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## Unit III

### Chapter 1

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## DIELECTRICS

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### INTRODUCTION:

By combining two words *Dia* and *electric* we get the word “dielectric”. The greek prefix *dia* means “Through”. *Electric* means “electric field”.

According to band theory of solids, solids can be classified into conductors, semiconductors and insulators. In insulators, the valence band is full while the conduction band is empty. Further the energy gap between valence band and conduction band is very large .i.e., greater than 3eV. So electrons cannot jump from the valence band into the conduction band. This means that there are no free electrons available for conduction in the insulators under normal conditions.

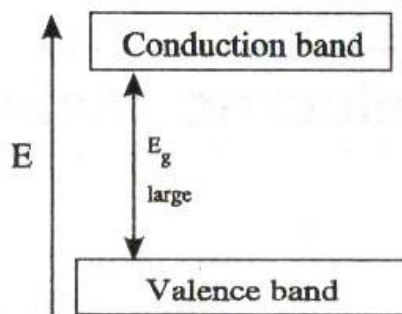


Fig 2.1 Energy band diagram of dielectrics

For this reason, the electrical conductivity is extremely small and may be nil under ordinary conditions. Though some of the insulators exhibit behavior of electric polarization when they are placed in an electric field such insulators are known as dielectrics. Therefore, a dielectric is an electrical insulator that can be polarized by an applied electric field.

In other words, if the main function of non- conducting materials is to provide electrical insulation, then they are called as *insulators*. On the other hand, if the main function of non- conducting materials is to store electrical charges, then they are called as *dielectrics*. Dielectrics are widely used in electrical applications.

### 3.1 Dielectrics

Dielectrics are non-conductors of electricity which do not contain free charge carriers.

**Examples:** air, mica, rubber, ceramics, glass, wood and plastic etc.

## Properties

1. They are insulators.
2. They have a very large energy gap (more than 3 eV)
3. All the electrons in the dielectrics are tightly bound to their parent nucleus.
4. As there are no free electrons to carry the current, the electrical conductivity of dielectrics is very low.
5. They have high specific resistance.
6. They have negative temperature coefficient of resistance.
7. They can be polarized by an electric field.
8. The main function of dielectric materials is to store electric energy.

## 3.2 Types of Dielectrics

Each atom/molecule of a dielectric is neutral. Depending on the atomic/molecular structure, dielectrics are classified into two types. They are

- i) Non-polar Molecules
- ii) Polar Molecules

### Non-polar Molecules

If the centers of gravity of positive and negative charges in the molecules coincide; so that no electric dipoles are formed, the molecules of the dielectrics are said to be non-polar molecules. The examples of non-polar molecules are  $H_2$ ,  $N_2$ ,  $O_2$ ,  $CH_4$  and  $CO_2$  etc

### Polar Molecules

If the centers of gravity of positive and negative charges in the molecules do not coincide, so that electric dipoles are formed, the molecules of the dielectrics are said to be polar molecules. The examples of polar molecules are  $H_2O$ ,  $HCl$ ,  $NH_3$ ,  $CH_3Cl$  and  $CO$  etc.

Though each molecule of a polar dielectric has its own dipole moment; but the molecular dipoles are randomly oriented in all directions, so that the net dipole moment as a whole is zero.

## 3.3 Basic definitions

### 1. Electric dipole

Two equal and opposite electric charges are separated by a small distance is called an electric dipole.

Consider two opposite charges of magnitudes  $+q$  and  $-q$  are separated by a small distance  $d$  as shown in Fig.2.1. These two electric charges constitute electric dipole. Some of the examples of the electric dipoles are HCl, CO<sub>2</sub> and Water etc.

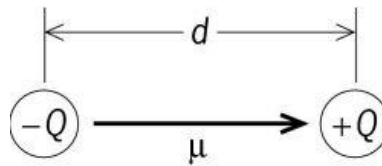


Fig.2.2. Electric dipole

## 2. Electric dipole moment ( $\mu$ )

If two opposite charges are separated by a certain distance, a dipole moment  $\mu$  arises. Mathematically, it is the product of magnitude of charge & distance of separation between the charges.

$$\mu = q \cdot d$$

→ (3.1)

It is a vector quantity pointing from a negative charge towards positive charge.

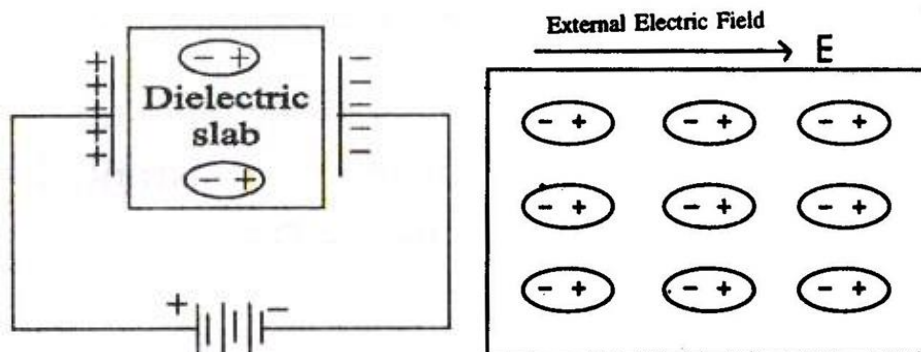
The S.I. unit of Dipole moment is coulomb-meter (C-meter) or Debye.

$$1 \text{ debye} = 3.3 \times 10^{-30} \text{ C-m}$$

## 3. Polarization in dielectrics

*The process of producing electric dipoles by an electric field is called polarization in dielectrics.*

When an electric field is applied to the dielectrics, the field exerts a force on each positive charge in its own direction, as a result the positive charges are displaced in the direction of the field while negative charges are displaced in the opposite direction. Consequently, the displacement of these charges produces electric dipoles throughout the dielectric material. This process is known as polarization in dielectrics.



(a)

(b)

Fig.2.3. Polarization in dielectrics.

**4. Polarizability( $\alpha$ )**

If the strength of the electric field  $E$  is increases, the strength of the induced dipole moment also increases. The induced dipole moment is proportional to the intensity of electric field.

$$\vec{\mu} \propto E$$

$$\vec{\mu} = \alpha E \rightarrow (3.2)$$

Where  $\alpha$  is the proportionality constant, called Polarizability.

**5. Polarisation vector ( $\vec{P}$ )**

If  $\mu$  is the average dipole moment per molecule and  $N$  is the number of molecules per unit volume, the polarization vector ( $\vec{P}$ ) is defined as dipole moment per unit volume of the dielectric material.

$$\vec{P} = N\vec{\mu} \rightarrow (3.3)$$

Units: Coulomb/m<sup>2</sup>

**6. Electric susceptibility ( $\chi_e$ )**

The electric susceptibility  $\chi_e$  of a dielectric material is a measure of how easily it polarizes in response to an electric field.

When a dielectric material is placed in an electric field  $E$ , then polarization takes place. The polarization vector  $\vec{P}$  is proportional to the electric field  $E$ .

$$\vec{P} \propto E$$

$$\vec{P} = \chi_e E \rightarrow (3.4)$$

Where  $\chi_e$  is called electric susceptibility.

Units: No units

**7. Permittivity**

It is defined as the ability of the material to permit the passage of electric field through it. The permittivity of any dielectric material can be represented as  $\epsilon = \epsilon_r \epsilon_0$ .

Where  $\epsilon_r$  is called the relative permittivity or dielectric constant of the dielectric material and  $\epsilon_0$  is permittivity of free space and is equal to  $8.85 \times 10^{-12}$  F/m.

It is a dimensionless quantity.

**8. Dielectric constant**

It is defined as the ratio of permittivity of medium to permittivity of free space.

$$\epsilon_r = \frac{\epsilon}{\epsilon_0} \rightarrow (3.5)$$

Units: No units

The dielectric constant can also be obtained from electric flux density ( $D$ ) and an applied electric field ( $E$ ). The number of electric force lines passing per unit area perpendicular to field is called electric flux density ( $D$ ). It is proportional to the applied electric field ( $E$ ).

$$D \propto E$$

$$D = \epsilon E \text{ (in medium)} \rightarrow (3.6)$$

Where  $\epsilon$  is the proportionality constant, called permittivity of medium.

$$D = \epsilon_0 E \text{ (in a free space)} \rightarrow (3.7)$$

### 3.4. Types of polarization mechanisms:

*When a dielectric material is placed in an external dc electric field, it gets polarized.*

*The four types of polarization which occur in dielectrics are:*

1. Electronic polarization,
2. Ionic polarization,
3. Orientation or dipole polarization,
4. Space charge or interfacial polarization

Let us discuss them one by one:

#### 3.4.1. Electronic polarization

*When electric field applied is applied on dielectric material then all the positive nuclei of atoms move in the field direction and the negative electron cloud of atoms move in opposite direction, hence dipoles will be formed. This phenomenon is known as electronic polarization.*

*Examples:* Mono atomic gases exhibit only electronic polarization: Helium (He), Neon (Ne), Argon (Ar), Krypton (Kr), Xenon (Xe) and Radon (Rn).

The induced dipole moment ( $\mu_e$ ) is proportional to the electrical field strength ( $E$ ).

$$\begin{aligned} \text{i.e., } \mu_e &\propto E \\ \mu_e &= \alpha_e E \end{aligned} \rightarrow (3.8)$$

Where  $\alpha_e$  is proportionality constant and it is known as electronic polarizability.

The electronic polarizability for a rare or noble gas atom is given by

$$\alpha_e = 4\pi\epsilon_0 R^3$$

## Calculation for electronic polarizability ( $\alpha_e$ )

(i) Without electric field ( $E=0$ )

Let us consider an atom of dielectric material with atomic number  $Z$ , then the charge on its nucleus is  $+Ze$ . The nucleus is surrounded by electron cloud of charge  $-Ze$  which is distributed over a sphere of radius  $R$  as shown in Fig .2.4. The centers of the electron cloud and the positive nucleus are at the same point and hence there is no dipole moment

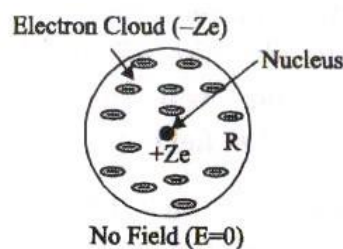


Fig.2.4. Atom without any electric field.

Therefore, the charge density for electron cloud is given by

$$\rho = \frac{\text{Total negative charge of electron cloud}}{\text{Volume of the atom}} = \frac{-Ze}{\frac{4}{3}\pi R^3} = \frac{-3}{4} \frac{Ze}{\pi R^3} \rightarrow (3.9)$$

(ii) With field ( $E \neq 0$ )

When the atom of dielectric material is subjected to electric field, two types of phenomena occur.

- (i) Lorentz force arises due to the electric field which separates the nucleus and the electron cloud from their equilibrium positions in opposite directions. Therefore, due to the Lorentz force the electron cloud and the nucleus move in opposite directions and they are separated by a distance ' $x$ ' where there is a formation of electrical dipole in the atom as shown in Fig.2.4 (b).

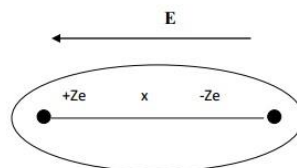


Fig.2.4. Atom with an electric field ( $E \neq 0$ ).

- (ii) Coulomb attractive force arises between the nucleus and the electron cloud after separation which tries to maintain the original equilibrium position. When these two forces are equal and opposite, there will be a new equilibrium between the nucleus and electron cloud of the atom.

$$\begin{aligned} \text{Lorentz force between the nucleus and the electron } F_L &= \text{charge} \times \text{electrical field} \\ &= -ZeE \end{aligned} \rightarrow (3.10)$$

*Coulomb attractive force* ( $F_C$ )

between the nucleus and the electron cloud being separated at a distance  $x$  is

$$= +Ze \times \frac{\text{Total negative charges (Q) enclosed in the sphere of radius } x}{4\pi\epsilon_0 x^2} \rightarrow (3.11)$$

The total negative charge enclosed in the sphere of radius  $x$

= Charge density of electrons ( $\rho$ )  $\times$  Volume of the sphere.

$$\begin{aligned} &= \frac{-3}{4} \frac{Ze}{\pi R^3} \times \frac{4}{3} \pi x^3 \\ &= -\frac{Zex^3}{R^3} \rightarrow (3.12) \end{aligned}$$

Substituting equation (2.11) in equation (2.10), we have

$$\begin{aligned} \text{Coulomb attractive force } (F_C) &= +Ze \times \frac{\frac{Zex^3}{R^3}}{4\pi\epsilon_0 x^2} \\ &= \frac{Z^2 e^2 x}{4\pi\epsilon_0 R^3} \rightarrow (3.13) \end{aligned}$$

At equilibrium, the Coulomb force and Lorentz force are equal and opposite. Hence

$$\begin{aligned} \text{i.e., } F_L &= F_C \\ -ZeE &= \frac{Z^2 e^2 x}{4\pi\epsilon_0 R^3} \\ x &= \frac{4\pi\epsilon_0 R^3 E}{Ze} \rightarrow (3.14) \end{aligned}$$

Therefore, the displacement of electron cloud ( $x$ ) is proportional to the applied electric field  $E$ .

Due to this displacement atom act as dipole.

Therefore, the induced dipole moment  $\mu_e$  is the product of magnitude of charge ( $Ze$ ) & distance of separation between the charges ( $x$ ).

$$\mu_e = Ze \cdot x \rightarrow (3.15)$$

Substituting equation (2.13) in equation (2.14), we have

$$\mu_e = Ze \cdot \frac{4\pi\epsilon_0 R^3 E}{Ze}$$

$$\mu_e = 4\pi\epsilon_0 R^3 E$$

$$(\text{or}) \mu_e \propto E$$

$$\mu_e = \alpha_e E \rightarrow (3.16)$$

Where  $\alpha_e = 4\pi\epsilon_0 R^3$  is called electronic polarizability, which is depending on the volume of the atom and is independent of temperature.

### 3.4.2. Ionic polarization

*When an electric field is applied on ionic dielectric material then positive ions move in the field direction where as negative ions move in the opposite direction, hence dipoles will be formed. This phenomenon is known as ionic polarization.*

Examples: Ionic solids (NaCl) exhibit ionic polarization.

The induced dipole moment ( $\mu_i$ ) due to ionic polarization is proportional to the electrical field strength (E).

$$\text{i.e., } \mu_i \propto E$$

$$\mu_i = \alpha_i E$$

Where  $\alpha_e$  is proportionality constant and it is known as electronic polarizability.

The ionic polarizability for an ionic solid is given by

$$\alpha_i = \frac{e^2}{\omega_0^2} \left( \frac{1}{m} + \frac{1}{M} \right)$$

### Calculation for ionic polarizability ( $\alpha_i$ )

Let us consider there are one cation and one anion present in each unit cell of NaCl ionic crystal. When an electrical field (E) is applied on an ionic dielectric crystal, there is a shift of one ion with respect to another from their mean positions. The positive ions displace in the direction of applied electrical field through the distance  $x_1$ . The negative ions displace in opposite direction through the distance  $x_2$  as shown in Fig. 2.5.

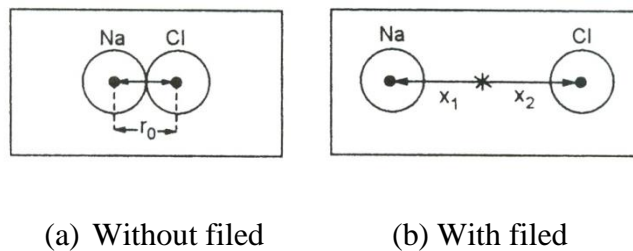


Fig 4.5. Ionic polarization of NaCl crystal



From Fig 4.5, the net distance between two ions  $x = x_1 + x_2$

Therefore, the resultant dipole moment per unit cell is  $\mu_i = e(x_1 + x_2) \rightarrow (3.17)$

When the ions are displaced from their mean positions in their respective directions then the restoring forces appear on the ions which tend to move the ions back to their mean position. The restoring force produced is proportional to the displacement.

*For Positive ion*

The restoring force acting on the positive ion  $F \propto x_1$

$$F = \beta_1 x_1 \rightarrow (3.18)$$

*For Negative ion*

The restoring force acting on the negative ion  $F \propto x_2$

$$F = \beta_2 x_2 \rightarrow (3.19)$$

Where  $\beta_1$  and  $\beta_2$  are restoring force constants which depend upon the masses of ions and angular frequency of the molecule in which ions are present.

If 'm' is the mass of positive ion and 'M' is the mass of negative ion and  $\omega_0$  is the angular frequency, then

$$\beta_1 = m\omega_0^2 \text{ and } \beta_2 = M\omega_0^2 \rightarrow (3.20)$$

At equilibrium the force and restoring force will be equal and opposite. Hence

$$F = \beta_1 x_1 = \beta_2 x_2$$

$$\text{Hence, } x_1 = \frac{F}{\beta_1} \text{ and } x_2 = \frac{F}{\beta_2} \rightarrow (3.21)$$

We know that,  $F = eE \rightarrow (3.22)$

Substituting equation (2.24) in equation (2.23), we have

$$\text{Thus, } x_1 = \frac{eE}{m\omega_0^2} \text{ and } x_2 = \frac{eE}{M\omega_0^2} \rightarrow (3.23)$$

Therefore, the resultant dipole moment per unit cell is  $\mu_i = e(x_1 + x_2)$

$$\begin{aligned} &= e \left( \frac{eE}{m\omega_0^2} + \frac{eE}{M\omega_0^2} \right) \\ &= \frac{e^2 E}{\omega_0^2} \left( \frac{1}{m} + \frac{1}{M} \right) \rightarrow (3.24) \end{aligned}$$

$$\text{But } \mu_i = \alpha_i E \rightarrow (3.25)$$

Comparing equations (2.26) and (2.27) we get

$$\alpha_i E = \frac{e^2 E}{\omega_0^2} \left( \frac{1}{m} + \frac{1}{M} \right)$$

$$\boxed{\alpha_i = \frac{e^2}{\omega_0^2} \left( \frac{1}{m} + \frac{1}{M} \right)} \rightarrow (3.26)$$

Thus, ionic polarisability is inversely proportional to the square of the natural frequency of the ionic molecule and directly proportional to its reduced mass  $\left(\frac{1}{m} + \frac{1}{M}\right)$ .

### 3.4.3 Orientation polarization (Dipolar polarization)

*Orientation polarization arises in dielectric materials which possess molecules with permanent dipole moment (i.e., in polar molecules).*

Examples: Dipolar polarization or orientation takes place only in polar molecules such as  $\text{H}_2\text{O}$ ,  $\text{HCl}$ ,  $\text{NH}_3$ ,  $\text{CH}_3\text{Cl}$  and  $\text{CO}$  etc.

The induced dipole moment ( $\mu_0$ ) due to orientation polarization is proportional to the electrical field strength ( $E$ ).

$$\text{i.e., } \mu_0 \propto E$$

$$\mu_0 = \alpha_0 E$$

Where  $\alpha_0$  is proportionality constant and it is known as orientation polarizability.

The orientation polarizability for polar dielectric medium is given by

$$\boxed{\alpha_0 = \frac{\mu^2}{3K_B T}}$$

### Explanation

(i) *Without electric field ( $E=0$ )*

Polar Molecules have permanent dipole moments even in the absence of an electric field and yet they have net zero dipole moment due to the random orientation of dipoles as shown in Fig 4.6.

(ii) *With field ( $E \neq 0$ )*

When an electric field is applied on the dielectric medium with polar molecules, the electric field tries to align the dipoles along its direction (Fig.4.6). Due to this there is a resultant dipole moment in the dielectric medium and this process is called orientation polarization. Orientation polarization depends on temperature, when the temperature is increased, thermal energy tends to disturb the alignment.

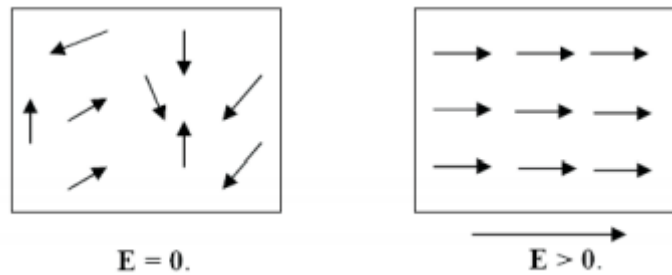


Fig.4.6.Orientation polarization

From the Langevin's theory of Para magnetism, the net intensity of magnetization  $= \frac{N\mu^2 B}{3K_B T}$

Since, the same principle can be applied to the application of electric field, we can write

$$\text{Orientation polarization } P_0 = \frac{N\mu^2 E}{3K_B T}$$

$$= N\alpha_0 E$$

Where  $\alpha_0$  is orientation polarizability and is given by  $\alpha_0 = \frac{\mu^2}{3K_B T}$

$$\boxed{\alpha_0 = \frac{\mu^2}{3K_B T}} \rightarrow (3.27)$$

Therefore, orientation polarizability is inversely proportional to the temperature of the material.

### 3.4.5 Space-charge polarization

*Space-charge polarization arises due to the accumulation of charges at the electrodes or at the interfaces of multiphase dielectric materials.*

Examples: Space-charge polarization occurs in ferrites and semiconductors

#### Explanation

(i) Without electric field ( $E=0$ )

In the absence of an electric field, ions are orderly arranged in the dielectric medium as shown in Fig.4.7.

(ii) With field ( $E \neq 0$ )

When the

dielectric material is subjected to an electrical field at high temperature, the charges get accumulated at the interfaces (Fig.4.7). These charges create dipoles. As a result, polarization is produced.

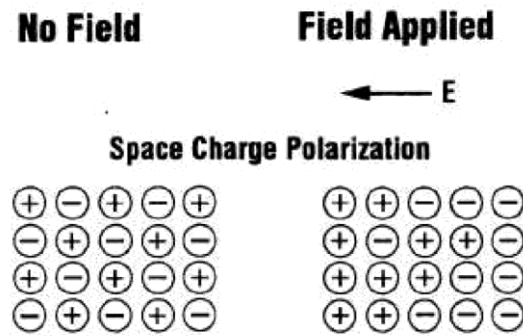


Fig.Space-charge polarization

### 3.4.6 Total polarization

The total polarization is the sum of electronic polarization, ionic polarization, orientation polarization and space-charge polarization. But space-charge polarization is very small when compared to other polarization mechanisms and it is not common in most of the dielectrics. So, it can be neglected.

Therefore, The total polarizability  $\alpha_T = \alpha_e + \alpha_i + \alpha_o$

$$= 4\pi\epsilon_0 R^3 + \frac{e^2}{\omega_0^2} \left( \frac{1}{m} + \frac{1}{M} \right) + \frac{\mu^2}{3K_B T}$$

We know that total polarization  $P = NE\alpha$

$$\therefore P = NE \left[ 4\pi\epsilon_0 R^3 + \frac{e^2}{\omega_0^2} \left( \frac{1}{m} + \frac{1}{M} \right) + \frac{\mu^2}{3K_B T} \right] \rightarrow (3.28)$$

This equation is called as Langevin-Debye equation.

### 3.5. Local (internal) field or Lorentz relation

#### Definition

When a dielectric material is subjected to an external electric field, each of the atoms develops a dipole moment and act as a electric dipole. Hence the resultant field at a given atom will be the sum of applied electric field and the electric field due to surrounding dipoles. This resultant field acting at an atom in a dielectric is called local field (or) internal field  $E_{int}$  and is different from the applied external field  $E_{app}$ . This was first calculated by Lorentz.

#### Calculation of local field (or) internal field ( $E_{int}$ )

To calculate an expression for local electric field on a dielectric molecule or an atom, we consider a dielectric material in the electric field of intensity  $E$ , between the capacitor

plates so that the material is uniformly polarized, as a result opposite type of charges are induced on the surface of the dielectric near the capacitor plates as shown in Fig.4.9.

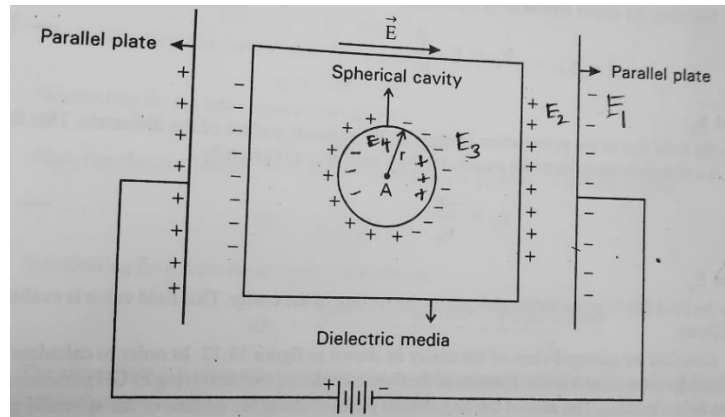


Fig.4.9. Local field (or) Internal field  $E_{int}$

Let us assume a small sphere region or cavity of radius 'r' around the atom A inside the dielectric at which the local field is to be calculated. It is also assumed that the radius of the cavity is large compared to the radius of the atom. i.e., there are many atomic dipoles within the sphere.

The electric field acting on the central atom of the sphere is called local field (or) internal field, which is arises due to sum of following fields.

$$E_{int} = E_1 + E_2 + E_3 + E_4 \rightarrow (2.29)$$

Where,

$E_1$  = Field at A due to the charges on the plates (externally applied).

$E_2$  = Field at A due to the polarized charges induced on the two sides of dielectric.

$E_3$  = Field at A due to the polarized charges induced on the surface of the spherical cavity.

$E_4$  = Field at A due to the atomic dipoles inside the spherical cavity.

Now, let us calculate  $E_1$ ,  $E_2$ ,  $E_3$ ,  $E_4$  values one by one as follows.

### Field $E_1$

When a dielectric medium is polarized due to an electric field E, the displacement vector D is given by

$$D = \epsilon_0 E + P \rightarrow (3.30)$$

From the field theory, the field at A due to the charges on the plates is given by

$$D = \epsilon_0 E_1 \rightarrow (3.31)$$

∴ Equating equation (2.31) and (2.32), we get

$$\epsilon_0 E_1 = \epsilon_0 E + P$$

Dividing the above equation by  $\epsilon_0$ , we get

$$E_1 = E + \frac{P}{\epsilon_0} \rightarrow (3.32)$$

### Field $E_2$

$E_2$  is the field at A due to the polarized charges induced on the two sides of dielectric. This field acts in a direction opposite to the external field. From the field theory, we have

$$E_2 = -\frac{P}{\epsilon_0} \rightarrow (3.33)$$

### Field $E_3$

$E_3$  is the field at A due to the polarized charges induced on the surface of the spherical cavity. This field value was calculated by Lorentz as given below.

Consider a magnified or enlarged view of the imagined spherical cavity as shown in Fig.4.10. A small elemental ring is cut with area 'dA' perpendicular to the field direction and is making between  $\theta$  and  $\theta + d\theta$ , where  $\theta$  represents the direction with respect to the applied field direction.

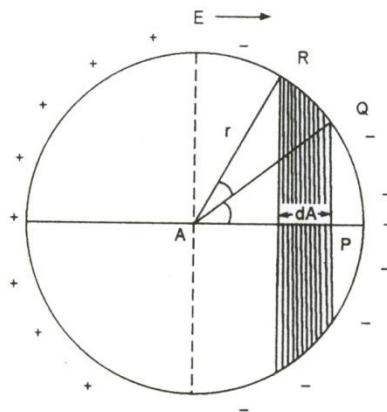


Fig.4.10. Enlarged view of the imagined spherical cavity

From the figure, the area of the small elemental ring along the surface of the sphere is given by

$$dA = 2\pi(PQ)(QR) \rightarrow (3.34)$$

From the triangle APQ,  $\sin\theta = \frac{PQ}{r}$  or  $PQ = r \sin\theta \rightarrow (3.35)$

and from the sector AQR,  $d\theta = \frac{QR}{r}$  or  $QR = r d\theta \rightarrow (3.36)$

Substituting equations (3.35) and (3.36) in equation (3.34)

$$dA = 2\pi(r \sin\theta)(r d\theta)$$

$$dA = 2\pi r^2 \sin\theta d\theta \rightarrow (3.37)$$

From the definition of polarization (i.e.,  $P = \frac{q}{A}$ ), The charge  $dq$  on the surface  $dA$  is equal to the product of the normal component of the polarization and the surface area.

$$\text{i.e., } dq = P_N \cdot dA \rightarrow (3.38)$$

The polarization  $P$  is parallel to  $E$ . Its component normal to  $dA$  is  $P_N = P \cos\theta$ .

$$dq = P \cos\theta dA \rightarrow (3.39)$$

Substituting  $dA$  value in equation (3.39)

$$dq = P \cos\theta \cdot 2\pi r^2 \sin\theta d\theta \rightarrow (3.40)$$

The field due to the charge  $dq$  is denoted by  $dE_3$  at 'A' is given by (Coulomb's law)

$$dE_3 = \frac{dq \cos\theta}{4\pi\epsilon_0 r^2} \rightarrow (3.41)$$

Substituting equation (3.40) in equation (3.41)

$$dE_3 = \frac{(P \cos\theta \cdot 2\pi r^2 \sin\theta d\theta) \cos\theta}{4\pi\epsilon_0 r^2} = \frac{P \cos^2\theta \sin\theta d\theta}{2\epsilon_0} \rightarrow (3.42)$$

Therefore, the field at a due to the surface charge on the cavity is obtained by integrating over the whole surface of the sphere.

$$\int_0^\pi dE_3 = \frac{P}{2\epsilon_0} \int_0^\pi \cos^2\theta \sin\theta d\theta \rightarrow (3.43)$$

$$E_3 = \frac{P}{2\epsilon_0} \int_0^\pi \cos^2\theta \sin\theta d\theta \rightarrow (3.44)$$

$$= \frac{P}{2\epsilon_0} \cdot \frac{2}{3} = \frac{P}{3\epsilon_0} \left( \because \int_0^\pi \cos^2\theta \sin\theta d\theta = \frac{2}{3} \right)$$

$$\therefore E_3 = \frac{P}{3\epsilon_0}, \rightarrow (3.45)$$

#### Field E<sub>4</sub>

E<sub>4</sub> is the field at A due to the atomic dipoles inside the spherical cavity which depends on the crystal structure. This field E<sub>4</sub> is zero for spherically symmetric system (cubic structure) because the dipoles will cancel with each other.

$$\therefore E_4 = 0 \rightarrow (3.46)$$

Hence, substituting all the four field values from the equations (3.32), (3.33), (3.45) and (3.46) in equation (3.29)

$$E_{\text{int}} = E + \frac{P}{\epsilon_0} - \frac{P}{\epsilon_0} + \frac{P}{3\epsilon_0} + 0 \rightarrow (3.47)$$

or

$$E_{\text{int}} = E + \frac{P}{3\epsilon_0} \rightarrow (3.48)$$

This equation is known as Local field or internal field. Thus, it is observed that the local field is greater than the electric field applied by an additional factor  $\frac{P}{3\epsilon_0}$ .

### 3.6. Claussius-Mosotti Equation

The relation between dielectric constant  $\epsilon_r$  (macroscopic quantity) and polarizability  $\alpha$  (microscopic quantity) of atoms in a dielectric is known as Claussius-Mosotti Equation.

$$\frac{N\alpha}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$$

This relation is known as Claussius-Mosotti Equation.

#### Proof

When a dielectric material is placed in an external electric field E, it gets polarized .ie., dipole moments are induced. If the strength of the electric field E is increased, the strength of the induced dipole moment also increases. The induced dipole moment is proportional to the intensity of electric field.

$$\vec{\mu} \propto E_{\text{int}}$$

$$\vec{\mu} = \alpha E_{\text{int}} \rightarrow (3.49)$$

Where  $\alpha$  is the proportionality constant, called Polarizability and E<sub>int</sub> is the local field.



If  $\mu$  is the average dipole moment per molecule and  $N$  is the number of molecules per unit volume, the polarization vector ( $\vec{P}$ ) is defined as dipole moment per unit volume of the dielectric material

$$P = N\vec{\mu} \rightarrow (3.50)$$

Substituting equation (2.49) in equation (2.50), we have

$$P = N\alpha E_{int} \rightarrow (3.51)$$

We know that

$$E_i = E + \frac{P}{3\epsilon_0}$$

$$P = N\alpha \left( E + \frac{P}{3\epsilon_0} \right)$$

$$P = N\alpha E + N\alpha \frac{P}{3\epsilon_0}$$

$$P - N\alpha \frac{P}{3\epsilon_0} = N\alpha E$$

$$\left( 1 - \frac{N\alpha}{3\epsilon_0} \right) P = N\alpha E$$

$$P = \frac{N\alpha E}{\left( 1 - \frac{N\alpha}{3\epsilon_0} \right)} \rightarrow (3.52)$$

We also know that

$$P = \epsilon_0 E (\epsilon_r - 1) \rightarrow (3.53)$$

Equating the equations (3.53) and (3.54)

$$\epsilon_0 E (\epsilon_r - 1) = \frac{N\alpha E}{\left( 1 - \frac{N\alpha}{3\epsilon_0} \right)}$$

$$1 - \frac{N\alpha}{3\epsilon_0} = \frac{N\alpha E}{\epsilon_0 E (\epsilon_r - 1)}$$

$$1 = \frac{N\alpha E}{\epsilon_0 E (\epsilon_r - 1)} + \frac{N\alpha}{3\epsilon_0}$$

$$1 = \frac{N\alpha}{3\epsilon_0} \left[ \frac{3}{(\epsilon_r - 1)} + 1 \right]$$

$$\frac{N\alpha}{3\epsilon_0} = \frac{1}{\frac{3}{(\epsilon_r - 1)} + 1}$$

$$\frac{N\alpha}{3\epsilon_0} = \frac{\epsilon_r - 1}{3 + \epsilon_r - 1}$$

$$\frac{N\alpha}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2} \rightarrow (3.54)$$

The above relation is known as Claussius-Mosotti equation.

### 3.7. Ferroelectricity

In general, when the dielectric materials are placed in an external electric field  $E$ , they get polarized. But some dielectric materials exhibit spontaneous polarization even in the absence of external electric field. This phenomenon is known as ferroelectricity and these dielectric materials are known as ferroelectrics.

The ferroelectric effect was first observed by Valasek in 1921, in the Rochelle salt. This has molecular formula  $\text{KNaC}_4\text{H}_4\text{O}_6 \cdot 4\text{H}_2\text{O}$ .

#### Examples:

1. Rochelle salt ( $\text{NaKC}_4\text{H}_4\text{O}_6 \cdot 4\text{H}_2\text{O}$ )
2. Potassium dihydrogen phosphate or KDP ( $\text{KH}_2\text{PO}_4$ )
3. Ammonium dihydrogen phosphate or ADP ( $\text{NH}_4\text{H}_2\text{PO}_4$ )
4. Potassium niobate ( $\text{KNbO}_3$ )
5. Lithium niobate ( $\text{LiNbO}_3$ )
6. Barium titanate ( $\text{BaTiO}_3$ )
7. Lithium titanate ( $\text{LiTiO}_3$ ) etc.

#### Properties

1. They are anisotropic crystals.
2. They exhibit spontaneous polarization.
3. As the temperature increases the Spontaneous polarization decreases and at a particular temperature, the spontaneous polarization vanishes. This temperature is known as Curie temperature ( $T_c$ ). The variation of spontaneous polarization with temperature as shown in Fig.
4. Below Curie temperature, the dielectric constant is function of electric field. Above Curie temperature dielectric constant varies with temperature. The variation of dielectric constant with temperature as shown in Fig.

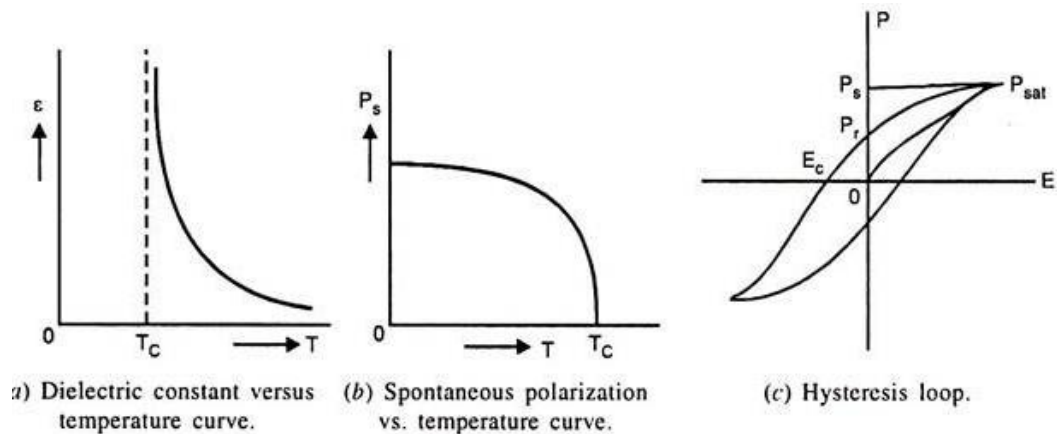
According to Curie – Weiss law,

$$\epsilon_r = \frac{C}{T - T_c}$$

Where  $C$  is called curie constant and  $T_c$  is called curie temperature.

5. Ferroelectric materials exhibit hysteresis, similar to that of ferro magnetic materials.

**Hysteresis:** When an electric field is applied on a ferroelectric material then polarization takes place. This polarization always lags behind the applied electric field. This phenomenon is known as hysteresis of ferroelectric material.



#### 4.11. Properties of ferro electrics

6. Ferroelectric materials exhibit piezoelectricity and Pyroelectricity.

*When a dielectric material acquires electric polarization due to the external mechanical pressure, it is called piezoelectricity and these materials are known as piezo electrics.*

*When a dielectric material acquires electric polarization due to the external thermal energy, it is called Pyroelectricity and these materials are known as pyro electrics.*

### 3.8. Applications of Ferroelectric Materials

Ferroelectric Materials can be used in various applications; some of the applications are given below.

1. Production of ultrasonics.
2. SONAR, strain gauges, etc.
3. Frequency stabilizers and crystal-controlled oscillators.
4. Magnetic amplifiers and dielectric amplifiers.
5. Magnetic information storage devices.
6. Magnetostrictive Transducers.
7. Capacitor microphones.
8. Gas filters.
9. Delay lines
10. High sensitive infra-red detectors.

### 3.9. Dielectric loss or loss tangent

When a dielectric material is subjected to an alternative voltage (a.c), some of the electrical energy is absorbed by the dielectric material and later it is dissipated in the form of heat. This dissipated energy is called dielectric loss.

#### Expression for dielectric loss

##### Case 1

When an ac voltage is applied to a pure dielectric material like vacuum or purified gases, there is no any absorption of energy, and then the resulting current leads applied voltage by  $90^\circ$ , as shown in Fig.

We know that, Power loss  $P = VI \cos\theta$

$$\text{When } \theta=90^\circ, P=0 \rightarrow (3.55)$$

Thus we can say that, if current (I) leads applied by voltage (V) exactly by  $90^\circ$ , then there is no lose of electrical energy.

##### Case 2

When an ac voltage is applied to commercial dielectrics, the resulting current lead applied voltage by  $(90^\circ-\theta)$  as shown in Fig. This angle  $\delta = (90^\circ-\theta)$  is the dielectric loss angle which measure of the power or heat dissipated in each cycle.

In this case, the power loss  $P = VI \cos\theta$

$$\text{When } \theta=90^\circ-\delta, P= VI \cos (90^\circ-\delta) = VI \sin \delta$$

We know that,  $V = IR$

$$I = \frac{V}{R}$$

If I is the current through the capacitor in amp, then

$$I = \frac{V}{X_C}$$

Where  $X_C = \frac{1}{2\pi fC}$  is the capacity reactance

$$\text{Therefore, } P = V \frac{V}{X_C} \sin \delta$$

$$= V^2 2\pi fC \sin \delta$$

But, since  $\sin \delta$  is negligibly small in most of the dielectrics,

$$\sin \delta = \tan \delta$$

$$\therefore P = V^2 2\pi fC \tan \delta \rightarrow (3.56)$$

Where  $\tan \delta$  is called the power factor or tangent loss.

The above equation (3.56) represents the expression for dielectric loss.

**Factors affecting dielectric loss**

The dielectric loss increases with

- 1) High frequency of the applied voltage
- 2) High applied voltage
- 3) High temperature and humidity