# Maximum likelihood estimation

## Definition and Purpose

Maximum likelihood estimation (MLE) is a method of **estimating the parameters of an assumed probability distribution, given some observed data.** If the likelihood function is differentiable, then we could use the derivative test for determining maxima value.

In mathematical notation, the goal of maximum likelihood estimation is to find the value of model parameters that maximize the likelihood function over the parameter space

Where is the observed data, is called the parameter space, is the likelihood function.

A picture containing chart

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Chart, scatter chart

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## Examples

If we assume the population distribution follows , while we observe , the likelihood of the data from our assumed distribution is

If we assume the population distribution as the likelihood function would be

We could treat the as the parameter and plot the parameter space

Chart

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Diagram

Description automatically generated with medium confidence

Diagram

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When we have multiple observations, the likelihood of those data is the join probability of the likelihood of all individuals, which is

For simplify the calculation, we take the log transformation on each likelihood function (since it is monotonous, which would not change the position of peak), so the equation becomes

And then, take the partial derivative of , we get

Let equal to 0, we get

take the partial derivative of , we get

Let equal to 0, we get

Diagram

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