Moran's I

Define the weight matrix

$$W_{ij} = \begin{cases} 1 \ \ if\ border \\ 0 \ \ otherwise \end{cases}$$

We could also use k-nearest neighbor weight matrix

If we use the weight matrix defined above, and our situation is blow

1	1	0	0
1	1	0	0

Then, we should let $W = \sum_i \sum_j w_{ij} = 20$, since we have 10 pairs border relationship. The dimension of our matrix is $(n-1)*(n-1)$.

Calculate the global mean

In this situation, our global mean is $\bar{x} = \frac{4}{8} = \frac{1}{2}$

Calculate the global variance

$$Var = \sum_{i=1}^{n}(x_i - \bar{x})^2 = 8 * \frac{1}{4} = 2$$

Calculate the Co-Variance by the weight matrix

For each pair of correlation, we calculate

$$\sum_{i=1}^{n}\sum_{j=1}^{n}(x_i - \bar{x})(x_j - \bar{x})$$

For

1	1	0	0
1	1	0	0

We have 4 pairs of 1-1 border relations, 2 pairs of 1-0 border relations, 4 pairs of 0-0 relations, then

$$Total = \sum_{i=1}^{n}\sum_{j=1}^{n}(x_i - \bar{x})(x_j - \bar{x}) = 8 * \frac{1}{4} + 8 * \frac{1}{4} + 4 * \left(-\frac{1}{4}\right) = 3$$

For

1	0	1	0
0	1	0	1

We have

$$Total = \sum_{i=1}^{n}\sum_{j=1}^{n}(x_i - \bar{x})(x_j - \bar{x}) = 20 * (-\frac{1}{4}) = 5$$

Calculate Moran's I

$$I = \frac{N}{W} * \frac{CoVar_{by\ Weight\ matrix}}{Var}$$

N is the total number of events we interested about, W is the sum of elements from weight matrix (how many relationships we care about), the fraction represents how many variance are contributed by location.

For

1	1	0	0
1	1	0	0

$$I = \frac{8}{20} * \frac{3}{2} = \frac{3}{5}$$

For

1	0	1	0
0	1	0	1

$$I = \frac{8}{20} * \frac{-5}{2} = -1$$

Notice: Moran 'I are limited from -1 to 1.