Review and Extension — Divisibility and Congruences

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1 Key concepts for this term

- Induction, strong induction, well-ordering
- If $a \mid b$ and $a \mid c$ then for any integers x and y, $a \mid bx + cy$
- Division algorithm, Euclid's algorithm, Bezout's identity
- Fundamental Theorem of Arithmetic
- GCD, LCM in terms of prime factorisations
- Factorise expressions
- Take out the GCD
- Basic properties of $\mathbb{Z}_p$: operations, inverses, Wilson, Fermat, GCD trick
- Multiplicative and completely multiplicative functions
- Formulas for d, σ and φ
- Prove a problem for prime powers first
- Basic properties of $\mathbb{Z}_n$: operations, inverses, Euler, GCD trick, Chinese Remainder Theorem, generalised Wilson
- How to choose a mod
- Modular contradictions
- Quadratic discriminant trick

2 Homework

Solve and submit any three problems from the problems sections of this term's lectures.