . Connect $t \rightarrow s$ with capacity ∞ and let the flow from S to T be f' . If $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, f' is the answer.
 - The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.
- Construct minimum vertex cover from maximum matching M on bipartite graph (X, Y)
 - Redirect every edge: $y \rightarrow x$ if $(x, y) \in M$, $x \rightarrow y$ otherwise.
 - Dfs from unmatched vertices in X .
 - $x \in X$ is chosen iff x is unvisited.
 - $y \in Y$ is chosen iff y is visited.
- Minimum cost cyclic flow
 - Construct super source S and sink T
 - For each edge (x, y, c) , connect $x \rightarrow y$ with $(cost, cap) = (c, 1)$ if $c > 0$, otherwise connect $y \rightarrow x$ with $(cost, cap) = (-c, 1)$
 - For each edge with $c < 0$, sum these cost as K , then increase $d(y)$ by 1, decrease $d(x)$ by 1
 - For each vertex v with $d(v) > 0$, connect $S \rightarrow v$ with $(cost, cap) = (0, d(v))$
 - For each vertex v with $d(v) < 0$, connect $v \rightarrow T$ with $(cost, cap) = (0, -d(v))$
 - Flow from S to T , the answer is the cost of the flow $C + K$
- Maximum density induced subgraph
 - Binary search on answer, suppose we're checking answer T
 - Construct a max flow model, let K be the sum of all weights
 - Connect source $s \rightarrow v$, $v \in G$ with capacity K
 - For each edge (u, v, w) in G , connect $u \rightarrow v$ and $v \rightarrow u$ with capacity w
 - For $v \in G$, connect it with sink $v \rightarrow t$ with capacity $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
 - T is a valid answer if the maximum flow $f < K|V|$
- Minimum weight edge cover
 - Change the weight of each edge to $\mu(u) + \mu(v) - w(u, v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v .
 - Let the maximum weight matching of the graph be x , the answer will be $\sum \mu(v) - x$.

5 Geometry

5.1 Point

```

using namespace::pi;
template<class T> inline constexpr T eps = numeric_limits<T>::
    epsilon() * 1E6;
using Real = long double;
struct Pt {
    Real x{}, y{};
    Pt operator+(Pt a) const { return {x + a.x, y + a.y}; }
    Pt operator-(Pt a) const { return {x - a.x, y - a.y}; }
    Pt operator*(Real k) const { return {x * k, y * k}; }
    Pt operator/(Real k) const { return {x / k, y / k}; }
    Real operator*(Pt a) const { return x * a.x + y * a.y; }
    Real operator^(Pt a) const { return x * a.x - y * a.y; }
    auto operator<=>(const Pt&) const = default;
    bool operator==(const Pt&) const = default;
};
int sgn(Real x) { return (x > -eps<Real>) - (x < eps<Real>); }
Real ori(Pt a, Pt b, Pt c) { return (b - a) ^ (c - a); }
bool argcmp(const Pt &a, const Pt &b) { // arg(a) < arg(b)
    int f = (Pt{a.y, -a.x} > Pt{} ? 1 : -1) * (a != Pt{});
    int g = (Pt{b.y, -b.x} > Pt{} ? 1 : -1) * (b != Pt{});
    return f == g ? (a ^ b) > 0 : f < g;
}
Pt rotate(Pt u) { return {-u.y, u.x}; }
Real abs2(Pt a) { return a * a; }
// floating point only
Pt rotate(Pt u, Real a) {
    Pt v{sinl(a), cosl(a)};
    return {u ^ v, u * v};
}
Real abs(Pt a) { return sqrtl(a * a); }
Real arg(Pt x) { return atan2l(x.y, x.x); }
Pt unit(Pt x) { return x / abs(x); }

```

5.2 Line

```
struct Line {
    Pt a, b;
    Pt dir() const { return b - a; }
};

int PtSide(Pt p, Line l) {
    return sgn(ori(l.a, l.b, p)); // for int
    return sgn(ori(l.a, l.b, p) / abs(l.a - l.b));
}

bool PtOnSeg(Pt p, Line l) {
    return PtSide(p, l) == 0 and sgn((p - l.a) * (p - l.b)) <= 0;
}

Pt proj(Pt p, Line l) {
    Pt dir = unit(l.b - l.a);
    return l.a + dir * (dir * (p - l.a));
}
```

5.3 Circle

```
struct Cir {
    Pt o;
    double r;
};

bool disjunct(const Cir &a, const Cir &b) {
    return sgn(abs(a.o - b.o) - a.r - b.r) >= 0;
}

bool contain(const Cir &a, const Cir &b) {
    return sgn(a.r - b.r - abs(a.o - b.o)) >= 0;
}
```

5.4 Point to Segment Distance

```
double PtSegDist(Pt p, Line l) {
    double ans = min(abs(p - l.a), abs(p - l.b));
    if (sgn(abs(l.a - l.b)) == 0) return ans;
    if (sgn((l.a - l.b) * (p - l.b)) < 0) return ans;
    if (sgn((l.b - l.a) * (p - l.a)) < 0) return ans;
    return min(ans, abs(ori(p, l.a, l.b)) / abs(l.a - l.b));
}

double SegDist(Line l, Line m) {
    return PtSegDist({0, 0}, {l.a - m.a, l.b - m.b});
}
```

5.5 Point In Polygon

```
int inPoly(Pt p, const vector<Pt> &P) {
    const int n = P.size();
    int cnt = 0;
    for (int i = 0; i < n; i++) {
        Pt a = P[i], b = P[(i + 1) % n];
        if (PtOnSeg(p, {a, b})) return 1; // on edge
        if ((sgn(a.y - p.y) == 1) ^ (sgn(b.y - p.y) == 1))
            cnt += sgn(ori(a, b, p));
    }
    return cnt == 0 ? 0 : 2; // out, in
}
```

5.6 Intersection of Line

```
bool isInter(Line l, Line m) {
    if (PtOnSeg(m.a, l) or PtOnSeg(m.b, l) or
        PtOnSeg(l.a, m) or PtOnSeg(l.b, m))
        return true;
    return PtSide(m.a, l) * PtSide(m.b, l) < 0 and
        PtSide(l.a, m) * PtSide(l.b, m) < 0;
}

Pt LineInter(Line l, Line m) {
    double s = ori(m.a, m.b, l.a), t = ori(m.a, m.b, l.b);
    return (l.b * s - l.a * t) / (s - t);
}

bool strictInter(Line l, Line m) {
    int la = PtSide(m.a, l);
    int lb = PtSide(m.b, l);
    int ma = PtSide(l.a, m);
    int mb = PtSide(l.b, m);
    if (la == 0 and lb == 0) return false;
    return la * lb < 0 and ma * mb < 0;
}
```

5.7 Intersection of Circles

```
vector<Pt> CircleInter(Cir a, Cir b) {
    double d2 = abs2(a.o - b.o), d = sqrt(d2);
    if (d < max(a.r, b.r) - min(a.r, b.r) || d > a.r + b.r)
        return {};
    Pt u = (a.o + b.o) / 2 + (a.o - b.o) * ((b.r * b.r - a.r * a.r) / (2 * d2));
```

```
double A = sqrt((a.r + b.r + d) * (a.r - b.r + d) * (a.r + b.r - d) * (-a.r + b.r + d));
Pt v = rotate(b.o - a.o) * A / (2 * d2);
if (sgn(v.x) == 0 and sgn(v.y) == 0) return {u};
return {u - v, u + v}; // counter clockwise of a
}
```

5.8 Intersection of Circle and Line

```
vector<Pt> CircleLineInter(Cir c, Line l) {
    Pt H = proj(c.o, l);
    Pt dir = unit(l.b - l.a);
    double h = abs(H - c.o);
    if (sgn(h - c.r) > 0) return {};
    double d = sqrt(max((double)0., c.r * c.r - h * h));
    if (sgn(d) == 0) return {H};
    return {H - dir * d, H + dir * d};
    // Counterclockwise
}
```

5.9 Area of Circle Polygon

```
double CirclePoly(Cir C, const vector<Pt> &P) {
    auto arg = [&](Pt p, Pt q) { return atan2(p ^ q, p * q); };
    double r2 = C.r * C.r / 2;
    auto tri = [&](Pt p, Pt q) {
        Pt d = q - p;
        auto a = (d * p) / abs2(d), b = (abs2(p) - C.r * C.r) / abs2(d);
        auto det = a * a - b;
        if (det <= 0) return arg(p, q) * r2;
        auto s = max(0., -a - sqrt(det)), t = min(1., -a + sqrt(det));
        if (t < 0 or 1 <= s) return arg(p, q) * r2;
        Pt u = p + d * s, v = p + d * t;
        return arg(p, u) * r2 + (u ^ v) / 2 + arg(v, q) * r2;
    };
    double sum = 0.0;
    for (int i = 0; i < P.size(); i++)
        sum += tri(P[i] - C.o, P[(i + 1) % P.size()] - C.o);
    return sum;
}
```

5.10 Convex Hull

```
vector<Pt> BuildHull(vector<Pt> pt) {
    sort(all(pt));
    pt.erase(unique(all(pt)), pt.end());
    if (pt.size() <= 2) return pt;
    vector<Pt> hull;
    int sz = 1;
    rep (t, 0, 2) {
        rep (i, t, ssize(pt)) {
            while (ssize(hull) > sz && ori(hull.end()[-2], pt[i],
                hull.back()) >= 0)
                hull.pop_back();
            hull.pb(pt[i]);
        }
        sz = ssize(hull);
        reverse(all(pt));
    }
    hull.pop_back();
    return hull;
}
```

5.11 Convex Trick

```
struct Convex {
    int n;
    vector<Pt> A, V, L, U;
    Convex(const vector<Pt> &A) : A(A), n(A.size()) { // n >= 3
        auto it = max_element(all(A));
        L.assign(A.begin(), it + 1);
        U.assign(it, A.end()), U.push_back(A[0]);
        rep (i, 0, n) {
            V.push_back(A[(i + 1) % n] - A[i]);
        }
    }

    int inside(Pt p, const vector<Pt> &h, auto f) {
        auto it = lower_bound(all(h), p, f);
        if (it == h.end()) return 0;
        if (it == h.begin()) return p == *it;
        return 1 - sgn(ori(*prev(it), p, *it));
    }

    // 0: out, 1: on, 2: in
    int inside(Pt p) {
```



```

    return min(inside(p, L, less{}), inside(p, U, greater{}));
}
static bool cmp(Pt a, Pt b) { return sgn(a ^ b) > 0; }
// A[i] is a far/closer tangent point
int tangent(Pt v, bool close = true) {
    assert(v != Pt{});
    auto l = V.begin(), r = V.begin() + L.size() - 1;
    if (v < Pt{}) l = r, r = V.end();
    if (close) return (lower_bound(l, r, v, cmp) - V.begin()) % n;
    return (upper_bound(l, r, v, cmp) - V.begin()) % n;
}
// closer tangent point
array<int, 2> tangent2(Pt p) {
    array<int, 2> t{-1, -1};
    if (inside(p) == 2) return t;
    if (auto it = lower_bound(all(L), p); it != L.end() and p == *it) {
        int s = it - L.begin();
        return {(s + 1) % n, (s - 1 + n) % n};
    }
    if (auto it = lower_bound(all(U), p, greater{}); it != U.end() and p == *it) {
        int s = it - U.begin() + L.size() - 1;
        return {(s + 1) % n, (s - 1 + n) % n};
    }
    for (int i = 0; i != t[0]; i = tangent((A[t[0] = i] - p), 0));
    for (int i = 0; i != t[1]; i = tangent((p - A[t[1] = i]), 1));
    return t;
}
int find(int l, int r, Line L) {
    if (r < l) r += n;
    int s = PtSide(A[l % n], L);
    return *ranges::partition_point(views::iota(l, r), [&](int m) {
        return PtSide(A[m % n], L) == s;
    }) - 1;
};
// Line A_x A_{x+1} intersect with L
vector<int> intersect(Line L) {
    int l = tangent(L.a - L.b, r = tangent(L.b - L.a);
    if (PtSide(A[l], L) * PtSide(A[r], L) >= 0) return {l, r};
    return {find(l, r, L) % n, find(r, l, L) % n};
}
};

```

5.12 Half Plane Intersection

```

bool cover(Line L, Line P, Line Q) {
    // return PtSide(LineInter(P, Q), L) <= 0; for double
    i128 u = (Q.a - P.a) ^ Q.dir();
    i128 v = P.dir() ^ Q.dir();
    i128 x = P.dir().x * u + (P.a - L.a).x * v;
    i128 y = P.dir().y * u + (P.a - L.a).y * v;
    return sgn(x * L.dir().y - y * L.dir().x) * sgn(v) >= 0;
}
vector<Line> HPI(vector<Line> P) {
    sort(all(P), [&](Line l, Line m) {
        if (argcmp(l.dir(), m.dir()) return true;
        if (argcmp(m.dir(), l.dir()) return false;
        return ori(m.a, m.b, l.a) > 0;
    });
    int n = P.size(), l = 0, r = -1;
    for (int i = 0; i < n; i++) {
        if (i and !argcmp(P[i - 1].dir(), P[i].dir())) continue;
        while (l < r and cover(P[i], P[r - 1], P[r])) r--;
        while (l < r and cover(P[i], P[l], P[l + 1])) l++;
        P[++r] = P[i];
    }
    while (l < r and cover(P[l], P[r - 1], P[r])) r--;
    while (l < r and cover(P[r], P[l], P[l + 1])) l++;
    if (r - l <= 1 or !argcmp(P[l].dir(), P[r].dir()))
        return {}; // empty
    if (cover(P[l + 1], P[l], P[r]))
        return {}; // infinity
    return vector(P.begin() + l, P.begin() + r + 1);
}

```

5.13 Minimal Enclosing Circle

```

struct Cir {
    Pt o;

```

```

    double r;
    bool inside(Pt p) {
        return sgn(r - abs(p - o)) >= 0;
    }
};
Pt Center(Pt a, Pt b, Pt c) {
    Pt x = (a + b) / 2;
    Pt y = (b + c) / 2;
    return LineInter({x, x + rotate(b - a)}, {y, y + rotate(c - b)});
}
Cir MEC(vector<Pt> P) {
    mt19937 rng(time(0));
    shuffle(all(P), rng);
    Cir C{};
    for (int i = 0; i < P.size(); i++) {
        if (C.inside(P[i])) continue;
        C = {P[i], 0};
        for (int j = 0; j < i; j++) {
            if (C.inside(P[j])) continue;
            C = {(P[i] + P[j]) / 2, abs(P[i] - P[j]) / 2};
            for (int k = 0; k < j; k++) {
                if (C.inside(P[k])) continue;
                C.o = Center(P[i], P[j], P[k]);
                C.r = abs(C.o - P[i]);
            }
        }
    }
    return C;
}

```

5.14 Minkowski

```

// P, Q, R(return) are counterclockwise order convex polygon
vector<Pt> Minkowski(vector<Pt> P, vector<Pt> Q) {
    assert(P.size() >= 2 && Q.size() >= 2);
    auto cmp = [&](Pt a, Pt b) {
        return Pt{a.y, a.x} < Pt{b.y, b.x};
    };
    auto reorder = [&](auto &R) {
        rotate(R.begin(), min_element(all(R), cmp), R.end());
        R.push_back(R[0]), R.push_back(R[1]);
    };
    const int n = P.size(), m = Q.size();
    reorder(P), reorder(Q);
    vector<Pt> R;
    for (int i = 0, j = 0, s; i < n || j < m; ) {
        R.push_back(P[i] + Q[j]);
        s = sgn((P[i + 1] - P[i]) ^ (Q[j + 1] - Q[j]));
        if (s >= 0) i++;
        if (s <= 0) j++;
    }
    return R; // May not be a strict convexhull
}

```

5.15 Point In Circumcircle

```

// p[0], p[1], p[2] should be counterclockwise order
int inCC(const array<Pt, 3> &p, Pt a) {
    i128 det = 0;
    for (int i = 0; i < 3; i++)
        det += i128(abs2(p[i]) - abs2(a)) * ori(a, p[(i + 1) % 3], p[(i + 2) % 3]);
    return (det > 0) - (det < 0); // in:1, on:0, out:-1
}

```

5.16 Tangent Lines of Circle and Point

```

vector<Line> CircleTangent(Cir c, Pt p) {
    vector<Line> z;
    double d = abs(p - c.o);
    if (sgn(d - c.r) == 0) {
        Pt i = rotate(p - c.o);
        z.push_back({p, p + i});
    } else if (d > c.r) {
        double o = acos(c.r / d);
        Pt i = unit(p - c.o);
        Pt j = rotate(i, o) * c.r;
        Pt k = rotate(i, -o) * c.r;
        z.push_back({c.o + j, p});
        z.push_back({c.o + k, p});
    }
    return z;
}

```

5.17 Tangent Lines of Circles

```
vector<Line> CircleTangent(Cir c1, Cir c2, int sign1) {
    // sign1 = 1 for outer tang, -1 for inter tang
    vector<Line> ret;
    double d_sq = abs2(c1.o - c2.o);
    if (sgn(d_sq) == 0) return ret;
    double d = sqrt(d_sq);
    Pt v = (c2.o - c1.o) / d;
    double c = (c1.r - sign1 * c2.r) / d;
    if (c * c > 1) return ret;
    double h = sqrt(max(0.0, 1.0 - c * c));
    for (int sign2 = 1; sign2 >= -1; sign2 -= 2) {
        Pt n = Pt(v.x * c - sign2 * h * v.y, v.y * c + sign2 * h *
            v.x);
        Pt p1 = c1.o + n * c1.r;
        Pt p2 = c2.o + n * (c2.r * sign1);
        if (sgn(p1.x - p2.x) == 0 && sgn(p1.y - p2.y) == 0)
            p2 = p1 + rotate(c2.o - c1.o);
        ret.push_back({p1, p2});
    }
    return ret;
}
```

5.18 Triangle Center

```
Pt TriangleCircumCenter(Pt a, Pt b, Pt c) {
    Pt res;
    double a1 = atan2(b.y - a.y, b.x - a.x) + pi / 2;
    double a2 = atan2(c.y - b.y, c.x - b.x) + pi / 2;
    double ax = (a.x + b.x) / 2;
    double ay = (a.y + b.y) / 2;
    double bx = (c.x + b.x) / 2;
    double by = (c.y + b.y) / 2;
    double r1 = (sin(a2) * (ax - bx) + cos(a2) * (by - ay)) / (sin
        (a1) * cos(a2) - sin(a2) * cos(a1));
    return Pt(ax + r1 * cos(a1), ay + r1 * sin(a1));
}
Pt TriangleMassCenter(Pt a, Pt b, Pt c) {
    return (a + b + c) / 3.0;
}
Pt TriangleOrthoCenter(Pt a, Pt b, Pt c) {
    return TriangleMassCenter(a, b, c) * 3.0 -
        TriangleCircumCenter(a, b, c) * 2.0;
}
Pt TriangleInnerCenter(Pt a, Pt b, Pt c) {
    Pt res;
    double la = abs(b - c);
    double lb = abs(a - c);
    double lc = abs(a - b);
    res.x = (la * a.x + lb * b.x + lc * c.x) / (la + lb + lc);
    res.y = (la * a.y + lb * b.y + lc * c.y) / (la + lb + lc);
    return res;
}
```

5.19 Union of Circles

```
// Area[i] : area covered by at least i circle
vector<double> CircleUnion(const vector<Cir> &C) {
    const int n = C.size();
    vector<double> Area(n + 1);
    auto check = [&](int i, int j) {
        if (!contain(C[i], C[j]))
            return false;
        return sgn(C[i].r - C[j].r) > 0 or (sgn(C[i].r - C[j].r) ==
            0 and i < j);
    };
    struct Teve {
        double ang; int add; Pt p;
        bool operator<(const Teve &b) { return ang < b.ang; }
    };
    auto ang = [&](Pt p) { return atan2(p.y, p.x); };
    for (int i = 0; i < n; i++) {
        int cov = 1;
        vector<Teve> event;
        for (int j = 0; j < n; j++) if (i != j) {
            if (check(j, i)) cov++;
            else if (!check(i, j) and !disjunct(C[i], C[j])) {
                auto I = CircleInter(C[i], C[j]);
                assert(I.size() == 2);
                double a1 = ang(I[0] - C[i].o), a2 = ang(I[1] - C[i].o)
                    ;
                event.push_back({a1, 1, I[0]});
                event.push_back({a2, -1, I[1]});
                if (a1 > a2) cov++;
            }
        }
    }
```

```
    }
    if (event.empty()) {
        Area[cov] += pi * C[i].r * C[i].r;
        continue;
    }
    sort(all(event));
    event.push_back(event[0]);
    for (int j = 0; j + 1 < event.size(); j++) {
        cov += event[j].add;
        Area[cov] += (event[j].p ^ event[j + 1].p) / 2.;
        double theta = event[j + 1].ang - event[j].ang;
        if (theta < 0) theta += 2 * pi;
        Area[cov] += (theta - sin(theta)) * C[i].r * C[i].r / 2.;
    }
    return Area;
}
```

6 Graph

6.1 Block Cut Tree

```
struct BlockCutTree {
    int n;
    vector<vector<int>> adj;
    BlockCutTree(int _n) : n(_n), adj(_n) {}
    void addEdge(int u, int v) {
        adj[u].push_back(v);
        adj[v].push_back(u);
    }
    pair<int, vector<pair<int, int>>> work() {
        vector<int> dfn(n, -1), low(n), stk;
        vector<pair<int, int>> edg;
        int cnt = 0, cur = 0;
        function<void(int)> dfs = [&](int x) {
            stk.push_back(x);
            dfn[x] = low[x] = cur++;
            for (auto y : adj[x]) {
                if (dfn[y] == -1) {
                    dfs(y);
                    low[x] = min(low[x], low[y]);
                    if (low[y] == dfn[x]) {
                        int v;
                        do {
                            v = stk.back();
                            stk.pop_back();
                            edg.emplace_back(n + cnt, v);
                        } while (v != y);
                        edg.emplace_back(x, n + cnt);
                        cnt++;
                    }
                } else {
                    low[x] = min(low[x], dfn[y]);
                }
            }
        };
        for (int i = 0; i < n; i++) {
            if (dfn[i] == -1) {
                stk.clear();
                dfs(i);
            }
        }
        return {cnt, edg};
    }
};
```

6.2 Count Cycles

```
// ord = sort by deg decreasing, rk[ord[i]] = i
// D: undirected to directed edge from rk small to rk big
vector<int> vis(n, 0);
int c3 = 0, c4 = 0;
for (int x : ord) { // c3
    for (int y : D[x]) vis[y] = 1;
    for (int y : D[x]) for (int z : D[y]) c3 += vis[z];
    for (int y : D[x]) vis[y] = 0;
}
for (int x : ord) { // c4
    for (int y : D[x]) for (int z : adj[y])
        if (rk[z] > rk[x]) c4 += vis[z]++;
    for (int y : D[x]) for (int z : adj[y])
        if (rk[z] > rk[x]) --vis[z];
}
```


6.3 Dominator Tree

```
vector<int> BuildDomTree(vector<vector<int>> adj, int rt) {
    int n = adj.size();

    // buckets: list of vertices y with sdom(y) = x
    vector<vector<int>> buckets(n), radj(n);

    // rev[dfn[x]] = x
    vector<int> dfn(n, -1), rev(n, -1), pa(n, -1);
    vector<int> sdom(n, -1), dom(n, -1);
    vector<int> fa(n, -1), val(n, -1);

    int stamp = 0;

    // re-number in DFS order
    auto dfs = [&](auto self, int u) -> void {
        rev[dfn[u]] = stamp;
        fa[stamp] = sdom[stamp] = val[stamp] = stamp;
        stamp++;
        for (int v : adj[u]) {
            if (dfn[v] == -1) {
                self(self, v);
                pa[dfn[v]] = dfn[u];
            }
            radj[dfn[v]].pb(dfn[u]);
        }
    };

    function<int(int, bool)> Eval = [&](int x, bool fir) {
        if (x == fa[x]) return fir ? x : -1;
        int p = Eval(fa[x], false);
        // x is one step away from the root
        if (p == -1) return x;
        if (sdom[val[x]] > sdom[val[fa[x]]]) val[x] = val[fa[x]];
        fa[x] = p;
        return fir ? val[x] : p;
    };

    auto Link = [&](int x, int y) -> void { fa[x] = y; };

    dfs(dfs, rt);

    // compute sdom in reversed DFS order
    for (int x = stamp - 1; x >= 0; --x) {
        for (int y : radj[x]) {
            // sdom[x] = min{y | (y, x) in E(G), y < x}, {sdom[z] |
            // (y, x) in E(G), z > x && z is y's ancestor})
            chmin(sdom[x], sdom[Eval(y, true)]);
        }
        if (x > 0) buckets[sdom[x]].pb(x);
        for (int u : buckets[x]) {
            int p = Eval(u, true);
            if (sdom[p] == x) dom[u] = x;
            else dom[u] = p;
        }
        if (x > 0) Link(x, pa[x]);
    }
    // idom[x] = -1 if x is unreachable from rt
    vector<int> idom(n, -1);
    idom[rt] = rt;
    rep(x, 1, stamp) {
        if (sdom[x] != dom[x]) dom[x] = dom[dom[x]];
    }
    rep(i, 1, stamp) idom[rev[i]] = rev[dom[i]];
    return idom;
}
```

6.4 Enumerate Planar Face

```
// 0-based
struct PlanarGraph{
    int n, m, id;
    vector<Pt<int>> v;
    vector<vector<pair<int, int>>> adj;
    vector<int> conv, nxt, vis;

    PlanarGraph(int n, int m, vector<Pt<int>> _v) :
        n(n), m(m), id(0),
        v(_v), adj(n),
        conv(m << 1), nxt(m << 1), vis(m << 1) {}

    void add_edge(int x, int y) {
        adj[x].push_back({y, id << 1});
        adj[y].push_back({x, id << 1 | 1});
        conv[id << 1] = x;
    }
}
```

```
conv[id << 1 | 1] = y;
id++;
}

vector<int> enumerate_face() {
    for (int i = 0; i < n; i++) {
        sort(all(adj[i]), [&](const auto &a, const auto &b) {
            return (v[a.first] - v[i]) < (v[b.first] - v[i]);
        });
        int sz = adj[i].size(), pre = sz - 1;
        for (int j = 0; j < sz; j++) {
            nxt[adj[i][pre].second] = adj[i][j].second ^ 1;
            pre = j;
        }
    }

    vector<int> ret;
    for (int i = 0; i < m * 2; i++) {
        if (!vis[i]) {
            int area = 0, now = i;
            vector<int> pt;
            while (!vis[now]) {
                vis[now] = true;
                pt.push_back(conv[now]);
                now = nxt[now];
            }
            pt.push_back(pt.front());
            for (int i = 0; i + 1 < ssize(pt); i++) {
                area -= (v[pt[i]] ^ v[pt[i + 1]]);
            }
            // pt = face boundary
            if (area > 0) {
                ret.push_back(area);
            } else {
                // pt is outer face
            }
        }
    }
    return ret;
}
```

6.5 Manhattan MST

```
// {w, u, v}
vector<tuple<int, int, int>> ManhattanMST(vector<Pt> P) {
    vector<int> id(P.size());
    iota(all(id), 0);
    vector<tuple<int, int, int>> edg;
    for (int k = 0; k < 4; k++) {
        sort(all(id), [&](int i, int j) {
            return (P[i] - P[j]).ff < (P[j] - P[i]).ss;
        });
        map<int, int> sweep;
        for (int i : id) {
            auto it = sweep.lower_bound(-P[i].ss);
            while (it != sweep.end()) {
                int j = it->ss;
                Pt d = P[i] - P[j];
                if (d.ss > d.ff) {
                    break;
                }
                edg.emplace_back(d.ff + d.ss, i, j);
                it = sweep.erase(it);
            }
            sweep[-P[i].ss] = i;
        }
        for (Pt &p : P) {
            if (k % 2) {
                p.ff = -p.ff;
            } else {
                swap(p.ff, p.ss);
            }
        }
    }
    return edg;
}
```

6.6 Matroid Intersection

```
/*
M1 = xx matroid, M2 = xx matroid
y<-s if I+y satisfies M1
y->t if I+y satisfies M2
x<-y if I-x+y satisfies M2
*/
```

```

x->y if I-x+y satisfies M1
交換圖點權：
-w[e] if e \in I
w[e] otherwise
*/
vector<int> I(, 0);
while (true) {
    vector<vector<int>> adj();
    int s = , t = s + 1;
    auto M1 = [&]() -> void { // xx matroid
        { // y<-s
        }
        { // x->y
        }
    };
    auto M2 = [&]() -> void { // xx matroid
        { // y->t
        }
        { // x<-y
        }
    };
    auto augment = [&]() -> bool { // 註解掉的是帶權版
        vector<int> vis( + 2, 0), dis( + 2, IINF), from( + 2, -1);
        queue<int> q;
        vis[s] = 1;
        dis[s] = 0;
        q.push(s);
        while (!q.empty()) {
            int u = q.front(); q.pop();
            // vis[u] = 0;
            for (int v : adj[u]) {
                int w = ; // no weight -> 1
                if (chmin(dis[v], dis[u] + w)) {
                    from[v] = u;
                    // if (!vis[v]) {
                    //     vis[v] = 1;
                    //     q.push(v);
                    // }
            }
        }
        if (from[t] == -1) return false;
        for (int cur = from[t]; cur = from[cur]) {
            if (cur == -1 || cur == s) break;
            I[cur] ^= 1;
        }
        return true;
    };
    M1(), M2();
    if (!augment()) break;
}

```

6.7 Maximum Clique

```

constexpr size_t kN = 150;
using bits = bitset<kN>;
struct MaxClique {
    bits G[kN], cs[kN];
    int ans, sol[kN], q, cur[kN], d[kN], n;
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) G[i].reset();
    }
    void addEdge(int u, int v) {
        G[u][v] = G[v][u] = 1;
    }
    void preDfs(vector<int> &v, int i, bits mask) {
        if (i < 4) {
            for (int x : v) d[x] = (G[x] & mask).count();
            sort(all(v), [&](int x, int y) {
                return d[x] > d[y];
            });
        }
        vector<int> c(v.size());
        cs[1].reset(), cs[2].reset();
        int l = max(ans - q + 1, 1), r = 2, tp = 0, k;
        for (int p : v) {
            for (k = 1;
                 (cs[k] & G[p]).any(); ++k);
            if (k >= r) cs[++r].reset();
        }
    }
};

```

```

        cs[k][p] = 1;
        if (k < l) v[tp++] = p;
    }
    for (k = l; k < r; ++k)
        for (auto p = cs[k]._Find_first(); p < kN; p = cs[k]._Find_next(p))
            v[tp] = p, c[tp] = k, ++tp;
    dfs(v, c, i + 1, mask);
}
void dfs(vector<int> &v, vector<int> &c, int i, bits mask) {
    while (!v.empty()) {
        int p = v.back();
        v.pop_back();
        mask[p] = 0;
        if (q + c.back() <= ans) return;
        cur[q++] = p;
        vector<int> nr;
        for (int x : v)
            if (G[p][x]) nr.push_back(x);
        if (!nr.empty()) preDfs(nr, i, mask & G[p]);
        else if (q > ans) ans = q, copy_n(cur, q, sol);
        c.pop_back();
        --q;
    }
}
int solve() {
    vector<int> v(n);
    iota(all(v), 0);
    ans = q = 0;
    preDfs(v, 0, bits(string(n, '1')));
    return ans;
}
} cliq;

```

6.8 Tree Hash

```

map<vector<int>, int> id;
vector<vector<int>> sub;
vector<int> siz;
int getid(const vector<int> &T) {
    if (id.count(T)) return id[T];
    int s = 1;
    for (int x : T) {
        s += siz[x];
    }
    sub.push_back(T);
    siz.push_back(s);
    return id[T] = id.size();
}
int dfs(int u, int f) {
    vector<int> S;
    for (int v : G[u]) if (v != f) {
        S.push_back(dfs(v, u));
    }
    sort(all(S));
    return getid(S);
}

```

6.9 Two-SAT

```

struct TwoSat {
    int n;
    vector<vector<int>> G;
    vector<bool> ans;
    vector<int> id, dfn, low, stk;
    TwoSat(int n) : n(n), G(2 * n) {}
    void addClause(int u, bool f, int v, bool g) { // (u = f) or (v = g)
        G[2 * u + !f].push_back(2 * v + g);
        G[2 * v + !g].push_back(2 * u + f);
    }
    void addImPLY(int u, bool f, int v, bool g) { // (u = f) -> (v = g)
        G[2 * u + f].push_back(2 * v + g);
        G[2 * v + !g].push_back(2 * u + !f);
    }
    int addVar() {
        G.emplace_back();
        G.emplace_back();
        return n++;
    }
    void addAtMostOne(const vector<pair<int, bool>> &li) {
        if (ssize(li) <= 1) return;
        int pu; bool pf; tie(pu, pf) = li[0];
        for (int i = 2; i < ssize(li); i++) {

```

```

    const auto &u, f] = li[i];
    int nxt = addVar();
    addClause(pu, !pf, u, !f);
    addClause(pu, !pf, nxt, true);
    addClause(u, !f, nxt, true);
    tie(pu, pf) = make_pair(nxt, true);
}
addClause(pu, !pf, li[1].first, !li[1].second);
}
int cur = 0, scc = 0;
void dfs(int u) {
    stk.push_back(u);
    dfn[u] = low[u] = cur++;
    for (int v : G[u]) {
        if (dfn[v] == -1) {
            dfs(v);
            chmin(low[u], low[v]);
        } else if (id[v] == -1) {
            chmin(low[u], dfn[v]);
        }
    }
    if (dfn[u] == low[u]) {
        int x;
        do {
            x = stk.back();
            stk.pop_back();
            id[x] = scc;
        } while (x != u);
        scc++;
    }
}
bool satisfiable() {
    ans.assign(n, 0);
    id.assign(2 * n, -1);
    dfn.assign(2 * n, -1);
    low.assign(2 * n, -1);
    for (int i = 0; i < n * 2; i++)
        if (dfn[i] == -1) {
            dfs(i);
        }
    for (int i = 0; i < n; ++i) {
        if (id[2 * i] == id[2 * i + 1]) {
            return false;
        }
        ans[i] = id[2 * i] > id[2 * i + 1];
    }
    return true;
}
};

```

6.10 Virtual Tree

```

// need LCA
vector<vector<int>> vir(n);
auto clear = [&](auto self, int u) -> void {
    for (int v : vir[u]) self(self, v);
    vir[u].clear();
};
auto build = [&](vector<int> &v) -> void { // be careful of the
    changes to the array
    // maybe dont need to sort when do it while dfs
    sort(all(v), [&](int a, int b) {
        return dfn[a] < dfn[b];
    });
    clear(clear, 0);
    if (v[0] != 0) v.insert(v.begin(), 0);
    int k = v.size();
    vector<int> st;
    rep(i, 0, k) {
        if (st.empty()) {
            st.push_back(v[i]);
            continue;
        }
        int p = lca(v[i], st.back());
        if (p == st.back()) {
            st.push_back(v[i]);
            continue;
        }
        while (st.size() >= 2 && dep[st.end()[-2]] >= dep[p]) {
            vir[st.end()[-2]].push_back(st.back());
            st.pop_back();
        }
        if (st.back() != p) {
            vir[p].push_back(st.back());

```

```

            st.pop_back();
            st.push_back(p);
        }
        st.push_back(v[i]);
    }
    while (st.size() >= 2) {
        vir[st.end()[-2]].push_back(st.back());
        st.pop_back();
    }
};

```

7 Math

7.1 Combinatoric

```

vector<mint> fac, inv;

inline void init (int n) {
    fac.resize(n + 1);
    inv.resize(n + 1);
    fac[0] = inv[0] = 1;
    rep(i, 1, n + 1) fac[i] = fac[i - 1] * i;
    inv[n] = fac[n].inv();
    for (int i = n; i > 0; --i) inv[i - 1] = inv[i] * i;
}

inline mint Comb(int n, int k) {
    if (k > n || k < 0) return 0;
    return fac[n] * inv[k] * inv[n - k];
}

inline mint H(int n, int m) {
    return Comb(n + m - 1, m);
}

inline mint catalan(int n) {
    return fac[2 * n] * inv[n + 1] * inv[n];
}

```

7.2 Discrete Log

```

int power(int a, int b, int p, int res = 1) {
    for (; b; b /= 2, a = 1LL * a * a % p) {
        if (b & 1) {
            res = 1LL * res * a % p;
        }
    }
    return res;
}

int exbsgs(int a, int b, int p) {
    a %= p;
    b %= p;
    if (b == 1 || p == 1) {
        return 0;
    }
    if (a == 0) {
        return b == 0 ? 1 : -1;
    }

    i64 g, k = 0, t = 1; // t : a ^ k / sum{d}
    while ((g = std::gcd(a, p)) > 1) {
        if (b % g) {
            return -1;
        }
        b /= g;
        p /= g;
        k++;
        t = t * (a / g) % p;
        if (t == b) {
            return k;
        }
    }

    const int n = std::sqrt(p) + 1;
    std::unordered_map<int, int> mp;
    mp[b] = 0;

    int x = b, y = t;
    int mi = power(a, n, p);
    for (int i = 1; i < n; i++) {
        x = 1LL * x * a % p;
        mp[x] = i;
    }

    for (int i = 1; i <= n; i++) {
        t = 1LL * t * mi % p;

```

```

    if (mp.contains(t)) {
        return 1LL * i * n - mp[t] + k;
    }
}

return -1; // no solution
}

```

7.3 Div Floor Ceil

```

int CEIL(int a, int b) {
    return (a >= 0 ? (a + b - 1) / b : a / b);
}

int FLOOR(int a, int b) {
    return (a >= 0 ? a / b : (a - b + 1) / b);
}

```

7.4 exCRT

```

i64 exgcd(i64 a, i64 b, i64 &x, i64 &y) {
    if (b == 0) {
        x = 1;
        y = 0;
        return a;
    }
    i64 g = exgcd(b, a % b, y, x);
    y -= a / b * x;
    return g;
}

// return {x, T}
// a: moduli, b: remainders
// x: first non-negative solution, T: minimum period
std::pair<i64, i64> exCRT(auto &a, auto &b) {
    auto [m1, r1] = std::tie(a[0], b[0]);
    for (int i = 1; i < std::ssize(a); i++) {
        auto [m2, r2] = std::tie(a[i], b[i]);
        i64 x, y;
        i64 g = exgcd(m1, m2, x, y);
        if ((r2 - r1) % g) { // no solution
            return {-1, -1};
        }
        x = (i128(x) * (r2 - r1) / g) % (m2 / g);
        if (x < 0) {
            x += (m2 / g);
        }
        r1 = m1 * x + r1;
        m1 = std::lcm(m1, m2);
    }
    r1 %= m1;
    if (r1 < 0) {
        r1 += m1;
    }
    return {r1, m1};
};

```

7.5 Factorization

```

ull modmul(ull a, ull b, ull M) {
    int ret = a * b - M * ull(1.L / M * a * b);
    return ret + M * (ret < 0) - M * (ret >= (int)M);
}

ull modpow(ull b, ull e, ull mod) {
    ull ans = 1;
    for (; e; b = modmul(b, b, mod), e /= 2)
        if (e & 1) ans = modmul(ans, b, mod);
    return ans;
}

bool isPrime(ull n) {
    if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
    ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
        s = __builtin_ctzll(n - 1), d = n >> s;
    for (ull a : A) {
        ull p = modpow(a % n, d, n), i = s;
        while (p != 1 && p != n - 1 && a % n && i--)
            p = modmul(p, p, n);
        if (p != n - 1 && i != s) return 0;
    }
    return 1;
}

ull pollard(ull n) {
    uniform_int_distribution<ull> unif(0, n - 1);
    ull c = 1;

```

```

    auto f = [n, &c](ull x) { return modmul(x, x, n) + c % n; };
    ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
    while (t++ % 40 || __gcd(prd, n) == 1) {
        if (x == y) c = unif(rng), x = ++i, y = f(x);
        if ((q = modmul(prd, max(x, y) - min(x, y), n))) prd = q;
        x = f(x), y = f(f(y));
    }
    return __gcd(prd, n);
}

```

```

vector<ull> factor(ull n) {
    if (n == 1) return {};
    if (isPrime(n)) return {n};
    ull x = pollard(n);
    auto l = factor(x), r = factor(n / x);
    l.insert(l.end(), r.begin(), r.end());
    return l;
}

```

7.6 Floor Sum

```

// \sum_0^n floor((a * x + b) / c) in log(n + m + a + b)
int floor_sum(int a, int b, int c, int n) { // add mod if needed
    int m = (a * n + b) / c;
    if (a >= c || b >= c)
        return (a / c) * (n * (n + 1) / 2) + (b / c) * (n + 1) +
            floor_sum(a % c, b % c, c, n);
    if (n < 0 || a == 0)
        return 0;
    return n * m - floor_sum(c, c - b - 1, a, m - 1);
}

```

7.7 FWT

```

void fwt(vector<ll> &f, bool inv = false) { // xor-convolution
    const int N = 31 - __builtin_clz(ssize(f)),
        inv2 = (MOD + 1) / 2;
    rep(i, 0, N) rep(j, 0, 1 << N) {
        if (j >> i & 1 ^ 1) {
            ll a = f[j], b = f[j | (1 << i)];
            if (inv) {
                f[j] = (a + b) * inv2 % MOD;
                f[j | (1 << i)] = (a - b + MOD) * inv2 % MOD;
            } else {
                f[j] = (a + b) % MOD;
                f[j | (1 << i)] = (a - b + MOD) % MOD;
            }
        }
    }
}

```

7.8 Gauss Elimination

```

template <class T>
constexpr T power(T a, u64 b, T res = 1) {
    for (; b != 0; b /= 2, a *= a) {
        if (b & 1) {
            res *= a;
        }
    }
    return res;
}

template <u32 P = 998244353>
struct ModInt {
    u32 v;
    ModInt(i64 x = 0) { norm(x % P + P); }
    ModInt &norm(u32 x) {
        v = x < P ? x : x - P;
        return *this;
    }
    ModInt inv() const { return power(*this, P - 2); }
    ModInt operator+(const ModInt &r) const { return ModInt().norm(v + r.v); }
    ModInt operator-(const ModInt &r) const { return ModInt().norm(v + P - r.v); }
    ModInt operator*(const ModInt &r) const { return ModInt().norm(u64(v) * r.v % P); }
    ModInt operator/(const ModInt &r) const { return *this * r.inv(); }
    ModInt &operator+=(const ModInt &r) { return *this = *this + r; }
    ModInt &operator-=(const ModInt &r) { return *this = *this - r; }
    ModInt &operator*=(const ModInt &r) { return *this = *this * r; }
}

```

```

ModInt &operator/=(const ModInt &r) { return *this = *this /
    r; }
bool operator==(const ModInt &r) const { return v == r.v; }
explicit operator bool() const { return v != 0; }
friend ostream &operator<<(std::ostream &os, const
    ModInt &r) {
    return os << r.v;
}
};

using Z = ModInt<998244353>;
// using F = long double;

using Matrix = std::vector<std::vector<Z>>;
// using Matrix = std::vector<std::vector<F>>; (double)
// using Matrix = std::vector<std::bitset<5000>>; (mod 2)

template <typename T>
auto gauss(Matrix &A, std::vector<T> &b, int n, int m) {
    assert(std::ssize(b) == n);
    int r = 0;
    std::vector<int> where(m, -1);

    for (int i = 0; i < m && r < n; i++) {
        int p = r; // pivot
        while (p < n && A[p][i] == T(0)) {
            p++;
        }
        if (p == n) {
            continue;
        }
        std::swap(A[r], A[p]);
        std::swap(b[r], b[p]);
        where[i] = r;

        // coef: mod 2 don't need this
        T inv = T(1) / A[r][i];
        for (int j = i; j < m; j++) {
            A[r][j] *= inv;
        }
        b[r] *= inv;

        for (int j = 0; j < n; j++) { // deduct: mod 2 don't need
            this
            if (j != r) {
                T x = A[j][i];
                for (int k = i; k < m; k++) {
                    A[j][k] -= x * A[r][k];
                }
                b[j] -= x * b[r];
            }
        }

        // for (int j = 0; j < n; ++j) { // (mod 2) -> coef and
        // deduct
        //     if (j != r && A[j][i]) {
        //         A[j] ^= A[r];
        //         b[j] ^= b[r];
        //     }
        // }

        r++;
    }

    for (int i = r; i < n; i++) {
        if (ranges::all_of(A[i] | views::take(m), [](auto x) {
            return x == 0; }) && b[i] != T(0)) {
            return std::vector<T>(); // no solution
        }
        // if (A[i].none() && b[i]) { // (mod 2)
        //     return std::vector<T>();
        // }
    }

    // if (r < m) { // infinite solution
    //     return std::vector<T>();
    // }

    std::vector<T> res(m);
    for (int i = 0; i < m; i++) {
        if (where[i] != -1) {
            res[i] = b[where[i]];
        }
    }
}

```

```

return res;
};

```

7.9 Lagrange Interpolation

```

struct Lagrange {
    int deg{};
    vector<int> C;
    Lagrange(const vector<int> &P) {
        deg = P.size() - 1;
        C.assign(deg + 1, 0);
        for (int i = 0; i <= deg; i++) {
            int q = inv[i] * inv[i - deg] % mod;
            if ((deg - i) % 2 == 1) {
                q = mod - q;
            }
            C[i] = P[i] * q % mod;
        }
    }
    int operator()(int x) { // 0 <= x < mod
        if (0 <= x and x <= deg) {
            int ans = fac[x] * fac[deg - x] % mod;
            if ((deg - x) % 2 == 1) {
                ans = (mod - ans);
            }
            return ans * C[x] % mod;
        }
        vector<int> pre(deg + 1), suf(deg + 1);
        for (int i = 0; i <= deg; i++) {
            pre[i] = (x - i);
            if (i) {
                pre[i] = pre[i] * pre[i - 1] % mod;
            }
        }
        for (int i = deg; i >= 0; i--) {
            suf[i] = (x - i);
            if (i < deg) {
                suf[i] = suf[i] * suf[i + 1] % mod;
            }
        }
        int ans = 0;
        for (int i = 0; i <= deg; i++) {
            ans += (i == 0 ? 1 : pre[i - 1]) * (i == deg ? 1 : suf[i
                + 1]) % mod * C[i];
            ans %= mod;
        }
        if (ans < 0) ans += mod;
        return ans;
    }
};

```

7.10 Linear Sieve

```

const int C = 1e6 + 5;

int mo[C], lp[C], phi[C], isp[C];
vector<int> prime;

void sieve() {
    mo[1] = phi[1] = 1;
    rep (i, 1, C) lp[i] = 1;
    rep (i, 2, C) {
        if (lp[i] == 1) {
            lp[i] = i;
            prime.pb(i);
            isp[i] = 1;
            mo[i] = -1;
            phi[i] = i - 1;
        }
        for (int p : prime) {
            if (i * p >= C) break;
            lp[i * p] = p;
            if (i % p == 0) {
                phi[p * i] = phi[i] * p;
                break;
            }
            phi[i * p] = phi[i] * (p - 1);
            mo[i * p] = mo[i] * mo[p];
        }
    }
}

```

7.11 Lucas

```

// comb(n, m) % M, M = p^k
// O(M)-O(log(n))

```



```

struct Lucas {
    const int p, M;
    vector<int> f;
    Lucas(int p, int M) : p(p), M(M), f(M + 1) {
        f[0] = 1;
        for (int i = 1; i <= M; i++) {
            f[i] = f[i - 1] * (i % p == 0 ? 1 : i) % M;
        }
    }
    int CountFact(int n) {
        int c = 0;
        while (n) c += (n /= p);
        return c;
    }
    // (n! without factor p) % p^k
    int ModFact(int n) {
        int r = 1;
        while (n) {
            r = r * power(f[M], n / M % 2, M) % M * f[n % M] % M;
            n /= p;
        }
        return r;
    }
    int ModComb(int n, int m) {
        if (m < 0 or n < m) return 0;
        int c = CountFact(n) - CountFact(m) - CountFact(n - m);
        int r = ModFact(n) * power(ModFact(m), M / p * (p - 1) - 1,
            M) % M
            * power(ModFact(n - m), M / p * (p - 1) - 1, M) %
            M;
        return r * power(p, c, M) % M;
    }
};

```

7.12 Mod Int

```

using u32 = unsigned int;
using u64 = unsigned long long;
template <class T>
constexpr T power(T a, u64 b, T res = 1) {
    for (; b != 0; b /= 2, a *= a) {
        if (b & 1) {
            res *= a;
        }
    }
    return res;
}

template <u32 P>
struct ModInt {
    u32 v;
    const static ModInt G;

    constexpr ModInt &norm(u32 x) {
        v = x < P ? x : x - P;
        return *this;
    }

    constexpr ModInt(i64 x = 0) { norm(x % P + P); }
    constexpr ModInt inv() const { return power(*this, P - 2); }
    constexpr ModInt operator-(const ModInt &r) const { return ModInt() - *this; }
    constexpr ModInt operator+(const ModInt &r) const { return
        ModInt().norm(v + r.v); }
    constexpr ModInt operator-(const ModInt &r) const { return
        ModInt().norm(v + P - r.v); }
    constexpr ModInt operator*(const ModInt &r) const { return
        ModInt().norm(u64(v) * r.v % P); }
    constexpr ModInt operator/(const ModInt &r) const { return *
        this * r.inv(); }
    constexpr ModInt &operator+=(const ModInt &r) { return *this
        = *this + r; }
    constexpr ModInt &operator-=(const ModInt &r) { return *this
        = *this - r; }
    constexpr ModInt &operator*=(const ModInt &r) { return *this
        = *this * r; }
    constexpr ModInt &operator/=(const ModInt &r) { return *this
        = *this / r; }
    constexpr bool operator==(const ModInt &r) const { return v
        == r.v; }
    constexpr bool operator!=(const ModInt &r) const { return v
        != r.v; }
    explicit constexpr operator bool() const { return v != 0; }
    friend std::ostream &operator<<(std::ostream &os, const
        ModInt &r) {
        return os << r.v;
    }
};

```

```

}
};
using mint = ModInt<998244353>;
template <> const mint mint::G = mint(3);

```

7.13 Primitive Root

```

ull primitiveRoot(ull p) {
    auto fac = factor(p - 1);
    sort(all(fac));
    fac.erase(unique(all(fac)), fac.end());
    auto test = [p, fac](ull x) {
        for(ull d : fac)
            if (modpow(x, (p - 1) / d, p) == 1)
                return false;
        return true;
    };
    uniform_int_distribution<ull> unif(1, p - 1);
    ull root;
    while(!test(root = unif(rng)));
    return root;
}

```

7.14 Simplex

```

// max{cx} subject to {Ax<=b, x>=0}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
// usage :
// x = simplex(A, b, c); (A <= 100 x 100)
vector<double> simplex(
    const vector<vector<double>> &a,
    const vector<double> &b,
    const vector<double> &c) {
    int n = (int)a.size(), m = (int)a[0].size() + 1;
    vector val(n + 2, vector<double>(m + 1));
    vector<int> idx(n + m);
    iota(all(idx), 0);
    int r = n, s = m - 1;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m - 1; ++j)
            val[i][j] = -a[i][j];
        val[i][m - 1] = 1;
        val[i][m] = b[i];
        if (val[r][m] > val[i][m])
            r = i;
    }
    copy(all(c), val[n].begin());
    val[n + 1][m - 1] = -1;
    for (double num; ; ) {
        if (r < n) {
            swap(idx[s], idx[r + m]);
            val[r][s] = 1 / val[r][s];
            for (int j = 0; j <= m; ++j) if (j != s)
                val[r][j] *= -val[r][s];
            for (int i = 0; i <= n + 1; ++i) if (i != r) {
                for (int j = 0; j <= m; ++j) if (j != s)
                    val[i][j] += val[r][j] * val[i][s];
                val[i][s] *= val[r][s];
            }
        }
        r = s = -1;
        for (int j = 0; j < m; ++j)
            if (s < 0 || idx[s] > idx[j])
                if (val[n + 1][j] > eps || val[n + 1][j] > -eps && val[
                    n][j] > eps)
                    s = j;
        if (s < 0) break;
        for (int i = 0; i < n; ++i) if (val[i][s] < -eps) {
            if (r < 0
                || (num = val[r][m] / val[r][s] - val[i][m] / val[i][s]
                    ) < -eps
                || num < eps && idx[r + m] > idx[i + m])
                r = i;
        }
        if (r < 0) {
            // Solution is unbounded.
            return vector<double>{};
        }
    }
    if (val[n + 1][m] < -eps) {
        // No solution.
        return vector<double>{};
    }
}

```

```
vector<double> x(m - 1);
for (int i = m; i < n + m; ++i)
    if (idx[i] < m - 1)
        x[idx[i]] = val[i - m][m];
return x;
}
```

7.15 Sqrt Mod

```
// the Jacobi symbol is a generalization of the Legendre symbol
// such that the bottom doesn't need to be prime.
// (n|p) -> same as legendre
// (n|ab) = (n|a)(n|b)
// work with long long
int Jacobi(int a, int m) {
    int s = 1;
    for (; m > 1; ) {
        a %= m;
        if (a == 0) return 0;
        const int r = __builtin_ctz(a);
        if ((r & 1) && ((m + 2) & 4)) s = -s;
        a >>= r;
        if (a & m & 2) s = -s;
        swap(a, m);
    }
    return s;
}

// 0: a == 0
// -1: a isn't a quad res of p
// else: return X with X^2 % p == a
// doesn't work with long long
int QuadraticResidue(int a, int p) {
    if (p == 2) return a & 1;
    if (int jc = Jacobi(a, p); jc <= 0) return jc;
    int b, d;
    for (; ; ) {
        b = rand() % p;
        d = (1LL * b * b + p - a) % p;
        if (Jacobi(d, p) == -1) break;
    }
    int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
    for (int e = (1LL + p) >> 1; e; e >>= 1) {
        if (e & 1) {
            tmp = (1LL * g0 * f0 + 1LL * d * (1LL * g1 * f1 % p)) % p;
            g1 = (1LL * g0 * f1 + 1LL * g1 * f0) % p;
            g0 = tmp;
        }
        tmp = (1LL * f0 * f0 + 1LL * d * (1LL * f1 * f1 % p)) % p;
        f1 = (2LL * f0 * f1) % p;
        f0 = tmp;
    }
    return g0;
}
```

7.16 FFT

```
template<typename C = complex<double>>
void FFT(vector<C> &P, C w, bool inv = 0) {
    int n = P.size(), lg = __builtin_ctz(n);
    assert(__builtin_popcount(n) == 1);

    for (int j = 1, i = 0; j < n - 1; ++j) {
        for (int k = n >> 1; k > (i ^ k); k >>= 1);
        if (j < i) swap(P[i], P[j]);
    }

    vector<C> ws = {inv ? C{1} / w : w};

    rep(i, 1, lg) ws.pb(ws[i - 1] * ws[i - 1]);
    reverse(all(ws));

    rep(i, 0, lg) {
        for (int k = 0; k < n; k += 2 << i) {
            C base = C{1};
            rep(j, k, k + (1 << i)) {
                auto t = base * P[j + (1 << i)];
                auto u = P[j];
                P[j] = u + t;
                P[j + (1 << i)] = u - t;
                base = base * ws[i];
            }
        }
    }
}
```

```
if (inv) rep(i, 0, n) P[i] = P[i] / C(n);
}

const int N = 1 << 21;
const double PI = acos(-1);
const auto w = exp(-complex<double>(0, 2.0 * PI / N));
```

7.17 Polynomial

```
std::mt19937_64 rng(std::chrono::steady_clock::now().
    time_since_epoch().count());

template <class mint>
void nft(bool type, std::vector<mint> &a) {
    int n = int(a.size()), s = 0;
    while ((1 << s) < n) {
        s++;
    }
    assert(1 << s == n);
    static std::vector<mint> ep, iep;
    while (int(ep.size()) <= s) {
        ep.push_back(power(mint::G, mint(-1).v / (1 << int(ep.size())
            )));
        iep.push_back(ep.back().inv());
    }
    std::vector<mint> b(n);
    for (int i = 1; i <= s; i++) {
        int w = 1 << (s - i);
        mint base = type ? iep[i] : ep[i], now = 1;
        for (int y = 0; y < n / 2; y += w) {
            for (int x = 0; x < w; x++) {
                auto l = a[y << 1 | x];
                auto r = now * a[y << 1 | x | w];
                b[y | x] = l + r;
                b[y | x | n >> 1] = l - r;
            }
            now *= base;
        }
        std::swap(a, b);
    }
}

template <class mint>
std::vector<mint> multiply(const std::vector<mint> &a, const
    std::vector<mint> &b) {
    int n = int(a.size()), m = int(b.size());
    if (!n || !m) return {};
    if (std::min(n, m) <= 8) {
        std::vector<mint> ans(n + m - 1);
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < m; j++) {
                ans[i + j] += a[i] * b[j];
            }
        }
        return ans;
    }
    int lg = 0;
    while ((1 << lg) < n + m - 1) {
        lg++;
    }
    int z = 1 << lg;
    auto a2 = a, b2 = b;
    a2.resize(z);
    b2.resize(z);
    nft(false, a2);
    nft(false, b2);
    for (int i = 0; i < z; i++) {
        a2[i] *= b2[i];
    }
    nft(true, a2);
    a2.resize(n + m - 1);
    mint iz = mint(z).inv();
    for (int i = 0; i < n + m - 1; i++) {
        a2[i] *= iz;
    }
    return a2;
}

template <class D>
struct Poly {
    std::vector<D> v;
    Poly(const std::vector<D> &v_ = {}) : v(v_) { shrink(); }
    void shrink() {
        while (v.size() && !v.back()) {

```

```

    v.pop_back();
}
}
int size() const { return int(v.size()); }
D freq(int p) const { return (p < size()) ? v[p] : D(0); }
Poly operator+(const Poly &r) const {
    auto n = std::max(size(), r.size());
    std::vector<D> res(n);
    for (int i = 0; i < n; i++) {
        res[i] = freq(i) + r.freq(i);
    }
    return res;
}
Poly operator-(const Poly &r) const {
    int n = std::max(size(), r.size());
    std::vector<D> res(n);
    for (int i = 0; i < n; i++) {
        res[i] = freq(i) - r.freq(i);
    }
    return res;
}
Poly operator*(const Poly &r) const { return {multiply(v, r.v)}; }
Poly operator*(const D &r) const {
    int n = size();
    std::vector<D> res(n);
    for (int i = 0; i < n; i++) {
        res[i] = v[i] * r;
    }
    return res;
}
Poly operator/(const D &r) const { return *this * r.inv(); }
Poly operator/(const Poly &r) const {
    if (size() < r.size()) return {{{}}};
    int n = size() - r.size() + 1;
    return (rev().pre(n) * r.rev().inv(n)).pre(n).rev();
}
Poly operator%(const Poly &r) const { return *this - *this / r * r; }
Poly operator<<(int s) const {
    std::vector<D> res(size() + s);
    for (int i = 0; i < size(); i++) {
        res[i + s] = v[i];
    }
    return res;
}
Poly operator>>(int s) const {
    if (size() <= s) {
        return Poly();
    }
    std::vector<D> res(size() - s);
    for (int i = 0; i < size() - s; i++) {
        res[i] = v[i + s];
    }
    return res;
}
Poly &operator+=(const Poly &r) { return *this = *this + r; }
Poly &operator-=(const Poly &r) { return *this = *this - r; }
Poly &operator*=(const Poly &r) { return *this = *this * r; }
Poly &operator*=(const D &r) { return *this = *this * r; }
Poly &operator/=(const Poly &r) { return *this = *this / r; }
Poly &operator/=(const D &r) { return *this = *this / r; }
Poly &operator%=(const Poly &r) { return *this = *this % r; }
Poly &operator<=<(const size_t &n) { return *this = *this << n; }
Poly &operator>>=(const size_t &n) { return *this = *this >> n; }
Poly pre(int le) const {
    return {{v.begin(), v.begin() + std::min(size(), le)}};
}
Poly rev(int n = -1) const {
    std::vector<D> res = v;
    if (n != -1) {
        res.resize(n);
    }
    std::reverse(res.begin(), res.end());
    return res;
}
Poly diff() const {
    std::vector<D> res(std::max(0, size() - 1));
    for (int i = 1; i < size(); i++) {
        res[i - 1] = freq(i) * i;
    }
}

return res;
}
Poly inte() const {
    std::vector<D> res(size() + 1);
    for (int i = 0; i < size(); i++) {
        res[i + 1] = freq(i) / (i + 1);
    }
    return res;
}
// f * f.inv() = 1 + g(x)x^m
Poly inv(int m) const {
    Poly res = Poly({D(1) / freq(0)});
    for (int i = 1; i < m; i *= 2) {
        res = (res * D(2) - res * res * pre(2 * i)).pre(2 * i);
    }
    return res.pre(m);
}
Poly exp(int n) const {
    assert(freq(0) == 0);
    Poly f({1}), g({1});
    for (int i = 1; i < n; i *= 2) {
        g = (g * 2 - f * g * g).pre(i);
        Poly q = diff().pre(i - 1);
        Poly w = (q + g * (f.diff() - f * q)).pre(2 * i - 1);
        f = (f + f * (*this - w.inte()).pre(2 * i)).pre(2 * i);
    }
    return f.pre(n);
}
Poly log(int n) const {
    assert(freq(0) == 1);
    auto f = pre(n);
    return (f.diff() * f.inv(n - 1)).pre(n - 1).inte();
}
Poly sqrt(int n) const {
    assert(freq(0) == 1);
    Poly f = pre(n + 1);
    Poly g({1});
    for (int i = 1; i < n; i *= 2) {
        g = (g + f.pre(2 * i) * g.inv(2 * i)) / 2;
    }
    return g.pre(n + 1);
}
Poly modpower(u64 n, const Poly &mod) {
    Poly x = *this, res = {{1}};
    for (; n; n /= 2, x = x * x % mod) {
        if (n & 1) {
            res = res * x % mod;
        }
    }
    return res;
}
friend std::ostream &operator<<(std::ostream &os, const Poly &p) {
    if (p.size() == 0) {
        return os << "0";
    }
    for (auto i = 0; i < p.size(); i++) {
        if (p.v[i]) {
            os << p.v[i] << "x^" << i;
            if (i != p.size() - 1) {
                os << "+";
            }
        }
    }
    return os;
}
};
template <class mint>
struct MultiEval {
    using NP = MultiEval *;
    NP l, r;
    int sz;
    Poly<mint> mul;
    std::vector<mint> que;
    MultiEval(const std::vector<mint> &que_, int off, int sz_) :
        sz(sz_) {
        if (sz <= 100) {
            que = {que_.begin() + off, que_.begin() + off + sz};
            mul = {{1}};
            for (auto x : que) {
                mul *= {{-x, 1}};
            }
            return;
        }
    }

```

```

    }
    l = new MultiEval(que_, off, sz / 2);
    r = new MultiEval(que_, off + sz / 2, sz - sz / 2);
    mul = l->mul * r->mul;
}
MultiEval(const std::vector<mint> &que_) : MultiEval(que_, 0,
    int(que_.size())) {}
void query(const Poly<mint> &pol_, std::vector<mint> &res)
    const {
    if (sz <= 100) {
        for (auto x : que_) {
            mint sm = 0, base = 1;
            for (int i = 0; i < pol_.size(); i++) {
                sm += base * pol_.freq(i);
                base *= x;
            }
            res.push_back(sm);
        }
        return;
    }
    auto pol = pol_ % mul;
    l->query(pol, res);
    r->query(pol, res);
}
std::vector<mint> query(const Poly<mint> &pol) const {
    std::vector<mint> res;
    query(pol, res);
    return res;
}
};
template <class mint>
Poly<mint> berlekampMassey(const std::vector<mint> &s) {
    int n = int(s.size());
    std::vector<mint> b = {mint(-1)}, c = {mint(-1)};
    mint y = mint(1);
    for (int ed = 1; ed <= n; ed++) {
        int l = int(c.size()), m = int(b.size());
        mint x = 0;
        for (int i = 0; i < l; i++) {
            x += c[i] * s[ed - l + i];
        }
        b.push_back(0);
        m++;
        if (!x) {
            continue;
        }
        mint freq = x / y;
        if (l < m) {
            // use b
            auto tmp = c;
            c.insert(begin(c), m - l, mint(0));
            for (int i = 0; i < m; i++) {
                c[m - 1 - i] -= freq * b[m - 1 - i];
            }
            b = tmp;
            y = x;
        } else {
            // use c
            for (int i = 0; i < m; i++) {
                c[l - 1 - i] -= freq * b[m - 1 - i];
            }
        }
    }
    return c;
}
template <class E, class mint = decltype(E().f)>
mint sparseDet(const std::vector<std::vector<E>> &g) {
    int n = int(g.size());
    if (n == 0) {
        return 1;
    }
    auto randV = [&]() {
        std::vector<mint> res(n);
        for (int i = 0; i < n; i++) {
            res[i] = mint(std::uniform_int_distribution<i64>(1, mint
                (-1).v)(rng)); // need rng
        }
        return res;
    };
    std::vector<mint> c = randV(), l = randV(), r = randV();
    // l * mat * r
    std::vector<mint> buf(2 * n);
    for (int fe = 0; fe < 2 * n; fe++) {
        for (int i = 0; i < n; i++) {

```

```

            buf[fe] += l[i] * r[i];
        }
    }
    for (int i = 0; i < n; i++) {
        r[i] *= c[i];
    }
    std::vector<mint> tmp(n);
    for (int i = 0; i < n; i++) {
        for (auto e : g[i]) {
            tmp[i] += r[e.to] * e.f;
        }
    }
    r = tmp;
}
auto u = berlekampMassey(buf);
if (u.size() != n + 1) {
    return sparseDet(g);
}
auto acdet = u.freq(0) * mint(-1);
if (n % 2) {
    acdet *= mint(-1);
}
if (!acdet) {
    return 0;
}
mint cdet = 1;
for (int i = 0; i < n; i++) {
    cdet *= c[i];
}
return acdet / cdet;
}
}

```

7.18 Theorem

- Pick's Theorem
 $A = i + \frac{b}{2} - 1$
 A : Area i : grid number in the inner b : grid number on the side
- Matrix-Tree theorem
 undirected graph
 $D_{ii}(G) = \deg(i), D_{ij} = 0, i \neq j$
 $A_{ij}(G) = A_{ji}(G) = \#e(i, j), i \neq j$
 $L(G) = D(G) - A(G)$
 $t(G) = \det L(G)_{(1,2,\dots,i-1,i+1,\dots,n)}^{(1,2,\dots,i-1,i+1,\dots,n)}$
 leaf to root
 $D_{ii}^{out}(G) = \deg^{out}(i), D_{ij}^{out} = 0, i \neq j$
 $A_{ij}(G) = \#e(i, j), i \neq j$
 $L^{out}(G) = D^{out}(G) - A(G)$
 $t^{root}(G, k) = \det L^{out}(G)_{(1,2,\dots,k-1,k+1,\dots,n)}^{(1,2,\dots,k-1,k+1,\dots,n)}$
 root to leaf
 $L^{in}(G) = D^{in}(G) - A(G)$
 $t^{leaf}(G, k) = \det L^{in}(G)_{(1,2,\dots,k-1,k+1,\dots,n)}^{(1,2,\dots,k-1,k+1,\dots,n)}$
- Derangement
 $D_n = (n-1)(D_{n-1} + D_{n-2}) = nD_{n-1} + (-1)^n$
- Möbius Inversion
 $f(n) = \sum_{d|n} g(d) \Leftrightarrow g(n) = \sum_{d|n} \mu\left(\frac{n}{d}\right) f(d)$
- Euler Inversion
 $\sum_{i|n} \varphi(i) = n$
- Binomial Inversion
 $f(n) = \sum_{i=0}^n \binom{n}{i} g(i) \Leftrightarrow g(n) = \sum_{i=0}^n (-1)^{n-i} \binom{n}{i} f(i)$
- Subset Inversion
 $f(S) = \sum_{T \subseteq S} g(T) \Leftrightarrow g(S) = \sum_{T \subseteq S} (-1)^{|S|-|T|} f(T)$
- Min-Max Inversion
 $\max_{i \in S} x_i = \sum_{T \subseteq S} (-1)^{|T|-1} \min_{j \in T} x_j$
- Ex Min-Max Inversion
 $kthmax_{i \in S} x_i = \sum_{T \subseteq S} (-1)^{|T|-k} \binom{|T|-1}{k-1} \min_{j \in T} x_j$
- Lcm-Gcd Inversion
 $lcm_{i \in S} x_i = \prod_{T \subseteq S} \left(\gcd_{j \in T} x_j \right)^{(-1)^{|T|-1}}$
- Sum of powers
 $\sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k^+ n^{m+1-k}$
 $\sum_{j=0}^m \binom{m+1}{j} B_j^- = 0$
 note: $B_1^+ = -B_1^-, B_i^+ = B_i^-$
- Cayley's formula
 number of trees on n labeled vertices: n^{n-2}
 Let $T_{n,k}$ be the number of labelled forests on n vertices with k connected components, such that vertices $1, 2, \dots, k$ all belong to different connected components. Then $T_{n,k} = k n^{n-k-1}$.

- High order residue

$$[d^{\frac{p-1}{n, p-1}}] \equiv 1]$$
- Packing and Covering
 $|\text{maximum independent set}| + |\text{minimum vertex cover}| = |V|$
- Kofig's theorem
 $|\text{maximum matching}| = |\text{minimum vertex cover}|$
- Dilworth's theorem
 $\text{width} = |\text{largest antichain}| = |\text{smallest chain decomposition}|$
- Mirsky's theorem
 $\text{height} = |\text{longest chain}| = |\text{smallest antichain decomposition}| = |\text{minimum antichain partition}|$
- Lucas' Theorem
For $n, m \in \mathbb{Z}^*$ and prime P , $\binom{m}{n} \bmod P = \prod \binom{m_i}{n_i}$ where m_i is the i -th digit of m in base P .
- Stirling approximation
 $n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n e^{\frac{1}{12n}}$
- 1st Stirling Numbers(permutation $|P| = n$ with k cycles)
 $S(n, k) = \text{coefficient of } x^k \text{ in } \prod_{i=0}^{n-1} (x+i)$
 $S(n+1, k) = nS(n, k) + S(n, k-1)$
- 2nd Stirling Numbers(Partition n elements into k non-empty set)

$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$

$$S(n+1, k) = kS(n, k) + S(n, k-1)$$
- Catalan number

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n-1}$$

$$\binom{n+m}{n} - \binom{n+m}{n+1} = (m+n)! \frac{n-m+1}{n+1} \quad \text{for } n \geq m$$

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1 \quad \text{and} \quad C_{n+1} = 2 \binom{2n+1}{n+2} C_n$$

$$C_0 = 1 \quad \text{and} \quad C_{n+1} = \sum_{i=0}^n C_i C_{n-i} \quad \text{for } n \geq 0$$
- Extended Catalan number

$$\frac{1}{(k-1)n+1} \binom{kn}{n}$$
- Calculate $c[i-j] += a[i] \times b[j]$ for $a[n], b[m]$
1. $a = \text{reverse}(a)$; $c = \text{mul}(a, b)$; $c = \text{reverse}(c[1:n])$;
2. $b = \text{reverse}(b)$; $c = \text{mul}(a, b)$; $c = \text{rshift}(c, m-1)$;
- Eulerian number (permutation $1 \sim n$ with m $a[i] > a[i-1]$)

$$A(n, m) = \sum_{i=0}^m (-1)^i \binom{n+1}{i} (m+1-i)^n$$

$$A(n, m) = (n-m)A(n-1, m-1) + (m+1)A(n-1, m)$$
- Hall's theorem
Let $G = (X + Y, E)$ be a bipartite graph. For $W \subseteq X$, let $N(W) \subseteq Y$ denotes the adjacent vertices set of W . Then, G has a X' -perfect matching (matching contains $X' \subseteq X$) iff $\forall W \subseteq X', |W| \leq |N(W)|$.
- Tutte Matrix:
For a graph $G = (V, E)$, its maximum matching = $\frac{\text{rank}(A)}{2}$ where $A_{ij} = ((i, j) \in E ? (i < j ? x_{ij} : -x_{ji}) : 0)$ and x_{ij} are random numbers.
- Erdoš–Gallai theorem
There exists a simple graph with degree sequence $d_1 \geq \dots \geq d_n$ iff
 $\sum_{i=1}^n d_i$ is even and $\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k), \forall 1 \leq k \leq n$
- Euler Characteristic
planar graph: $V - E + F - C = 1$
convex polyhedron: $V - E + F = 2$
 V, E, F, C : number of vertices, edges, faces(regions), and components
- Burnside Lemma
 $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$
- Polya theorem
 $|Y^x/G| = \frac{1}{|G|} \sum_{g \in G} m^{c(g)}$
 $m = |Y|$: num of colors, $c(g)$: num of cycle
- Cayley's Formula
Given a degree sequence d_1, \dots, d_n of a labeled tree, there are $\frac{(n-2)!}{(d_1-1)! \dots (d_n-1)!}$ spanning trees.
- Find a Primitive Root of n :
 n has primitive roots iff $n = 2, 4, p^k, 2p^k$ where p is an odd prime.
1. Find $\phi(n)$ and all prime factors of $\phi(n)$, says $P = \{p_1, \dots, p_m\}$
2. $\forall g \in [2, n]$, if $g^{p_i} \not\equiv 1, \forall p_i \in P$, then g is a primitive root.
3. Since the smallest one isn't too big, the algorithm runs fast.
4. n has exactly $\phi(\phi(n))$ primitive roots.
- Taylor series

$$f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \frac{f'''(c)}{3!}(x-c)^3 + \dots$$
- Lagrange Multiplier
 $\min f(x, y)$, subject to $g(x, y) = 0$
 $\frac{\partial f}{\partial x} + \lambda \frac{\partial g}{\partial x} = 0$
 $\frac{\partial f}{\partial y} + \lambda \frac{\partial g}{\partial y} = 0$
 $g(x, y) = 0$
- Calculate $f(x+n)$ where $f(x) = \sum_{i=0}^{n-1} a_i x^i$

$$f(x+n) = \sum_{i=0}^{n-1} a_i (x+n)^i = \sum_{i=0}^{n-1} x^i \cdot \frac{1}{i!} \sum_{j=i}^{n-1} \frac{a_j}{j!} \cdot \frac{n^{j-i}}{(j-i)!}$$
- Bell 數 (有 n 個人, 把他們拆組的方法總數)
 $B_0 = 1$
 $B_n = \sum_{k=0}^n s(n, k)$ (second - stirling)
 $B_{n+1} = \sum_{k=0}^n \binom{n}{k} B_k$
- Wilson's theorem
 $(p-1)! \equiv -1 \pmod{p}$

$$(p^q)! \equiv \begin{cases} 1, & (p=2) \wedge (q \geq 3), \\ -1, & \text{otherwise.} \end{cases} \pmod{p^q}$$
- Fermat's little theorem
 $a^p \equiv a \pmod{p}$
- Euler's theorem

$$a^b \equiv \begin{cases} a^{b \bmod \varphi(m)}, & \gcd(a, m) = 1, \\ a^b, & \gcd(a, m) \neq 1, b < \varphi(m), \\ a^{(b \bmod \varphi(m)) + \varphi(m)}, & \gcd(a, m) \neq 1, b \geq \varphi(m). \end{cases} \pmod{m}$$
- 環狀著色 (相鄰塗異色)
 $(k-1)(-1)^n + (k-1)^n$

8 Stringology

8.1 Aho–Corasick AM

```

struct ACM {
    int idx = 0;
    vector<array<int, 26>> tr;
    vector<int> cnt, fail, id;

    void clear() {
        tr.resize(1, array<int, 26>{});
        cnt.resize(1, 0);
        fail.resize(1, 0);
    }

    ACM() {
        clear();
    }

    int newnode() {
        tr.push_back(array<int, 26>{});
        cnt.push_back(0);
        fail.push_back(0);
        return ++idx;
    }

    void insert(string &s, int i) {
        int u = 0;
        id.push_back(i);
        for (char c : s) {
            c -= 'a';
            if (tr[u][c] == 0) tr[u][c] = newnode();
            u = tr[u][c];
        }
        cnt[u]++;
    }

    void build() {
        queue<int> q;
        rep (i, 0, 26) if (tr[0][i]) q.push(tr[0][i]);
        while (!q.empty()) {
            int u = q.front(); q.pop();
            rep (i, 0, 26) {
                if (tr[u][i]) {
                    fail[tr[u][i]] = tr[fail[u]][i];
                    cnt[tr[u][i]] += cnt[fail[tr[u][i]]];
                    q.push(tr[u][i]);
                } else {
                    tr[u][i] = tr[fail[u]][i];
                }
            }
        }
    }

    int query(string &s) {

```



```

    };
    sort(all(suf), cmp);
    auto tmp = rk;
    tmp[suf[0]] = 0;
    for (int i = 1; i < n; i++) {
        tmp[suf[i]] = tmp[suf[i - 1]] + cmp(suf[i - 1], suf[i])
    };
    rk.swap(tmp);
}
};

```

8.7 Z-value

```

struct zvalue {
    vector<int> z;
    int operator[] (const int &x) const {
        return z[x];
    }
    zvalue(string s) {
        int n = s.size();
        z.resize(n);
        z[0] = 0;
        for (int i = 1, l = 1, r = 0; i < n; i++) {
            z[i] = min(z[i - 1], max<int>(0, r - i));
            while (i + z[i] < n && s[i + z[i]] == s[z[i]]) z[i]++;
            if (i + z[i] > r) l = i, r = i + z[i];
        }
    }
};

```