

# Contents

<b>1 Basic</b>	<b>1</b>	<b>8 Stringology</b>	<b>20</b>
1.1 createFile	1	8.1 Aho–Corasick AM	20
1.2 run	1	8.2 Double String	20
1.3 tem	1	8.3 Lyndon Factorization	20
1.4 debug	2	8.4 Manacher	21
1.5 run.bat	2	8.5 SA-IS	21
1.6 random	2	8.6 Suffix Array	21
1.7 TempleHash	2	8.7 Z-value	21
<b>2 Misc</b>	<b>2</b>	<b>1 Basic</b>	
2.1 FastIO	2	1.1 createFile	
2.2 stress.sh	2	// Linux	
2.3 Timer	2	for i in {A..Z}; do cp tem.cpp \$i.cpp; done	
2.4 MinPlusConvolution	2	// Windows	
2.5 PyTrick	2	'A'..'Z'   % { cp tem.cpp "\$_.cpp" }	
<b>3 Data Structure</b>	<b>3</b>	<b>1.2 run</b>	
3.1 Fenwick Tree	3	g++ -std=c++20 -DPEPPA -Wall -Wextra -Wshadow -O2 -fsanitize=	
3.2 Li Chao	3	address,undefined \$1.cpp -o \$1 && ./\$1	
3.3 PBDS	3	<b>1.3 tem</b>	
3.4 ODT	3	#include <bits/stdc++.h>	
3.5 Sparse Table	4	using namespace std;	
3.6 Splay	4	using i64 = long long;	
3.7 Treap	4	#define int i64	
<b>4 Matching and Flow</b>	<b>5</b>	#define all(a) a.begin(), a.end()	
4.1 Dinic	5	#define rep(a, b, c) for (int a = b; a < c; a++)	
4.2 General Matching	5	#ifdef PEPPA	
4.3 KM	5	template <typename R>	
4.4 MCMF	6	concept I = ranges::range<R> && !std::same_as<ranges::	
4.5 Model	6	range_value_t<R>, char>;	
<b>5 Geometry</b>	<b>7</b>	template <typename A, typename B>	
5.1 Point	7	std::ostream &operator<<(std::ostream &o, const std::pair<A, B>	
5.2 Line	7	&p) {	
5.3 Circle	7	return o << "(" << p.first << ", " << p.second << ");	
5.4 Point to Segment Distance	7	}	
5.5 Point In Polygon	7	template <I T>	
5.6 Intersection of Line	7	std::ostream &operator<<(std::ostream &o, const T &v) {	
5.7 Intersection of Circles	7	o << "{";	
5.8 Intersection of Circle and Line	7	int f = 0;	
5.9 Area of Circle Polygon	7	for (auto &i : v) o << (f++ ? " " : "") << i;	
5.10 Convex Hull	7	return o << "}";	
5.11 Convex Trick	8	}	
5.12 Half Plane Intersection	8	void debug__(int c, auto &&...a) {	
5.13 Minimal Enclosing Circle	8	std::cerr << "\e[1;" << c << "m";	
5.14 Minkowski	8	(..., (std::cerr << a << " "));	
5.15 Point In Circumcircle	9	std::cerr << "\e[0m" << std::endl;	
5.16 Tangent Lines of Circle and Point	9	}	
5.17 Tangent Lines of Circles	9	#define debug_(c, x...) debug__(c, __LINE__, "[" + std::string	
5.18 Triangle Center	9	(#x) + "]", x)	
5.19 Union of Circles	9	#define debug(x...) debug_(93, x)	
<b>6 Graph</b>	<b>10</b>	#else	
6.1 Block Cut Tree	10	#define debug(x...) void(0)	
6.2 Count Cycles	10	#endif	
6.3 Dominator Tree	10	bool chmin(auto& a, auto b) { return (b < a and (a = b, true));	
6.4 Enumerate Planar Face	10	}	
6.5 Manhattan MST	11	bool chmax(auto& a, auto b) { return (a < b and (a = b, true));	
6.6 Matroid Intersection	11	}	
6.7 Maximum Clique	11	void solve() {	
6.8 Tree Hash	11	//	
6.9 Two-SAT	12	}	
6.10 Virtual Tree	12	int32_t main() {	
<b>7 Math</b>	<b>12</b>	std::ios::sync_with_stdio(false);	
7.1 Combinatoric	12	std::cin.tie(nullptr);	
7.2 Discrete Log	13	int t = 1;	
7.3 Div Floor Ceil	13	std::cin >> t;	
7.4 exCRT	13	while (t--) {	
7.5 Factorization	13	solve();	
7.6 Floor Sum	13	}	
7.7 FWT	13	return 0;	
7.8 Gauss Elimination	14	}	
7.9 Lagrange Interpolation	14		
7.10 Linear Sieve	14		
7.11 Lucas	15		
7.12 Mod Int	15		
7.13 Primitive Root	15		
7.14 Simplex	15		
7.15 Sqrt Mod	16		
7.16 PiCount	16		
7.17 ModMin	16		
7.18 FFT	16		
7.19 NTT prime	16		
7.20 Polynomial	17		
7.21 Theorem	19		

## 1.4 debug

```
#ifdef PEPPA
template <typename R>
concept I = ranges::range<R> && !std::same_as<ranges::
    range_value_t<R>, char>;
template <typename A, typename B>
std::ostream& operator<<(std::ostream& o, const std::pair<A, B
    >& p) {
    return o << "(" << p.first << ", " << p.second << ")";
}
template <I T>
std::ostream& operator<<(std::ostream& o, const T& v) {
    o << "{";
    int f = 0;
    for (auto &&i : v) o << (f++ ? " " : "") << i;
    return o << "}";
}
void debug__(int c, auto&&... a) {
    std::cerr << "\e[1;" << c << "m";
    (... , (std::cerr << a << " "));
    std::cerr << "\e[0m" << std::endl;
}
#define debug_(c, x...) debug__(_c, __LINE__, "[" + std::string
    (#x) + "]", x)
#define debug(x...) debug_(93, x)
#else
#define debug(x...) void(0)
#endif
```

## 1.5 run.bat

```
@echo off
g++ -std=c++23 -DPEPPA -Wall -Wextra -Wshadow -O2 %1.cpp -o %1.
    exe
if "%2" == "" ("%1.exe") else ("%1.exe" < "%2")
```

## 1.6 random

```
std::mt19937_64 rng(std::chrono::steady_clock::now().
    time_since_epoch().count());
inline i64 rand(i64 l, i64 r) { return std::
    uniform_int_distribution<i64>(l, r)(rng); }
```

## 1.7 TempleHash

```
cat file.cpp | cpp -dD -P -fpreprocessed | tr -d "[:space:]" |
    md5sum | cut -c-6
```

# 2 Misc

## 2.1 FastIO

```
#include <unistd.h>
int OP;
char OB[65536];
inline char RC() {
    static char buf[65536], *p = buf, *q = buf;
    return p == q && (q = (p = buf) + read(0, buf, 65536)) == buf
        ? -1 : *p++;
}
inline int R() {
    static char c;
    while ((c = RC()) < '0');
    int a = c ^ '0';
    while ((c = RC()) >= '0') a *= 10, a += c ^ '0';
    return a;
}
inline void W(int n) {
    static char buf[12], p;
    if (n == 0) OB[OP++] = '0';
    p = 0;
    while (n) buf[p++] = '0' + (n % 10), n /= 10;
    for (--p; p >= 0; --p) OB[OP++] = buf[p];
    if (OP > 65520) write(1, OB, OP), OP = 0;
}
// another FastIO
char buf[1 << 21], *p1 = buf, *p2 = buf;
inline char getc() {
    return p1 == p2 && (p2 = (p1 = buf) + fread(buf, 1, 1 << 21,
        stdin), p1 == p2) ? 0 : *p1++;
}
template<typename T> void Cin(T &a) {
    T res = 0; int f = 1;
    char c = getc();
    for (; c < '0' || c > '9'; c = getc()) {
        if (c == '-') f = -1;
```

```
    }
    for (; c >= '0' && c <= '9'; c = getc()) {
        res = res * 10 + c - '0';
    }
    a = f * res;
}
template<typename T, typename... Args> void Cin(T &a, Args &...
    args) {
    Cin(a), Cin(args...);
}
template<typename T> void Cout(T x) { // there's no '\n' in
    output
    if (x < 0) putchar('-'), x = -x;
    if (x > 9) Cout(x / 10);
    putchar(x % 10 + '0');
}
```

## 2.2 stress.sh

```
#!/usr/bin/env bash
g++ $1.cpp -o $1
g++ $2.cpp -o $2
g++ $3.cpp -o $3
for i in {1..100} ; do
    ./$3 > input.txt
    # st=$(date +%s%N)
    ./$1 < input.txt > output1.txt
    # echo "$(((date +%s%N) - $st)/1000000))ms"
    ./$2 < input.txt > output2.txt
    if cmp --silent -- "output1.txt" "output2.txt" ; then
        continue
    fi
    echo Input:
    cat input.txt
    echo Your Output:
    cat output1.txt
    echo Correct Output:
    cat output2.txt
    exit 1
done
echo OK!
./stress.sh main good gen
```

## 2.3 Timer

```
struct Timer {
    int t;
    bool enable = false;

    void start() {
        enable = true;
        t = std::clock();
    }
    int msec() {
        assert(enable);
        return (std::clock() - t) * 1000 / CLOCKS_PER_SEC;
    }
};
```

## 2.4 MinPlusConvolution

```
// a is convex a[i+1]-a[i] <= a[i+2]-a[i+1]
vector<int> min_plus_convolution(vector<int> &a, vector<int> &b
    ) {
    int n = ssize(a), m = ssize(b);
    vector<int> c(n + m - 1, INF);
    auto dc = [&](auto Y, int l, int r, int jl, int jr) {
        if (l > r) return;
        int mid = (l + r) / 2, from = -1, &best = c[mid];
        for (int j = jl; j <= jr; ++j)
            if (int i = mid - j; i >= 0 && i < n)
                if (best > a[i] + b[j])
                    best = a[i] + b[j], from = j;
        Y(Y, l, mid - 1, jl, from), Y(Y, mid + 1, r, from, jr);
    };
    return dc(dc, 0, n - 1 + m - 1, 0, m - 1), c;
}
```

## 2.5 PyTrick

```
import sys
input = sys.stdin.readline

from itertools import permutations
op = ['+', '-', '*', '/']
a, b, c, d = input().split()
ans = set()
for (x,y,z,w) in permutations([a, b, c, d]):
```

```

for op1 in op:
    for op2 in op:
        for op3 in op:
            val = eval(f'{{{x}}}{op1}{{{y}}}{op2}{{{z}}}{op3}{{{w}}}')
            if (op1 == '' and op2 == '' and op3 == '') or val < 0:
                continue
            ans.add(val)
print(len(ans))

map(int,input().split())
arr2d = [ [ list(map(int,input().split())) ] for i in range(N)
           ] # N*M

from decimal import *
from fractions import *
s = input()
n = int(input())
f = Fraction(s)
g = Fraction(s).limit_denominator(n)
h = f * 2 - g
if h.numerator <= n and h.denominator <= n and h < g:
    g = h
print(g.numerator, g.denominator)

from fractions import Fraction
x = Fraction(1, 2), y = Fraction(1)
print(x.as_integer_ratio()) # print 1/2
print(x.is_integer())
print(x.__round__())
print(float(x))

r = Fraction(input())
N = int(input())
r2 = r - 1 / Fraction(N) ** 2
ans = r.limit_denominator(N)
ans2 = r2.limit_denominator(N)
if ans2 < ans and 0 <= ans2 <= 1 and abs(ans - r) >= abs(ans2 -
    r):
    ans = ans2
print(ans.numerator, ans.denominator)

```

## 3 Data Structure

### 3.1 Fenwick Tree

```

template<class T>
struct Fenwick {
    int n;
    vector<T> a;
    Fenwick(int _n) : n(_n), a(_n) {}
    void add(int p, T x) {
        for (int i = p; i < n; i = i | (i + 1)) {
            a[i] = a[i] + x;
        }
    }
    T qry(int p) { // sum [0, p]
        T s{};
        for (int i = p; i >= 0; i = (i & (i + 1)) - 1) {
            s = s + a[i];
        }
        return s;
    }
    T qry(int l, int r) { // sum [l, r]
        return qry(r - 1) - qry(l - 1);
    }
    pair<int, T> select(T k) { // [first position >= k, sum [0, p
    )
        T s{};
        int p = 0;
        for (int i = 1 << __lg(n); i; i >>= 1) {
            if (p + i <= n and s + a[p + i - 1] < k) {
                p += i;
                s = s + a[p - 1];
            }
        }
        return {p, s};
    }
};

```

### 3.2 Li Chao

```

struct Line {
    // y = ax + b
    i64 a{0}, b{-inf<i64>};
    i64 operator()(i64 x) {
        return a * x + b;
    }
};

```

```

}
};
// max LiChao
struct Seg {
    int l, r;
    Seg *ls{}, *rs{};
    Line f{};
    Seg(int l, int r) : l(l), r(r) {}
    void add(Line g) {
        int m = (l + r) / 2;
        if (g(m) > f(m)) {
            swap(g, f);
        }
        if (g.b == -inf<i64> or r - l == 1) {
            return;
        }
        if (g.a < f.a) {
            if (!ls) {
                ls = new Seg(l, m);
            }
            ls->add(g);
        } else {
            if (!rs) {
                rs = new Seg(m, r);
            }
            rs->add(g);
        }
    }
    i64 qry(i64 x) {
        if (f.b == -inf<i64>) {
            return -inf<i64>;
        }
        int m = (l + r) / 2;
        i64 y = f(x);
        if (x < m and ls) {
            chmax(y, ls->qry(x));
        } else if (x >= m and rs) {
            chmax(y, rs->qry(x));
        }
        return y;
    }
};

```

### 3.3 PBDS

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template<typename T> using RBT = tree<T, null_type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;
/*
.find_by_order(k) 回傳第 k 小的值 (based-0)
.order_of_key(k) 回傳有多少元素比 k 小
*/
struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }

    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM = chrono::steady_clock::
            now().time_since_epoch().count();
        return splitmix64(x + FIXED_RANDOM);
    }
};
// gp_hash_table<int, int, custom_hash> ss;

```

### 3.4 ODT

```

map<int, int> odt;
// initialize edges odt[1] and odt[n + 1]
auto split = [&](const int &x) -> void {
    const auto it = prev(odt.upper_bound(x));
    odt[x] = it->second;
};
auto merge = [&](const int &l, const int &r) -> void {
    auto itl = odt.lower_bound(l), itr = odt.lower_bound(r + 1);
    for (; itl != itr; itl = odt.erase(itl)) {
        // do something
    }
    // assign value to odt[l]
};

```

### 3.5 Sparse Table

```
template<class T>
struct SparseTable{
    function<T(T, T)> F;
    vector<vector<T>> sp;
    SparseTable(vector<T> &a, const auto &f) {
        F = f;
        int n = a.size();
        sp.resize(n, vector<T>(__lg(n) + 1));
        for (int i = n - 1; i >= 0; i--) {
            sp[i][0] = a[i];
            for (int j = 1; i + (1 << j) <= n; j++) {
                sp[i][j] = F(sp[i][j - 1], sp[i + (1 << j - 1)][j - 1]);
            }
        }
    }
    T query(int l, int r) { // [l, r)
        int k = __lg(r - l);
        return F(sp[l][k], sp[r - (1 << k)][k]);
    }
};
```

### 3.6 Splay

```
struct Node {
    Node *ch[2]{{}, *p{}};
    Info info{}, sum{};
    Tag tag{};
    int size{};
    bool rev{};
} pool[1E5 + 10], *top = pool;
Node *newNode(Info a) {
    Node *t = top++;
    t->info = t->sum = a;
    t->size = 1;
    return t;
}
int size(const Node *x) { return x ? x->size : 0; }
Info get(const Node *x) { return x ? x->sum : Info{}; }
int dir(const Node *x) { return x->p->ch[1] == x; }
bool nroot(const Node *x) { return x->p and x->p->ch[dir(x)] == x; }
void reverse(Node *x) { if (x) x->rev = !x->rev; }
void update(Node *x, const Tag &t) {
    if (!x) return;
    f(x->tag);
    f(x->info);
    f(x->sum);
}
void push(Node *x) {
    if (x->rev) {
        swap(x->ch[0], x->ch[1]);
        reverse(x->ch[0]);
        reverse(x->ch[1]);
        x->rev = false;
    }
    update(x->ch[0], x->tag);
    update(x->ch[1], x->tag);
    x->tag = Tag{};
}
void pull(Node *x) {
    x->size = size(x->ch[0]) + 1 + size(x->ch[1]);
    x->sum = get(x->ch[0]) + x->info + get(x->ch[1]);
}
void rotate(Node *x) {
    Node *y = x->p, *z = y->p;
    push(y);
    int d = dir(x);
    push(x);
    Node *w = x->ch[d ^ 1];
    if (nroot(y)) {
        z->ch[dir(y)] = x;
    }
    if (w) {
        w->p = y;
    }
    (x->ch[d ^ 1] = y)->ch[d] = w;
    (y->p = x)->p = z;
    pull(y);
    pull(x);
}
void splay(Node *x) {
    while (nroot(x)) {
```

```
        Node *y = x->p;
        if (nroot(y)) {
            rotate(dir(x) == dir(y) ? y : x);
        }
        rotate(x);
    }
    Node *nth(Node *x, int k) {
        assert(size(x) > k);
        while (true) {
            push(x);
            int left = size(x->ch[0]);
            if (left > k) {
                x = x->ch[0];
            } else if (left < k) {
                k -= left + 1;
                x = x->ch[1];
            } else {
                break;
            }
        }
        splay(x);
        return x;
    }
    Node *split(Node *x) {
        assert(x);
        push(x);
        Node *l = x->ch[0];
        if (l) l->p = x->ch[0] = nullptr;
        pull(x);
        return l;
    }
    Node *join(Node *x, Node *y) {
        if (!x or !y) return x ? x : y;
        y = nth(y, 0);
        push(y);
        y->ch[0] = x;
        if (x) x->p = y;
        pull(y);
        return y;
    }
    Node *find_first(Node *x, auto &&pred) {
        Info pre{};
        while (true) {
            push(x);
            if (pred(pre + get(x->ch[0]))) {
                x = x->ch[0];
            } else if (pred(pre + get(x->ch[0]) + x->info) or !x->ch[1]) {
                break;
            } else {
                pre = pre + get(x->ch[0]) + x->info;
                x = x->ch[1];
            }
        }
        splay(x);
        return x;
    }
}
```

### 3.7 Treap

```
struct Treap {
    Treap *l, *r;
    int key, size;
    Treap(int k) : l(nullptr), r(nullptr), key(k), size(1) {}
    void pull();
    void push() {}
};
inline int SZ(Treap *p) {
    return p == nullptr ? 0 : p->size;
}
void Treap::pull() {
    size = 1 + SZ(l) + SZ(r);
}
Treap *merge(Treap *a, Treap *b) {
    if (!a || !b) return a ? a : b;
    if (rand() % (SZ(a) + SZ(b)) < SZ(a)) {
        return a->push(), a->r = merge(a->r, b), a->pull(), a;
    }
    return b->push(), b->l = merge(a, b->l), b->pull(), b;
}
// <= k, > k
void split(Treap *p, Treap *&a, Treap *&b, int k) { // by key
    if (!p) return a = b = nullptr, void();
```

```

p->push();
if (p->key <= k) {
    a = p, split(p->r, a->r, b, k), a->pull();
} else {
    b = p, split(p->l, a, b->l, k), b->pull();
}
}
// k, n - k
void split2(Treap *p, Treap *a, Treap *b, int k) { // by size
    if (!p) return a = b = nullptr, void();
    p->push();
    if (SZ(p->l) + 1 <= k) {
        a = p, split2(p->r, a->r, b, k - SZ(p->l) - 1);
    } else {
        b = p, split2(p->l, a, b->l, k);
    }
    p->pull();
}
void insert(Treap *p, int k) {
    Treap *l, *r;
    p->push(), split(p, l, r, k);
    p = merge(merge(l, new Treap(k)), r);
    p->pull();
}
bool erase(Treap *p, int k) {
    if (!p) return false;
    if (p->key == k) {
        Treap *t = p;
        p->push(), p = merge(p->l, p->r);
        delete t;
        return true;
    }
    Treap *t = k < p->key ? p->l : p->r;
    return erase(t, k) ? p->pull(), true : false;
}
int Rank(Treap *p, int k) { // # of key < k
    if (!p) return 0;
    if (p->key < k) return SZ(p->l) + 1 + Rank(p->r, k);
    return Rank(p->l, k);
}
Treap *kth(Treap *p, int k) { // 1-base
    if (k <= SZ(p->l)) return kth(p->l, k);
    if (k == SZ(p->l) + 1) return p;
    return kth(p->r, k - SZ(p->l) - 1);
}
// pref: kth(Rank(x)), succ: kth(Rank(x)+1)
tuple<Treap*, Treap*, Treap*> interval(Treap *s, int l, int r)
{ // 1-based
    Treap *a, *b, *c; // b: [l, r]
    split2(o, a, b, l - 1), split2(b, b, c, r - l + 1);
    return make_tuple(a, b, c);
}

```

## 4 Matching and Flow

### 4.1 Dinic

```

template <typename T>
struct Dinic {
    const T INF = numeric_limits<T>::max() / 2;
    struct edge {
        int v, r; T rc;
    };
    vector<vector<edge>> adj;
    vector<T> dis, it;
    Dinic(int n) : adj(n), dis(n), it(n) {}
    void add_edge(int u, int v, T c) {
        adj[u].pb({v, adj[v].size(), c});
        adj[v].pb({u, adj[u].size() - 1, 0});
    }
    bool bfs(int s, int t) {
        fill(all(dis), INF);
        queue<int> q;
        q.push(s);
        dis[s] = 0;
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (const auto& [v, r, rc] : adj[u]) {
                if (dis[v] < INF || rc == 0) continue;
                dis[v] = dis[u] + 1;
                q.push(v);
            }
        }
    }
}

```

```

return dis[t] < INF;
}

T dfs(int u, int t, T cap) {
    if (u == t || cap == 0) return cap;
    for (int &i = it[u]; i < (int)adj[u].size(); ++i) {
        auto &[v, r, rc] = adj[u][i];
        if (dis[v] != dis[u] + 1) continue;
        T tmp = dfs(v, t, min(cap, rc));
        if (tmp > 0) {
            rc -= tmp;
            adj[v][r].rc += tmp;
            return tmp;
        }
    }
    return 0;
}

T flow(int s, int t) {
    T ans = 0, tmp;
    while (bfs(s, t)) {
        fill(all(it), 0);
        while ((tmp = dfs(s, t, INF)) > 0) {
            ans += tmp;
        }
    }
    return ans;
}

bool inScut(int u) { return dis[u] < INF; }
};

```

### 4.2 General Matching

```

struct GeneralMatching { // n <= 500
    const int BLOCK = 10;
    int n;
    vector<vector<int>> g;
    vector<int> hit, mat;
    priority_queue<pair<int, int>, vector<pair<int, int>>,
        greater<pair<int, int>>> unmat;
    GeneralMatching(int _n) : n(_n), g(_n), mat(n, -1), hit(n) {}
    void add_edge(int a, int b) { // 0 <= a != b < n
        g[a].push_back(b);
        g[b].push_back(a);
    }
    int get_match() {
        for (int i = 0; i < n; i++) if (!g[i].empty()) {
            unmat.emplace(0, i);
        }
        // If WA, increase this
        // there are some cases that need >= 1.3 * n^2 steps for BLOCK
        // = 1
        // no idea what the actual bound needed here is.
        const int MAX_STEPS = 10 + 2 * n + n * n / BLOCK / 2;
        mt19937 rng(random_device{}());
        for (int i = 0; i < MAX_STEPS; ++i) {
            if (unmat.empty()) break;
            int u = unmat.top().second;
            unmat.pop();
            if (mat[u] != -1) continue;
            for (int j = 0; j < BLOCK; j++) {
                ++hit[u];
                auto &e = g[u];
                const int v = e[rng() % e.size()];
                mat[u] = v;
                swap(u, mat[v]);
                if (u == -1) break;
            }
            if (u != -1) {
                mat[u] = -1;
                unmat.emplace(hit[u] * 100ULL / (g[u].size() + 1), u);
            }
        }
        int siz = 0;
        for (auto e : mat) siz += (e != -1);
        return siz / 2;
    }
};

```

### 4.3 KM

```

template<class T>
T KM(const vector<vector<T>> &w) {
    const T INF = numeric_limits<T>::max() / 2;
}

```

```

const int n = w.size();
vector<T> lx(n), ly(n);
vector<int> mx(n, -1), my(n, -1), pa(n);
auto augment = [&](int y) {
    for (int x, z; y != -1; y = z) {
        x = pa[y];
        z = mx[x];
        my[y] = x;
        mx[x] = y;
    }
};
auto bfs = [&](int s) {
    vector<T> sy(n, INF);
    vector<bool> vx(n), vy(n);
    queue<int> q;
    q.push(s);
    while (true) {
        while (q.size()) {
            int x = q.front();
            q.pop();
            vx[x] = 1;
            for (int y = 0; y < n; y++) {
                if (vy[y]) continue;
                T d = lx[x] + ly[y] - w[x][y];
                if (d == 0) {
                    pa[y] = x;
                    if (my[y] == -1) {
                        augment(y);
                        return;
                    }
                    vy[y] = 1;
                    q.push(my[y]);
                } else if (chmin(sy[y], d)) {
                    pa[y] = x;
                }
            }
        }
        T cut = INF;
        for (int y = 0; y < n; y++)
            if (!vy[y])
                chmin(cut, sy[y]);
        for (int j = 0; j < n; j++) {
            if (vx[j]) lx[j] -= cut;
            if (vy[j]) ly[j] += cut;
            else sy[j] -= cut;
        }
        for (int y = 0; y < n; y++)
            if (!vy[y] and sy[y] == 0) {
                if (my[y] == -1) {
                    augment(y);
                    return;
                }
                vy[y] = 1;
                q.push(my[y]);
            }
    }
};
for (int x = 0; x < n; x++)
    lx[x] = ranges::max(w[x]);
for (int x = 0; x < n; x++)
    bfs(x);
T ans = 0;
for (int x = 0; x < n; x++)
    ans += w[x][mx[x]];
return ans;
}

```

#### 4.4 MCMF

```

template<class T>
struct MCMF {
    const T INF = numeric_limits<T>::max() / 2;
    struct edge { int v, r; T f, w; };
    vector<vector<edge>> adj;
    const int n;
    MCMF(int n) : n(n), adj(n) {}
    void addEdge(int u, int v, T f, T c) {
        adj[u].push_back({v, ssize(adj[v]), f, c});
        adj[v].push_back({u, ssize(adj[u]) - 1, 0, -c});
    }
    vector<T> dis;
    vector<bool> vis;
    bool spfa(int s, int t) {
        queue<int> que;

```

```

        dis.assign(n, INF);
        vis.assign(n, false);
        que.push(s);
        vis[s] = 1;
        dis[s] = 0;
        while (!que.empty()) {
            int u = que.front(); que.pop();
            vis[u] = 0;
            for (auto [v, _, f, w] : adj[u])
                if (f && chmin(dis[v], dis[u] + w))
                    if (!vis[v]) {
                        que.push(v);
                        vis[v] = 1;
                    }
        }
        return dis[t] != INF;
    }
    T dfs(int u, T in, int t) {
        if (u == t) return in;
        vis[u] = 1;
        T out = 0;
        for (auto &[v, rev, f, w] : adj[u])
            if (f && !vis[v] && dis[v] == dis[u] + w) {
                T x = dfs(v, min(in, f), t);
                in -= x;
                out += x;
                f -= x;
                adj[v][rev].f += x;
                if (!in) break;
            }
        if (in) dis[u] = INF;
        vis[u] = 0;
        return out;
    }
    pair<T, T> flow(int s, int t) { // {flow, cost}
        T a = 0, b = 0;
        while (spfa(s, t)) {
            T x = dfs(s, INF, t);
            a += x;
            b += x * dis[t];
        }
        return {a, b};
    }
};

```

#### 4.5 Model

- Maximum/Minimum flow with lower bound / Circulation problem
  - Construct super source  $S$  and sink  $T$ .
  - For each edge  $(x, y, l, u)$ , connect  $x \rightarrow y$  with capacity  $u - l$ .
  - For each vertex  $v$ , denote by  $in(v)$  the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
  - If  $in(v) > 0$ , connect  $S \rightarrow v$  with capacity  $in(v)$ , otherwise, connect  $v \rightarrow T$  with capacity  $-in(v)$ .
    - To maximize, connect  $t \rightarrow s$  with capacity  $\infty$  (skip this in circulation problem), and let  $f$  be the maximum flow from  $S$  to  $T$ . If  $f \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise, the maximum flow from  $s$  to  $t$  is the answer.
    - To minimize, let  $f$  be the maximum flow from  $S$  to  $T$ . Connect  $t \rightarrow s$  with capacity  $\infty$  and let the flow from  $S$  to  $T$  be  $f'$ . If  $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise,  $f'$  is the answer.
  - The solution of each edge  $e$  is  $l_e + f_e$ , where  $f_e$  corresponds to the flow of edge  $e$  on the graph.
- Construct minimum vertex cover from maximum matching  $M$  on bipartite graph  $(X, Y)$ 
  - Redirect every edge:  $y \rightarrow x$  if  $(x, y) \in M$ ,  $x \rightarrow y$  otherwise.
  - DFS from unmatched vertices in  $X$ .
  - $x \in X$  is chosen iff  $x$  is unvisited.
  - $y \in Y$  is chosen iff  $y$  is visited.
- Minimum cost cyclic flow
  - Construct super source  $S$  and sink  $T$
  - For each edge  $(x, y, c)$ , connect  $x \rightarrow y$  with  $(cost, cap) = (c, 1)$  if  $c > 0$ , otherwise connect  $y \rightarrow x$  with  $(cost, cap) = (-c, 1)$
  - For each edge with  $c < 0$ , sum these cost as  $K$ , then increase  $d(y)$  by 1, decrease  $d(x)$  by 1
  - For each vertex  $v$  with  $d(v) > 0$ , connect  $S \rightarrow v$  with  $(cost, cap) = (0, d(v))$
  - For each vertex  $v$  with  $d(v) < 0$ , connect  $v \rightarrow T$  with  $(cost, cap) = (0, -d(v))$
  - Flow from  $S$  to  $T$ , the answer is the cost of the flow  $C + K$
- Maximum density induced subgraph
  - Binary search on answer, suppose we're checking answer  $T$
  - Construct a max flow model, let  $K$  be the sum of all weights
  - Connect source  $s \rightarrow v, v \in G$  with capacity  $K$
  - For each edge  $(u, v, w)$  in  $G$ , connect  $u \rightarrow v$  and  $v \rightarrow u$  with capacity  $w$



5. For  $v \in G$ , connect it with sink  $v \rightarrow t$  with capacity  $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
  6.  $T$  is a valid answer if the maximum flow  $f < K|V|$
- Minimum weight edge cover
    1. Change the weight of each edge to  $\mu(u) + \mu(v) - w(u, v)$ , where  $\mu(v)$  is the cost of the cheapest edge incident to  $v$ .
    2. Let the maximum weight matching of the graph be  $x$ , the answer will be  $\sum \mu(v) - x$ .

## 5 Geometry

### 5.1 Point

```
using numbers::pi;
template<class T> inline constexpr T eps = numeric_limits<T>::
    epsilon() * 1E6;
using Real = long double;
struct Pt {
    Real x, y;
    Pt operator+(Pt a) const { return {x + a.x, y + a.y}; }
    Pt operator-(Pt a) const { return {x - a.x, y - a.y}; }
    Pt operator*(Real k) const { return {x * k, y * k}; }
    Pt operator/(Real k) const { return {x / k, y / k}; }
    Real operator*(Pt a) const { return x * a.x + y * a.y; }
    Real operator^(Pt a) const { return x * a.y - y * a.x; }
    auto operator<=>(const Pt&) const = default;
    bool operator==(const Pt&) const = default;
};
int sgn(Real x) { return (x > -eps<Real>) - (x < eps<Real>); }
Real ori(Pt a, Pt b, Pt c) { return (b - a) ^ (c - a); }
bool argcmp(const Pt &a, const Pt &b) { // arg(a) < arg(b)
    int f = (Pt{a.y, -a.x} > Pt{b.y, -b.x}) ? 1 : -1;
    int g = (Pt{b.y, -b.x} > Pt{a.y, -a.x}) ? 1 : -1;
    return f == g ? (a ^ b) > 0 : f < g;
}
Pt rotate(Pt u) { return {-u.y, u.x}; }
Real abs2(Pt a) { return a * a; }
// Floating point only
Pt rotate(Pt u, Real a) {
    Pt v{sinl(a), cosl(a)};
    return {u ^ v, u * v};
}
Real abs(Pt a) { return sqrtl(a * a); }
Real arg(Pt x) { return atan2l(x.y, x.x); }
Pt unit(Pt x) { return x / abs(x); }
```

### 5.2 Line

```
struct Line {
    Pt a, b;
    Pt dir() const { return b - a; }
};
int PtSide(Pt p, Line l) {
    return sgn(ori(l.a, l.b, p)); // for int
    return sgn(ori(l.a, l.b, p) / abs(l.a - l.b));
}
bool PtOnSeg(Pt p, Line l) {
    return PtSide(p, l) == 0 and sgn((p - l.a) * (p - l.b)) <= 0;
}
Pt proj(Pt p, Line l) {
    Pt dir = unit(l.b - l.a);
    return l.a + dir * (dir * (p - l.a));
}
```

### 5.3 Circle

```
struct Cir {
    Pt o;
    double r;
};
bool disjunct(const Cir &a, const Cir &b) {
    return sgn(abs(a.o - b.o) - a.r - b.r) >= 0;
}
bool contain(const Cir &a, const Cir &b) {
    return sgn(a.r - b.r - abs(a.o - b.o)) >= 0;
}
```

### 5.4 Point to Segment Distance

```
double PtSegDist(Pt p, Line l) {
    double ans = min(abs(p - l.a), abs(p - l.b));
    if (sgn(abs(l.a - l.b)) == 0) return ans;
    if (sgn((l.a - l.b) * (p - l.b)) < 0) return ans;
    if (sgn((l.b - l.a) * (p - l.a)) < 0) return ans;
    return min(ans, abs(ori(p, l.a, l.b)) / abs(l.a - l.b));
}
```

```
double SegDist(Line l, Line m) {
    return PtSegDist({0, 0}, {l.a - m.a, l.b - m.b});
}
```

### 5.5 Point In Polygon

```
int inPoly(Pt p, const vector<Pt> &P) {
    const int n = P.size();
    int cnt = 0;
    for (int i = 0; i < n; i++) {
        Pt a = P[i], b = P[(i + 1) % n];
        if (PtOnSeg(p, {a, b})) return 1; // on edge
        if ((sgn(a.y - p.y) == 1) ^ (sgn(b.y - p.y) == 1))
            cnt += sgn(ori(a, b, p));
    }
    return cnt == 0 ? 0 : 2; // out, in
}
```

### 5.6 Intersection of Line

```
bool isInter(Line l, Line m) {
    if (PtOnSeg(m.a, l) or PtOnSeg(m.b, l) or
        PtOnSeg(l.a, m) or PtOnSeg(l.b, m))
        return true;
    return PtSide(m.a, l) * PtSide(m.b, l) < 0 and
        PtSide(l.a, m) * PtSide(l.b, m) < 0;
}
Pt LineInter(Line l, Line m) {
    double s = ori(m.a, m.b, l.a), t = ori(m.a, m.b, l.b);
    return (l.b * s - l.a * t) / (s - t);
}
bool strictInter(Line l, Line m) {
    int la = PtSide(m.a, l);
    int lb = PtSide(m.b, l);
    int ma = PtSide(l.a, m);
    int mb = PtSide(l.b, m);
    if (la == 0 and lb == 0) return false;
    return la * lb < 0 and ma * mb < 0;
}
```

### 5.7 Intersection of Circles

```
vector<Pt> CircleInter(Cir a, Cir b) {
    double d2 = abs2(a.o - b.o), d = sqrt(d2);
    if (d < max(a.r, b.r) - min(a.r, b.r) || d > a.r + b.r)
        return {};
    Pt u = (a.o + b.o) / 2 + (a.o - b.o) * ((b.r * b.r - a.r * a.r) / (2 * d2));
    double A = sqrt((a.r + b.r + d) * (a.r - b.r + d) * (a.r + b.r - d) * (-a.r + b.r + d));
    Pt v = rotate(b.o - a.o) * A / (2 * d2);
    if (sgn(v.x) == 0 and sgn(v.y) == 0) return {u};
    return {u - v, u + v}; // counter clockwise of a
}
```

### 5.8 Intersection of Circle and Line

```
vector<Pt> CircleLineInter(Cir c, Line l) {
    Pt H = proj(c.o, l);
    Pt dir = unit(l.b - l.a);
    double h = abs(H - c.o);
    if (sgn(h - c.r) > 0) return {};
    double d = sqrt(max((double)0., c.r * c.r - h * h));
    if (sgn(d) == 0) return {H};
    return {H - dir * d, H + dir * d};
    // Counterclockwise
}
```

### 5.9 Area of Circle Polygon

```
double CirclePoly(Cir C, const vector<Pt> &P) {
    auto arg = [&](Pt p, Pt q) { return atan2(p ^ q, p * q); };
    double r2 = C.r * C.r / 2;
    auto tri = [&](Pt p, Pt q) {
        Pt d = q - p;
        auto a = (d * p) / abs2(d), b = (abs2(p) - C.r * C.r) / abs2(d);
        auto det = a * a - b;
        if (det <= 0) return arg(p, q) * r2;
        auto s = max(0., -a - sqrt(det)), t = min(1., -a + sqrt(det));
        if (t < 0 or 1 <= s) return arg(p, q) * r2;
        Pt u = p + d * s, v = p + d * t;
        return arg(p, u) * r2 + (u ^ v) / 2 + arg(v, q) * r2;
    };
    double sum = 0.0;
    for (int i = 0; i < P.size(); i++)
```

```
sum += tri(P[i] - C.o, P[(i + 1) % P.size()] - C.o);
return sum;
}
```

## 5.10 Convex Hull

```
vector<Pt> BuildHull(vector<Pt> pt) {
    sort(all(pt));
    pt.erase(unique(all(pt)), pt.end());
    if (pt.size() <= 2) return pt;
    vector<Pt> hull;
    int sz = 1;
    rep (t, 0, 2) {
        rep (i, t, ssize(pt)) {
            while (ssize(hull) > sz && ori(hull.end()[-2], pt[i],
                hull.back()) >= 0)
                hull.pop_back();
            hull.pb(pt[i]);
        }
        sz = ssize(hull);
        reverse(all(pt));
    }
    hull.pop_back();
    return hull;
}
```

## 5.11 Convex Trick

```
struct Convex {
    int n;
    vector<Pt> A, V, L, U;
    Convex(const vector<Pt> &A) : A(A), n(A.size()) { // n >= 3
        auto it = max_element(all(A));
        L.assign(A.begin(), it + 1);
        U.assign(it, A.end()), U.push_back(A[0]);
        rep (i, 0, n) {
            V.push_back(A[(i + 1) % n] - A[i]);
        }
    }
    int inside(Pt p, const vector<Pt> &h, auto f) {
        auto it = lower_bound(all(h), p, f);
        if (it == h.end()) return 0;
        if (it == h.begin()) return p == *it;
        return 1 - sgn(ori(*prev(it), p, *it));
    }
    // 0: out, 1: on, 2: in
    int inside(Pt p) {
        return min(inside(p, L, less{}), inside(p, U, greater{}));
    }
    static bool cmp(Pt a, Pt b) { return sgn(a ^ b) > 0; }
    // A[i] is a far/closer tangent point
    int tangent(Pt v, bool close = true) {
        assert(v != Pt{});
        auto l = V.begin(), r = V.begin() + L.size() - 1;
        if (v < Pt{}) l = r, r = V.end();
        if (close) return (lower_bound(l, r, v, cmp) - V.begin()) % n;
        return (upper_bound(l, r, v, cmp) - V.begin()) % n;
    }
    // closer tangent point
    array<int, 2> tangent2(Pt p) {
        array<int, 2> t{-1, -1};
        if (inside(p) == 2) return t;
        if (auto it = lower_bound(all(L), p); it != L.end() and p == *it) {
            int s = it - L.begin();
            return {(s + 1) % n, (s - 1 + n) % n};
        }
        if (auto it = lower_bound(all(U), p, greater{}); it != U.end() and p == *it) {
            int s = it - U.begin() + L.size() - 1;
            return {(s + 1) % n, (s - 1 + n) % n};
        }
        for (int i = 0; i != t[0]; i = tangent((A[t[0] = i] - p), 0));
        for (int i = 0; i != t[1]; i = tangent((p - A[t[1] = i]), 1));
        return t;
    }
    int find(int l, int r, Line L) {
        if (r < l) r += n;
        int s = PtSide(A[l % n], L);
        return *ranges::partition_point(views::iota(l, r),
            [&](int m) {
```

```
        return PtSide(A[m % n], L) == s;
    }) - 1;
};
// Line A_x A_x+1 intersect with L
vector<int> intersect(Line L) {
    int l = tangent(L.a - L.b), r = tangent(L.b - L.a);
    if (PtSide(A[l], L) * PtSide(A[r], L) >= 0) return {};
    return {find(l, r, L) % n, find(r, l, L) % n};
}
};
```

## 5.12 Half Plane Intersection

```
bool cover(Line L, Line P, Line Q) {
    // for double, i128 => Real
    i128 u = (Q.a - P.a) ^ Q.dir();
    i128 v = P.dir() ^ Q.dir();
    i128 x = P.dir().x * u + (P.a - L.a).x * v;
    i128 y = P.dir().y * u + (P.a - L.a).y * v;
    return sgn(x * L.dir().y - y * L.dir().x) * sgn(v) >= 0;
}
vector<Line> HPI(vector<Line> P) {
    sort(all(P), [&](Line l, Line m) {
        if (argcmp(l.dir(), m.dir()) return true;
        if (argcmp(m.dir(), l.dir()) return false;
        return ori(m.a, m.b, l.a) > 0;
    });
    int n = P.size(), l = 0, r = -1;
    for (int i = 0; i < n; i++) {
        if (i and !argcmp(P[i - 1].dir(), P[i].dir())) continue;
        while (l < r and cover(P[i], P[r - 1], P[r])) r--;
        while (l < r and cover(P[i], P[l], P[l + 1])) l++;
        P[++r] = P[i];
    }
    while (l < r and cover(P[l], P[r - 1], P[r])) r--;
    while (l < r and cover(P[r], P[l], P[l + 1])) l++;
    if (r - l <= 1 or !argcmp(P[l].dir(), P[r].dir()))
        return {}; // empty
    if (cover(P[l + 1], P[l], P[r]))
        return {}; // infinity
    return vector(P.begin() + l, P.begin() + r + 1);
}
```

## 5.13 Minimal Enclosing Circle

```
struct Cir {
    Pt o;
    double r;
    bool inside(Pt p) {
        return sgn(r - abs(p - o)) >= 0;
    }
};
Pt Center(Pt a, Pt b, Pt c) {
    Pt x = (a + b) / 2;
    Pt y = (b + c) / 2;
    return LineInter({x, x + rotate(b - a)}, {y, y + rotate(c - b)});
}
Cir MEC(vector<Pt> P) {
    mt19937 rng(time(0));
    shuffle(all(P), rng);
    Cir C{};
    for (int i = 0; i < P.size(); i++) {
        if (C.inside(P[i])) continue;
        C = {P[i], 0};
        for (int j = 0; j < i; j++) {
            if (C.inside(P[j])) continue;
            C = {(P[i] + P[j]) / 2, abs(P[i] - P[j]) / 2};
            for (int k = 0; k < j; k++) {
                if (C.inside(P[k])) continue;
                C.o = Center(P[i], P[j], P[k]);
                C.r = abs(C.o - P[i]);
            }
        }
    }
    return C;
}
```

## 5.14 Minkowski

```
// P, Q, R(return) are counterclockwise order convex polygon
vector<Pt> Minkowski(vector<Pt> P, vector<Pt> Q) {
    assert(P.size() >= 2 && Q.size() >= 2);
    auto cmp = [&](Pt a, Pt b) {
```



```

    return Pt{a.y, a.x} < Pt{b.y, b.x};
};
auto reorder = [&](auto &R) {
    rotate(R.begin(), min_element(all(R), cmp), R.end());
    R.push_back(R[0]), R.push_back(R[1]);
};
const int n = P.size(), m = Q.size();
reorder(P), reorder(Q);
vector<Pt> R;
for (int i = 0, j = 0, s; i < n || j < m; ) {
    R.push_back(P[i] + Q[j]);
    s = sgn((P[i + 1] - P[i]) ^ (Q[j + 1] - Q[j]));
    if (s >= 0) i++;
    if (s <= 0) j++;
}
return R; // May not be a strict convexhull
}

```

## 5.15 Point In Circumcircle

```

// p[0], p[1], p[2] should be counterclockwise order
int inCC(const array<Pt, 3> &p, Pt a) {
    i128 det = 0;
    for (int i = 0; i < 3; i++)
        det += i128(abs2(p[i]) - abs2(a)) * ori(a, p[(i + 1) % 3],
            p[(i + 2) % 3]);
    return (det > 0) - (det < 0); // in:1, on:0, out:-1
}

```

## 5.16 Tangent Lines of Circle and Point

```

vector<Line> CircleTangent(Cir c, Pt p) {
    vector<Line> z;
    double d = abs(p - c.o);
    if (sgn(d - c.r) == 0) {
        Pt i = rotate(p - c.o);
        z.push_back({p, p + i});
    } else if (d > c.r) {
        double o = acos(c.r / d);
        Pt i = unit(p - c.o);
        Pt j = rotate(i, o) * c.r;
        Pt k = rotate(i, -o) * c.r;
        z.push_back({c.o + j, p});
        z.push_back({c.o + k, p});
    }
    return z;
}

```

## 5.17 Tangent Lines of Circles

```

vector<Line> CircleTangent(Cir c1, Cir c2, int sign1) {
    // sign1 = 1 for outer tang, -1 for inner tang
    vector<Line> ret;
    double d_sq = abs2(c1.o - c2.o);
    if (sgn(d_sq) == 0) return ret;
    double d = sqrt(d_sq);
    Pt v = (c2.o - c1.o) / d;
    double c = (c1.r - sign1 * c2.r) / d;
    if (c * c > 1) return ret;
    double h = sqrt(max(0.0, 1.0 - c * c));
    for (int sign2 = 1; sign2 >= -1; sign2 -= 2) {
        Pt n = Pt(v.x * c - sign2 * h * v.y, v.y * c + sign2 * h *
            v.x);
        Pt p1 = c1.o + n * c1.r;
        Pt p2 = c2.o + n * (c2.r * sign1);
        if (sgn(p1.x - p2.x) == 0 && sgn(p1.y - p2.y) == 0)
            p2 = p1 + rotate(c2.o - c1.o);
        ret.push_back({p1, p2});
    }
    return ret;
}

```

## 5.18 Triangle Center

```

Pt TriangleCircumCenter(Pt a, Pt b, Pt c) {
    Pt res;
    double a1 = atan2(b.y - a.y, b.x - a.x) + pi / 2;
    double a2 = atan2(c.y - b.y, c.x - b.x) + pi / 2;
    double ax = (a.x + b.x) / 2;
    double ay = (a.y + b.y) / 2;
    double bx = (c.x + b.x) / 2;
    double by = (c.y + b.y) / 2;
    double r1 = (sin(a2) * (ax - bx) + cos(a2) * (by - ay)) / (sin
        (a1) * cos(a2) - sin(a2) * cos(a1));
    return Pt(ax + r1 * cos(a1), ay + r1 * sin(a1));
}

```

```

Pt TriangleMassCenter(Pt a, Pt b, Pt c) {
    return (a + b + c) / 3.0;
}
Pt TriangleOrthoCenter(Pt a, Pt b, Pt c) {
    return TriangleMassCenter(a, b, c) * 3.0 -
        TriangleCircumCenter(a, b, c) * 2.0;
}
Pt TriangleInnerCenter(Pt a, Pt b, Pt c) {
    Pt res;
    double la = abs(b - c);
    double lb = abs(a - c);
    double lc = abs(a - b);
    res.x = (la * a.x + lb * b.x + lc * c.x) / (la + lb + lc);
    res.y = (la * a.y + lb * b.y + lc * c.y) / (la + lb + lc);
    return res;
}

```

## 5.19 Union of Circles

```

// Area[i] : area covered by at least i circle
vector<double> CircleUnion(const vector<Cir> &C) {
    const int n = C.size();
    vector<double> Area(n + 1);
    auto check = [&](int i, int j) {
        if (!contain(C[i], C[j]))
            return false;
        return sgn(C[i].r - C[j].r) > 0 or (sgn(C[i].r - C[j].r) ==
            0 and i < j);
    };
    struct Teve {
        double ang; int add; Pt p;
        bool operator<(const Teve &b) { return ang < b.ang; }
    };
    auto ang = [&](Pt p) { return atan2(p.y, p.x); };
    for (int i = 0; i < n; i++) {
        int cov = 1;
        vector<Teve> event;
        for (int j = 0; j < n; j++) if (i != j) {
            if (check(j, i)) cov++;
            else if (!check(i, j) and !disjunct(C[i], C[j])) {
                auto I = CircleInter(C[i], C[j]);
                assert(I.size() == 2);
                double a1 = ang(I[0] - C[i].o), a2 = ang(I[1] - C[i].o);
                event.push_back({a1, 1, I[0]});
                event.push_back({a2, -1, I[1]});
                if (a1 > a2) cov++;
            }
        }
        if (event.empty()) {
            Area[cov] += pi * C[i].r * C[i].r;
            continue;
        }
        sort(all(event));
        event.push_back(event[0]);
        for (int j = 0; j + 1 < event.size(); j++) {
            cov += event[j].add;
            Area[cov] += (event[j].p ^ event[j + 1].p) / 2.;
            double theta = event[j + 1].ang - event[j].ang;
            if (theta < 0) theta += 2 * pi;
            Area[cov] += (theta - sin(theta)) * C[i].r * C[i].r / 2.;
        }
    }
    return Area;
}

```

## 6 Graph

### 6.1 Block Cut Tree

```

struct BlockCutTree {
    int n;
    vector<vector<int>> adj;
    BlockCutTree(int _n) : n(_n), adj(_n) {}
    void addEdge(int u, int v) {
        adj[u].push_back(v);
        adj[v].push_back(u);
    }
    pair<int, vector<pair<int, int>>> work() {
        vector<int> dfn(n, -1), low(n), stk;
        vector<pair<int, int>> edg;
        int cnt = 0, cur = 0;
        function<void(int)> dfs = [&](int x) {
            stk.push_back(x);
            dfn[x] = low[x] = cur++;

```

```

    for (auto y : adj[x]) {
        if (dfn[y] == -1) {
            dfs(y);
            low[x] = min(low[x], low[y]);
            if (low[y] == dfn[x]) {
                int v;
                do {
                    v = stk.back();
                    stk.pop_back();
                    edg.emplace_back(n + cnt, v);
                } while (v != y);
                edg.emplace_back(x, n + cnt);
                cnt++;
            }
        } else {
            low[x] = min(low[x], dfn[y]);
        }
    }
};

for (int i = 0; i < n; i++) {
    if (dfn[i] == -1) {
        stk.clear();
        dfs(i);
    }
}

return {cnt, edg};
};

```

## 6.2 Count Cycles

```

// ord = sort by deg decreasing, rk[ord[i]] = i
// D: undirected to directed edge from rk small to rk big
vector<int> vis(n, 0);
int c3 = 0, c4 = 0;
for (int x : ord) { // c3
    for (int y : D[x]) vis[y] = 1;
    for (int y : D[x]) for (int z : D[y]) c3 += vis[z];
    for (int y : D[x]) vis[y] = 0;
}

for (int x : ord) { // c4
    for (int y : D[x]) for (int z : adj[y])
        if (rk[z] > rk[x]) c4 += vis[z]++;
    for (int y : D[x]) for (int z : adj[y])
        if (rk[z] > rk[x]) --vis[z];
}

```

## 6.3 Dominator Tree

```

vector<int> BuildDomTree(vector<vector<int>> adj, int rt) {
    int n = adj.size();

    // buckets: list of vertices y with sdom(y) = x
    vector<vector<int>> buckets(n, radj(n));

    // rev[dfn[x]] = x
    vector<int> dfn(n, -1), rev(n, -1), pa(n, -1);
    vector<int> sdom(n, -1), dom(n, -1);
    vector<int> fa(n, -1), val(n, -1);

    int stamp = 0;

    // re-number in DFS order
    auto dfs = [&](auto self, int u) -> void {
        rev[dfn[u] = stamp] = u;
        fa[stamp] = sdom[stamp] = val[stamp] = stamp;
        stamp++;
        for (int v : adj[u]) {
            if (dfn[v] == -1) {
                self(self, v);
                pa[dfn[v]] = dfn[u];
            }
            radj[dfn[v]].pb(dfn[u]);
        }
    };

    function<int(int, bool)> Eval = [&](int x, bool fir) {
        if (x == fa[x]) return fir ? x : -1;
        int p = Eval(fa[x], false);
        // x is one step away from the root
        if (p == -1) return x;
        if (sdom[val[x]] > sdom[val[fa[x]]]) val[x] = val[fa[x]];
        fa[x] = p;
        return fir ? val[x] : p;
    };
};

```

```

auto Link = [&](int x, int y) -> void { fa[x] = y; };

dfs(dfs, rt);

// compute sdom in reversed DFS order
for (int x = stamp - 1; x >= 0; --x) {
    for (int y : radj[x]) {
        // sdom[x] = min({y | (y, x) in E(G), y < x}, {sdom[z] |
        // (y, x) in E(G), z > x && z is y's ancestor})
        chmin(sdom[x], sdom[Eval(y, true)]);
    }
    if (x > 0) buckets[sdom[x]].pb(x);
    for (int u : buckets[x]) {
        int p = Eval(u, true);
        if (sdom[p] == x) dom[u] = x;
        else dom[u] = p;
    }
    if (x > 0) Link(x, pa[x]);
}

// idom[x] = -1 if x is unreachable from rt
vector<int> idom(n, -1);
idom[rt] = rt;
rep(x, 1, stamp) {
    if (sdom[x] != dom[x]) dom[x] = dom[dom[x]];
}
rep(i, 1, stamp) idom[rev[i]] = rev[dom[i]];
return idom;
};

```

## 6.4 Enumerate Planar Face

```

// 0-based
struct PlanarGraph {
    int n, m, id;
    vector<Pt<int>> v;
    vector<vector<pair<int, int>>> adj;
    vector<int> conv, nxt, vis;

    PlanarGraph(int n, int m, vector<Pt<int>> _v) :
        n(n), m(m), id(0),
        v(_v), adj(n),
        conv(m << 1), nxt(m << 1), vis(m << 1) {}

    void add_edge(int x, int y) {
        adj[x].push_back({y, id << 1});
        adj[y].push_back({x, id << 1 | 1});
        conv[id << 1] = x;
        conv[id << 1 | 1] = y;
        id++;
    }

    vector<int> enumerate_face() {
        for (int i = 0; i < n; i++) {
            sort(all(adj[i]), [&](const auto &a, const auto &b) {
                return (v[a.first] - v[i]) < (v[b.first] - v[i]);
            });
            int sz = adj[i].size(), pre = sz - 1;
            for (int j = 0; j < sz; j++) {
                nxt[adj[i][pre].second] = adj[i][j].second ^ 1;
                pre = j;
            }
        }

        vector<int> ret;
        for (int i = 0; i < m * 2; i++) {
            if (!vis[i]) {
                int area = 0, now = i;
                vector<int> pt;
                while (!vis[now]) {
                    vis[now] = true;
                    pt.push_back(conv[now]);
                    now = nxt[now];
                }
                pt.push_back(pt.front());
                for (int i = 0; i + 1 < ssize(pt); i++) {
                    area -= (v[pt[i]] ^ v[pt[i + 1]]);
                }
                // pt = face boundary
                if (area > 0) {
                    ret.push_back(area);
                } else {
                    // pt is outer face
                }
            }
        }
    }
};

```

```

    }
    return ret;
}
};

```

## 6.5 Manhattan MST

```

// {w, u, v}
vector<tuple<int, int, int>> ManhattanMST(vector<Pt> P) {
    vector<int> id(P.size());
    iota(all(id), 0);
    vector<tuple<int, int, int>> edg;
    for (int k = 0; k < 4; k++) {
        sort(all(id), [&](int i, int j) {
            return (P[i] - P[j]).ff < (P[j] - P[i]).ss;
        });
        map<int, int> sweep;
        for (int i : id) {
            auto it = sweep.lower_bound(-P[i].ss);
            while (it != sweep.end()) {
                int j = it->ss;
                Pt d = P[i] - P[j];
                if (d.ss > d.ff) {
                    break;
                }
                edg.emplace_back(d.ff + d.ss, i, j);
                it = sweep.erase(it);
            }
            sweep[-P[i].ss] = i;
        }
        for (Pt &p : P) {
            if (k % 2) {
                p.ff = -p.ff;
            } else {
                swap(p.ff, p.ss);
            }
        }
    }
    return edg;
}

```

## 6.6 Matroid Intersection

```

/*
M1 = xx matroid, M2 = xx matroid
y<-s if I+y satisfies M1
y->t if I+y satisfies M2
x<-y if I-x+y satisfies M2
x->y if I-x+y satisfies M1
交換圖點權：
-w[e] if e \in I
w[e] otherwise
*/
vector<int> I(, 0);
while (true) {
    vector<vector<int>> adj();
    int s = , t = s + 1;
    auto M1 = [&]() -> void { // xx matroid
        { // y<-s
        }
        { // x->y
        }
    };
    auto M2 = [&]() -> void { // xx matroid
        { // y->t
        }
        { // x<-y
        }
    };
    auto augment = [&]() -> bool { // 註解掉的是帶權版
        vector<int> vis(+ 2, 0), dis(+ 2, IINF), from(+ 2, -1);
        queue<int> q;
        vis[s] = 1;
        dis[s] = 0;
        q.push(s);
        while (!q.empty()) {
            int u = q.front(); q.pop();
            // vis[u] = 0;
            for (int v : adj[u]) {
                int w = ; // no weight -> 1
                if (chmin(dis[v], dis[u] + w)) {
                    from[v] = u;

```

```

                // if (!vis[v]) {
                //     vis[v] = 1;
                //     q.push(v);
                // }
            }
        }
        if (from[t] == -1) return false;
        for (int cur = from[t]; cur = from[cur]) {
            if (cur == -1 || cur == s) break;
            I[cur] ^= 1;
        }
        return true;
    };
    M1(), M2();
    if (!augment()) break;
}

```

## 6.7 Maximum Clique

```

constexpr size_t kN = 150;
using bits = bitset<kN>;
struct MaxClique {
    bits G[kN], cs[kN];
    int ans, sol[kN], q, cur[kN], d[kN], n;
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) G[i].reset();
    }
    void addEdge(int u, int v) {
        G[u][v] = G[v][u] = 1;
    }
    void preDfs(vector<int> &v, int i, bits mask) {
        if (i < 4) {
            for (int x : v) d[x] = (G[x] & mask).count();
            sort(all(v), [&](int x, int y) {
                return d[x] > d[y];
            });
        }
        vector<int> c(v.size());
        cs[1].reset(), cs[2].reset();
        int l = max(ans - q + 1, 1), r = 2, tp = 0, k;
        for (int p : v) {
            for (k = 1;
                (cs[k] & G[p]).any(); ++k);
            if (k >= r) cs[++r].reset();
            cs[k][p] = 1;
            if (k < l) v[tp++] = p;
        }
        for (k = l; k < r; ++k)
            for (auto p = cs[k]._Find_first(); p < kN; p = cs[k]._Find_next(p))
                v[tp] = p, c[tp] = k, ++tp;
        dfs(v, c, i + 1, mask);
    }
    void dfs(vector<int> &v, vector<int> &c, int i, bits mask) {
        while (!v.empty()) {
            int p = v.back();
            v.pop_back();
            mask[p] = 0;
            if (q + c.back() <= ans) return;
            cur[q++] = p;
            vector<int> nr;
            for (int x : v)
                if (G[p][x]) nr.push_back(x);
            if (!nr.empty()) preDfs(nr, i, mask & G[p]);
            else if (q > ans) ans = q, copy_n(cur, q, sol);
            c.pop_back();
            --q;
        }
    }
    int solve() {
        vector<int> v(n);
        iota(all(v), 0);
        ans = q = 0;
        preDfs(v, 0, bits(string(n, '1')));
        return ans;
    }
} cliq;

```

## 6.8 Tree Hash

```

map<vector<int>, int> id;
vector<vector<int>> sub;
vector<int> siz;

```

```

int getid(const vector<int> &T) {
    if (id.count(T)) return id[T];
    int s = 1;
    for (int x : T) {
        s += siz[x];
    }
    sub.push_back(T);
    siz.push_back(s);
    return id[T] = id.size();
}

int dfs(int u, int f) {
    vector<int> S;
    for (int v : G[u]) if (v != f) {
        S.push_back(dfs(v, u));
    }
    sort(all(S));
    return getid(S);
}

```

## 6.9 Two-SAT

```

struct TwoSat {
    int n;
    vector<vector<int>> G;
    vector<bool> ans;
    vector<int> id, dfn, low, stk;
    TwoSat(int n) : n(n), G(2 * n) {}
    void addClause(int u, bool f, int v, bool g) { // (u = f) or
        (v = g)
        G[2 * u + !f].push_back(2 * v + g);
        G[2 * v + !g].push_back(2 * u + f);
    }
    void addImply(int u, bool f, int v, bool g) { // (u = f) -> (
        v = g)
        G[2 * u + f].push_back(2 * v + g);
        G[2 * v + !g].push_back(2 * u + !f);
    }
    int addVar() {
        G.emplace_back();
        G.emplace_back();
        return n++;
    }
    void addAtMostOne(const vector<pair<int, bool>> &li) {
        if (ssize(li) <= 1) return;
        int pu; bool pf; tie(pu, pf) = li[0];
        for (int i = 2; i < ssize(li); i++) {
            const auto &[u, f] = li[i];
            int nxt = addVar();
            addClause(pu, !pf, u, !f);
            addClause(pu, !pf, nxt, true);
            addClause(u, !f, nxt, true);
            tie(pu, pf) = make_pair(nxt, true);
        }
        addClause(pu, !pf, li[1].first, !li[1].second);
    }
    int cur = 0, scc = 0;
    void dfs(int u) {
        stk.push_back(u);
        dfn[u] = low[u] = cur++;
        for (int v : G[u]) {
            if (dfn[v] == -1) {
                dfs(v);
                chmin(low[u], low[v]);
            } else if (id[v] == -1) {
                chmin(low[u], dfn[v]);
            }
        }
        if (dfn[u] == low[u]) {
            int x;
            do {
                x = stk.back();
                stk.pop_back();
                id[x] = scc;
            } while (x != u);
            scc++;
        }
    }
    bool satisfiable() {
        ans.assign(n, 0);
        id.assign(2 * n, -1);
        dfn.assign(2 * n, -1);
        low.assign(2 * n, -1);
        for (int i = 0; i < n * 2; i++)
            if (dfn[i] == -1) {

```

```

                dfs(i);
            }
            for (int i = 0; i < n; ++i) {
                if (id[2 * i] == id[2 * i + 1]) {
                    return false;
                }
            }
            ans[i] = id[2 * i] > id[2 * i + 1];
        }
        return true;
    }
};

```

## 6.10 Virtual Tree

```

// need LCA
vector<vector<int>> vir(n);
auto clear = [&](auto self, int u) -> void {
    for (int v : vir[u]) self(self, v);
    vir[u].clear();
};
auto build = [&](vector<int> &v) -> void { // be careful of the
    changes to the array
    // maybe dont need to sort when do it while dfs
    sort(all(v), [&](int a, int b) {
        return dfn[a] < dfn[b];
    });
    clear(clear, 0);
    if (v[0] != 0) v.insert(v.begin(), 0);
    int k = v.size();
    vector<int> st;
    rep(i, 0, k) {
        if (st.empty()) {
            st.push_back(v[i]);
            continue;
        }
        int p = lca(v[i], st.back());
        if (p == st.back()) {
            st.push_back(v[i]);
            continue;
        }
        while (st.size() >= 2 && dep[st.end()[-2]] >= dep[p]) {
            vir[st.end()[-2]].push_back(st.back());
            st.pop_back();
        }
        if (st.back() != p) {
            vir[p].push_back(st.back());
            st.pop_back();
            st.push_back(p);
        }
        st.push_back(v[i]);
    }
    while (st.size() >= 2) {
        vir[st.end()[-2]].push_back(st.back());
        st.pop_back();
    }
};

```

## 7 Math

### 7.1 Combinatoric

```

vector<mint> fac, inv;

inline void init(int n) {
    fac.resize(n + 1);
    inv.resize(n + 1);
    fac[0] = inv[0] = 1;
    rep(i, 1, n + 1) fac[i] = fac[i - 1] * i;
    inv[n] = fac[n].inv();
    for (int i = n; i > 0; --i) inv[i - 1] = inv[i] * i;
}

inline mint Comb(int n, int k) {
    if (k > n || k < 0) return 0;
    return fac[n] * inv[k] * inv[n - k];
}

inline mint H(int n, int m) {
    return Comb(n + m - 1, m);
}

inline mint catalan(int n) {
    return fac[2 * n] * inv[n + 1] * inv[n];
}

```

## 7.2 Discrete Log

```
int power(int a, int b, int p, int res = 1) {
    for (; b; b /= 2, a = 1LL * a * a % p) {
        if (b & 1) {
            res = 1LL * res * a % p;
        }
    }
    return res;
}

int exbsgs(int a, int b, int p) {
    a %= p;
    b %= p;
    if (b == 1 || p == 1) {
        return 0;
    }
    if (a == 0) {
        return b == 0 ? 1 : -1;
    }

    i64 g, k = 0, t = 1; // t : a ^ k / sum{d}
    while ((g = std::gcd(a, p)) > 1) {
        if (b % g) {
            return -1;
        }
        b /= g;
        p /= g;
        k++;
        t = t * (a / g) % p;
        if (t == b) {
            return k;
        }
    }

    const int n = std::sqrt(p) + 1;
    std::unordered_map<int, int> mp;
    mp[b] = 0;

    int x = b, y = t;
    int mi = power(a, n, p);
    for (int i = 1; i < n; i++) {
        x = 1LL * x * a % p;
        mp[x] = i;
    }

    for (int i = 1; i <= n; i++) {
        t = 1LL * t * mi % p;
        if (mp.contains(t)) {
            return 1LL * i * n - mp[t] + k;
        }
    }

    return -1; // no solution
}
```

## 7.3 Div Floor Ceil

```
int CEIL(int a, int b) {
    return (a >= 0 ? (a + b - 1) / b : a / b);
}

int FLOOR(int a, int b) {
    return (a >= 0 ? a / b : (a - b + 1) / b);
}
```

## 7.4 exCRT

```
i64 exgcd(i64 a, i64 b, i64 &x, i64 &y) {
    if (b == 0) {
        x = 1;
        y = 0;
        return a;
    }
    i64 g = exgcd(b, a % b, y, x);
    y -= a / b * x;
    return g;
}

// return {x, T}
// a: moduli, b: remainders
// x: first non-negative solution, T: minimum period
std::pair<i64, i64> exCRT(auto &a, auto &b) {
    auto [m1, r1] = std::tie(a[0], b[0]);
    for (int i = 1; i < std::ssize(a); i++) {
        auto [m2, r2] = std::tie(a[i], b[i]);
        i64 x, y;
        i64 g = exgcd(m1, m2, x, y);

```

```
        if ((r2 - r1) % g) { // no solution
            return {-1, -1};
        }
        x = (i128(x) * (r2 - r1) / g) % (m2 / g);
        if (x < 0) {
            x += (m2 / g);
        }
        r1 = m1 * x + r1;
        m1 = std::lcm(m1, m2);
    }
    r1 %= m1;
    if (r1 < 0) {
        r1 += m1;
    }
    return {r1, m1};
};
```

## 7.5 Factorization

```
ull modmul(ull a, ull b, ull M) {
    i64 ret = a * b - M * ull(1.L / M * a * b);
    return ret + M * (ret < 0) - M * (ret >= (i64)M);
}

ull modpow(ull b, ull e, ull mod) {
    ull ans = 1;
    for (; e; b = modmul(b, b, mod), e /= 2) {
        if (e & 1) ans = modmul(ans, b, mod);
    }
    return ans;
}

bool isPrime(ull n) {
    if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
    ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
        s = __builtin_ctzll(n - 1), d = n >> s;
    for (ull a : A) {
        ull p = modpow(a % n, d, n), i = s;
        while (p != 1 && p != n - 1 && a % n && i--) {
            p = modmul(p, p, n);
            if (p != n - 1 && i != s) return 0;
        }
        return 1;
    }
}

ull pollard(ull n) {
    uniform_int_distribution<ull> unif(0, n - 1);
    ull c = 1;
    auto f = [n, &c](ull x) { return modmul(x, x, n) + c % n; };
    ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
    while (t++ % 40 || __gcd(prd, n) == 1) {
        if (x == y) c = unif(rng), x = ++i, y = f(x);
        if ((q = modmul(prd, max(x, y) - min(x, y), n))) prd = q;
        x = f(x), y = f(f(y));
    }
    return __gcd(prd, n);
}

vector<ull> factor(ull n) {
    if (n == 1) return {};
    if (isPrime(n)) return {n};
    ull x = pollard(n);
    auto l = factor(x), r = factor(n / x);
    l.insert(l.end(), r.begin(), r.end());
    return l;
}
```

## 7.6 Floor Sum

```
// \sum_0^n floor((a * x + b) / c) in log(n + m + a + b)
int floor_sum(int a, int b, int c, int n) { // add mod if needed
    int m = (a * n + b) / c;
    if (a >= c || b >= c) {
        return (a / c) * (n * (n + 1) / 2) + (b / c) * (n + 1) +
            floor_sum(a % c, b % c, c, n);
    }
    if (n < 0 || a == 0) {
        return 0;
    }
    return n * m - floor_sum(c, c - b - 1, a, m - 1);
}
```

## 7.7 FWT

```
void fwt(vector<ll> &f, bool inv = false) { // xor-convolution
    const int N = 31 - __builtin_clz(ssize(f)),
        inv2 = (MOD + 1) / 2;
    rep(i, 0, N) rep(j, 0, 1 << N) {
        if (j >> i & 1 ^ 1) {

```

```

    ll a = f[j], b = f[j | (1 << i)];
    if (inv) {
        f[j] = (a + b) * inv2 % MOD;
        f[j | (1 << i)] = (a - b + MOD) * inv2 % MOD;
    } else {
        f[j] = (a + b) % MOD;
        f[j | (1 << i)] = (a - b + MOD) % MOD;
    }
}
}
}

```

## 7.8 Gauss Elimination

```

using Z = ModInt<998244353>;
// using F = long double;

using Matrix = std::vector<std::vector<Z>>>;
// using Matrix = std::vector<std::vector<F>>>; (double)
// using Matrix = std::vector<std::bitset<5000>>>; (mod 2)

template <typename T>
auto gauss(Matrix &A, std::vector<T> &b, int n, int m) {
    assert(std::ssize(b) == n);
    int r = 0;
    std::vector<int> where(m, -1);

    for (int i = 0; i < m && r < n; i++) {
        int p = r; // pivot
        while (p < n && A[p][i] == T(0)) {
            p++;
        }
        if (p == n) {
            continue;
        }
        std::swap(A[r], A[p]);
        std::swap(b[r], b[p]);
        where[i] = r;

        // coef: mod 2 don't need this
        T inv = T(1) / A[r][i];
        for (int j = i; j < m; j++) {
            A[r][j] *= inv;
        }
        b[r] *= inv;

        for (int j = 0; j < n; j++) { // deduct: mod 2 don't need this
            if (j != r) {
                T x = A[j][i];
                for (int k = i; k < m; k++) {
                    A[j][k] -= x * A[r][k];
                }
                b[j] -= x * b[r];
            }
        }

        // for (int j = 0; j < n; ++j) { // (mod 2) -> coef and deduct
        //     if (j != r && A[j][i]) {
        //         A[j] ^= A[r];
        //         b[j] ^= b[r];
        //     }
        // }

        r++;
    }

    for (int i = r; i < n; i++) {
        if (ranges::all_of(A[i] | views::take(m), [](auto x) {
            return x == 0; }) && b[i] != T(0)) {
            return std::vector<T>(); // no solution
        }
        // if (A[i].none() && b[i]) { // (mod 2)
        //     return std::vector<T>();
        // }
    }

    // if (r < m) { // infinite solution
    //     return std::vector<T>();
    // }

    std::vector<T> res(m);
    for (int i = 0; i < m; i++) {
        if (where[i] != -1) {

```

```

            res[i] = b[where[i]];
        }
    }
    return res;
};

```

## 7.9 Lagrange Interpolation

```

struct Lagrange {
    int deg{};
    vector<int> C;
    Lagrange(const vector<int> &P) {
        deg = P.size() - 1;
        C.assign(deg + 1, 0);
        for (int i = 0; i <= deg; i++) {
            int q = inv[i] * inv[i - deg] % mod;
            if ((deg - i) % 2 == 1) {
                q = mod - q;
            }
            C[i] = P[i] * q % mod;
        }
    }
    int operator()(int x) { // 0 <= x < mod
        if (0 <= x and x <= deg) {
            int ans = fac[x] * fac[deg - x] % mod;
            if ((deg - x) % 2 == 1) {
                ans = (mod - ans);
            }
            return ans * C[x] % mod;
        }
        vector<int> pre(deg + 1), suf(deg + 1);
        for (int i = 0; i <= deg; i++) {
            pre[i] = (x - i);
            if (i) {
                pre[i] = pre[i] * pre[i - 1] % mod;
            }
        }
        for (int i = deg; i >= 0; i--) {
            suf[i] = (x - i);
            if (i < deg) {
                suf[i] = suf[i] * suf[i + 1] % mod;
            }
        }
        int ans = 0;
        for (int i = 0; i <= deg; i++) {
            ans += (i == 0 ? 1 : pre[i - 1]) * (i == deg ? 1 : suf[i + 1]) % mod * C[i];
            ans %= mod;
        }
        if (ans < 0) ans += mod;
        return ans;
    }
};

```

## 7.10 Linear Sieve

```

const int C = 1e6 + 5;

int mo[C], lp[C], phi[C], isp[C];
vector<int> prime;

void sieve() {
    mo[1] = phi[1] = 1;
    rep(i, 1, C) lp[i] = 1;
    rep(i, 2, C) {
        if (lp[i] == 1) {
            lp[i] = i;
            prime.pb(i);
            isp[i] = 1;
            mo[i] = -1;
            phi[i] = i - 1;
        }
        for (int p : prime) {
            if (i * p >= C) break;
            lp[i * p] = p;
            if (i % p == 0) {
                phi[p * i] = phi[i] * p;
                break;
            }
            phi[i * p] = phi[i] * (p - 1);
            mo[i * p] = mo[i] * mo[p];
        }
    }
}

```



## 7.11 Lucas

```
// comb(n, m) % M, M = p^k
// O(M)-O(log(n))
struct Lucas {
    const int p, M;
    vector<int> f;
    Lucas(int p, int M) : p(p), M(M), f(M + 1) {
        f[0] = 1;
        for (int i = 1; i <= M; i++) {
            f[i] = f[i - 1] * (i % p == 0 ? 1 : i) % M;
        }
    }
    int CountFact(int n) {
        int c = 0;
        while (n) c += (n /= p);
        return c;
    }
    // (n! without factor p) % p^k
    int ModFact(int n) {
        int r = 1;
        while (n) {
            r = r * power(f[M], n / M % 2, M) % M * f[n % M] % M;
            n /= p;
        }
        return r;
    }
    int ModComb(int n, int m) {
        if (m < 0 or n < m) return 0;
        int c = CountFact(n) - CountFact(m) - CountFact(n - m);
        int r = ModFact(n) * power(ModFact(m), M / p * (p - 1) - 1,
            M) % M
            * power(ModFact(n - m), M / p * (p - 1) - 1, M) %
            M;
        return r * power(p, c, M) % M;
    }
};
```

## 7.12 Mod Int

```
using u32 = unsigned int;
using u64 = unsigned long long;
template <class T>
constexpr T power(T a, u64 b, T res = 1) {
    for (; b != 0; b /= 2, a *= a) {
        if (b & 1) {
            res *= a;
        }
    }
    return res;
}

template <u32 P>
struct ModInt {
    u32 v;
    const static ModInt G;

    constexpr ModInt &norm(u32 x) {
        v = x < P ? x : x - P;
        return *this;
    }

    constexpr ModInt(i64 x = 0) { norm(x % P + P); }
    constexpr ModInt inv() const { return power(*this, P - 2); }
    constexpr ModInt operator-() const { return ModInt() - *this; }
    }

    constexpr ModInt operator+(const ModInt &r) const { return
        ModInt().norm(v + r.v); }
    constexpr ModInt operator-(const ModInt &r) const { return
        ModInt().norm(v + P - r.v); }
    constexpr ModInt operator*(const ModInt &r) const { return
        ModInt().norm(u64(v) * r.v % P); }
    constexpr ModInt operator/(const ModInt &r) const { return *
        this * r.inv(); }
    constexpr ModInt operator+=(const ModInt &r) { return *this
        = *this + r; }
    constexpr ModInt operator-=(const ModInt &r) { return *this
        = *this - r; }
    constexpr ModInt operator*=(const ModInt &r) { return *this
        = *this * r; }
    constexpr ModInt operator/=(const ModInt &r) { return *this
        = *this / r; }
    constexpr bool operator==(const ModInt &r) const { return v
        == r.v; }
    constexpr bool operator!=(const ModInt &r) const { return v
        != r.v; }
    explicit constexpr operator bool() const { return v != 0; }
```

```
friend std::ostream &operator<<(std::ostream &os, const
    ModInt &r) {
    return os << r.v;
}
};
using mint = ModInt<998244353>;
template <> const mint mint::G = mint(3);
```

## 7.13 Primitive Root

```
ull primitiveRoot(ull p) {
    auto fac = factor(p - 1);
    sort(all(fac));
    fac.erase(unique(all(fac)), fac.end());
    auto test = [p, fac](ull x) {
        for(ull d : fac)
            if (modpow(x, (p - 1) / d, p) == 1)
                return false;
        return true;
    };
    uniform_int_distribution<ull> unif(1, p - 1);
    ull root;
    while(!test(root = unif(rng)));
    return root;
}
```

## 7.14 Simplex

```
// max{cx} subject to {Ax<=b, x>=0}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
// usage :
// x = simplex(A, b, c); (A <= 100 x 100)
vector<double> simplex(
    const vector<vector<double>> &a,
    const vector<double> &b,
    const vector<double> &c) {

    int n = (int)a.size(), m = (int)a[0].size() + 1;
    vector val(n + 2, vector<double>(m + 1));
    vector<int> idx(n + m);
    iota(all(idx), 0);
    int r = n, s = m - 1;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m - 1; ++j)
            val[i][j] = -a[i][j];
        val[i][m - 1] = 1;
        val[i][m] = b[i];
        if (val[r][m] > val[i][m])
            r = i;
    }
    copy(all(c), val[n].begin());
    val[n + 1][m - 1] = -1;
    for (double num; ; ) {
        if (r < n) {
            swap(idx[s], idx[r + m]);
            val[r][s] = 1 / val[r][s];
            for (int j = 0; j <= m; ++j) if (j != s)
                val[r][j] *= -val[r][s];
            for (int i = 0; i <= n + 1; ++i) if (i != r) {
                for (int j = 0; j <= m; ++j) if (j != s)
                    val[i][j] += val[r][j] * val[i][s];
                val[i][s] *= val[r][s];
            }
        }
        r = s = -1;
        for (int j = 0; j < m; ++j)
            if (s < 0 || idx[s] > idx[j])
                if (val[n + 1][j] > eps || val[n + 1][j] > -eps && val[
                    n][j] > eps)
                    s = j;
        if (s < 0) break;
        for (int i = 0; i < n; ++i) if (val[i][s] < -eps) {
            if (r < 0
                || (num = val[r][m] / val[r][s] - val[i][m] / val[i][s]
                    ) < -eps
                || num < eps && idx[r + m] > idx[i + m])
                r = i;
        }
        if (r < 0) {
            // Solution is unbounded.
            return vector<double>{};
        }
        if (val[n + 1][m] < -eps) {
            // No solution.
            return vector<double>{};
        }
    }
}
```

```

    return vector<double>{};
}
vector<double> x(m - 1);
for (int i = m; i < n + m; ++i)
    if (idx[i] < m - 1)
        x[idx[i]] = val[i - m][m];
return x;
}

```

## 7.15 Sqrt Mod

```

// the Jacobi symbol is a generalization of the Legendre symbol
// such that the bottom doesn't need to be prime.
// (n|p) -> same as legendre
// (n|ab) = (n|a)(n|b)
// work with long long
int Jacobi(int a, int m) {
    int s = 1;
    for (; m > 1; ) {
        a %= m;
        if (a == 0) return 0;
        const int r = __builtin_ctz(a);
        if ((r & 1) && ((m + 2) & 4)) s = -s;
        a >>= r;
        if (a & m & 2) s = -s;
        swap(a, m);
    }
    return s;
}

// 0: a == 0
// -1: a isn't a quad res of p
// else: return X with X^2 % p == a
// doesn't work with long long
int QuadraticResidue(int a, int p) {
    if (p == 2) return a & 1;
    if (int jc = Jacobi(a, p); jc <= 0) return jc;
    int b, d;
    for (; ; ) {
        b = rand() % p;
        d = (1LL * b * b + p - a) % p;
        if (Jacobi(d, p) == -1) break;
    }
    int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
    for (int e = (1LL + p) >> 1; e; e >>= 1) {
        if (e & 1) {
            tmp = (1LL * g0 * f0 + 1LL * d * (1LL * g1 * f1 % p)) % p;
            ;
            g1 = (1LL * g0 * f1 + 1LL * g1 * f0) % p;
            g0 = tmp;
        }
        tmp = (1LL * f0 * f0 + 1LL * d * (1LL * f1 * f1 % p)) % p;
        f1 = (2LL * f0 * f1) % p;
        f0 = tmp;
    }
    return g0;
}

```

## 7.16 PiCount

```

i64 PrimeCount(i64 n) { // n ~ 10^13 => < 2s
    if (n <= 1) return 0;
    int v = sqrt(n), s = (v + 1) / 2, pc = 0;
    vector<int> smalls(v + 1), skip(v + 1), roughs(s);
    vector<i64> larges(s);
    for (int i = 2; i <= v; ++i) smalls[i] = (i + 1) / 2;
    for (int i = 0; i < s; ++i) {
        roughs[i] = 2 * i + 1;
        larges[i] = (n / (2 * i + 1) + 1) / 2;
    }
    for (int p = 3; p <= v; ++p) {
        if (smalls[p] > smalls[p - 1]) {
            int q = p * p;
            ++pc;
            if (1LL * q * q > n) break;
            skip[p] = 1;
            for (int i = q; i <= v; i += 2 * p) skip[i] = 1;
            int ns = 0;
            for (int k = 0; k < s; ++k) {
                int i = roughs[k];
                if (skip[i]) continue;
                i64 d = 1LL * i * p;
                larges[ns] = larges[k] - (d <= v ? larges[smalls[d] - pc] : smalls[n / d]) + pc;
                roughs[ns++] = i;
            }
        }
    }
}

```

```

}
s = ns;
for (int j = v / p; j >= p; --j) {
    int c = smalls[j] - pc, e = min(j * p + p, v + 1);
    for (int i = j * p; i < e; ++i) smalls[i] -= c;
}
}
}
for (int k = 1; k < s; ++k) {
    const i64 m = n / roughs[k];
    i64 t = larges[k] - (pc + k - 1);
    for (int l = 1; l < k; ++l) {
        int p = roughs[l];
        if (1LL * p * p > m) break;
        t -= smalls[m / p] - (pc + l - 1);
    }
    larges[0] -= t;
}
return larges[0];
}

```

## 7.17 ModMin

```

// min{k | l <= ((ak) mod m) <= r}, no solution -> -1
int mod_min(int a, int m, int l, int r) {
    if (a == 0) return l ? -1 : 0;
    if (int k = (l + a - 1) / a; k * a <= r)
        return k;
    int b = m / a, c = m % a;
    if (int y = mod_min(c, a, a - r % a, a - l % a))
        return (l + y * c + a - 1) / a + y * b;
    return -1;
}

```

## 7.18 FFT

```

template<typename C = complex<double>>
void FFT(vector<C> &P, C w, bool inv = 0) {
    int n = P.size(), lg = __builtin_ctz(n);
    assert(__builtin_popcount(n) == 1);

    for (int j = 1, i = 0; j < n - 1; ++j) {
        for (int k = n >> 1; k > (i ^ k); k >>= 1);
        if (j < i) swap(P[i], P[j]);
    }

    vector<C> ws = {inv ? C{1} / w : w};

    rep (i, 1, lg) ws.pb(ws[i - 1] * ws[i - 1]);
    reverse(all(ws));

    rep (i, 0, lg) {
        for (int k = 0; k < n; k += 2 << i) {
            C base = C{1};
            rep (j, k, k + (1 << i)) {
                auto t = base * P[j + (1 << i)];
                auto u = P[j];
                P[j] = u + t;
                P[j + (1 << i)] = u - t;
                base = base * ws[i];
            }
        }
    }

    if (inv) rep (i, 0, n) P[i] = P[i] / C(n);
}

```

```

const int N = 1 << 21;
const double PI = acos(-1);
const auto w = exp(-complex<double>(0, 2.0 * PI / N));

```

## 7.19 NTT prime

• P: 7681, Rt: 17	P: 12289, Rt: 11
• P: 40961, Rt: 3	P: 65537, Rt: 3
• P: 786433, Rt: 10	P: 5767169, Rt: 3
• P: 7340033, Rt: 3	P: 23068673, Rt: 3
• P: 469762049, Rt: 3	P: 2061584302081, Rt: 7
• P: 2748779069441, Rt: 3	P: 167772161, Rt: 3
• P: 104857601, Rt: 3	P: 985661441, Rt: 3
• P: 998244353, Rt: 3	P: 1107296257, Rt: 10
• P: 2013265921, Rt: 31	P: 2810183681, Rt: 11
• P: 2885681153, Rt: 3	P: 605028353, Rt: 3
• P: 1945555039024054273, Rt: 5	P: 9223372036737335297, Rt: 3

## 7.20 Polynomial

```
std::mt19937_64 rng(std::chrono::steady_clock::now().
    time_since_epoch().count());

template <class mint>
void nft(bool type, std::vector<mint> &a) {
    int n = int(a.size()), s = 0;
    while ((1 << s) < n) {
        s++;
    }
    assert(1 << s == n);
    static std::vector<mint> ep, iep;
    while (int(ep.size()) <= s) {
        ep.push_back(power(mint::G, mint(-1).v / (1 << int(ep.size())
            )));
        iep.push_back(ep.back().inv());
    }
    std::vector<mint> b(n);
    for (int i = 1; i <= s; i++) {
        int w = 1 << (s - i);
        mint base = type ? iep[i] : ep[i], now = 1;
        for (int y = 0; y < n / 2; y += w) {
            for (int x = 0; x < w; x++) {
                auto l = a[y << 1 | x];
                auto r = now * a[y << 1 | x | w];
                b[y | x] = l + r;
                b[y | x | n >> 1] = l - r;
            }
            now *= base;
        }
        std::swap(a, b);
    }
}

template <class mint>
std::vector<mint> multiply(const std::vector<mint> &a, const
    std::vector<mint> &b) {
    int n = int(a.size()), m = int(b.size());
    if (!n || !m) return {};
    if (std::min(n, m) <= 8) {
        std::vector<mint> ans(n + m - 1);
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < m; j++) {
                ans[i + j] += a[i] * b[j];
            }
        }
        return ans;
    }
    int lg = 0;
    while ((1 << lg) < n + m - 1) {
        lg++;
    }
    int z = 1 << lg;
    auto a2 = a, b2 = b;
    a2.resize(z);
    b2.resize(z);
    nft(false, a2);
    nft(false, b2);
    for (int i = 0; i < z; i++) {
        a2[i] *= b2[i];
    }
    nft(true, a2);
    a2.resize(n + m - 1);
    mint iz = mint(z).inv();
    for (int i = 0; i < n + m - 1; i++) {
        a2[i] *= iz;
    }
    return a2;
}

template <class D>
struct Poly {
    std::vector<D> v;
    Poly(const std::vector<D> &v_) : v(v_) { shrink(); }
    void shrink() {
        while (v.size() > 1 && !v.back()) {
            v.pop_back();
        }
    }
    int size() const { return int(v.size()); }
    D freq(int p) const { return (p < size()) ? v[p] : D(0); }
    Poly operator+(const Poly &r) const {
        auto n = std::max(size(), r.size());
        std::vector<D> res(n);
        for (int i = 0; i < n; i++) {
            res[i] = freq(i) + r.freq(i);
        }
        return res;
    }
    Poly operator-(const Poly &r) const {
        auto n = std::max(size(), r.size());
        std::vector<D> res(n);
        for (int i = 0; i < n; i++) {
            res[i] = freq(i) - r.freq(i);
        }
        return res;
    }
    Poly operator*(const Poly &r) const { return {multiply(v, r.v)}; }
    Poly operator*(const D &r) const {
        int n = size();
        std::vector<D> res(n);
        for (int i = 0; i < n; i++) {
            res[i] = v[i] * r;
        }
        return res;
    }
    Poly operator/(const D &r) const { return *this * r.inv(); }
    Poly operator/(const Poly &r) const {
        if (size() < r.size()) return {};
        int n = size() - r.size() + 1;
        return (rev().pre(n) * r.rev().inv(n)).pre(n).rev();
    }
    Poly operator%(const Poly &r) const { return *this - *this /
        r * r; }
    Poly operator<<(int s) const {
        std::vector<D> res(size() + s);
        for (int i = 0; i < size(); i++) {
            res[i + s] = v[i];
        }
        return res;
    }
    Poly operator>>(int s) const {
        if (size() <= s) {
            return Poly();
        }
        std::vector<D> res(size() - s);
        for (int i = 0; i < size() - s; i++) {
            res[i] = v[i + s];
        }
        return res;
    }
    Poly &operator+=(const Poly &r) { return *this = *this + r; }
    Poly &operator-=(const Poly &r) { return *this = *this - r; }
    Poly &operator*=(const Poly &r) { return *this = *this * r; }
    Poly &operator*=(const D &r) { return *this = *this * r; }
    Poly &operator/=(const Poly &r) { return *this = *this / r; }
    Poly &operator/=(const D &r) { return *this = *this / r; }
    Poly &operator%=(const Poly &r) { return *this = *this % r; }
    Poly &operator<<=(const size_t &n) { return *this = *this <<
        n; }
    Poly &operator>>=(const size_t &n) { return *this = *this >>
        n; }
    Poly pre(int le) const {
        return {v.begin(), v.begin() + std::min(size(), le)};
    }
    Poly rev(int n = -1) const {
        std::vector<D> res = v;
        if (n != -1) {
            res.resize(n);
        }
        std::reverse(res.begin(), res.end());
        return res;
    }
    Poly diff() const {
        std::vector<D> res(std::max(0, size() - 1));
        for (int i = 1; i < size(); i++) {
            res[i - 1] = freq(i) * i;
        }
        return res;
    }
    Poly inte() const {
        std::vector<D> res(size() + 1);
        for (int i = 0; i < size(); i++) {
            res[i + 1] = freq(i) / (i + 1);
        }
        return res;
    }
}
```

```

    res[i] = freq(i) + r.freq(i);
}
return res;
}
Poly operator-(const Poly &r) const {
    int n = std::max(size(), r.size());
    std::vector<D> res(n);
    for (int i = 0; i < n; i++) {
        res[i] = freq(i) - r.freq(i);
    }
    return res;
}
Poly operator*(const Poly &r) const { return {multiply(v, r.v)}; }
Poly operator*(const D &r) const {
    int n = size();
    std::vector<D> res(n);
    for (int i = 0; i < n; i++) {
        res[i] = v[i] * r;
    }
    return res;
}
Poly operator/(const D &r) const { return *this * r.inv(); }
Poly operator/(const Poly &r) const {
    if (size() < r.size()) return {};
    int n = size() - r.size() + 1;
    return (rev().pre(n) * r.rev().inv(n)).pre(n).rev();
}
Poly operator%(const Poly &r) const { return *this - *this /
    r * r; }
Poly operator<<(int s) const {
    std::vector<D> res(size() + s);
    for (int i = 0; i < size(); i++) {
        res[i + s] = v[i];
    }
    return res;
}
Poly operator>>(int s) const {
    if (size() <= s) {
        return Poly();
    }
    std::vector<D> res(size() - s);
    for (int i = 0; i < size() - s; i++) {
        res[i] = v[i + s];
    }
    return res;
}
Poly &operator+=(const Poly &r) { return *this = *this + r; }
Poly &operator-=(const Poly &r) { return *this = *this - r; }
Poly &operator*=(const Poly &r) { return *this = *this * r; }
Poly &operator*=(const D &r) { return *this = *this * r; }
Poly &operator/=(const Poly &r) { return *this = *this / r; }
Poly &operator/=(const D &r) { return *this = *this / r; }
Poly &operator%=(const Poly &r) { return *this = *this % r; }
Poly &operator<<=(const size_t &n) { return *this = *this <<
    n; }
Poly &operator>>=(const size_t &n) { return *this = *this >>
    n; }
Poly pre(int le) const {
    return {v.begin(), v.begin() + std::min(size(), le)};
}
Poly rev(int n = -1) const {
    std::vector<D> res = v;
    if (n != -1) {
        res.resize(n);
    }
    std::reverse(res.begin(), res.end());
    return res;
}
Poly diff() const {
    std::vector<D> res(std::max(0, size() - 1));
    for (int i = 1; i < size(); i++) {
        res[i - 1] = freq(i) * i;
    }
    return res;
}
Poly inte() const {
    std::vector<D> res(size() + 1);
    for (int i = 0; i < size(); i++) {
        res[i + 1] = freq(i) / (i + 1);
    }
    return res;
}
}
```

```

// f * f.inv() = 1 + g(x)x^m
Poly inv(int m) const {
    Poly res = Poly({D(1) / freq(0)});
    for (int i = 1; i < m; i *= 2) {
        res = (res * D(2) - res * res * pre(2 * i)).pre(2 * i);
    }
    return res.pre(m);
}

Poly exp(int n) const {
    assert(freq(0) == 0);
    Poly f({1}), g({1});
    for (int i = 1; i < n; i *= 2) {
        g = (g * 2 - f * g * g).pre(i);
        Poly q = diff().pre(i - 1);
        Poly w = (q + g * (f.diff() - f * q)).pre(2 * i - 1);
        f = (f + f * (*this - w.inte()).pre(2 * i)).pre(2 * i);
    }
    return f.pre(n);
}

Poly log(int n) const {
    assert(freq(0) == 1);
    auto f = pre(n);
    return (f.diff() * f.inv(n - 1)).pre(n - 1).inte();
}

Poly pow(int n, i64 k) const {
    int m = 0;
    while (m < n && freq(m) == 0) m++;
    Poly f(std::vector<D>(n, 0));
    if (k && m && (k >= n || k * m >= n)) return f;
    f.v.resize(n);
    if (m == n) return f.v[0] = 1, f;
    int le = m * k;
    Poly g({v.begin() + m, v.end()});
    D base = power<D>(g.freq(0), k), inv = g.freq(0).inv();
    g = ((g * inv).log(n - m) * D(k)).exp(n - m);
    for (int i = le; i < n; i++) f.v[i] = g.freq(i - le) * base;
    return f;
}

Poly Getsqrt(int n) const {
    if (size() == 0) return {{0}};
    int z = QuadraticResidue(freq(0).v, 998244353);
    if (z == -1) return Poly{};
    Poly f = pre(n + 1);
    Poly g({z});
    for (int i = 1; i < n; i *= 2) {
        g = (g + f.pre(2 * i) * g.inv(2 * i)) / 2;
    }
    return g.pre(n + 1);
}

Poly sqrt(int n) const {
    int m = 0;
    while (m < n && freq(m) == 0) m++;
    if (m == n) return {{0}};
    if (m & 1) return Poly{};
    Poly s = Poly(std::vector<D>(v.begin() + m, v.end())).
        Getsqrt(n);
    if (s.size() == 0) return Poly{};
    std::vector<D> res(n);
    for (int i = 0; i + m / 2 < n; i++) res[i + m / 2] = s.freq(i);
    return Poly(res);
}

Poly modpower(u64 n, const Poly &mod) {
    Poly x = *this, res = {{1}};
    for (; n; n /= 2, x = x * x % mod) {
        if (n & 1) {
            res = res * x % mod;
        }
    }
    return res;
}

friend std::ostream &operator<<(std::ostream &os, const Poly
    &p) {
    if (p.size() == 0) {
        return os << "0";
    }
    for (auto i = 0; i < p.size(); i++) {
        if (p.v[i]) {
            os << p.v[i] << "x^" << i;
            if (i != p.size() - 1) {
                os << "+";
            }
        }
    }
}

}

}

return os;
}
};

template <class mint>
struct MultiEval {
    using NP = MultiEval *;
    NP l, r;
    int sz;
    Poly<mint> mul;
    std::vector<mint> que;
    MultiEval(const std::vector<mint> &que_, int off, int sz_) :
        sz(sz_) {
        if (sz <= 100) {
            que = {que_.begin() + off, que_.begin() + off + sz};
            mul = {{1}};
            for (auto x : que) {
                mul *= {-x, 1};
            }
            return;
        }
        l = new MultiEval(que_, off, sz / 2);
        r = new MultiEval(que_, off + sz / 2, sz - sz / 2);
        mul = l->mul * r->mul;
    }
    MultiEval(const std::vector<mint> &que_) : MultiEval(que_, 0,
        int(que_.size())) {}
    void query(const Poly<mint> &pol_, std::vector<mint> &res)
        const {
        if (sz <= 100) {
            for (auto x : que) {
                mint sm = 0, base = 1;
                for (int i = 0; i < pol_.size(); i++) {
                    sm += base * pol_.freq(i);
                    base *= x;
                }
                res.push_back(sm);
            }
            return;
        }
        auto pol = pol_ % mul;
        l->query(pol, res);
        r->query(pol, res);
    }
    std::vector<mint> query(const Poly<mint> &pol) const {
        std::vector<mint> res;
        query(pol, res);
        return res;
    }
};

template <class mint>
Poly<mint> berlekampMassey(const std::vector<mint> &s) {
    int n = int(s.size());
    std::vector<mint> b = {mint(-1)}, c = {mint(-1)};
    mint y = mint(1);
    for (int ed = 1; ed <= n; ed++) {
        int l = int(c.size()), m = int(b.size());
        mint x = 0;
        for (int i = 0; i < l; i++) {
            x += c[i] * s[ed - l + i];
        }
        b.push_back(0);
        m++;
        if (!x) {
            continue;
        }
        mint freq = x / y;
        if (l < m) {
            // use b
            auto tmp = c;
            c.insert(begin(c), m - l, mint(0));
            for (int i = 0; i < m; i++) {
                c[m - 1 - i] -= freq * b[m - 1 - i];
            }
            b = tmp;
            y = x;
        } else {
            // use c
            for (int i = 0; i < m; i++) {
                c[l - 1 - i] -= freq * b[m - 1 - i];
            }
        }
    }
    return c;
}

```

```

}
template <class E, class mint = decltype(E().f)>
mint sparseDet(const std::vector<std::vector<E>> &g) {
    int n = int(g.size());
    if (n == 0) {
        return 1;
    }
    auto randV = [&]() {
        std::vector<mint> res(n);
        for (int i = 0; i < n; i++) {
            res[i] = mint(std::uniform_int_distribution<i64>(1, mint
            (-1).v)(rng)); // need rng
        }
        return res;
    };
    std::vector<mint> c = randV(), l = randV(), r = randV();
    // l * mat * r
    std::vector<mint> buf(2 * n);
    for (int fe = 0; fe < 2 * n; fe++) {
        for (int i = 0; i < n; i++) {
            buf[fe] += l[i] * r[i];
        }
        for (int i = 0; i < n; i++) {
            r[i] *= c[i];
        }
        std::vector<mint> tmp(n);
        for (int i = 0; i < n; i++) {
            for (auto e : g[i]) {
                tmp[i] += r[e.to] * e.f;
            }
        }
        r = tmp;
    }
    auto u = berlekampMassey(buf);
    if (u.size() != n + 1) {
        return sparseDet(g);
    }
    auto acdet = u.freq(0) * mint(-1);
    if (n % 2) {
        acdet *= mint(-1);
    }
    if (!acdet) {
        return 0;
    }
    mint cdet = 1;
    for (int i = 0; i < n; i++) {
        cdet *= c[i];
    }
    return acdet / cdet;
}

```

## 7.21 Theorem

- Pick's Theorem  
 $A = i + \frac{b}{2} - 1$   
 $A$ : Area  $\setminus i$ : grid number in the inner  $\setminus b$ : grid number on the side
- Matrix-Tree theorem  
 undirected graph  
 $D_{ii}(G) = \deg(i), D_{ij} = 0, i \neq j$   
 $A_{ij}(G) = A_{ji}(G) = \#e(i, j), i \neq j$   
 $L(G) = D(G) - A(G)$   
 $t(G) = \det L(G)_{1,2,\dots,i-1,i+1,\dots,n}$   
 leaf to root  
 $D_{ii}^{out}(G) = \deg^{out}(i), D_{ij}^{out} = 0, i \neq j$   
 $A_{ij}(G) = \#e(i, j), i \neq j$   
 $L^{out}(G) = D^{out}(G) - A(G)$   
 $t^{root}(G, k) = \det L^{out}(G)_{1,2,\dots,k-1,k+1,\dots,n}$   
 root to leaf  
 $L^{in}(G) = D^{in}(G) - A(G)$   
 $t^{leaf}(G, k) = \det L^{in}(G)_{1,2,\dots,k-1,k+1,\dots,n}$
- Derangement  
 $D_n = (n-1)(D_{n-1} + D_{n-2}) = nD_{n-1} + (-1)^n$
- Möbius Inversion  
 $f(n) = \sum_{d|n} g(d) \Leftrightarrow g(n) = \sum_{d|n} \mu\left(\frac{n}{d}\right) f(d)$
- Euler Inversion  
 $\sum_{i|n} \varphi(i) = n$
- Binomial Inversion  
 $f(n) = \sum_{i=0}^n \binom{n}{i} g(i) \Leftrightarrow g(n) = \sum_{i=0}^n (-1)^{n-i} \binom{n}{i} f(i)$
- Subset Inversion  
 $f(S) = \sum_{T \subseteq S} g(T) \Leftrightarrow g(S) = \sum_{T \subseteq S} (-1)^{|S|-|T|} f(T)$

- Min-Max Inversion  
 $\max_{i \in S} x_i = \sum_{T \subseteq S} (-1)^{|T|-1} \min_{j \in T} x_j$
- Ex Min-Max Inversion  
 $\text{kthmax}_{i \in S} x_i = \sum_{T \subseteq S} (-1)^{|T|-k} \binom{|T|-1}{k-1} \min_{j \in T} x_j$
- Lcm-Gcd Inversion  
 $\text{lcm}_{i \in S} x_i = \prod_{T \subseteq S} \left( \gcd_{j \in T} x_j \right)^{(-1)^{|T|-1}}$
- Sum of powers  
 $\sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k^+ n^{m+1-k}$   
 $\sum_{j=0}^m \binom{m+1}{j} B_j^- = 0$   
 note:  $B_1^+ = -B_1^-, B_i^+ = B_i^-$
- Cayley's formula  
 number of trees on  $n$  labeled vertices:  $n^{n-2}$   
 Let  $T_{n,k}$  be the number of labelled forests on  $n$  vertices with  $k$  connected components, such that vertices  $1, 2, \dots, k$  all belong to different connected components. Then  $T_{n,k} = k n^{n-k-1}$ .
- High order residue  
 $\left[ d^{\frac{p-1}{p-1}} \equiv 1 \right]$
- Packing and Covering  
 $|\text{maximum independent set}| + |\text{minimum vertex cover}| = |V|$
- Kořig's theorem  
 $|\text{maximum matching}| = |\text{minimum vertex cover}|$
- Dilworth's theorem  
 $\text{width} = |\text{largest antichain}| = |\text{smallest chain decomposition}|$
- Mirsky's theorem  
 $\text{height} = |\text{longest chain}| = |\text{smallest antichain decomposition}| = |\text{minimum anticlique partition}|$
- Lucas' Theorem  
 For  $n, m \in \mathbb{Z}^*$  and prime  $P$ ,  $\binom{m}{n} \bmod P = \prod \binom{m_i}{n_i}$  where  $m_i$  is the  $i$ -th digit of  $m$  in base  $P$ .
- Stirling approximation  
 $n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n e^{\frac{1}{12n}}$
- 1st Stirling Numbers(permutation  $|P| = n$  with  $k$  cycles)  
 $S(n, k) = \text{coefficient of } x^k \text{ in } \Pi_{i=0}^{n-1} (x+i)$   
 $S(n+1, k) = nS(n, k) + S(n, k-1)$
- 2nd Stirling Numbers(Partition  $n$  elements into  $k$  non-empty set)  
 $S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$   
 $S(n+1, k) = kS(n, k) + S(n, k-1)$
- Catalan number  
 $C_n = \frac{1}{n+1} \binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n-1}$   
 $\binom{n+m}{n} - \binom{n+m}{n+1} = (m+n)! \frac{n-m+1}{n+1}$  for  $n \geq m$   
 $C_n = \frac{1}{n+1} \binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}$   
 $C_0 = 1$  and  $C_{n+1} = 2 \binom{2n+1}{n+2} C_n$   
 $C_0 = 1$  and  $C_{n+1} = \sum_{i=0}^n C_i C_{n-i}$  for  $n \geq 0$
- Extended Catalan number  
 $\frac{1}{(k-1)n+1} \binom{kn}{n}$
- Calculate  $c[i-j] + = a[i] \times b[j]$  for  $a[n], b[m]$   
 1.  $a = \text{reverse}(a); c = \text{mul}(a, b); c = \text{reverse}(c[n]);$   
 2.  $b = \text{reverse}(b); c = \text{mul}(a, b); c = \text{rshift}(c, m-1);$
- Eulerian number (permutation  $1 \sim n$  with  $m$   $a[i] > a[i-1]$ )  
 $A(n, m) = \sum_{i=0}^m (-1)^i \binom{n+1}{i} (m+1-i)^n$   
 $A(n, m) = (n-m)A(n-1, m-1) + (m+1)A(n-1, m)$
- Hall's theorem  
 Let  $G = (X+Y, E)$  be a bipartite graph. For  $W \subseteq X$ , let  $N(W) \subseteq Y$  denotes the adjacent vertices set of  $W$ . Then,  $G$  has a  $X'$ -perfect matching (matching contains  $X' \subseteq X$ ) iff  $\forall W \subseteq X', |W| \leq |N(W)|$ .
- Tutte Matrix:  
 For a graph  $G = (V, E)$ , its maximum matching =  $\frac{\text{rank}(A)}{2}$  where  $A_{ij} = ((i, j) \in E ? (i < j ? x_{ij} : -x_{ji}) : 0)$  and  $x_{ij}$  are random numbers.
- Erdoš-Gallai theorem  
 There exists a simple graph with degree sequence  $d_1 \geq \dots \geq d_n$  iff  
 $\sum_{i=1}^n d_i$  is even and  $\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k), \forall 1 \leq k \leq n$
- Euler Characteristic  
 planar graph:  $V - E + F - C = 1$   
 convex polyhedron:  $V - E + F = 2$   
 $V, E, F, C$ : number of vertices, edges, faces(regions), and components
- Burnside Lemma  
 $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$

- Polyá theorem  
 $|Y^x/G| = \frac{1}{|G|} \sum_{g \in G} m^{c(g)}$   
 $m = |Y|$  : num of colors,  $c(g)$  : num of cycle
- Cayley's Formula  
 Given a degree sequence  $d_1, \dots, d_n$  of a labeled tree, there are  $\frac{(n-2)!}{(d_1-1)! \dots (d_n-1)!}$  spanning trees.
- Find a Primitive Root of  $n$ :  
 $n$  has primitive roots iff  $n = 2, 4, p^k, 2p^k$  where  $p$  is an odd prime.  
 1. Find  $\phi(n)$  and all prime factors of  $\phi(n)$ , says  $P = \{p_1, \dots, p_m\}$   
 2.  $\forall g \in [2, n]$ , if  $g^{\frac{\phi(n)}{p_i}} \neq 1, \forall p_i \in P$ , then  $g$  is a primitive root.  
 3. Since the smallest one isn't too big, the algorithm runs fast.  
 4.  $n$  has exactly  $\phi(\phi(n))$  primitive roots.
- Taylor series  

$$f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \frac{f'''(c)}{3!}(x-c)^3 + \dots$$
- Lagrange Multiplier  
 $\min f(x, y)$ , subject to  $g(x, y) = 0$   
 $\frac{\partial f}{\partial x} + \lambda \frac{\partial g}{\partial x} = 0$   
 $\frac{\partial f}{\partial y} + \lambda \frac{\partial g}{\partial y} = 0$   
 $g(x, y) = 0$
- Calculate  $f(x+n)$  where  $f(x) = \sum_{i=0}^{n-1} a_i x^i$   

$$f(x+n) = \sum_{i=0}^{n-1} a_i (x+n)^i = \sum_{i=0}^{n-1} a_i x^i \cdot \frac{1}{i!} \sum_{j=i}^{n-1} \frac{a_j}{j!} \cdot \frac{n^{j-i}}{(j-i)!}$$
- Bell 數 (有  $n$  個人, 把他們拆組的方法總數)  
 $B_0 = 1$   
 $B_n = \sum_{k=0}^n s(n, k)$  (second – stirling)  
 $B_{n+1} = \sum_{k=0}^n \binom{n}{k} B_k$
- Wilson's theorem  
 $(p-1)! \equiv -1 \pmod{p}$   
 $(p^q)! \equiv \begin{cases} 1, & (p=2) \wedge (q \geq 3), \\ -1, & \text{otherwise.} \end{cases} \pmod{p^q}$
- Fermat's little theorem  
 $a^p \equiv a \pmod{p}$
- Euler's theorem  
 $a^b \equiv \begin{cases} a^{b \bmod \varphi(m)}, & \gcd(a, m) = 1, \\ a^b, & \gcd(a, m) \neq 1, b < \varphi(m), \\ a^{(b \bmod \varphi(m)) + \varphi(m)}, & \gcd(a, m) \neq 1, b \geq \varphi(m). \end{cases} \pmod{m}$
- 環狀著色 (相鄰塗異色)  
 $(k-1)(-1)^n + (k-1)^n$

## 8 Stringology

### 8.1 Aho–Corasick AM

```
struct ACM {
    int idx = 0;
    vector<array<int, 26>> tr;
    vector<int> cnt, fail;

    void clear() {
        tr.resize(1, array<int, 26>{});
        cnt.resize(1, 0);
        fail.resize(1, 0);
    }

    ACM() {
        clear();
    }

    int newnode() {
        tr.push_back(array<int, 26>{});
        cnt.push_back(0);
        fail.push_back(0);
        return ++idx;
    }

    void insert(string &s) {
        int u = 0;
        for (char c : s) {
            c -= 'a';
            if (tr[u][c] == 0) tr[u][c] = newnode();
            u = tr[u][c];
        }
        cnt[u]++;
    }

    void build() {
        queue<int> q;
```

```
    rep (i, 0, 26) if (tr[0][i]) q.push(tr[0][i]);
    while (!q.empty()) {
        int u = q.front(); q.pop();
        rep (i, 0, 26) {
            if (tr[u][i]) {
                fail[tr[u][i]] = tr[fail[u]][i];
                cnt[tr[u][i]] += cnt[fail[tr[u][i]]];
                q.push(tr[u][i]);
            } else {
                tr[u][i] = tr[fail[u]][i];
            }
        }
    }
}

int query(string &s) {
    int u = 0, res = 0;
    for (char c : s) {
        c -= 'a';
        u = tr[u][c];
        res += cnt[u];
    }
    return res;
}
};
```

### 8.2 Double String

```
// need zvalue
int ans = 0;
auto dc = [&](auto self, string cur) -> void {
    int m = cur.size();
    if (m <= 1) return;
    string _s = cur.substr(0, m / 2), _t = cur.substr(m / 2, m);
    self(self, _s);
    self(self, _t);
    rep (T, 0, 2) {
        int m1 = _s.size(), m2 = _t.size();
        string s = _t + "$" + _s, t = _s;
        reverse(all(t));
        zvalue z1(s), z2(t);
        auto get_z = [&](zvalue &z, int x) -> int {
            if (0 <= x && x < z.z.size()) return z[x];
            return 0;
        };
        rep (i, 0, m1) if (_s[i] == _t[0]) {
            int len = m1 - i;
            int L = m1 - min(get_z(z2, m1 - i), len - 1);
            R = get_z(z1, m2 + 1 + i);
            if (T == 0) R = min(R, len - 1);
            R = i + R;
            ans += max(0, R - L + 1);
        }
        swap(_s, _t);
        reverse(all(_s));
        reverse(all(_t));
    }
};
dc(dc, str);
```

### 8.3 Lyndon Factorization

```
// partition s = w[0] + w[1] + ... + w[k-1],
// w[0] >= w[1] >= ... >= w[k-1]
// each w[i] strictly smaller than all its suffix
// min rotate: last < n of duval_min(s + s)
// max rotate: last < n of duval_max(s + s)
// min suffix: last of duval_min(s)
// max suffix: last of duval_max(s + -1)
vector<int> duval(const auto &s) {
    int n = s.size(), i = 0;
    vector<int> pos;
    while (i < n) {
        int j = i + 1, k = i;
        while (j < n and s[k] <= s[j]) { // >=
            if (s[k] < s[j]) k = i; // >
            else k++;
            j++;
        }
        while (i <= k) {
            pos.push_back(i);
            i += j - k;
        }
    }
    pos.push_back(n);
```



```
    return pos;
}
```

## 8.4 Manacher

```
/* center i: radius z[i * 2 + 1] / 2
   center i, i + 1: radius z[i * 2 + 2] / 2
   both aba, abba have radius 2 */
vector<int> manacher(const string &tmp) { // 0-based
    string s = "%";
    int l = 0, r = 0;
    for (char c : tmp) s += c, s += '%';
    vector<int> z(ssize(s));
    for (int i = 0; i < ssize(s); i++) {
        z[i] = r > i ? min(z[2 * l - i], r - i) : 1;
        while (i - z[i] >= 0 && i + z[i] < ssize(s) && s[i - z[i]]
            == s[i + z[i]])
            ++z[i];
        if (z[i] + i > r) r = z[i] + i, l = i;
    }
    return z;
}
```

## 8.5 SA-IS

```
auto sais(const auto &s) {
    const int n = (int)s.size(), z = ranges::max(s) + 1;
    if (n == 1) return vector{0};
    vector<int> c(z); for (int x : s) ++c[x];
    partial_sum(all(c), begin(c));
    vector<int> sa(n); auto I = views::iota(0, n);
    vector<bool> t(n); t[n - 1] = true;
    for (int i = n - 2; i >= 0; i--)
        t[i] = (s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i + 1]);
    auto is_lms = views::filter([&t](int x) {
        return x && t[x] & !t[x - 1];
    });
    auto induce = [&] {
        for (auto x = c; int y : sa)
            if (y-- and !t[y]) sa[x[s[y] - 1]++] = y;
        for (auto x = c; int y : sa | views::reverse)
            if (y-- and t[y]) sa[--x[s[y]]] = y;
    };
    vector<int> lms, q(n); lms.reserve(n);
    for (auto x = c; int i : I | is_lms) {
        q[i] = int(lms.size());
        lms.push_back(sa[--x[s[i]]] = i);
    }
    induce(); vector<int> ns(lms.size());
    for (int j = -1, nz = 0; int i : sa | is_lms) {
        if (j >= 0) {
            int len = min({n - i, n - j, lms[q[i] + 1] - i});
            ns[q[i]] = nz += lexicographical_compare(
                s.begin() + j, s.begin() + j + len,
                s.begin() + i, s.begin() + i + len
            );
        }
        j = i;
    }
    ranges::fill(sa, 0); auto nsa = sais(ns);
    for (auto x = c; int y : nsa | views::reverse)
        y = lms[y], sa[--x[s[y]]] = y;
    return induce(), sa;
}

// sa[i]: sa[i]-th suffix is the
// i-th lexicographically smallest suffix.
// lcp[i]: LCP of suffix sa[i] and suffix sa[i + 1].
struct Suffix {
    int n;
    vector<int> sa, rk, lcp;
    Suffix(const auto &s) : n(s.size()),
        lcp(n - 1), rk(n) {
        vector<int> t(n + 1); // t[n] = 0
        copy(all(s), t.begin()); // s shouldn't contain 0
        sa = sais(t); sa.erase(sa.begin());
        for (int i = 0; i < n; i++) rk[sa[i]] = i;
        for (int i = 0, h = 0; i < n; i++) {
            if (!rk[i]) { h = 0; continue; }
            for (int j = sa[rk[i] - 1];
                i + h < n and j + h < n
                and s[i + h] == s[j + h];) ++h;
            lcp[rk[i] - 1] = h ? h-- : 0;
        }
    }
};
```

## 8.6 Suffix Array

```
struct SuffixArray {
    int n;
    vector<int> suf, rk, S;
    SuffixArray(vector<int> _S) : S(_S) {
        n = S.size();
        suf.assign(n, 0);
        rk.assign(n * 2, -1);
        iota(all(suf), 0);
        for (int i = 0; i < n; i++) rk[i] = S[i];
        for (int k = 2; k < n + n; k *= 2) {
            auto cmp = [&](int a, int b) -> bool {
                return rk[a] == rk[b] ? (rk[a + k / 2] < rk[b + k / 2])
                    : (rk[a] < rk[b]);
            };
            sort(all(suf), cmp);
            auto tmp = rk;
            tmp[suf[0]] = 0;
            for (int i = 1; i < n; i++) {
                tmp[suf[i]] = tmp[suf[i - 1]] + cmp(suf[i - 1], suf[i]);
            }
            rk.swap(tmp);
        }
    }
};
```

## 8.7 Z-value

```
struct zvalue {
    vector<int> z;
    int operator[] (const int &x) const {
        return z[x];
    }
    zvalue(string s) {
        int n = s.size();
        z.resize(n);
        z[0] = 0;
        for (int i = 1, l = 1, r = 0; i < n; i++) {
            z[i] = min(z[i - l], max<int>(0, r - i));
            while (i + z[i] < n && s[i + z[i]] == s[z[i]]) z[i]++;
            if (i + z[i] > r) l = i, r = i + z[i];
        }
    }
};
```







