Camera Calibration

Let $K=\begin{pmatrix}\alpha & s & u_0\\ 0 & \beta & v_0\\ 0 & 0 & 1\end{pmatrix}$ $R=\begin{pmatrix}r_1^T\\ r_2^T\\ r_3^T\end{pmatrix}$ and $T=[t_x t_y t_z]^T$

$$P=K[RT]=\begin{pmatrix}\alpha & s & u_0\\ 0 & \beta & v_0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}r_1^T & t_x\\ r_2^T & t_y\\ r_3^T & t_z\end{pmatrix}=\begin{pmatrix}\alpha r_1^T+s r_2^T+u_0 r_3^T & \alpha t_x+st_y+u_0 t_z\\ \beta r_2^T+v_0 r_3^T & \beta t_y+v_0 t_z\\ r_3^T & t_z\end{pmatrix}$$

$$p=[p_{11}\,p_{12}\,p_{13}\,p_{14}\,p_{21}\,p_{22}\,p_{23}\,p_{24}\,p_{31}\,p_{32}\,p_{33}\,p_{34}]^T$$

$$P=\begin{pmatrix}p_{11} & p_{12} & p_{13} & p_{14}\\ p_{21} & p_{22} & p_{23} & p_{24}\\ p_{31} & p_{32} & p_{33} & p_{34}\end{pmatrix} \qquad P33=\begin{pmatrix}p_{11} & p_{12} & p_{13}\\ p_{21} & p_{22} & p_{23}\\ p_{31} & p_{32} & p_{33}\end{pmatrix}=\begin{pmatrix}a_1^T\\ a_2^T\\ a_3^T\end{pmatrix}=\begin{pmatrix}\alpha r_1^T+s r_2^T+u_0 r_3^T\\ \beta r_2^T+v_0 r_3^T\\ r_3^T\end{pmatrix}$$

As P and λP will have the same effect, from the solution from the homogeneous system, we normalize the $P33$ in the way that the last row has the norm of 1.

let $\rho=\dfrac{\pm 1}{norm(a_3)}$ then $\rho P33=\begin{pmatrix}\rho a_1^T\\ \rho a_2^T\\ \rho a_3^T\end{pmatrix}=\begin{pmatrix}a'^T_1\\ a'^T_2\\ a'^T_3\end{pmatrix}$

$$u_0=a'^T_1 * a'_3 \qquad v_0=a'^T_2 * a'_3 \qquad \beta=norm(a'_2\, x\, a'_3) \qquad s=\frac{a'^T_1 * a'_2-u_0 v_0}{\beta}$$

$$\alpha=\sqrt{a'^T_1 * a_1-s^2-u_0^2} \qquad r_1=\frac{a'_2\, x a'_3}{norm(a'_2 x a'_3)} \qquad r_3=a'_3 \qquad r_2=r_3 x r_1 \qquad T=\rho K^{-1}\begin{pmatrix}p_{14}\\ p_{24}\\ p_{34}\end{pmatrix}$$

As there are two possible ways to normalize the matrix P33, there will be two solutions. From the mathematical analysis, from one solution, we can obtain the other one.

$K_2=K$ `the intrinsic parameters remain unchanged.`

$R_2=\begin{pmatrix}r_1^T\\ -r_2^T\\ -r_3^T\end{pmatrix}$ and $T_2=-T$