UNIT -I: WAVE OPTICS

Introduction

Have you ever observed that the beautiful colors in a soap bubbles, mica and film of oil floating on the surface of water when a sun light falls on them? Did you ever try to find out the reason? This is due to the phenomenon of interference in thin films by reflection of light**.** This was first explained by Thomas young on basis of Hugens' wave concept and principle of superposition.

Principle of Superposition of Waves

When two or more light waves traveling through a medium superimpose one another then the resultant displacement at any point is equal to the algebraic sum of the individual displacements at that point. This is called principle of superposition.

i.e., $$\vec{y} = \vec{y_1} \pm \vec{y_2} \pm \vec{y_3} \pm \ldots\ldots \qquad \rightarrow(1.1)$$

where $\vec{y}$ is the resultant displacement and $\vec{y_1}, \vec{y_2}, \vec{y_3}$ ... are the displacements of individual waves.

$+$ ve sign is taken when displacements are in same direction.

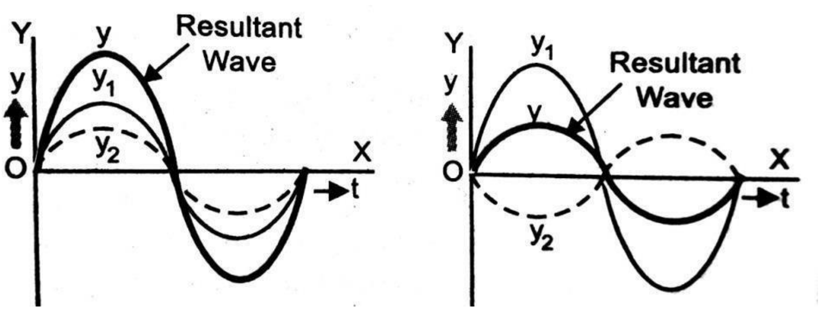
$-$ ve sign is taken when displacements are in opposite direction.

(i) If two light waves are in same phase, the resultant displacement is the sum of displacements of two waves. This is called constructive interference.

i.e., $$\vec{y} = \vec{y_1} + \vec{y_2} \qquad \rightarrow(1.2)$$

(ii) If two light waves are out of phase, the resultant displacement is the difference of displacements of two waves. This is called destructive interference

i.e., $$\vec{y} = \vec{y_1} - \vec{y_2} \qquad \rightarrow(1.3)$$



(a) (b)

Fig.1.1.Superposition of waves

Interference of light

When two or more light waves traveling through a medium superimpose one another then the resultant intensity at different points of the medium undergoes change from point to point. The change in the intensity of light in a medium from point to point is called interference of light.

Thus, the phenomenon of modification or change in uniform redistribution of light energy or intensity or amplitude due to the super position of two or more light waves is called interference.

Types of Interference

There are two methods are used to produce the phenomenon of interference.

1. By Division of Wave front 2. By Division of Amplitude

Division of Wave front

In this method, a single (point or narrow slit) source emits wave fronts. A Wave front is divided into two parts by using mirrors, biprisms or lenses. These two wave fronts travel different distances and are then superimposed to produce interference.

Examples: Young's double slit Fresnel biprism, Fresnel mirrors, Lloyd's mirror, and lasers, etc.

Division of Amplitude

In this method, the amplitude of the incident beam is divided into two or more parts either by partial reflection or refraction. Thus we have coherent beams produced by division of amplitude. These beams travel different paths and are finally brought together to produce interference.

Examples: Interference in thin films, Newton's rings, Michelson's interferometer, and Fabry-Perot's interferometer,etc.

Theory of interference

Let us consider a monochromatic source of light S as shown in Fig.1.2. Let S_1 and S_2 are two narrow slits equidistant from S and they act as coherent sources. The waves emitted from S_1 and S_2 superimpose each other at point P on the screen XY.

Let a_1, a_2 be the amplitudes of the waves from S_1 and S_2.

Let $\emptyset$ be the phase difference the two waves reaching the point P.

Let y_1 and y_2 be the displacement of the two waves, arriving at point P.

$y_1 = a_1 \sin \omega t$ →(1.4)

$y_2 = a_2 \sin (\omega t + \phi)$ →(1.5)

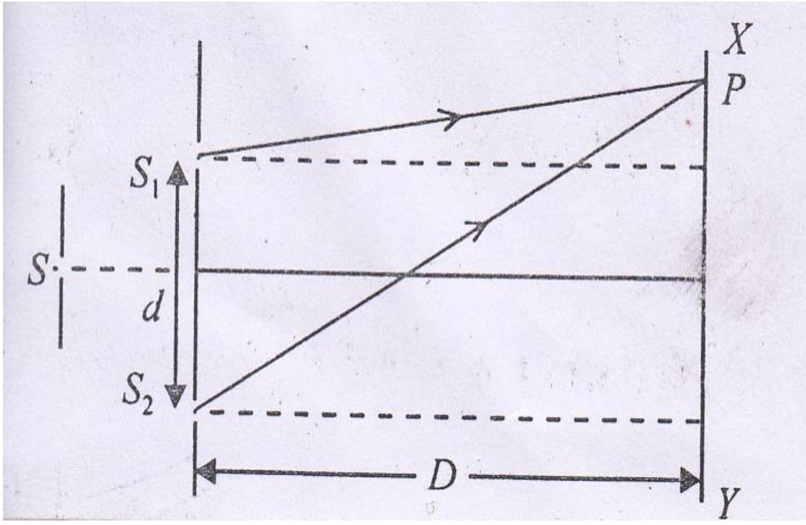


Fig.1.2.Schematic representation of Interference due to two slits.

According to principle of superposition of waves, the resultant displacement of the waves at P is given by;

$y = y_1+y_2 = a_1 \sin \omega t + a_2 \sin (\omega t+\phi)$

$= a_1 \sin \omega t + a_2 \sin \omega t \cos \phi + a_2 \cos \omega t \sin \phi$

$= \sin \omega t\, (a_1 + a_2 \cos \phi) + \cos \omega t\, (a_2 \sin \phi)$ $\longrightarrow$(1.6)

Let $a_1 + a_2 \cos \phi = A \cos\theta$ $\longrightarrow$(1.7)

and $a_2 \sin \phi = A \sin\theta$ $\longrightarrow$(1.8)

Substituting in equation (1.6)

$y = \sin \omega t\, A \cos\theta + \cos \omega t\, A \sin\theta$

$= A \sin (\omega t + \theta)$ $\longrightarrow$(1.9)

Squaring and equations (1.7) and (1.8), we have

$A^2 \cos^2\theta + A^2 \sin^2\theta = (a_1 + a_2 \cos \phi)^2 + a_2^2 \sin^2 \phi$

$A^2 (\cos^2\theta + \sin^2\theta) = a_1^2 + 2a_1 a_2 \cos \phi + a_2^2 \cos^2 \phi + a_2^2 \sin^2 \phi$

$A^2 = a_1^2 + 2a_1 a_2 \cos \phi + a_2^2 (\cos^2 \phi + \sin^2 \phi)$

$A^2 = a_1^2 + 2a_1 a_2 \cos \phi + a_2^2$

The resultant amplitude of waves is

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi} \quad \longrightarrow(1.10)$$

The resultant intensity at point ‘P’ is given by the square of the amplitude

i.e., $I = A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\phi$ $\longrightarrow$(1.11)

Case1. Condition for constructive interference

If the two waves are in phase with each other, then they undergo constructive interference producing maximum intensity of light called bright band or fringe.

The resultant intensity will be maximum when $cos\emptyset = +1$

i.e., $\emptyset$ =0,2π,4π, 6π ... ..

From equation (1.1), $\emptyset = n(2\pi)$ where n= 0,1, 2, 3…

$$I_{max} = a_1^2 + a_2^2 + 2a_1a_2 \quad \longrightarrow(1.12)$$

$$\boxed{I_{max} = (a_1 + a_2)^2} \quad \longrightarrow(1.13)$$

Let $a_1 = a_2 = a$ (if two light waves are coherent

$$I_{max} = I_o = (4a)^2 \quad \longrightarrow(1.14)$$

Therefore, the intensity is maximum when the phase difference is an integral multiple of 2π or path difference is an integral multiple of λ because phase difference of 2π corresponds to a path difference of λ.

Path difference Δ =nλ or
Phase difference $\emptyset$ =2nπ

⟶(1.15)

Case: 2.Condition for destructive interference

If the two waves are out of phase, then they undergo destructive interference producing zero intensity of light called dark band or fringe.

The resultant intensity will be maximum when $\cos\emptyset = -1$

i.e., $\emptyset$ =π,3π, 5π ... ..

$\emptyset = (2n - 1)\pi$

Where n= 1, 2, 3…

From equation (1.1)

$$I_{min} = a_1^2 + a_2^2 - 2a_1a_2 \quad \longrightarrow(1.16)$$

$$\boxed{I_{min} = (a_1 - a_2)^2} \quad \longrightarrow(1.17)$$

Let $a_1 = a_2 = a$ (if two light waves are coherent)

$$I_{min} = 0 \quad \longrightarrow(1.18)$$

Therefore, the intensity is minimum when the phase difference is an odd multiple of π or path difference is an odd multiple of $\lambda/2$ because phase difference of π corresponds to a path difference of $\lambda/2$.

Path difference $\Delta = (2n - 1)\frac{\lambda}{2}$ or
Phase difference $\emptyset = (2n - 1)\pi$

⟶(1.19)

1.5.1. Intensity distribution graph

The variation of intensity (I) with the phase difference ∅ is as shown Fig.1.3.The intensity is maximum at bright bands and is minimum at dark bands. From the graph, it is observed that the intensity is uniformly redistributed in the interference pattern.

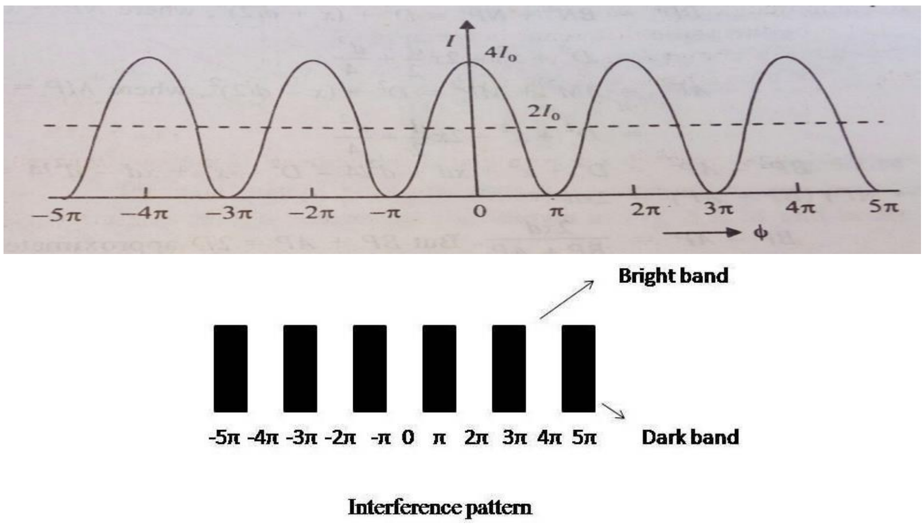


Fig.1.3. Intensity distribution graph

Conditions for sustained interference

To observe a well defined interference pattern, the following conditions must be fulfilled.

a) Conditions for Sustained Interference

i. To produce interference, we require two coherent light sources. i.e., the two sources must emit light of same frequency or wavelength, same amplitude and in the same phase or with a constant phase.

ii. The two sources must be perfectly monochromatic, emitting light of a single wavelength or frequency.

b) Conditions for good visibility or observation

i. The distance between the two sources (2d) must be small.

ii. The distance between the two coherent sources and the screen (D) must be large.

c) Conditions for good contrast

i. The amplitudes of the two waves must be nearly the same or equal.

ii. The two sources must be narrow and parallel.

iii. The two sources must emit light in same direction

iv. To view interference fringes, the background must be dark

Interference in thin films

If a light incident on the thin film, a small part of light is reflected from the top surface and the reaming portion is transmitted into the film. Again, a small part of the transmitted component light is reflected back into the film by the bottom surface and emerges through the top surface. A small portion of the light thus is reflected partially several times in succession within the film. These reflected light waves superimpose with each other, producing interference and forming interference patterns.

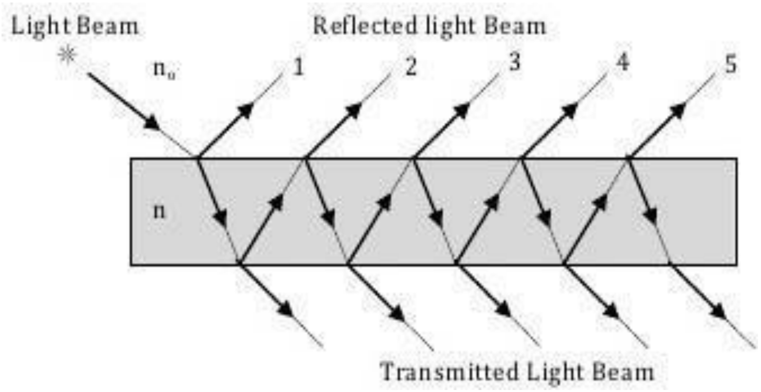


Fig.1.4.Interference in thin films by reflection

Interference in thin films by reflection of light

When light is incident on plane parallel thin film, some portion of light gets reflected from the top surface and the reaming portion is transmitted into the film. Again, some portion of the transmitted light is reflected back into the film by the bottom surface and emerges through the top surface. These reflected light waves superimpose with each other, producing interference and forming interference patterns. This is the principle of interference in thin films by reflection of light.

- Let us consider a thin transparent film of uniform thickness t with refractive index μ and is surrounded by air on both sides. Let the refractive index of the air be μ_{air} as shown in Fig 1.5.

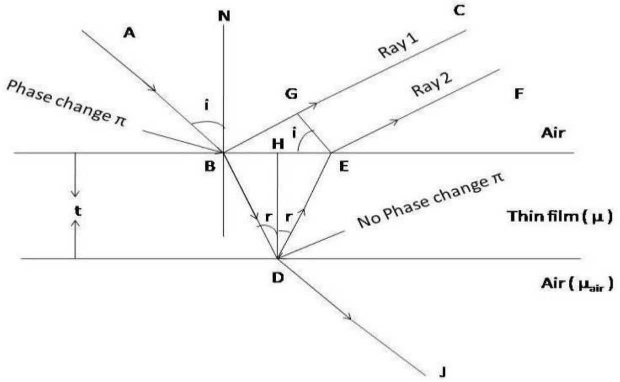


Fig1.5.Interference in thin films by reflection of light

- When a monochromatic light ray AB is incident on the top surface at an angle *i*, it is partly reflected along BC and partly refracted into the film along BD makes an angle ***'r'*** with the normal DH and meets the lower surface.
- At D, it is again partly reflected back into the film along DE and emerges along EF which is parallel to the ray BC. The major portion of light refracts into the outer medium along DJ.
- Since the two reflected light rays BC and EF are derived from the incident ray AB, so they act as coherent light rays. These two coherent light rays superimpose and produce interference patterns. Condition of interference depends on the optical path difference between the rays 1 and 2.
- To calculate let us draw normals from EG to BC and DH to BE and the rays GC and EF equal distances.

- The Optical path difference (OPD**)** between the rays (1) and (2) is

$\Delta = \mu$ x geometrical path BDE in film- μ_{air} x geometrical path BG in air

$= \mu\,(BD+DE) - \mu_{air}\,(BG)$ [Since $\Delta = \mu\, L$ and $\mu_{air} = 1$]

$= \mu\,(BD+DE) - BG$ $\longrightarrow(1.20)$

Step 1: Calculation of geometrical path BDE in film

Let us calculate path BD+ DE in film,

From fig, ΔBDH, $\cos r = \frac{DH}{BD}$

$BD = \frac{t}{\cos r}$ $\longrightarrow(1.21)$

Similarly,

From ΔEDH, $\cos r = \frac{DH}{DE}$

$DE = \frac{t}{\cos r}$ $\longrightarrow(1.22)$

$\therefore BD+DE = \frac{t}{\cos r} + \frac{t}{\cos r}$

$= \frac{2t}{\cos r}$ $\longrightarrow(1.23)$

Step 2: Calculation of geometrical path BG in film

To calculate BG air, first BE which is equal to (BH+HE) has to be obtained.

From ΔBDH, $\tan r = \frac{BH}{DH} = \frac{BH}{t}$

$BH = t.\tan r$ $\longrightarrow(1.24)$

Similarly,

FromΔEDH, $\tan r = \frac{HE}{DH} = \frac{HE}{t}$

$HE = t.\tan r$ $\longrightarrow$(1.25)

$\therefore BE=BH+HE = t.\tan r + t.\tan r$

$= 2t \tan r$ $\longrightarrow$(1.26)

From ΔBGE, $\sin i = \frac{BG}{BE}$

$BG = BE \sin i$

$= 2t \tan r \sin i$ $\longrightarrow$(1.27)

From Snell's law, $\mu = \frac{\sin i}{\sin r}$

$\sin i = \mu \sin r$ $\longrightarrow$(1.28)

From the equations (1.27) and (1.28)

$BG = 2t \tan r\ \mu \sin r$

$= 2\mu t \tan r. \sin r$

$= 2\mu t \frac{\sin r}{\cos r}. \sin r$

$= 2\mu t \frac{\sin^2 r}{\cos r}$ $\longrightarrow$(1.29)

Substituting the equations (1.23) and (1.29) in equation (1.20); we have

$$\Delta = \frac{2\mu t}{\cos r} - 2\mu t \frac{\sin^2 r}{\cos r}$$

$$= \frac{2\mu t}{\cos r}(1 - \sin^2 r)$$

$$= \frac{2\mu t}{\cos r} \cos^2 r$$

$\Delta = 2\mu t \cos r$ $\longrightarrow$(1.30)

This is called Cosine law.

According to stoke principle, if a light wave traveling from rarer medium to denser medium under goes a phase of π or path change $\frac{\lambda}{2}$ (i.e., the wave loses half of wavelength) when it gets reflected at the boundary of a rarer to denser medium.

$$\boxed{\Delta = 2\mu t \cos r - \frac{\lambda}{2}}$$ $\longrightarrow$(1.31)

Case 1. Condition for constructive interference or bright band

Bright band occurs when the path difference $\Delta = n\lambda$ $\longrightarrow(1.32)$

From the above equations (1.31) and (1.32); we have

$$2\mu t \operatorname{Cos} r - \frac{\lambda}{2} = n\lambda$$

$$2\mu t \operatorname{Cos} r = n\lambda + \frac{\lambda}{2}$$

$$2\mu t \operatorname{Cos} r = (2n+1)\frac{\lambda}{2} \quad \longrightarrow(1.33)$$

where n= 0,1, 2, 3…

This is the condition for constructive interference. The film appears bright under this condition.

Case2. Condition for destructive interference or dark band

Dark band occurs when the path difference $\Delta = (2n-1)\frac{\lambda}{2}$ $\longrightarrow(1.34)$

From the above equations (1.31) and (1.34); we have

$$2\mu t \operatorname{Cos} r - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$$

$$2\mu t \operatorname{Cos} r = (2n-1)\frac{\lambda}{2} + \frac{\lambda}{2}$$

$$2\mu t \operatorname{Cos} r = n\lambda \quad \text{where } n=0, 1, 2, 3 \quad \longrightarrow(1.35)$$

This is the condition for destructive interference. The film appears dark under this condition.

Interference in Non-uniform thin films - Newton's Rings

Interference in non-uniform thin films by reflection of light was first experimentally observed by Newton.

Definition

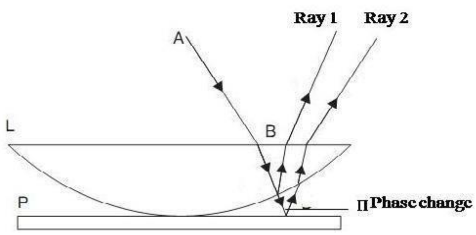
When a Plano-convex lens of large radius of curvature is placed convex surface on a plane glass plate, an air film is formed between the lower surface of the Plano-convex lens and the upper surface of the glass plate. The thickness of the air film is zero at point of contact and gradually increases from the point of contact outwards. If a monochromatic light is allowed to fall normally on this air film, then interference pattern is formed in the form of alternate concentric bright and dark circular rings are formed in the air film due to superposition of reflected light, known as Newton's rings because these rings were discovered by Newton.

Principle

Newton's rings are formed due to interference between the light rays reflected from the top and bottom surfaces of air film between the glass plate and the Plano-convex lens.

Formation of Newton's rings

- The formation of Newton's rings can be explained with help of Fig 1.6.
- When a light ray (AB) is incident on the systems, a part of the light is reflected by the curved surface of the lens (L) and a part is transmitted which is reflected back from the plane surface of the glass plate (P).
- Ray 1 undergoes no phase change but ray 2 under goes a phase of π or path change $\frac{\lambda}{2}$ (i.e., the wave loses half of wavelength) when it gets reflected at the boundary of a rare to denser medium.
- These reflected light rays superimpose with each other producing interference and forming interference and forming interference patterns in the form of bright dark circular rings.



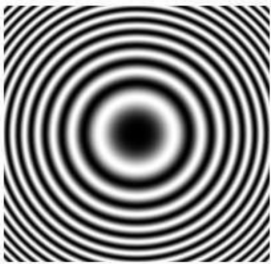


Fig.1.6 (a) Formation of Newton's rings (b) Newton's rings

- We know that, the path difference between two reflected rays in uniform thin film(plane parallel thin film) is

$$\Delta = 2\mu t \cos r - \frac{\lambda}{2} \qquad \longrightarrow (1.36)$$

- *Similarly for non-uniform thin film,*

  Let refractive index of air film $\mu = 1$ and r =0 for normal incidence.

Thus the path difference $\boxed{\Delta = 2t - \frac{\lambda}{2}}$ $\longrightarrow$(1.37)

This is the path difference between two reflected rays in non-uniform thin film.

Case 1. Dark central spot

At the point of contact 'O' of the lens and glass plate, the thickness of air film is approximately is zero.

i.e., $t \approx 0$

$$\therefore \text{The path difference } \Delta = 2t - \frac{\lambda}{2}$$

$$= 2(0) - \frac{\lambda}{2}$$

$$\Delta = -\frac{\lambda}{2} \qquad \longrightarrow (1.38)$$

Thus, at point of contact two waves are out of phase and interference destructively. Hence dark spot is formed at centre.

Case 2.Condition for bright band

Bright band occurs when the path difference Δ = nλ →(1.39)

From equations (1.37) and (1.39), we have

$$2t - \frac{\lambda}{2} = n\lambda$$

$$2t = n\lambda + \frac{\lambda}{2}$$

$$2t = (2n+1)\frac{\lambda}{2} \qquad \longrightarrow (1.40)$$

where n= 0,1, 2, 3…

This is the condition for maxima. The film appears bright under this condition.

Case 3.Condition for dark band

Dark band occurs when the path difference Δ = (2n-1) $\frac{\lambda}{2}$) →(1.41)

From the above equations (1.37) and (1.41), we have

$$2t - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$$

$$2t = (2n-1)\frac{\lambda}{2} + \frac{\lambda}{2}$$

$$2t = n\lambda \qquad \longrightarrow (1.42)$$

where n=0, 1, 2, 3…

This is the condition for minima. The film appears dark under this condition.

Experimental Arrangement

- The experimental arrangement for producing Newton rings is as shown in Fig 1.7.
- Keep the convex surface of the Plano-convex lens over the plane glass plate and arrange glass plate G at an angle of 45^0 over the base set.
- Switch on the monochromatic light source 'S' (Sodium vapor lamp) and it is focus on the Double convex lens (L). This sends parallel beam of light. This beam of light falls on the glass plate B at 45^0.
- The glass plate 'G' reflects a part of light towards the air film enclosed by the Plano-convex lens and the plane glass plate.
- A part of the light is reflected by the curved surface of the Plano-convex lens (P) and a part is transmitted which is reflected back from the plane surface of the plane glass plate (E).
- These reflected light rays superimpose with each other producing interference and forming interference and forming interference patterns in the form of bright dark circular rings.
- These rings are seen with a microscope (M) focused on the air film

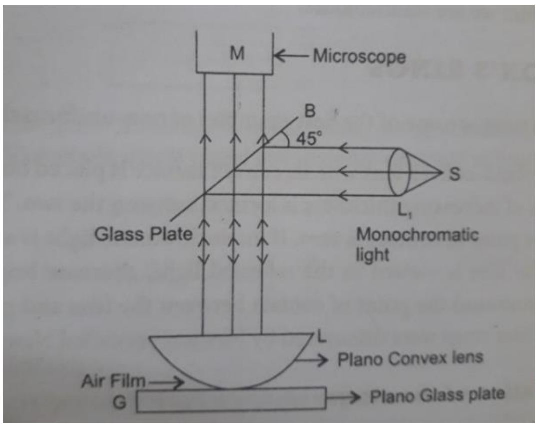


Fig.1.7.The experimental arrangement of Newton rings

Theory of Newton's rings

To find the diameters (radii) of dark and bight rings, let 'L' be a Plano convex lens placed on a glass plate P. The convex surface of the lens is the part of spherical surface with centre at C as shown in Fig.1.8.

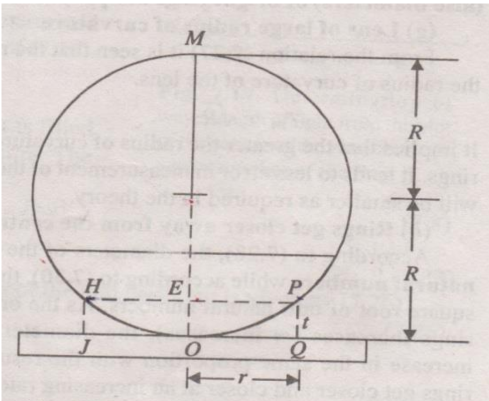


Fig.1.8.Theory of Newton's rings

Case1. Diameters (Radii) of the dark rings

Let 'R' be the radius of curvature of the lens.

Let a dark be located at the point Q is PQ= t.

The radius of the ring at Q is OQ= r.

By the theorem of intersecting chords,

$$EP \times HE = OE \times EM \qquad \longrightarrow(1.43)$$

But EP=OQ=HE= r; OE=PQ= t

and EM= OM-OE =2R-t

From equation (1.43)

$$r \times r = t \times (2R-t)$$

$$\text{(Or)} \qquad r^2 = 2Rt-t^2$$

As $2Rt \gg t^2$; t^2 can be neglected.

$$r^2 = 2Rt$$

Thus, the radius of the nth dark ring will be given by $r_n^2 = 2Rt \qquad \longrightarrow (1.44)$

We know that, the condition of the dark ring is $2t = n\lambda \qquad \longrightarrow (1.45)$

From the above equations (1.44) and (1.45)

$$r_n^2 = n\lambda R$$

$$r_n = \sqrt{n\lambda R} \qquad \longrightarrow (1.46)$$

This is the condition for radii of the dark rings.

The diameter of the dark ring is therefore given by $D_n = 2r_n$

$$D_n = 2\sqrt{n\lambda R} \qquad \longrightarrow (1.47)$$

∴ The radii (diameters) of the dark rings is directly proportional to (i) $\sqrt{n(\text{ natural nuber})}$

(ii) $\sqrt{\lambda}$

(iii) $\sqrt{R}$

Case2. Diameters (Radii) of the bright rings

Let us now suppose that a bright ring be located at the point Q.

Therefore, the radius of the nth bright ring will be given by $r_n^2 = 2Rt \qquad \longrightarrow (1.48)$

We know that, the condition of the bright band is $2t = (2n+1)\frac{\lambda}{2} \qquad \longrightarrow (1.49)$

From the above equations (1.48) and (1.49)

$$r_n^2 = (2n+1)\frac{\lambda}{2}R = (n+\frac{1}{2})\lambda R$$

$$r_n = \sqrt{(n+\frac{1}{2})\lambda R} \qquad \longrightarrow (1.50)$$

This is the condition for radii of the bright rings.

The diameter of the bright ring is therefore given by $D_n = 2r_n$

$$D_n = 2\sqrt{(n+\frac{1}{2})\lambda R} \qquad \longrightarrow (1.51)$$

∴ The radii (diameters) of the bright rings is directly proportional to (i) $\sqrt{n+\frac{1}{2}}$

(ii) $\sqrt{\lambda}$

(iii) $\sqrt{R}$

Applications

The theory of Newton's rings can be used

(i) To determine the wavelength of monochromatic light

(ii) To determine the radius of curvature of Plano-convex lens and

(iii) To determine the refractive index of a given liquid.

Determination of wavelength of monochromatic light source.

Let 'R' be the radius of curvature of a Plano-convex lens, λ be the wavelength of monochromatic light source.

Let D_m and D_n are the diameters of m^{th} and n^{th} dark rings respectively.

Then,
$$D^2_m = 4m\lambda R,$$
$$D^2_n = 4n\lambda R,$$
and
$$D^2_n - D^2_m = 4n\lambda R - 4m\lambda R$$
$$= 4(n-m)\lambda R$$

$$\boxed{\lambda = \frac{D_n^2 - D_m^2}{4(n-m)R}} \longrightarrow (1.52)$$

A graph is drawn with the number of rings on the - axis and the square of the diameter of the ring (D^2) on the y-axis. The graph is straight line passing through the origin as shown in Fig.1.9. Form the graph the values of D_m^2 and D_n^2 corresponding to n^{th} and m^{th} rings are found. From the graph, the slope is calculated.

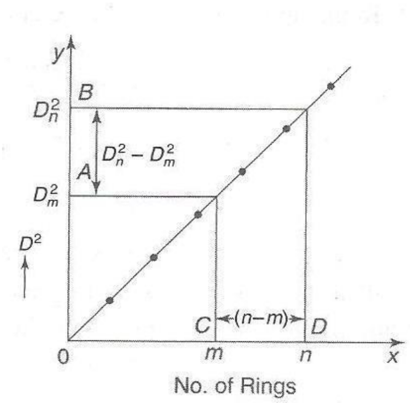


Fig.1.9. A graph between (D^2) and number of rings

From graph, the slope

$$\frac{D_n^2 - D_m^2}{(n-m)} = \frac{AB}{CD} \longrightarrow (1.53)$$

The radius of curvature 'R' of the Plano-convex lens is found by Boy's method or spherometer. Substituting the values of R and $D^2_n - D^2_m$ in the above formula (1.53) λ can be calculated.

Determination of refractive index of a given liquid

To find the refractive index of a given liquid, the Plano convex lens and glass plate set up is placed in a small container 'C'. The transparent liquid of refractive index 'μ'is introduced between the lens L and the glass Plate 'G' as shown as Fig 1.10. Then a film of liquid formed between the lens and glass plate. The diameters of m^{th} and n^{th} dark rings are determined with help of travelling microscope.

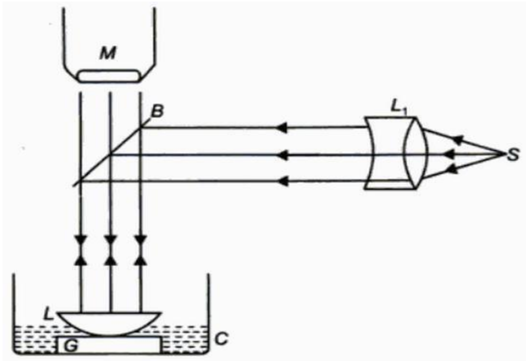


Fig.1.10.Determination of refractive index of a given liquid

We know that,

The diameters of m^{th} and n^{th} dark rings in the air film are given by

$$D^2_m = 4m\lambda R,$$

$$D^2_n = 4n\lambda R,$$

$$\text{and } D^2_n - D^2_m = 4n\lambda R - 4m\lambda R$$

$$= 4(n-m)\lambda R \qquad \longrightarrow (1.54)$$

With liquid film, the diameters of m^{th} and n^{th} dark rings are determined;

$$D^2_m{}^I = \frac{4m\lambda R}{\mu}$$

$$D^2_n{}^I = \frac{4n\lambda R}{\mu}$$

$$\text{and } D^2_n{}^I - D^2_m{}^I = \frac{4n\lambda R}{\mu} - \frac{4m\lambda R}{\mu}$$

$$= \frac{4\lambda R(n-m)}{\mu} \qquad \longrightarrow (1.55)$$

From the above equations (1.54) and (1.55), $D^2_n{}^I - D^2_m{}^I = \frac{D^2_n - D^2_m}{\mu}$

$$\mu = \frac{D^2_n - D^2_m}{D^2_n{}^I - D^2_m{}^I}$$

. Using the above formula μ can be calculated.

Applications of Interference

The phenomenon of interference can be used in various applications; some of the applications are given below.

i. It is used in anti-reflecting coatings.
ii. It is used to determination of thickness of a thin film coating.
iii. It is used in testing of flatness of surfaces.
iv. It is used in interference filters to transmit a very narrow range of wavelengths.

DIFFRACTION

Introduction

Diffraction is one of the natural phenomena. The effect of diffraction is usually seen in everyday life. For example, the iridescent colors of peacock feathers, butterfly wings and some other insects, rainbowlike diffraction are produced by CD OR DVD.

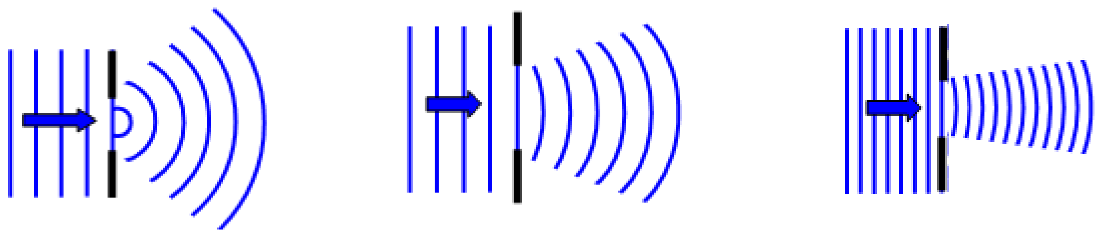
The English word diffraction was coined by the *diffractus* which means to spread out or to break out. This phenomenon was first discovered by Francesco Grimaldi.

When light waves encounter an obstacle they bend at the corners or edges of the obstacle and spreading into the geometrical shadow of an obstacle. This phenomenon is known as diffraction.

(or)

The phenomenon of redistribution of light intensity due to the superposition of secondary wavelets from the same wavefront is called diffraction.

The effect of diffraction depends on the size of the obstacle (d) and the wavelength (λ) of light wave as illustrated in Figure 1.10. Light waves are very small in wavelength, i.e., from 4×10^{-7} m to 7×10^{-7} m. If the size of opening or obstacle is near to this limit, only then we can observe the phenomenon of diffraction.



(a) $d \approx \lambda$ (b) $d > \lambda$ (c) $\lambda \ll d$

Figure 1.11. Illustration of diffraction phenomenon

From Fig.1.11, it is observed that

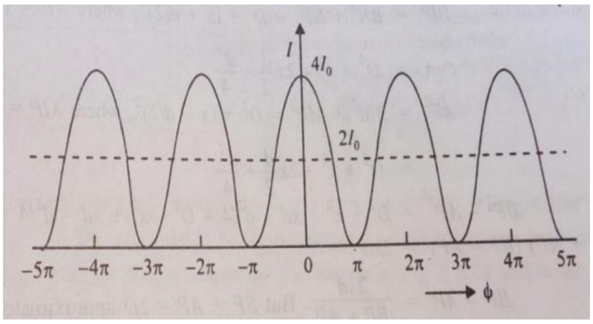
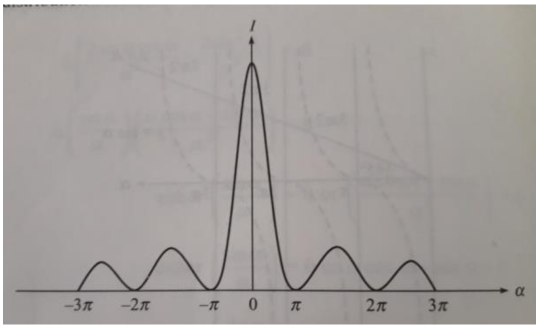
i. The effect of diffraction is more noticeable if the size of the obstacle is equal to the wavelength of light $(i.e. d \approx \lambda)$.
ii. The effect of diffraction is not noticeable if the size of the obstacle is greater than the wavelength of light $(i.e. d > \lambda)$.
iii. The effect of diffraction is less noticeable if the size of the obstacle is smaller than the wavelength of light $(i.e. \lambda \ll d)$.

Conditions for diffraction

To observe diffraction

i. Light must be monochromatic and
ii. The wavelength of light must be comparable to the obstacle. $(\lambda \approx d)$

Differences between interference and diffraction

S.No	Interference	Diffraction
1	It is arises due to the superposition of waves from the two coherent sources.	It is arises due to the superposition of secondary wavelets from the different parts of the same wavefront.
2	It is formed due to the uniform distribution of light intensity.	It is formed due to the non uniform distribution of light intensity.
3	In interference pattern, all bright fringes are of the same intensity	In diffraction pattern, all bright fringes are not of the same intensity
4	The intensity of all dark fringes is zero.	The intensity of all dark fringes is no zero.
5	In interference pattern, the width of fringes is equal.	In diffraction pattern, the width of fringes is not equal.
6	The intensity distribution graph of interference as shown in Fig.1.13.  	The intensity distribution graph of diffraction as shown in Fig1.14. 

Types of Diffraction

There are two types of diffraction.

a) Fresnel diffraction and
b) Fraunhofer diffraction.

(a) Fresnel diffraction

- In this class of diffraction, the source of light and the screen are finite from the diffraction slit (or) aperture.
- No double convex lenses (or) mirrors are used to observe diffraction effect.
- The diffraction pattern is the image of the slit.
- The centre of diffraction pattern is either bright (or) dark.
- The incident wave front is spherical (or) cylindrical.
- It is very easy to observe experimentally and complex mathematically.

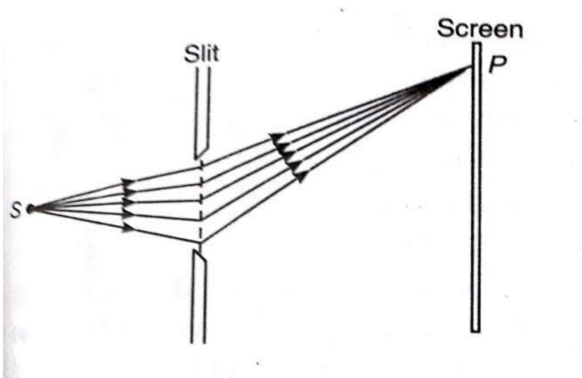


Fig.1.12. Fresnel diffraction

(b) Fraunhofer diffraction

- In this class of diffraction, the source of light and the screen are infinite distance from the diffraction slit (or) aperture.
- Two double convex lenses (or) mirrors are used to observe diffraction effect.
- The incident wave front is plane.
- The diffraction pattern is the image of source of itself.
- The centre of the diffraction pattern is bright.
- It is very simple and easy to observe both experimentally and mathematically.

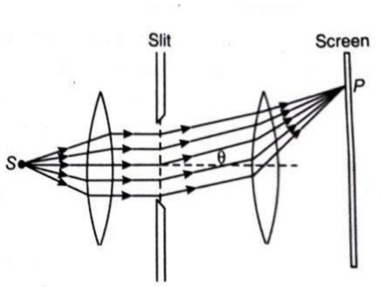


Fig.1.13.Fraunhofer diffraction

.Fraunhofer diffraction due to single slit

The arrangement of fraunhofer diffraction due to single slit as shown in Fig 1.14.

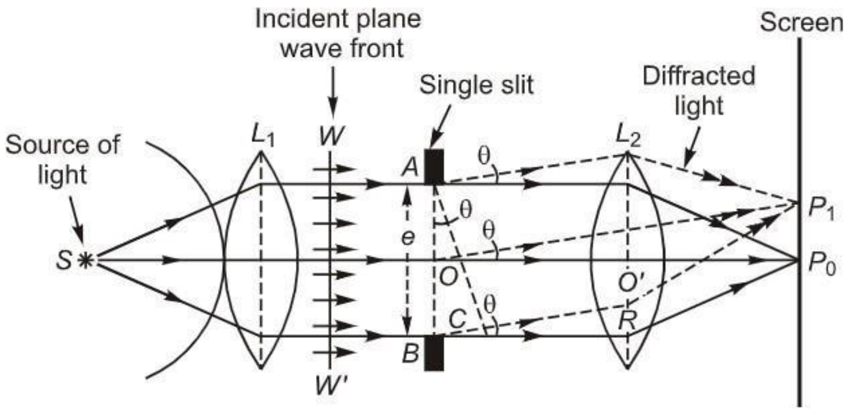


Fig 1.14.Fraunhofer diffraction due to single slit

- In Fig S is a point source of monochromatic light with wave length λ and AB is the single slit of width "e"
- Let WW' be the plan wave front and is incident on the single slit AB.
- According to Hygen's principle, each point on wave front is a source of fresh disturbance called secondary wavelet that spread out to right angles in all directions.
  Without diffraction $(\theta = 0)$
- Let us consider the wave front WW' travelling along OP_0 through the slit AB without diffraction and focus at P_0 by using lens L_2. Since all these wave fronts have same phase and no path difference between them. Therefore, they undergo constructive interference and producing maximum intensity called central maximum or principal maximum.

With diffraction ($\theta \neq 0$)

- Let us consider the plain wave front WW'(secondary wave lets) travelling along OP_1 and diffract at an angle θ and focused at P_1 . The intensity at P_1 either maximum or minimum depending up on the path difference between the two wavelets (A and B). To calculate path difference let us draw a normal AC to BR as shown in fig.

From fig, ΔABC,

$$\text{Sin}\,\theta = \frac{BC}{AB}$$

$$BC = AB\ \text{Sin}\theta$$

$$\therefore \text{The Path difference } BC = e\ \text{Sin}\theta \qquad \longrightarrow (1.56)$$

And corresponding phase difference $= \frac{2\pi}{\lambda} \times$ path difference

$$= \frac{2\pi}{\lambda} \times e\sin\theta \qquad \longrightarrow (1.57)$$

Let us consider the width of slit AB is divided into n parts and amplitude of each part is "a."
Therefore, the phase difference between any two successive parts is

$$\frac{1}{n} \times \text{phase difference} = \frac{1}{n} \times \text{path difference}\,e$$

$$\frac{1}{n} \times \text{phase difference} = \frac{1}{n}\left[\frac{2\pi}{\lambda} \times e\sin\theta\right] = d \qquad \longrightarrow (1.58)$$

By using the method of vector addition of amplitudes,

The resultant amplitude is given by $R = \frac{a\ \text{Sin}\frac{nd}{2}}{\text{Sin}\frac{d}{2}}$ $\longrightarrow$ (1.59)

Substituting equation (1.58) in equation (1.59)

$$R = \frac{a\ \text{Sin}\, n \times \frac{1}{n}\left(\frac{\frac{2\pi}{\lambda} \times e\text{Sin}\theta}{2}\right)}{\text{Sin} \times \frac{1}{n}\left(\frac{\frac{2\pi}{\lambda} \times e\text{Sin}\theta}{2}\right)}$$

$$= \frac{a\ \text{Sin}\left(\frac{\pi e\ \text{Sin}\theta}{\lambda}\right)}{\text{Sin}\left(\frac{\pi e\ \text{Sin}\theta}{n\lambda}\right)}$$

$$R = \frac{a\ \text{Sin}\alpha}{\text{Sin}\frac{\alpha}{n}}$$

$$\text{where } \alpha = \frac{\pi e \sin\theta}{\lambda} \qquad \longrightarrow(1.60)$$

Since $\text{Sin}\frac{\alpha}{n}$ is very small , $\text{Sin}\frac{\alpha}{n} \simeq \frac{\alpha}{n}$

$$= \frac{a\,\text{Sin}\alpha}{\frac{\alpha}{n}}$$

$$= \frac{na\,\text{Sin}\alpha}{\alpha} \qquad [\because na = A]$$

$$\boxed{R = A\left(\frac{\sin\alpha}{\alpha}\right)} \qquad \longrightarrow(1.61)$$

This equation represents the resultant amplitude of the wave fronts at P_1.

And intensity $I = R^2$

$$I = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2$$

$$\boxed{I = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2} \qquad \longrightarrow(1.62)$$

Case 1.Principle maximum or Central maximum at P_0

The resultant amplitude (R) can be expressed in ascending power of α,

$$R = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \ldots\ldots\ldots\ldots\right]$$

$$= \frac{A}{\alpha} \times \alpha\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \ldots\ldots\ldots.\right] \qquad \longrightarrow(1.63)$$

Intensity at P_0 will be maximum when the values of R is maximum.

For maximum value of R the negative terms must be vanish in equation (1.63)

$$\text{i.e., } \alpha = 0 \qquad \text{[From the equation (1.60)]}$$

$$\frac{\pi e\,\text{Sin}\theta}{\lambda} = 0$$

$$\pi e\,\text{Sin}\theta = 0$$

$$\text{Sin}\theta = 0$$

$$\boxed{\theta = 0}$$

Thus we can say that, the condition $\theta=0$ means that the wave fronts passing through the slit AB without diffraction and focus at P_0.Since all these wave fronts have same phase and no path difference between them. Therefore, they undergo constructive interference and producing maximum intensity called central maximum or principal maximum (i.e., $I_{max} = R^2 = A^2 = I_0$).

Case.2.Minimum intensity positions

Intensity I will be minimum when Sinα = 0.

$$\text{i.e.,} \ \alpha = \pm m\pi$$

Where m = 1, 2, 3 …

$$\frac{\pi e \sin\theta}{\lambda} = \pm m\pi$$

$$\boxed{e\sin\theta = \pm m\lambda} \qquad \longrightarrow(1.64)$$

This is the condition for minimum intensity. Thus, we obtain minimum intensity positions either side of the principle maximum.

Case.3.Maximum intensity positions (or) Secondary maximum

In addition to principle maximum, there exist week secondary maximum between equally spaced minima. These secondary maximum intensity positions can be obtained by differentiating the expression of intensity with respect to "α" and equating to zero.

$$I = A^2 \left(\frac{\sin\alpha}{\alpha}\right)^2$$

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left[A^2 \left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$$

$$= A^2 . \frac{d}{d\alpha}\left(\frac{\sin\alpha}{\alpha}\right)^2 = 0 \qquad \left[\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}\right]$$

$$= A^2 . \frac{2\sin\alpha}{\alpha} . \frac{d}{d\alpha}\left(\frac{\sin\alpha}{\alpha}\right) = 0$$

$$= A^2 . \frac{2\sin\alpha}{\alpha} \times \frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0 \qquad \longrightarrow(1.65)$$

In the above equation (1.65) either Sinα=0 (or) α Cosα−sinα=0

We know that, Sinα = 0 gives the minimum intensity positions. Thus αcosα−sinα=0 gives the maximum intensity positions.

i. e, α cosα−sinα=0

α cosα= sinα

$$\boxed{\alpha = \tan\alpha} \qquad \longrightarrow(1.66)$$

The value of α satisfying the above equations are obtained graphically by plotting the curves y=α and y= tanα on the same graph as shown in Fig 1.15.

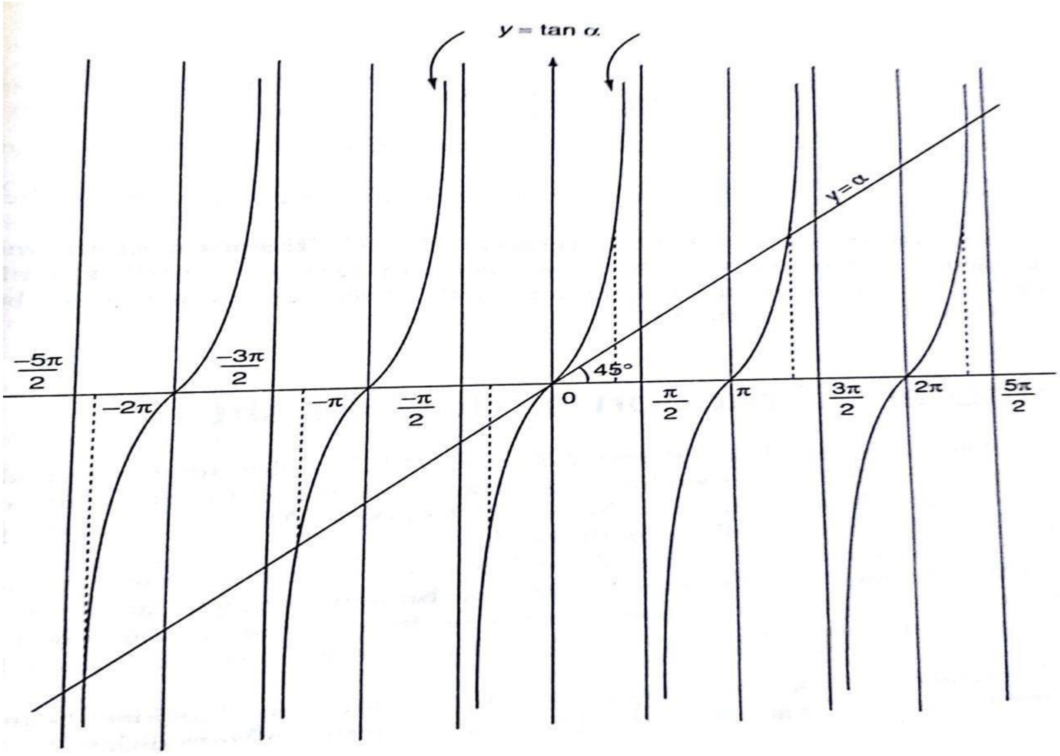


Fig 1.15.The plots of y=α and y= tanα

The points of intersections of the two curves gives the value of α which satisfying the above equation(1.66). The points of intersections are α= 0, $\pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}$……………

The first value of α gives the principle maximum and remaining values of α give the secondary maxima.

The values are α= $\pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}$……………

$$\alpha = \pm(2m+1)\frac{\pi}{2} \qquad \longrightarrow(1.67)$$

where m = 1,2,3,4…………..

∴ From the equations (1.60) & (1.67)

$$\frac{\pi e \sin\theta}{\lambda} = \pm(2m+1)\frac{\pi}{2}$$

$$\boxed{e \sin\theta = \pm(2m+1)\frac{\lambda}{2}} \qquad \longrightarrow (1.68)$$

This is the condition for maxima.

(i) For the principle maximum α = 0

$$\therefore \text{The intensity I} = R^2 = A^2 = \left(\frac{\sin\alpha}{\alpha}\right)^2$$

$$I = R^2 = A^2 = I_0$$

(ii) For the secondary maxima $\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}$……………

The intensity of first secondary maxima, $\alpha = \pm\frac{3\pi}{2}$

$$\therefore \text{Intensity } (I_1) = A^2\left(\frac{\sin\left(\frac{3\pi}{2}\right)}{\frac{3\pi}{2}}\right)^2$$

$$= \frac{4}{9}\pi^2 . A^2$$

$$= \frac{A^2}{22}$$

$$I_1 = \frac{I_0}{22}$$

$$\frac{I_1}{I_0} = \frac{1}{22} = 4.5\%$$

∴ The intensity of first secondary maxima is about $\frac{1}{22}$ of intensity of the principle maximum.

$$\boxed{i.e., I_1 \text{ is } 4.5\% \text{ of } I_0}$$

The intensity of second secondary maxima $\alpha = \pm\frac{5\pi}{2}$

$$I_2 = A^2\left(\frac{\sin\frac{5\pi}{2}}{\frac{5\pi}{2}}\right)^2$$

$$= A^2\left(\frac{4}{25\pi^2}\right)$$

$$I_2 = \frac{A^2}{62} = \frac{I_0}{62}$$

$$\frac{I_2}{I_0} = \frac{1}{62} = 1.61\%$$

∴ The intensity of second secondary maxima is about $\frac{1}{62}$ of intensity of princiole maxima.

$$\boxed{i.e., I_2 \text{ is } 1.61\% \text{ of } I_0}$$

∴ In the same way the intensity of secondary maxima goes on decreasing very rapidly.

Intensity distribution graph

A graph is plotted between intensity of light (I) and α is shown in Fig1.16.

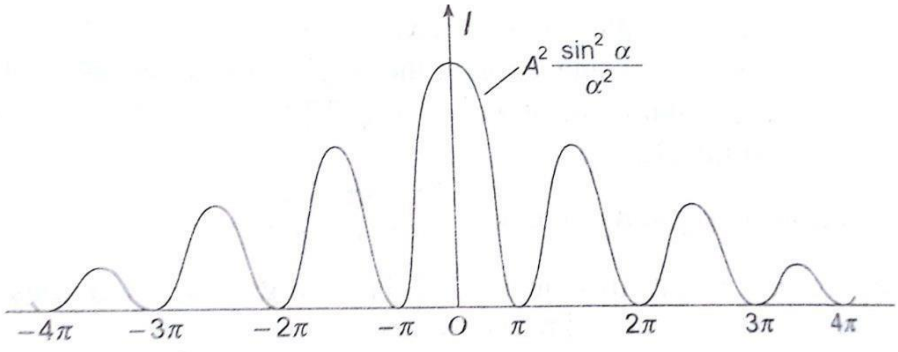


Fig1.16.The intensity of diffraction pattern due to single slit.

From the graph the following conclusions can be observed,

- At α=0, the intensity is very high called principle maximum.
- At $\alpha=\pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}$........................., the intensity is maximum called secondary maxima.
- At $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi$ ....,the intensity is minimum.
- The most of the light is concentrated in the principle maximum & the intensity of secondary maximum decreasing on either sides of principle maximum as shown in Fig 1.16.

Fraunhofer diffraction due to double slit

The arrangement of fraunhofer diffraction due to double slit as shown in Fig 1.17.

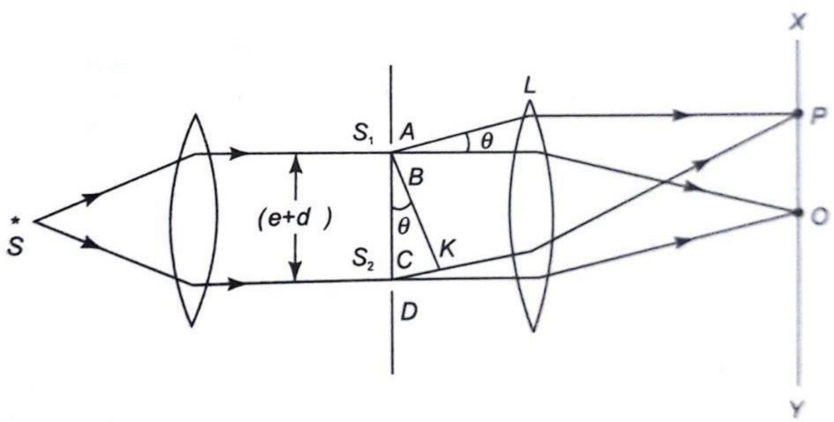


Fig 1.17.Fraunhofer diffraction due to double slit

- In Figure1.16, S is a point source of monochromatic light with wavelength λ and s_1 and s_2 are the two slits of same widthe "e" and separated by a distance 'd'.
- Let WW' be the plan wave front and is incident on the two slits s_1 and s_2.

- According to Hygen's principle, each point on wave front is a source of fresh disturbance called secondary wavelet that spread out to right angles in all directions.

  Without diffraction ($\theta = 0$)
- Let us consider the plane wave front ww' travelling along OP_0, through the slits s_1 and s_2 without diffraction ($\theta = 0^0$) and focused at P_0 by using lens L_2.Since all these wave fronts have same phase and no path difference between them .So they under go constructive interference and producing maximum intensity at P_0 called central maximum or principle maximum.

  With diffraction ($\theta \neq 0$)
- Let us consider the plain wave front ww' (secondary wavelets) travelling along OP_1 through the slits s_1 and s_2 and diffract at an angle θ with respect to the normal and focused at P_1.The intensity at P_1 is the combination of both interference and diffraction.
- *Interference:* It is due to the super position of secondary diffracted wavelets from the two slits s_1 and s_2.
- *Diffraction:* It is due to the single slit, the intensity at P_1 either maximum or minimum depending up on the path difference between secondary wavelets s_1 and s_2.

From Δ s_1s_2k,

$$\sin\theta = \frac{s_2k}{s_1s_2}$$

$$s_1s_2sin\theta = s_2k \qquad \longrightarrow(1.69)$$

∴ The path difference $s_2k = (e+d)sin\theta$ and corresponding phase difference

$$\delta = \frac{2\pi}{\lambda}(e+d)sin\theta \qquad \longrightarrow(1.70)$$

To find out resultant amplitude at P_1 we use vector addition method (as shown in figure) in which two sides of a triangle are represented by the amplitudes of s_1 and s_2 and third side gives the resultant amplitude.

According to vector addition method,

From the figure,

$$(OH)^2 = (OG)^2 + (GH)^2 + 2(OG)(GH)Cos\,\delta$$

$$R^2 = A^2\left(\frac{Sin\,\alpha}{\alpha}\right)^2 + A^2\left(\frac{Sin\,\alpha}{\alpha}\right)^2 + 2.A\left(\frac{sin\alpha}{\alpha}\right).A\left(\frac{sin\alpha}{\alpha}\right)Cos\,\delta$$

$$= 2A^2\left(\frac{Sin\,\alpha}{\alpha}\right)^2 + 2A^2\left(\frac{Sin\,\alpha}{\alpha}\right)^2 Cos\,\delta$$

$$= 2A^2\left(\frac{Sin\,\alpha}{\alpha}\right)^2[1 + Cos\,\delta\,]$$

$$= 2A^2 \left(\frac{Sin\,\alpha}{\alpha}\right)^2 2Cos^2\frac{\delta}{2}$$

$$= 4A^2 \left(\frac{Sin\,\alpha}{\alpha}\right)^2 Cos^2\frac{\delta}{2}$$

$$= 4A^2 \left(\frac{Sin\,\alpha}{\alpha}\right)^2 Cos^2\left(\frac{\frac{2\pi}{\lambda}(e+d)sin\theta}{2}\right)$$

$$= 4A^2 \left(\frac{SIN\alpha}{\alpha}\right)^2 Cos^2\frac{\pi}{\lambda}(e+d)sin\theta$$

$$\boxed{R^2 = 4A^2 \left(\frac{SIN\alpha}{\alpha}\right)^2 Cos^2\beta} \qquad \longrightarrow(1.71)$$

Where $\beta = \frac{\pi}{\lambda}(e+d)sin\theta$ $\longrightarrow$(1.72)

The above equation (1.71) represents the resultant amplitude of the wave fronts S_1 and S_2 at P_1.

And intensity I = R^2

$$\boxed{I = R^2 = 4A^2 \left(\frac{SIN\alpha}{\alpha}\right)^2 Cos^2\beta} \qquad \longrightarrow(1.73)$$

From the above equation (1.73), we can say that the resultant intensity (I) at P_1 is the product of two factors.

(i) $A^2 \left(\frac{SIN\alpha}{\alpha}\right)^2$ which represents the diffraction pattern due to a single slit.

(ii) $Cos^2\beta$ which represents the interference pattern due to the super position of secondary diffracted wavelets from two slits s_1 and s_2 .

Therefore, the resultant intensity is the combination of both diffraction and interference effects.

Diffraction effect

In the above equation (1.73), $A^2 \left(\frac{SIN\alpha}{\alpha}\right)^2$ gives the principle maximum at the centre of diffraction pattern with alternative minima and secondary maxima positions on the either side of it (Fig.1.18).

Case (i) The principle maximum occurs at $\alpha = 0$

$$\boxed{\text{i.e., } I = R^2 = A^2 = I_0}$$

Case (ii) The minimum intensity positions occurs at

$$\alpha = \pm m\pi \text{ , where m=1, 2, 3,4.........}$$

$$\frac{\pi e sin\theta}{\lambda} = \pm m\pi$$

$$\boxed{\text{e sin}\theta == \pm m\lambda} \longrightarrow(1.74)$$

Case (iii) The secondary maxima positions occurs at

$$\alpha = \pm(2m + 1)\frac{\pi}{2} \text{ , where m=1,2,3,4.......}$$

$$\frac{\pi e sin\theta}{\lambda} = \pm(2m + 1)\frac{\pi}{2}$$

$$\boxed{\text{e sin}\theta = \pm(2m + 1)\frac{\lambda}{2}} \longrightarrow(1.75)$$

Interference effect

In the above equation (1.73), $Cos^2\beta$ gives the alternative maximum and minimum intensity positions with equal magnitude of intensity.

Case (i) The Minimum intensity positions will occur at

$$Cos^2\beta = 0 \text{ When } \beta = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2} \dots\dots\dots\dots$$

$$\therefore \beta = \pm(2m + 1)\frac{\pi}{2} \longrightarrow(1.76)$$

where m= 0,1,2,3,4........

But $\beta = \frac{\pi}{\lambda}(e + d)sin\theta$ $\longrightarrow$(1.77)

From the equations (1.76) & (1.77)

$$\frac{\pi}{\lambda}(e + d)sin\theta = \pm(2m + 1)\frac{\pi}{2}$$

$$\boxed{(e + d)sin\theta = \pm(2m + 1)\frac{\lambda}{2}} \longrightarrow(1.78)$$

Case (ii) The maximum intensity positions will occur at

$$Cos^2\beta = 1 \text{ When } \beta = 0, \pm\pi, \pm2\pi, \pm3\pi, \pm4\pi, \dots\dots\dots\dots \pm m\pi$$

$\therefore \beta = \pm m\pi$ $\longrightarrow$(1.79)

But $\beta = \frac{\pi}{\lambda}(e + d)sin\theta$ $\longrightarrow$(1.80)

From the equations (1.79)&(1.80)

$$\frac{\pi(e+d)sin\theta}{\lambda} = \pm m\pi$$

$$\boxed{(e + d)sin\theta = \pm m\lambda} \longrightarrow(1.81)$$

Intensity distribution graph

Fig 1.18 shows the resultant intensity distribution due to the combination of both diffraction and interference effects. From the graph, it is observed that the resultant minima are not equal to zero; still they have some minimum intensity due to interference effect.

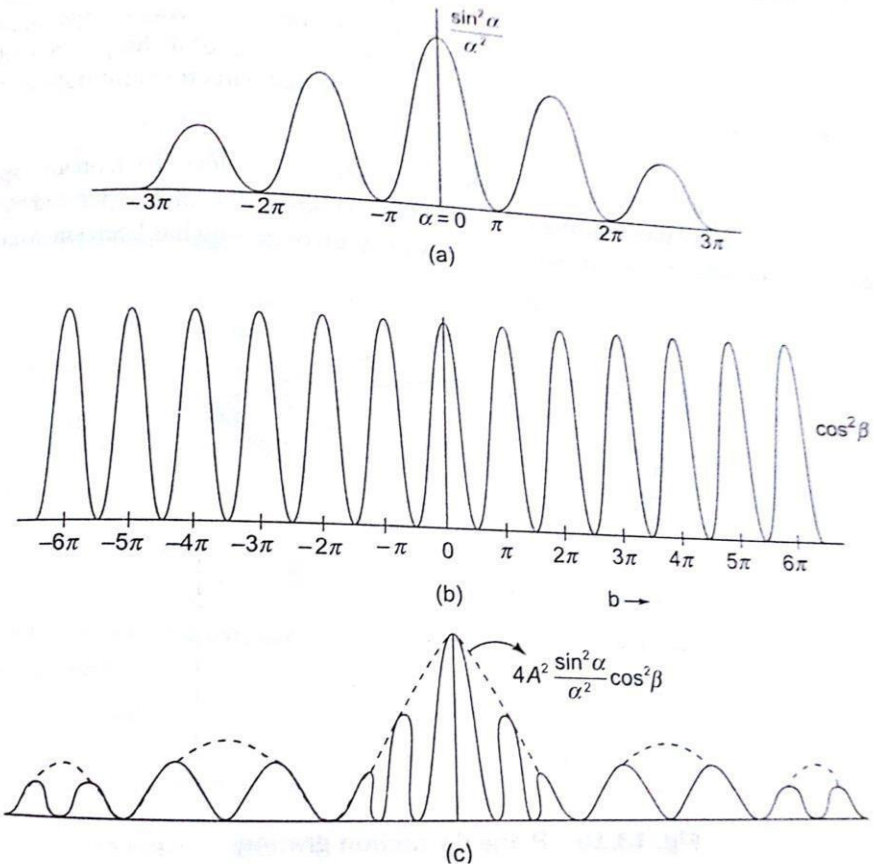


Fig.1.18.Intensity distribution due to diffraction double slit

Diffraction Grating

A diffraction grating is an optical device consisting of a large number of equidistant narrow parallel, rectangular slits of equal width and separated by opaque portions. This was first constructed by the German physicist Joseph von Fraunhofer in 1821.

Construction

- A diffraction grating is prepared by ruling a large number of equidistant narrow, parallel lines on an optically plane glass plate with a fine diamond point.
- The ruled lines are opaque to light called opaque portions while the space between any two successive ruled lines is transparent to light called transparent portions and act as slits.
- The distance between any two successive slits (or) ruled lines is called grating element as shown in Fig.1.19.

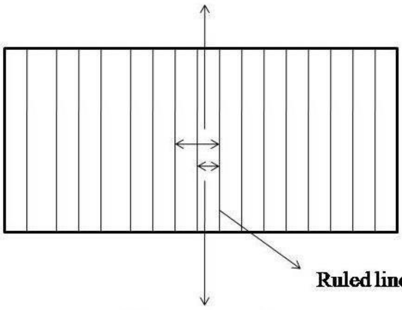


Fig.1.19. Construction of diffraction grating

- The Commercial gratings are made up a thin layer of colloidal solution (ex: Solution of cellulose Acetate) is poured on the ruled surface and allowed to dry to form a thin film.
- The colloidal thin film is peeled carefully from the ruled surface. The film retains impression of the ruling of ruled surface.
- The ruled lines acts as opaque portions where as the space between them act as transparent portions which transmit incident light. The film is mounted between two glass plates called grating.
- When the light is incident on the grating surface then light is transmitted through transmission portions or slits and obstructed by the opaque portions or ruled lines. Such a grating is known as transmission grating.

Theory

Let' e' be the width of each ruled line (or) opaque portion and 'd' be the width of transparent portion (or) slit then (e + d) represents the grating element. The relation between grating element and the angles of diffracted beams of light is known as grating equation. It is given by

$$(e+d)sin\theta = n\lambda, \quad \text{Where n = 0, 1, 2, 3....} \qquad \longrightarrow(1.82)$$

This expression is known as grating equation.

$$sin\theta = \left(\frac{1}{e+d}\right) n\lambda$$

$$sin\theta = \text{N}\, n\lambda$$

Where $\frac{1}{e+d}$ = N is the number of grating elements or lines per unit width of the grating.

$$\text{N}\,(e+d) = \text{1inch} = 2.54 \text{ cm}$$

$$e+d = \frac{2.54}{\text{N}} \text{ cm} \qquad \longrightarrow(1.83)$$

Grating Spectrum

The diffraction pattern formed with a grating is known as grating spectrum.

The positions of principle maxima in grating are given by

$$(e + d)sin\theta = n\lambda \text{ , Where n = 0, 1, 2, 3….}$$

This expression is known as grating equation.

For a particular wavelength λ , the diffraction angle is different for different for principal maxima of different orders.

The first order maxima is obtained for n=1, then $(e + d)sin\theta_1 = \lambda$

The second order maxima is obtained for n=2, then $(e + d)sin\theta_2 = 2\lambda$

The different orders of principal maxima are obtained on both sides of zero order maxima as shown Fig.1.20.

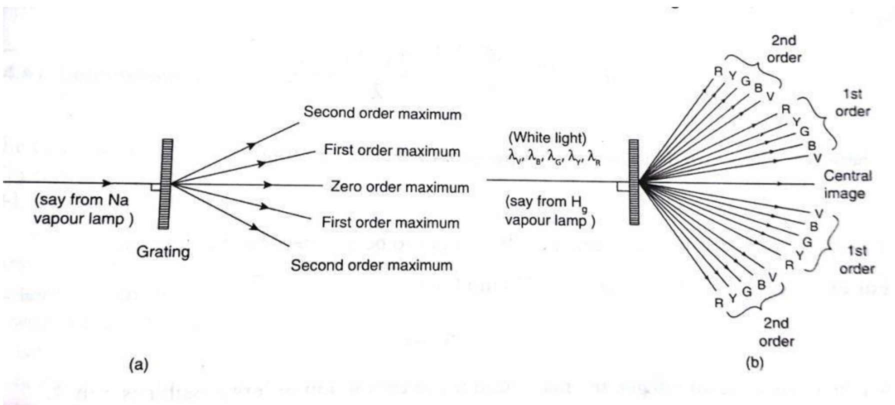


Fig.1.20. Grating spectrum (a) for monochromatic source (b) for white light source

Determination of Wavelength (λ) of given source of light

The positions of principle maxima in grating are given by

$$(e + d)sin\theta = n\lambda \text{ , Where n = 0, 1, 2, 3….}$$

$$sin\theta = \left(\frac{1}{e + d}\right) n\lambda$$

$$sin\theta = \mathrm{N}\, n\lambda$$

$$\lambda = \frac{sin\theta}{\mathrm{N}\, n} \qquad \longrightarrow(1.84)$$

Where $\frac{1}{e+d}$ = N is the number of grating elements or lines per unit width of the grating.

By knowing the values of angle of diffraction, order of the principal maxima and grating element, the wavelength can be determined.

FRAUNHOFER DIFFRACTION DUE TO GRATING (N PARALLEL SLITS):

Consider a grating AB considering of N parallel slits with grating elements (e+d) .Let a plane wavefront ω ω' of monochromatic light of wavelength λ be incident normally on the grating .the diffracted light through the N parallel slits is focused by means of convex lens on a screen placed in the focal plane of the lens . The secondary wavelets travelling normal to the slit or along the direction OP_0 are brought to focus at P_0 will be the central maximum . The secondary wavelets travelling at an angle at an angle θ with the direction of the incident light are focussed at a point P_1 on the screen.

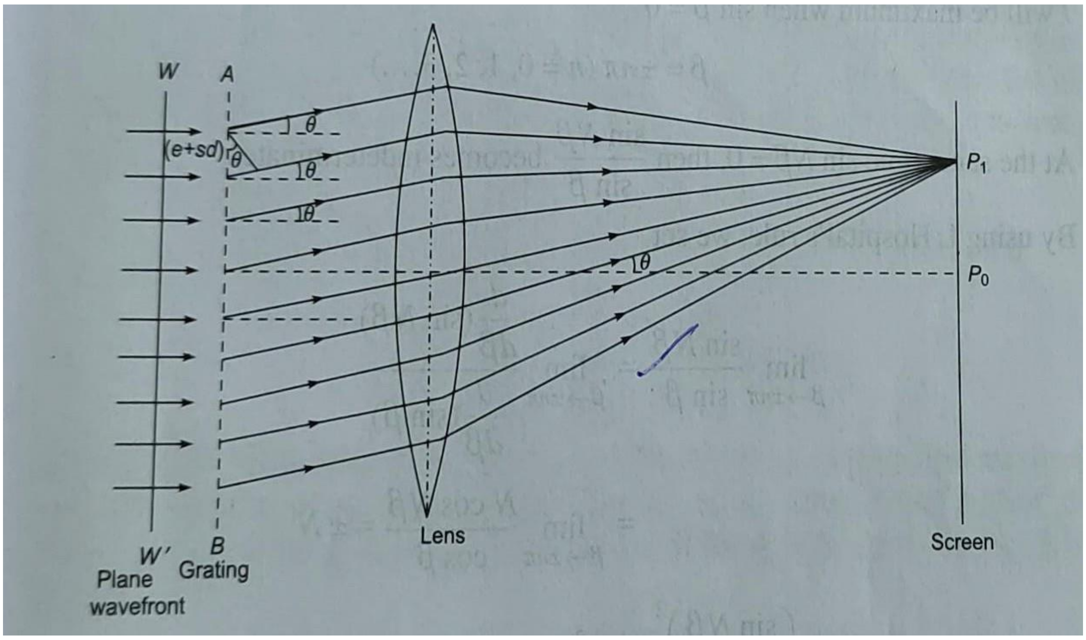


Figure 1:Fraunhofer diffraction -grating

Since the grating consists of N parallel slits from each slit are light wave emerges and we have N waves of amplitude $\frac{Asin\alpha}{\alpha}$ where $\alpha=\frac{\pi esin\theta}{N}$ emerges in the direction of θ.

The path difference between the waves emerging through two successive slits

$$=(e+d)\sin\theta$$

The corresponding phase difference

$$=\frac{2\pi}{\lambda}(e+d)\sin\theta=2\beta$$

From the method of vector additions of amplitudes , the resultant amplitude is given by

$$R=\frac{asinnd/2}{sind/2}$$

For the grating $a=\frac{Asin\alpha}{\alpha}$,n=N and d=2β

$$R=\frac{Asin\alpha}{\alpha}\frac{sinN\beta}{sin\beta}$$

And $$I=R^2=[\frac{Asin\alpha}{\alpha}]^2\ [\frac{sinN\beta}{sin\beta}]^2$$

The factor $(\frac{Asin\alpha}{\alpha})^2$ gives the diffraction effect due to single slit and $[\frac{sinN\beta}{sin\beta}]^2$ represents the combined effects of interference and diffraction due to all slits of grating.

PRINCIPAL MAXIMA

I will be maximum when sinβ=0

$$B=\pm n\pi(n=0,1,2,3....)$$

At the same time sinNβ=0,then $\frac{sinN\beta}{\sin\beta}$ becomes intermediate.

By using L Hospital's rule ,we set

$$\lim_{\beta\to\pm n\pi}\frac{sinN\beta}{sin\beta}=\lim_{\beta\to\pm n\pi}\frac{\frac{d}{d\beta}sinN\beta}{\frac{d}{d\beta}sin\beta}$$

$$=\lim_{\beta\to\pm n\pi}\frac{NcosN\beta}{cos\beta}=\pm N$$

$$\lim_{\beta\to\pm n\pi}(\frac{sinN\beta}{sin\beta})^2=N^2$$

The above intensity represents principal maximum and are obtained for $\beta=\pm n\pi$

$$\frac{\pi}{\lambda}(e+d)sin\theta=\pm n\pi$$

$$(e+d)sin\theta=\pm n\,\lambda$$

Where n=0,1,2,3….

N=0 corresponds to zero order principal maximum . for n=1,2,3,..., we get first ,second , third,... order principal maxima . the $\pm$sign shows that there are two principal maxima of the same order lying on either side of zero order maximum.

MINIMA A series of minima occur , when

$$\sin N\beta=0 \text{ but } \sin\beta\neq 0$$

for minima sin Nβ=0 or Nβ= $\pm$mπ

$$N\frac{\pi}{\lambda}(e+d)\sin\theta=\pm m\pi$$

$$N(e+d)\sin\theta=\pm m\,\lambda$$

Where m has all integral values expect 0,N,2N,...nN,because for these values sin β becomes zero and we get principal maxima. Thus , m=1,2,3...(N-1).

Hence , there are adjacent principal maxima.

SECONDARY MAXIMA:

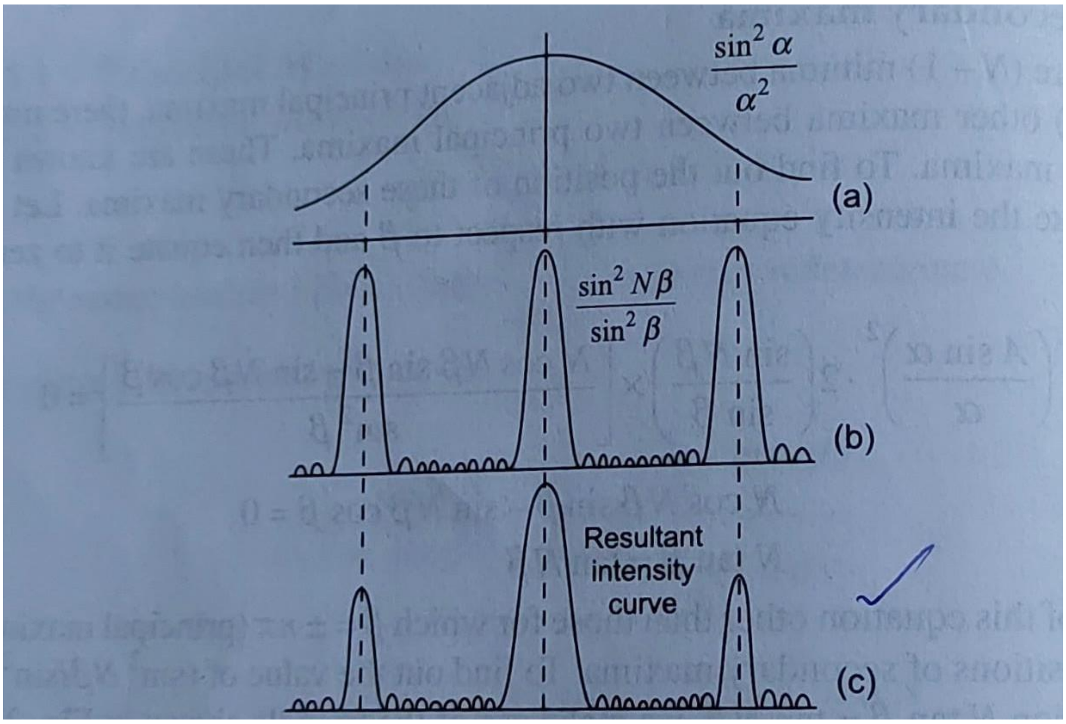
As there are (N-1)minima between 2 adjacent maxima ther must be (N-2) other maxima in between two principle maxima.

The condition for secondary maxima is

$$\tan N\beta=N\tan\beta$$

Therefore ,intensity(I_s)=$\frac{I}{1+(N^2-1)sin^2\beta}$

As N increases , the intensity of secondary maxima relative to principal maxima decrease and becomes negligible when N becomes large.


$\frac{\sin^2 \alpha}{\alpha^2}$
(a)
$\frac{\sin^2 N\beta}{\sin^2 \beta}$
(b)
Resultant
intensity
curve
(c)

Applications of diffraction

The phenomenon of diffraction can be used in various engineering applications; some of the applications are given below.

1. It is used in x-ray diffraction studies of crystals.
2. It is also used in holography for reconstructing 3D images of objects using laser light.
3. Diffraction gratings are used in spectroscopes to separate a light source into its component wavelengths.
4. Diffraction grating can be chosen to specifically analyze a wavelength of light emitted by molecules in diseased cells in a biopsy sample.
5. Diffraction gratings are used in optical fiber technologies where fibers are designed to provide optimum performance at specific wavelengths.

POLARIZATION

2.11.1 Introduction

The phenomena of interference and diffraction tells us that light is a form of wave. But they do not tell us whether light is transverse or longitudinal waves in nature. The phenomenon polarization confirms that light waves are transverse in nature.

In general, light waves are electromagnetic waves and contain vibrations of electric and magnetic field vectors (E&B) which are perpendicular to each other and also perpendicular to the direction of propagation of light waves .i.e., transverse waves. Therefore, light is a transverse wave motion. It has been found experimentally that the electric field vector (E) component of light is mainly responsible for polarization and other optical effects but not due to the magnetic field vector. Therefore, the electric field vector E is also called the light vector and vibration of light means vibrations of electric field vector of light.

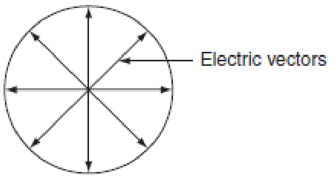
The word polarization comes from the Greek word *"polos"* which means orientation. *Therefore the phenomenon of restricting or orienting the vibrations of the electric field vector in a particular direction is called polarization of light.*

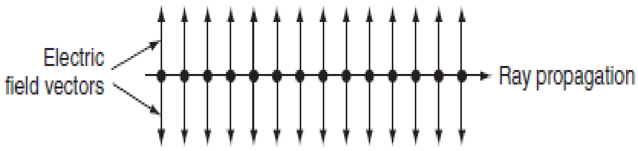
Unpolarized light

If the light vector (electric field vector E) vibrates in all possible directions which are perpendicular to the direction of propagation then the light is said to be unpolarized light.

Let us consider an ordinary light ray passing perpendicular to the plane of the paper and into the paper. The electric field vectors are perpendicular to the ray propagating with equal amplitude in all possible directions as shown in Fig.1.21.

This is the nature of unpolarized light. A double headed arrows show the to and fro vibrations of the electric vector.





(a) (b)

Fig 1.21. Representation of unpolarized light

Polarized light

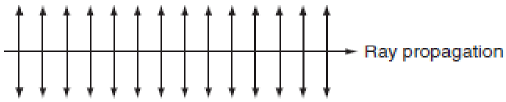
The phenomenon of restricting the vibrations of the light vector or electric field vector (E) in a particular direction is called polarization of light. The polarized light can be classified into three types. They are

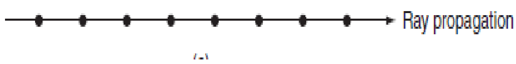
i. Linearly (or) Plane polarized light
ii. Circularly polarized light and
iii. Elliptically polarized light

Linearly (or) Plane polarized light

If the vibrations of light vector (electric field vector E) restricting along straight line only in a plane perpendicular to the direction of propagation of light, it is said to be plane polarized light.

The linearly polarized light is shown in 1.22. The arrowed lines represent in the plane of paper and the dots, the vibrations at right angles (perpendicular) to the paper. In Fig. 1.22(a), the vibrations of electric field vectors lie in the plane of the paper and in Fig. 1.22(b) the vibrations of electric field vectors are perpendicular to the plane of the paper.





(a) (b)

Fig 1.22. Representation of plane polarized light

Circularly polarized light and elliptically polarized light

When two plane polarized light waves are superimposed each other then under certain conditions the resultant light vector may rotates with a constant magnitude in a plane perpendicular to the direction of propagation of light.

If the magnitude of the resultant light vector remains constant while its orientation varies regularly, the tip of the vector traces a circle. Thus the light is said to be circularly polarized.

If however, both magnitude and orientation of light vector vary, the tip of the vector traces an ellipse. Thus the light is said to be elliptically polarized.

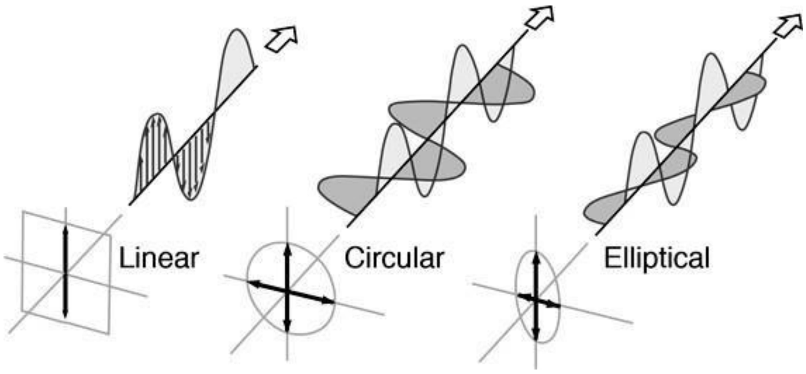


Fig 1.23. Representation of plane, circular and elliptically polarized light

Methods of producing polarized light

Polarized light can be produced by any one of the following.

i. Polarization by reflection
ii. Polarization by refraction
iii. Polarization by double refraction
iv. Polarization by scattering
v. Polarization by selective absorption

Present section we only discuss only polarization by double refraction.

Polarization by double refraction

When a light ray is passed through a glass plate, we get only one refracted ray. But when a light ray is passed through a certain crystals like calcite or quartz, we get two refracted rays, one refracted light ray which obeys the laws of refraction is called ordinary or *o*-ray. The other ray does not obey the laws of refraction and is extraordinary ray or *e*-ray. This phenomenon is called double refraction or birefringence. This was first discovered by Erasmus Bartholinus in 1869.

Explanation

When an ink dot is marked on a white paper and is seen through a calcite crystal, then two images (dots) are observed. If the crystal is rotated slowly with incident ray as vertical axis, then it is observed that one image remains fixed and the other image rotates with the rotation of the calcite crystal. The fixed image is called ordinary image and its refracted ray is called *o*-ray which has vibrations perpendicular to plane of the white paper. The other image is called extraordinary image and its refracted ray is called *e*-ray which has vibrations in the plane of the paper as shown in Fig .1.24.

Inside the crystal the *o*-ray travels with the same velocity in all direction. But the *e*-ray has different velocities in different directions. If the incident ray strikes the crystal along a certain direction called the **optical axis** of the crystal, there will be no double refraction. Hence optical axis of the crystal is a direction along which both *o*-ray and *e*-ray travel in the same direction with the same velocity.

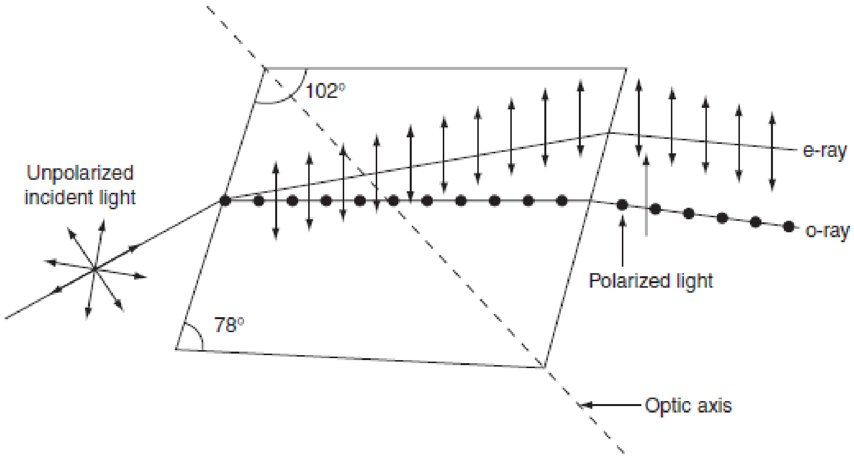


Fig.1.24. Double refraction in calcite crystal

Double refracting crystals

The crystals which exhibit double refraction are called double refracting crystals.
There are two types of double refracting crystals

i. uni-axial crystals
ii. biaxial crystals

Uni-axial crystals

If only one optic axis is present in the crystals, then they are called uni-axial crystals.
Examples: Calcite, quartz and tourmaline etc.

These crystals further dived into two crystals

i. Uni-axial negative crystals
ii. Uni-axial positive crystals

Uni-axial negative crystals

If the velocity of e-ray is more than that of o-ray in the crystals, then they are called negative crystals.

i.e., $v_e > v_o$ and $\mu_e < \mu_o$

Examples: Calcite, tourmaline, ruby and emerald etc

Uni-axial positive crystals

If the velocity of e-ray is less than that of o-ray in the crystals, then they are called negative crystals.

i.e., $v_e < v_o$ and $\mu_e > \mu_o$

Examples: Quartz and iron oxide.

Biaxial crystals

If two optic axes are present in the crystals, then they are called biaxial crystals.

Examples: Topaz and aragonite.

Note:

Optical axis

The direction in which the ray of transmitted light does not suffer double refraction inside the crystal is known as the optic axis. Along this axis both the velocities of o-ray and e-ray are the same and also refractive indices are same. i.e., $v_e = v_o$ and $\mu_e = \mu_0$. So, there no double refraction. Optical axis is a direction but not line.

Wave Plates or Retardation Plates

Wave plates are the doubly refracting uniaxial (calcite) crystals whose refracting faces are cut parallel to the direction of the optic axis and are used to produce a phase difference between ordinary and extraordinary rays when they emerging from the doubly refracting crystals.

These are of mainly two types

i. Quater-wave plate
ii. Half-wave plate

Quarter-wave plate

A quater-wave plate is a thin double refracting crystal (calcite) having a thickness ‘t’, cut and polished with its refracting faces parallel to the direction of optic axis such that it produces a path difference of $\lambda/4$ or phase difference of $\pi/2$ between the o-ray and e-ray when plane polarized light incident normally on the surface and passes through the plate.

Consider a calcite crystal of thickness of ‘t’. When a plane polarized light is incident normally on the surface of calcite crystal, then the light will split up into o-ray and e-ray. These rays travel with different velocities in the crystal. As a result, when o-ray and e-ray emerging from the crystal, they have a phase or path difference between them due to varaition in their velocities.

Let $\mu_o$and μ_e are the refractive indices of o-ray and e-ray respectively. Let ‘t’ be the thickness of the crystal. Hence the path difference between the two rays is

Δ = Optical path for o-ray $-$ Optical path for e-ray

$= \mu_o t - \mu_e\, t$

$= (\mu_o - \mu_e\,)t$ $\longrightarrow$(1.85)

As the crystal is a quater-wave plate, it introduces a path difference of $\lambda/4$ between o-ray and e-ray.

$\Delta = \lambda/4$ $\longrightarrow$(1.86)

From Eqs.(1.57) and (1.58),we get

$$(\mu_o - \mu_e)t = \frac{\lambda}{4}$$

$$\text{Therefore } t = \frac{\lambda}{4(\mu_o - \mu_e)}$$

$$\text{For positive crystal } \mu_e > \mu_o, \text{ then } t = \frac{\lambda}{4(\mu_e - \mu_o)}$$

$$\text{For negative crystal } \mu_o > \mu_e, \text{ then } t = \frac{\lambda}{4(\mu_o - \mu_e)}$$

Applications

1. A quarter wave plate is used to produce circularly and elliptically polarized light.
2. Quarter wave plate converts plane-polarized light into elliptically or circularly polarized light depending upon the angle that the incident light vector makes with the optic axis of the quarter wave plate.

Half-wave plate

A half-wave plate is a thin double refracting crystal (calcite) having a thickness 't', cut and polished with its refracting faces parallel to the direction of optic axis such that it produces a path difference of $\lambda/2$ or phase difference of π between the o-ray and e-ray when plane polarized light incident normally on the surface and passes through the plate.

As the crystal is a haf-wave plate, it introduces a path difference of $\lambda/2$ between o-ray and e-ray.

$$(\mu_e - \mu_o)t = \frac{\lambda}{2}$$

$$\text{Therefore } t = \frac{\lambda}{2(\mu_e - \mu_o)}$$

$$\text{For positive crystal } \mu_e > \mu_o, \text{ then } t = \frac{\lambda}{2(\mu_e - \mu_o)}$$

$$\text{For negative crystal } \mu_o > \mu_e, \text{ then } t = \frac{\lambda}{2(\mu_o - \mu_e)}$$

Applications

1. A half wave plate is used to produce plane polarized light.
2. It produces a phase difference of π between the ordinary and extraordinary ray.

Nicol's Prism

Nicol prism is an optical device used to produce and analyze plane polarized light. This was invented by William Nicol in the year 1828 and is known as Nicol prism. It is made from a double refracting calcite crystal.

Principle

It is based on the phenomenon of double refraction. When a light ray is passed through a calcite crystal, it splits up into two refracted rays such as ordinary (O-ray) and extraordinary ray (E-ray). Nicol prism transmits the extraordinary rays and eliminates ordinary rays with the help of the phenomenon of total internal reflection.

Construction

- It is constructed from the calcite crystal ABCD having length three times of its width.
- The end faces AB and CD are cut down such that the angles of principal section are 68^0 and 112^0 instead of 71^0 and 109^0.
- The crystal is then cut diagonally into two parts. The surfaces of these parts are ground to make optically flat and then these are polished.
- These polished surfaces are connecting together with special cement known as Canada balsam which is a transparent liquid material.
- Canada balsam is optically (refractive index) more dense than e-ray and less dense for o-ray ($\mu_o = 1.6584, \mu_{ca} = 1.55\ and\ \mu_e\ = 1.4864$).

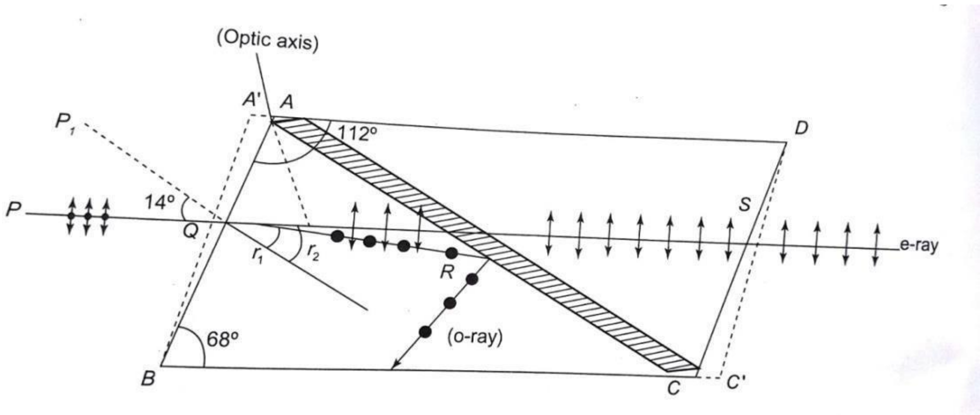


Fig.1.25.Nicol prism

Action or Working

- When a beam of unpolarized light PQ is incident on the face of AB, it splits into two refracted rays o-ray (QR) and e-ray (QS) due to double refraction of calcite crystal.
- These two rays are plane polarized.

- From the refractive index values, we know that the Canada balsam acts as a rarer medium for the ordinary ray and it acts as a denser medium for extraordinary ray.
- When *o*-ray of light travels in the calcite crystal and enters the Canada balsam cement, it passes from denser to rarer medium. When the angle of incidence for ordinary ray on the Canada balsam is greater than the critical angle then the incident ordinary ray is totally internally reflected from the crystal and only e-ray is transmitted through the prism and emerges out of Nicol prism.
- In this way, plane polarized light is produced.

Nicol prism as polarizer and analyzer

- Nicol prism can be used as polarizer and analyzer.
- In order to produce and analyse the plane polarized light, two Nicol prisms are arranged adjacently as shown in Fig.1.26.
- The first Nicol prism is used to produce plane polarized light and is called polarizer. The second Nicol prism is used to test the emerging light and is called analyzer.
- In the parallel positions, the extraordinary ray passes through both the prisms as shown in Fig.1.29 (a). In this case, the intensity of emergent extraordinary light is maximum.
- If the second prism is slowly rotated, then the intensity of the extraordinary ray decreases. When they are perpendicular to each other, no light come out of the second prism because the e-ray that comes out from first prism will enter into the second prism and act as an ordinary ray. So, this light is reflected in the second prism as shown in Fig1.29(b).

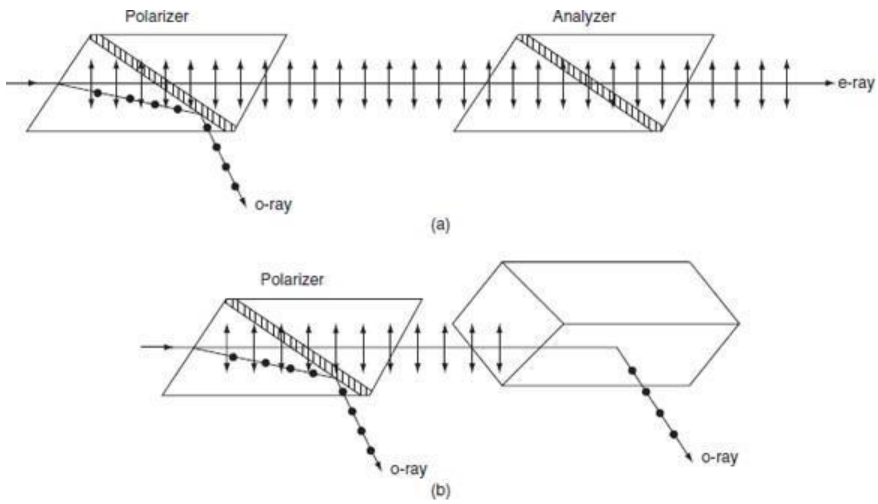


Fig.1.26. Nicol prism as polarizer and analyzer

Applications of Polarization

The phenomenon of Polarization can be used in various engineering applications; some of the applications are given below.

1. Polaroid glasses are used to reduce the amount of light that is approachable to eye.
2. Polarization is useful in receiving and transmitting wave signals.
3. Laser is an outcome of polarization of waves.
4. 3D movies (or stereoscopic movies) are possible because of polarization of light.
5. Photographic filters.
6. Photo elasticity – To study the objects with irregular boundaries and stress analysis.
7. LCD’s.