Problem Set 2 IO

Question 2

First, we set up the model by declaring parameters, variables and functions. Then, we run regressions and discussed results.

The specification is
$$\log(\text{Herfindahl}) = \alpha + \beta \log(\text{Lerner}) + \epsilon$$

And the hypothesis testing H_0 is $\beta = 1$

Setting up the model 2 a) b)

```
In [507]: rm(list=ls())

          setwd("P:/PHD/Courses/02 - Second Year/ECON 761 (IO)/Problem Sets")

          set.seed(3)
```

```
In [508]: ##### Parameters

          #demand
          c0 = 1
          c1 = 0.9
          xi = 0
          #costs
          f = 1
          b0 = 1
          b1 = 0
          eta = 0

          # others
          number_cities = 1000
          n =  ceiling(10*runif(number_cities))
          #first 500 collude if number firsms smaller than 8
          cutoff = number_cities/2
          collusion = as.numeric(c((n[1:cutoff]<=8), rep(FALSE, cutoff)))
          # division city types
```

```
In [509]: # Definitions

          #equil lerner index for N cournot
          lerner_cn = c1/n
          #herfindahl for symmetric firms
          herf = 1/n
          # equilibrium index for monopoly
          elasticity = rep(1/c1 , number_cities)
          # equilibrium index for monopoly
          lerner_m = c1
          # lerner index depends on conduct
          lerner = collusion*lerner_m+ (1-collusion)*lerner_cn
          # lerner log
          lerner_log = log(lerner)+0.1*(runif(number_cities)-0.5)
          # log herfindahl
          log_herf = log(herf)
```

```
In [510]: # Functions

          lerner_func = function(p, mc){
            return (p-mc)/p
          }

          herf_func = function(q,q_tot){
            share_i = q/q_tot
            return (sum(share_i^2))
          }
```

```
In [511]: # Creating data set
          data = data.frame(lerner_log, log_herf, collusion)
```

Regressions

```
In [512]: coll_reg = lm(lerner_log~log_herf, data=data[1:cutoff,])
          courn_reg = lm(lerner_log~log_herf, data=data[cutoff+1:1000,])
          full_reg = lm(lerner_log~log_herf, data=data)
```

Question 2 c)

Below are the tables corresponding to the regression of the Lerner Index on the Herfindahl Index for the case of a linear demand and fixed market function for each case **(Antitrust, No Antitrust, Whole Sample).**

```
In [513]:  #Hypothesis testing
           #Collusion
           sum_col =  summary(coll_reg )$coefficients
           t_col   = abs(sum_col [2,1]-1)/sum_col [2,2]
           reject_col = (t_col>=qnorm(0.95))
           sum_col
```

A matrix: 2 × 4 of type dbl

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.5538758	0.08440148	6.562394	1.330884e-10
log_herf	0.7512105	0.05057847	14.852379	1.423744e-41

We find that on the sample of cities **without antitrust** that collude, an increase of 100 \% in the Herfindahl index increases the Lerner index by 68 \%, and the results are significant. As competition decreases (more Herfindahl, thus more concentration), markups increase less than proportionally. Out t-test for the hypothesis that the coefficient of the herfindahl β is equal to 1, give us the followind result statistic

```
In [514]:  print(t_col)
           reject_col
```

[1] 4.918881

TRUE

So, we reject the null hypothesis, meaning that under collusion the relationship between the Lernex Index and the Herfindahl Index is less than proportional.

In the case **with antitrust**, we find that an increase of 100 \% in the Herfindahl index increases the Lerner index by 99.9 \%, and the results are significant. This results suggest that as competition decreases (more concentration), we have increase markups proportionally.

Out t-test for the hypothesis that the coefficient of the Herfindahl -β- is equal to 1, give us the following result statistic

```
In [515]: #Cournot
          sum_cou = summary(courn_reg)$coefficients
          t_cour = abs(sum_cou[2,1]-1)/sum_cou[2,2]
          reject_cou = (t_cour>=qnorm(0.95))
          sum_cou
```

A matrix: 2 × 4 of type dbl

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-0.1023314	0.003135829	-32.63295	9.049242e-126
log_herf	1.0008733	0.001867018	536.08119	0.000000e+00

Comparing Cournot with collusion, we can see that the positive effect on markups is higher in Cournot, this is because competition actually intensifies the effect on markups, wheras with collusion, this effect is limited. More concentration still yields more markups, but the impact is tampered by less competition.

```
In [516]: print(t_cour)
          reject_cou
```

[1] 0.4677326

FALSE

So, we do **not** reject the null hypothesis ($\beta = 1$), meaning that under Cournot competition the relationship between the Lernex Index and the Herfindahl Index is proportional and in the same direction.

In the **whole sample (pool)** , an increase of 100 \% in the Herfindahl index increases the Lerner index by 84\%, and the results are significant. Out t-test for the hypothesis that the coefficient of the herfindahl β is equal to 1, give us the followind result statistic

```
In [517]: # All cities
          sum_full = summary(full_reg)$coefficients
          t_full = abs(sum_full[2,1]-1)/sum_full[2,2]
          rejectall = (t_full>=qnorm(0.95))
          sum_full
```

A matrix: 2 × 4 of type dbl

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.2339360	0.05824884	4.016148	6.359424e-05
log_herf	0.8816055	0.03479274	25.338781	9.061150e-110

```
In [518]: print(t_full)
          rejectall
```

[1] 3.402851

TRUE

So, we **reject** the null hypothesis, meaning that in the whole sample (Cournot and collusion) the relationship between the Lerner Index and the Herfindahl Index is not proportional, because of the weight that the collusion cases have on the sample. However, the effect is smaller than Cournot and larger than with collusion, as the intuition indicates.

See that the price elasticity of demand is 1.1, i.e. demand is elastic, and despite this, firms can have higher profits. This is related to an exogenous number of firms, there are no potential entrants.

Question 2 d)

Question 2 d)

First we simulate for the 1000 cities, and finds equilibria according to the case.

```
In [519]:  # More constants
           a0 = 3
           a1 = 1
           v = 0

           lerner = NULL
           herf = NULL
           elasticity = NULL
```

In [520]:
```r
# Functions
find_equilibria_ex <- function() {

    lerner <<- NULL
    herf <<- NULL
    elasticity <<- NULL

    for(i in 1:number_cities){

        if(collusion[i]==T){

            Q = (a0+v-b0-eta)/(2*a1)
            q = Q/n[i]
            q = rep(q, n[i])
            p = (a0 + v + b0 + eta)/2

            lerner[i] <<- lerner_func(p=p, mc= b0 +eta)
            herf[i] <<- herf_func(q = q,  q_tot = Q)
            elasticity <<- rbind( elasticity,(1/a1)*((a0-a1*Q+v)/Q)) #same ela
sticity formula than cournot
        } else {

            q = (a0+v-b0-eta)/(a1*(n[i]+1))
            Q = n[i]*q
            q = rep(q, n[i])
            p = (a0+v+n[i]*(b0+eta))/(n[i]+1)

            lerner[i] <<- lerner_func(p =p , mc = b0 + eta)
            herf[i] <<- herf_func(q = q, q_tot = Q)
            elasticity <<- rbind(elasticity,abs((-1/a1)*((a0-a1*Q+v)/Q)))
        }
    }

}
```

In [521]:
```r
find_equilibria_ex()
```

In [522]:
```r
# Regression vars
lerner_log = log(lerner) + runif(number_cities,-0.05, 0.05)
herf_log = log(herf)
data = data.frame(lerner_log, herf_log)

#Regressions
coll_reg_d = lm(lerner_log~herf_log, data=data[1:cutoff,])
courn_reg_d = lm(lerner_log~herf_log, data=data[cutoff+1:number_cities,])
full_reg_d = lm(lerner_log~herf_log, data=data)
```

We obtain different results, for this set of simulations. Below we show tables with results of the regression of the Lerner index on Herfindahl index for a linear demand and exogenous market structure, for each case: **Antitrust, No Antitrust, Whole Sample.**.

We find that on the sample of cities **without antitrust** that collude, an increase of 100 \% in the Herfindahl index increases the Lerner index by 55 \%, and the results are significant. With these parameters, we get an even smaller result than before. As competition decreases (more Herfindahl, thus more concentration), markups increase less than proportionally. Out t-test for the hypothesis that the coefficient of the Herfindahl β is equal to 1, give us the following result statistic

```
In [523]: #Collusion
          sum_col =  summary(coll_reg_d)$coefficients
          t_col=abs(sum_col [2,1]-1)/sum_col [2,2]
          reject_col=(t_col>=qnorm(0.95))
          sum_col
```

A matrix: 2 × 4 of type dbl

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.4861460	0.06210369	7.827974	3.002188e-14
herf_log	0.5527455	0.03721628	14.852249	1.425663e-41

```
In [524]: print(t_col)
          reject_col
```

[1] 12.01771

TRUE

So, we **reject** the null hypothesis, meaning that under collusion the relationship between the Lerner Index and the Herfindahl Index is less than proportional.

In the case **with antitrust**, we find that an increase of 100 \% in the Herfindahl index increases the Lerner index by 75.2 \%, and the results are significant. These results suggest that as competition decreases (more concentration), we have increase markups proportionally.

Out t-test for the hypothesis that the coefficient of the Herfindahl -β- is equal to 1, give us the following result statistic

```
In [525]:  #Cournot
           sum_cou = summary(courn_reg_d)$coefficients
           t_cour = abs(sum_cou[2,1]-1)/sum_cou[2,2]
           reject_cou = (t_cour>=qnorm(0.95))
           sum_cou
```

A matrix: 2 × 4 of type dbl

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.08661314	0.005542366	15.62747	4.370602e-45
herf_log	0.75636883	0.003299828	229.21463	0.000000e+00

```
In [526]:  print(t_cour)
           reject_cou
```

[1] 73.83148

TRUE

So, we **reject** the null hypothesis, meaning that in this case, under Cournot competition the relationship between the Lerner Index and the Herfindahl Index is not proportional. This is different from c), and indicates that depending on the demand curve, we might or not be able to easily detect collusion

Using the **whole pool**, we find that an increase of 100 \% in the Herfindahl index increases the Lerner index by 65\%, and the results are significant. This effect is smaller than in d) and it is between the collusion sample and the Cournot sample results, which is what we would expect. Out t-test for the hypothesis that the coefficient of the Herfindahl β is equal to 1, give us the following result statistic

```
In [527]:  # All cities
           sum_full = summary(full_reg_d)$coefficients
           t_full = abs(sum_full[2,1]-1)/sum_full[2,2]
           rejectall = (t_full>=qnorm(0.95))
           sum_full
```

A matrix: 2 × 4 of type dbl

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.2924632	0.04163133	7.025073	3.955986e-12
herf_log	0.6587182	0.02486690	26.489765	1.596474e-117

```
In [484]:  print(t_full)
           rejectall
```

[1] 14.1418

TRUE

Likewise, we reject the null hypothesis.

Question 2 e)

We learned that if we suspect collusion in a set of markets, we can regress the Lerner index on the Herfindahl index, and test for the null hypothesis $\beta = 1$. If we reject the hypothesis, we might have collusion in those markets.

We also learn that the functional form of the demand matters on the impact of collusion in markets. Firms have different abilities to exert market power and thus have higher markups when colluding. Depending on how the elasticity varies along the demand curve, collusion and Competition have a different effect on markups. With constant elasticity, firms face similar constraints along the demand curve, while with varying elasticity, the more elastic parts yield smaller markups.

Question 3

We need to modify the simulation to calculate exogenously the market structure and to only play Cournot.

```
In [485]: # Setting params
          a0 = 5
```

```
In [486]: # Exogenous number of cities
          n_endog <- function(){
              n <<-NULL
              for(i in 1:number_cities){
                  n <<- rbind(n,((a0+v[i]-b0-eta[i])/sqrt(a1*f))-1)
              }
          }


          # Simulation
          find_equilibria_end <- function() {

              lerner <<- NULL
              herf <<- NULL
              elasticity <<- NULL

              for(i in 1:number_cities){
                  q = (a0+v-b0-eta[i])/(a1*(n[i]+1))
                  Q = n[i]*q
                  q = rep(q, n[i])
                  p = (a0+v[i]+n[i]*(b0+eta[i]))/(n[i]+1)

              # using endogenous formulas
                  lerner[i] <<- (sqrt(a1*f)/(sqrt(a1*f)+b0+eta[i]))
                  herf[i] <<- (sqrt(a1*f)/(a0+v[i]-b0-eta[i]- sqrt(a1*f)) )
                  elasticity[i] <<- ((sqrt(a1*f)+b0+eta[i])/(a0+v[i]-b0-eta[i]-sqrt(a1*f
          )))
              }

          }
```

Question 3 a)

Case where v is distributed $U[-1, 1]$ and $\eta = 0$: **variation in the willingness to pay**.

```
In [487]: # case eta = 0, v in [-1,1]
          eta =   rep (0,number_cities)
          v = runif(number_cities,-1,1)

          # number of cities
          n_endog()

          find_equilibria_end()

          lerner_log = log(lerner) + runif(number_cities,-0.05, 0.05)
          herf_log = log(herf)
          data = data.frame(lerner_log, herf_log)
          elast_3a <- elasticity

          #Regressions
          full_reg_3a = lm(lerner_log~herf_log, data=data)
```

```
In [488]: # All cities regression
          summary(full_reg_3a)$coefficients
```

A matrix: 2 × 4 of type dbl

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-0.697566454	0.005029198	-138.703312	0.0000000
herf_log	-0.003039956	0.004598978	-0.661007	0.5087604

The previous table shows the regression of the Lerner Index on the Herfindahl index for the case of **endogenous number of firms**, $v \sim U[-1, 1]$ and $\eta = 0$.

Question 3 b)

Case where η is distributed $U[-1, 1]$ and $v = 0$: **variation in firms marginal cost**.

```
In [443]: # case v = 0, eta in [-1,1]
          v =  rep (0,number_cities)
          eta = runif(number_cities,-1,1)

          # number of cities
          n_endog()

          find_equilibria_end()

          lerner_log = log(lerner) + runif(number_cities,-0.05, 0.05)
          herf_log = log(herf)
          data = data.frame(lerner_log, herf_log)
          elast_3b <- elasticity

          #Regressions
          full_reg_3b = lm(lerner_log~herf_log, data=data)
```

```
In [444]: # All cities regression
          #library(stargazer)
          summary(full_reg_3b)$coefficients
```

A matrix: 2 × 4 of type dbl

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-2.309528	0.01340157	-172.3327	0
herf_log	-1.538601	0.01213797	-126.7593	0

The previous table shows the regression of the Lerner Index on the Herfindahl index for the case of **endogenous number of firms**, $\eta \sim U[-1, 1]$ and $v = 0$.

3 c)

When there is variation in consumers' willingness to pay, we estimate that the effect on markups is very small and positive, $\beta = 0.004$ However this coefficient is not significant, the absolute value of the t value is smaller than 2, which indicates a high p-value This is a very small effect, which can be explained by the difficulty of identifying a pattern do to variations. If we compare this result with what we found in question 2, which was a positive and large effect on markups for every case.

Uncertainty in marginal cost across cities results also in a small but negative coefficient, $\beta = -1.5$, but this time we get a large and significant coefficient. The interpretation of this coefficient is that more concentration in the industry brings markups down. The optimal endogenous number of firms is 2 and 4, and the elasticities in the case where $\eta = 0$ are less sparse (ranging from 0.4 to 1) than when $v = 0$ (ranging from 0.2 to 1.5). Moreover, as expected, markets with fewer firms that are more concentrated have higher elasticities. In particular, the only markets where firms face elastic demands are those markets where firms have higher marginal costs, and thus there are fewer firms. So, the existence of a high number of markets where demand is elastic ($\epsilon > 1$), helps explain why the ability to set higher markups is limited when $v = 0$. The demand is more responsive, so when there is variation in the marginal costs, this shifts the equilibrium to a more elastic part of the demand, decreasing quantities whenever there is less competition. The fact that this happens in some markets in b) and in zero markets in a) helps to explain the difference in effects.

Note

Table results might be slightly different to coefficients in text, because every simulation is generated from a distribution, so every regression gives slightly different results.

Appendix

```
In [450]: hist(n, main= "Distribution of the number of firms 3 b)",xlab = "elasticities"
          ,col=rainbow(14))
```

Distribution of the number of firms 3 b)
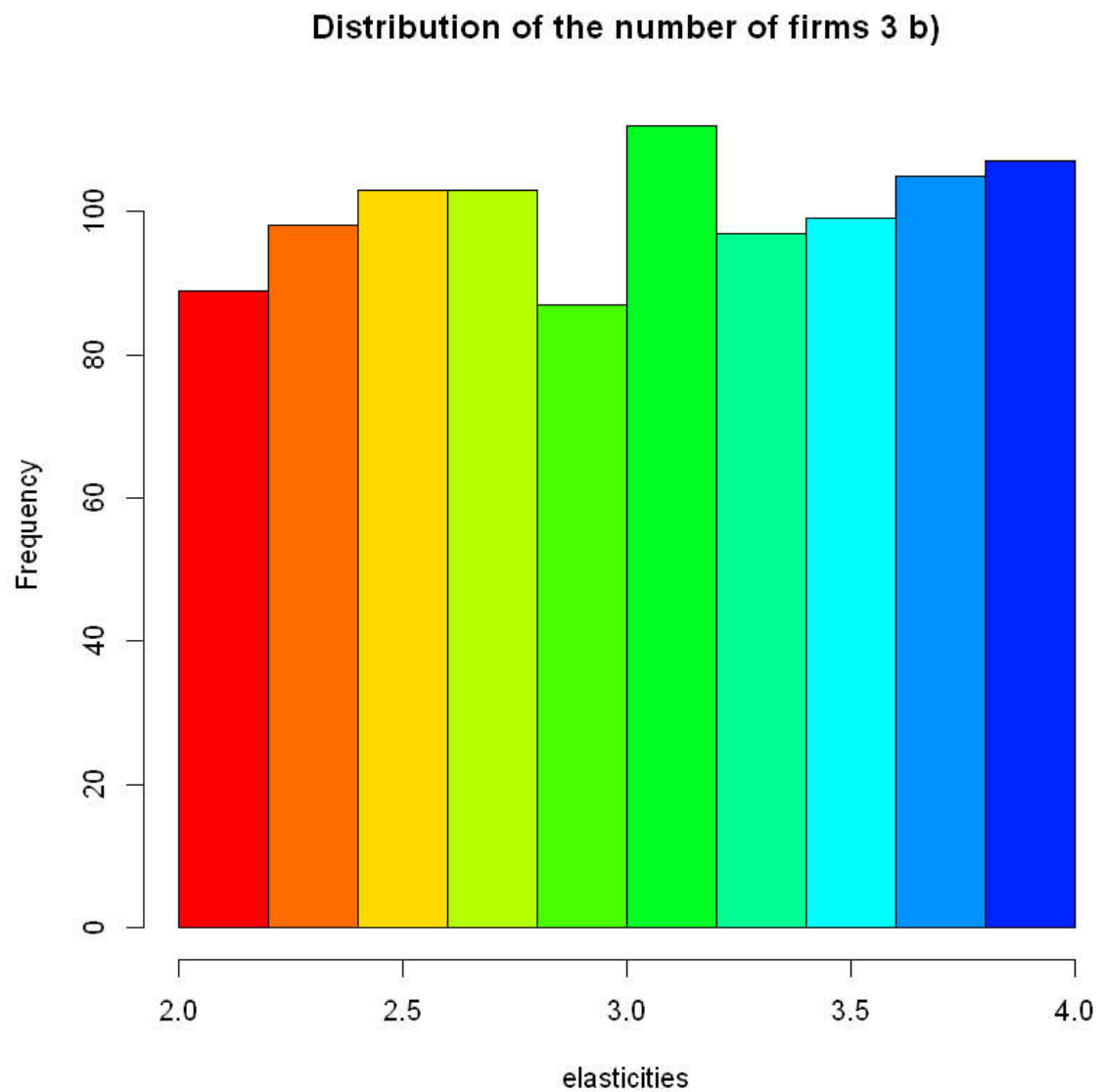
```
In [451]: hist(elast_3a, main= "Distribution of the elasticities 3 a)" ,xlab = "elastici
          ties",col=rainbow(14))
```

Distribution of the elasticities 3 a)
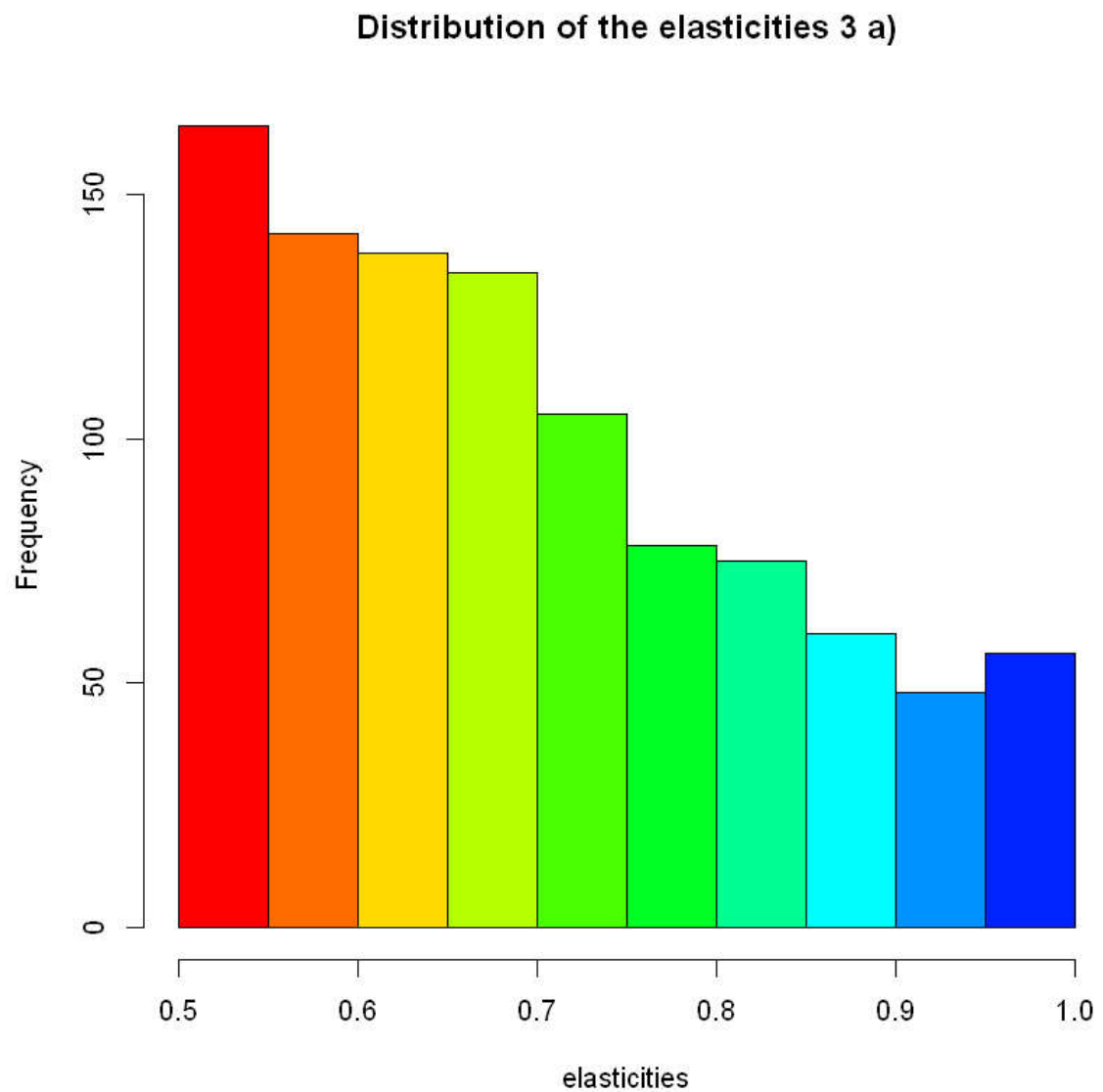
In [452]:
```
hist(elast_3b, main= "Distribution of the elasticities 3 b)",xlab = "elasticit
ies", col=rainbow(14))
```

Distribution of the elasticities 3 b)