Review and Extension — Divisibility

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1 Key concepts for this term

- Induction, strong induction and well-ordering.
- If $a \mid b$ and $a \mid c$ then for any integers x and y, $a \mid bx + cy$.
- Factorisations:

$$axy + bx + cy = d \iff (ax + c)(ay + b) = ad + bc$$
$$a^k - b^k = (a - b)\left(a^{k-1} + a^{k-2}b + \cdots + b^{k-1}\right)$$

- Division Algorithm: for any integer a and positive integer b there exist unique integers q and r such that $a = qb + r$ and $0 \leq r < b$.
- Euclid's Algorithm: if $a = qb + r$ then $\gcd(a, b) = \gcd(b, r)$.
- Bezout's Identity: for any integers a and b there exist integers x and y such that $\gcd(a, b) = ax + by$.
- Fundamental Theorem of Arithmetic: prime factorisations are unique.
- When dealing with divisibility problems it's often more convenient to think of numbers in terms of prime factorisations.

2 Arithmetic Functions

We define:

- The number of positive divisors function $d(n)$.
- The sum of positive divisors function $\sigma(n)$.

Find formulae for $d(n)$ and $\sigma(n)$ where $n = p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}$.

3 Homework

Solve and submit any three problems from the Problems sections of this term's handouts that weren't covered in class.