

# Diophantine Equations

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## 1 Techniques

- Factorisations
- Mods
- Quadratic discriminant trick
- Take out GCD

## 2 Problems

1. Find all positive integers  $x, y$  such that  $x^2 - 3xy + 2y^2 = 2023$ .
2. Determine all integers  $n \geq 2$  such that  $\sqrt{n - a^2}$  is an integer which divides  $n$ , where  $a$  is the smallest prime divisor of  $n$ .
3. Prove that there are infinitely many positive integers which are not the sum of a square and a prime.
4. Let  $m$  be a positive integer for which there exists a positive integer  $n$  such that  $mn$  is a perfect square and  $m - n$  is prime. Prove that  $4m = (m - n + 1)^2$ .
5. Find all pairs of integers  $x, y$  such that  $x^4 + 2x^2y + y^3 = 0$ .
6. Find all positive integers  $a, b, c$  such that  $a! \times b! = a! + b! + c!$ .
7. Prove that the equation  $y^2 = x^3 + 7$  has no integer solutions.
8. Given are positive integers  $n > 20$  and  $k > 1$ , such that  $k^2$  divides  $n$ . Prove that there exist positive integers  $a, b, c$  such that  $n = ab + bc + ca$ .

### 3 Homework

1. Prove that the equation  $x^3 + 3 = 4y(y + 1)$  has no integer solutions.
2. Find all triples of positive integers  $x, y, z$  such that  $x^3 + y^3 + z^3 - 3xyz$  is prime.
3. Do there exist primes  $x, y, z$  such that  $x^2 + y^3 = z^4$ ?