	GEETHANJALI INSTITUTE OF SCIENCE & TECHNOLOGY (AN AUTONOMOUS INSTITUTION) (Approved by AICTE, New Delhi & Affiliated to JNTUA, Ananthapuramu) (Accredited byNAACwith"A"Grade, NBA (EEE,ECE & ME)&ISO9001:2008CertifiedInstitution)

QUESTION BANK (DESCRIPTIVE)

Subject Name with Code: Differential Equations &Vector Calculus - 22A0002T
Course & Branch: B. Tech & Common to All
Year& Semester: I B. Tech II Sem **Regulation:** RG22

UNIT- I

***** Linear Diferential Equations of Second Order*****

1	a)	Solve $(D^2+a^2)y = cosax$	
	b)	Solve $(D-2)^2y = 8(e^{2x}+sin2x+x^2)$	
2	a)	Solve $\frac{d^2y}{dx^2}+\frac{dy}{dx}+y = (1-e^x)^2$	
	b)	Solve $y''+4y'+4y = 3sinx+4cosx\ y(0) = 1, y'(0) = 0$	
3	a)	Solve $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y = e^{3x}+sin2x$	
	b)	Solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y = \ xe^xsinx$	
4	(a)	Solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+3y = e^xcosx$	
	(b)	Using the method of variation of Parameters, solve $\frac{d^2y}{dx^2}+4y = tan2x$	
5	(a)	Solve $\frac{d^3y}{dx^3}+y = 3+e^{-x}+5e^{2x}$	
	(b)	Solve by the method of variation of Parameters $y''+y = cosecx$	
6		Solve the Simultaneous equations $\frac{dx}{dt}+2y+sint = 0, \frac{dy}{dt}-2x-cost = 0$. Given that $x = 0\ and\ y = 1\ when\ t = 0$	
7		Solve the Simultaneous equations $\frac{dx}{dt}+\frac{dy}{dt}-2y = 2cost-7sint, \frac{dx}{dt}-\frac{dy}{dt}+2x = 4cost-3sint$	
8		In an L-C-R circuit, the charge q on a plate of a condenser is given by $L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C} = \ Esinpt$. The circuit is tuned to resonance so that$p^2 = \frac{1}{LC}$. If initially the current i and the charge q be zero. Show that for small values of $\frac{R}{L}$,The current in the circuit at	

		time t is given by $\left(\frac{Et}{2L}\right) sinpt$.	
9		A condenser of capacity C discharged through an inductance L and resistance R in series and the charge q at time t satisfies the equation $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0. Given\ that\ L = 0.25\ heneries, R = 250\ ohms, C = 2 \times 10^{-6}$farads and that when $t = 0$, charge q is 0.0020 coulombs and the current $\frac{dq}{dt} = 0$, obtain the value of q in terms of t.	
10		A body weighing 10kg is hung from a string. A pull of 20kg, weight will stretch the spring to 10cm. The body is pulled down to 20cm below the static equilibrium position and then released. Find the displacement of the body from its equilibrium position at time t second, the maximum velocity and the period of oscillation.	

UNIT- II

*** Partial Differential Equtations***

1	a)	Form the PDE by eliminating arbitrary function from z = $x+y+ f(xy)$.	
	b)	Solve $(y^2 + z^2) p + x(yq - z) = 0$.	
2	a)	Form the PDE by eliminating arbitrary functions from z= $f(y/x) + \emptyset(xy)$.	
	b)	Solve $p - q = \log(x + y)$	
3	a)	Form the PDE by eliminating arbitrary function from $z = y^2 + 2f(\frac{1}{x} + \log y)$.	
	b)	Solve $x^2(y - z) p + y^2 (z - x) q = z^2 (x - y)$.	
4	a)	Form the PDE by eliminating arbitrary constants a, b from $2z = x^2/a^2 + y^2/b^2$.	
	b)	Solve $z = p^2 + q^2$.	
5	a)	Form the PDE by eliminating arbitrary constants a,b from $z = xy + y\sqrt{x^2 - a^2} + b$.	
	b)	Solve $p^2 + pq = z^2$	
6	a)	Solve $(y + z)p - (z + x)q = x - y$.	
	b)	Solve .$p^2 + q^2 = x + y$	
7	a)	Form the PDE by eliminating arbitrary constants a,b,c, from $$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$	
	b)	Solve $p^2 + q^2 = x^2 + y^2$	
8	a)	Solve $3p^2 - 2q^2 = 4pq$	
	b)	Form the PDE by eliminating arbitrary function from $xyz = f(x + y + z)$	
9	a)	Solve $(x^2 - y^2 - z^2) p + 2xy\, q = 2xz$.	

	b)	Solve $xp - y^2q^2 = 1.$	
10	a)	Solve $xp - yq = y^2 - x^2$	
	b)	Solve $q = px + p^2$	

		UNIT- III	
		*****Applications of Partial Differential Equations*****	
1	(a)	Classify the following partial differential equation of second order $(x^2 + y^2 - 1)\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + (x^2 + y^2 - 1)\frac{\partial^2 u}{\partial y^2}$	
	(b)	Solve by the method of separation of variables $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x,0) = 6e^{-3x}$	
2		Using the method of separation of variables, solve the partial differential equation $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$ given that $u(0,y) = 4e^{-y} - e^{-5y}$	
3	(a)	Classify the following partial differential equation of second order $xyr - (x^2 - y^2)s - xyt + pq - qx - 2(x^2 - y^2) = 0$	
	(b)	Solve $y^3\frac{\partial z}{\partial x} + x^2\frac{\partial z}{\partial y} = 0$ by the method of separation of variables	
4	(a)	Classify the following partial differential equation of second order $yu_{xx} - 4u_{xy} + 4xu_{yy} = 0$	
	(b)	Solve $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$ & $u = e^{-5y}$ when $x = 0$	
5	(a)	Classify the following partial differential equations of second order $(i)\ y^2r - 2xys + x^2t = \frac{y^2p}{x} + \frac{x^2q}{y}$ $(ii)\ x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial y^2} = 0$	
	(b)	Solve $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$ by the method of separation of variables, where $u(0,y) = 8e^{-3y}$	
6		Find the solution of the one-dimensional wave equation $\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$ by the method of separation of variables.	
7		A string is stretched and fastened to two points $x = 0$ and $x = l$ apart. Motion is started by displacing the string into the form $y = y_0 sin\frac{\pi x}{l}$ from which it is released at time t=0. Show that the displacement of any point at a distance x from one end at time t is given by $y(x,t) = y_0 sin\frac{\pi x}{l} cos\frac{\pi at}{l}$	
8		A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y(x,0) = y_0\, sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement y at any distance x from one end at any time t.	
9		A string is stretched and fastened to two points $x = 0$ and $x = l$ apart. Motion is started by displacing the string into the form $y = k(lx - x^2)$ from which is it is released at time t = 0. Find the displacement of any point on the string at a distance of x from one end at time t.	

10		A tightly stretched string of length l with fixed ends is initially in equilibrium position. It is set vibrating by giving each point a velocity $v_0\, sin^3\left(\frac{\pi x}{l}\right)$. Find the displacement $y(x,t)$.	

UNIT- IV			
***** Vector Differentiation*****			
1	a)	Let $\bar{F} = (x+y+1)\bar{\imath} + \bar{\jmath} - (x+y)\bar{k}$ .Show that $\bar{F}.curl\bar{F} = 0$	
	b)	Find the angle between the surfaces $x^2+y^2+z^2 = 9$ and $z = x^2+y^2-3$ at the point (2,–1,2).	
2	a)	Show that the vector $\bar{F} = (x+y)\bar{\imath} + (y-3z)\bar{\jmath} + (x-2z)\bar{k}$ is solenoidal.	
	b)	Find the directional derivative of $\emptyset = x^2yz + 4xz^2$ at (1,–2,–1) along $2\bar{\imath} - \bar{\jmath} - 2\bar{k}$.	
3	a)	Find the directional derivative of $f = x^2 - y^2 + 2z^2$ at the point $P(1,2,3)$ in the direction of the PQ where Q is the point (5,0,4). Also calculate the magnitude of the maximum directional derivative.	
	b)	Find div$\bar{F}$ and curl$\bar{F}$ where $\bar{F} = \nabla(x^3+y^3+z^3-3xyz)$	
4	a)	Show that the following field $\bar{F}$ is a potential field and find its scalar potential: $\bar{F} = 2xyz^2\,\bar{\imath} + (x^2z^2 + z\,cosyz)\bar{\jmath} + (2x^2yz + ycosyz)\bar{k}$	
	b)	Prove that $div(r^n\bar{r}) = (n+3)r^n$. Hence show that $\frac{\bar{r}}{r^3}$ is solenoidal.	
5	a)	Find the constants a, b, c so that $\bar{F} = (x+2y+az)\bar{\imath} + (bx-3y-z)\bar{\jmath} + (4x+cy+2z)\bar{k}$ is irrotational.	
	b)	Prove that $div(\emptyset\bar{F}) = grad\emptyset.\bar{F} + \emptyset div\bar{F}$	
6		Find the directional derivative of $\emptyset = 5x^2y - 5y^2z + 2.5z^2x$ at the point (1,1,1) in the direction of the line $\frac{x-1}{2} = \frac{y-3}{-2} = z$.	
7	a)	If $f = (x^2+y^2+z^2)^{-n}$ find $div(gradf)$ and determine n if $div\,(gradf) = 0$	
	b)	If $u = x^2yz, v = xy - 3z^2$, find:(i) $\nabla(\nabla u.\nabla v)(ii)\nabla.(\nabla u \times \nabla v)$	
8	a)	What is the directional derivative of $\emptyset = xy^2 + yz^3$ at $(2,-1,1)$ in the direction of the normal to the Surface $xlogz - y^2 = -4$ at $(1,-2,1)$.	
	b)	Prove that curl$\emptyset\bar{F}$ = grad$\emptyset$X$\bar{F}$ + $\emptyset$curl$\bar{F}$	
9	a)	If $\bar{F}$ is a solenoidal vector then show that $\nabla X\nabla X\nabla X(\nabla X\bar{F}) = \nabla^4\bar{F}$.	
	b)	Show that $r^n\bar{r}$ is irrotational vector for any value of *n* but is solenoidal if *n*+3=0, where $\bar{r} = x\bar{\imath} + y\bar{\jmath} + z\bar{k}$ and r is the magnitude of $\bar{r}$.	
10	(a)	Find the value of c if $\bar{A} = (5x+y)\bar{\imath} + (cy-z)\bar{\jmath} + (3x-7z)\bar{k}$ is a solenoidal vector.	
	(b)	Show that vector $(x^2+yz)\bar{\imath} + (y^2+zx)\bar{\jmath} + (z^2+xy)\bar{k}$ is irrotatinal and find scalar potential.	

UNIT- V

*** Vector Integration***

1	a)	If $\bar{F} = x^2\ \bar{\imath} + xy\ \bar{\jmath}$, evaluate $\int_c \bar{F}.\overline{dr}$ from (0,0) to (1,1) along the: (i) line $y = x$. (ii) parabola $y = \sqrt{x}$.	
	b)	Use Green's theorem in a plane to evaluate $\int (x^2 + y^2)dx + 3x^2y dy$ where C is the circle $x^2 + y^2 = 4$ traced in the positive sense.	
2	(a)	Verify Stokes theorem for $\bar{F} = (2x - y)\bar{\imath} + yz^2\bar{\jmath} + y^2z\bar{k}$ over the upper half surface of $x^2 + y^2 + z^2 = 1$ bounded by its projection on the xy plan.	
	(b)	Show that $\int_S (ax\bar{\imath} + by\bar{\jmath} + cz\bar{k}).\hat{n}ds = \frac{4}{3}\pi(a + b + c)$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$.	
3	a)	If $\bar{F} = xy\bar{\imath} + yz\bar{\jmath} + zx\bar{k}$, evaluate $\int \bar{F}.d\bar{r}$ where c is the curve represented by $x = t, y = t^2, z = t^3, -1 \le t \le 1$.	
	b)	Use Stoke's theorem to evaluate $\int_C [(x + y)dx + (2x - z)dy + (y + z)dz]$ where C is the boundary of the triangle with vertices $(2,0,0), (0,3,0)$ & $(0,0,6)$.	
4		Verify Green's theorem for $\int_c [(xy + y^2)dx + x^2dy]$, where C is bounded by $y = x$ and $y = x^2$	
5		Verify Stoke's theorem for $\bar{F} = (x^2 + y^2)\bar{\imath} - 2xy\bar{\jmath}$ taken around the rectangle bounded by the lines $x = \pm a, y = 0$ and $y = b$.	
6	(a)	Evaluate $\int_S \bar{F}.\hat{n}\ ds$ where $\bar{F} = 4xz\bar{\imath} - y^2\bar{\jmath} + 4z\bar{k}$ and S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.	
	(b)	If V is the volume enclosed by a closed surface S and $\bar{F} = x\bar{\imath} + 2y\bar{\jmath} + 3z\bar{k}$, show that $\int_S \bar{F}.\hat{n}\ ds = 6v$.	
7		Verify Green's theorem for $\int_c [(3x - 8y^2)dx + (4y - 6xy)dy]$, where C is the boundary of the region bounded by $x = 0$, $y = 0$ and $x + y = 1$.	
8		Verify divergence theorem for $\bar{F} = (x^2 - yz)\bar{\imath} + (y^2 - zx)\bar{\jmath} + (z^2 - xy)\bar{k}$ taken over the rectangular parallelepiped $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$.	
9		Verify Gauss divergence theorem for the function $\bar{F} = y\bar{\imath} + x\bar{\jmath} + z^2\bar{k}$ over the cylindrical region bounded by $x^2 + y^2 = 9$, $z = 0$ & $z = 2$.	
10	(a)	Using Green 's theorem, evaluate $\int_c (y - sinx)dx + cosx\ dy$ where C is the plane triangle enclosed by the lines $y = 0, x = \frac{\pi}{2}$ and $y = \frac{2}{\pi}x$.	

	(b)	By transforming to triple integral, evaluate $\iint (x^3 dydz + x^2 ydzdx + x^2 zdxdy)$ where S is the closed surface consisting of the cylinder $x^2 + y^2 = 4$ and the circular discs z= 0 and $z = 3$.	

GEETHANJALI INSTITUTE OF SCIENCE & TECHNOLOGY
(AN AUTONOMOUS INSTITUTION)
(Approved by AICTE, New Delhi & Affiliated to JNTUA, Ananthapuramu)
(Accredited byNAACwith"A"Grade, NBA(EEE,ECE,ME)&ISO9001:2008CertifiedInstitution)

QUESTIONBANK (2 Marks)

Subject Name with Code: Differential Equations &Vector Calculus - 22A0002T
Course & Branch: B.Tech& Common to All
Year& Semester: I B. Tech II Sem **Regulation:**RG22

UNIT – I

*** Linear Diferential Equations of Second Order ***

1	Find the complete solution of $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0.$	
2	Solve$(D - 1)y = cos2x.$	
3	Let$y_1 = cos2x, y_2 = sin2x. Find\ the\ Wronskian\ of\ y_1, y_2.$	
4	Solve$\frac{d^2y}{dx^2} + 9y = 0.$	
5	Find the Particular integral of $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4.$	

UNIT – II

*** Partial Differential Equations***

1	Form the partial differential equation by eliminating the arbitrary constants from $z = ax + by + ab.$	
2	Form the partial differential equation by eliminating the arbitrary function from $z = f(x^2 + y^2)$	
3	Solve $xp + yq = 3z.$	
4	Find the complete integral of $pq = 3$	
5	Solve $pqz = p^2(qx + p^2) + q^2(py + q^2)$	

UNIT – III

Applications of Partial Differential Equations

1	Classify the partial differential equation $u_{xx} + 3u_{yy} - 2u_x + 24u_y + 5u = 0$	
2	Classify the partial differential equation $4r + 5s + t + p + q = 0$	
3	Write the general form of second order partial differential equation and classify it.	
4	Verify whether the following partial differential equation is elliptic or hyperbolic $(x+1)u_{xx} - 2(x+2)u_{xy} + (x+3)u_{yy} = 0$	

5	Write the standard form of one dimensional wave equation with boundary and initial conditions.	

UNIT – IV

*** Vector Differntiation***

1	Find the directional derivative of $(x, y, z) = xy^3 + yz^3$ the point (2,-1,2) in the direction of vector $\bar{\imath} + 2\bar{\jmath} + 2\bar{k}$.	
2	If $\bar{F} = grad(x^3 + y^3 + z^3 - 3xyz)$ then find $div(\bar{F})$.	
3	Find the directional derivative of $\emptyset = xy^2 + yz^3$ at $(2,-1,1)$ along $\bar{\imath} + 2\bar{\jmath} + 2\bar{k}$.	
4	Find $curl(curl\ \bar{F})$ given that $\bar{F} = xy\bar{\imath} + y^2z\bar{\jmath} + z^2y\bar{k}$.	
5	Find the angle between the normals to the surfaces $xy = z^2$ at (4,1,2) and (3,3,-3)	

UNIT – V

*** Vector Integration***

1	State Green's theorem in the plane.	
2	State Stoke’s theorem.	
3	State Gauss divergence theorem.	
4	If $\bar{F} = 3xy\bar{\imath} - y^2\bar{\jmath}$ evaluate $\int \bar{F}.\overline{dr}$ along the curve in xy-plane $y = 2x^2$ from (0, 0) to (1, 2).	
5	Show that $\frac{1}{3}\int \bar{r}.\hat{n}ds = v$ where v is the volume of enclosed by surface S.	

PreparedBy : (_______________)