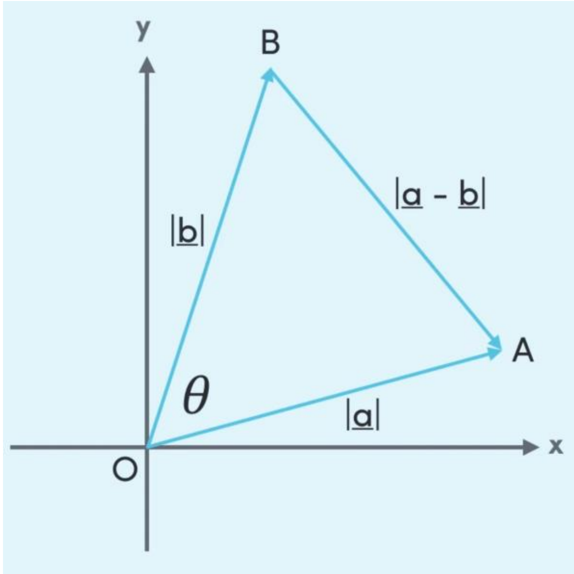
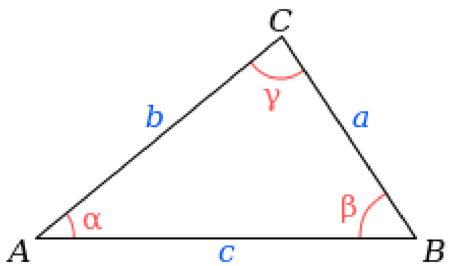
# Dot Product

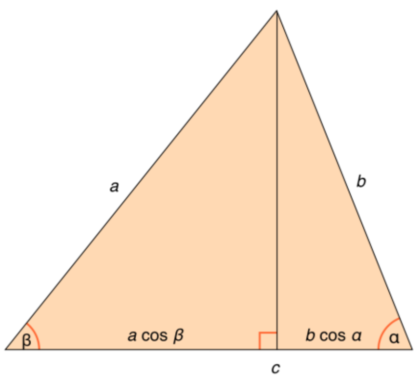
## Definition and prove

$$\vec{A} * \vec{B} = |A| * |B| \cos\theta = x_1x_2 + y_1y_2$$




For prove this, we need to use cosine rule, $c^2 = a^2 + b^2 - 2ab\cos\gamma$




In here, we do a quick review of cosine rule

$c = \text{a}\cos\beta + b\cos\alpha\ , so, c^2 = ac\cos\beta + bc\cos\alpha$. Based on the same theory, we get $a^2 = ab\cos\gamma + ac\cos\beta,\ b^2 = ab\cos\gamma + bc\cos\alpha$. Therefore,

$$
\begin{aligned}
a^2 + b^2 &= ab\cos\gamma + ac\cos\beta + ab\cos\gamma + bc\cos\alpha \\
&= ac\cos\beta + bc\cos\alpha + 2ab\cos\gamma \\
&= c^2 + +2ab\cos\gamma
\end{aligned}
$$

According to the cosine rule, we get $|a-b|^2 = |a|^2 + |b|^2 - 2|a||b|\cos\theta$ , in here, $\overrightarrow{|a|} = \sqrt{x_1^2 + y_1^2}$, so we get

$$|a-b|^2 = (x_1 - x_2)^2 + \ (y_1 - y_2)^2$$

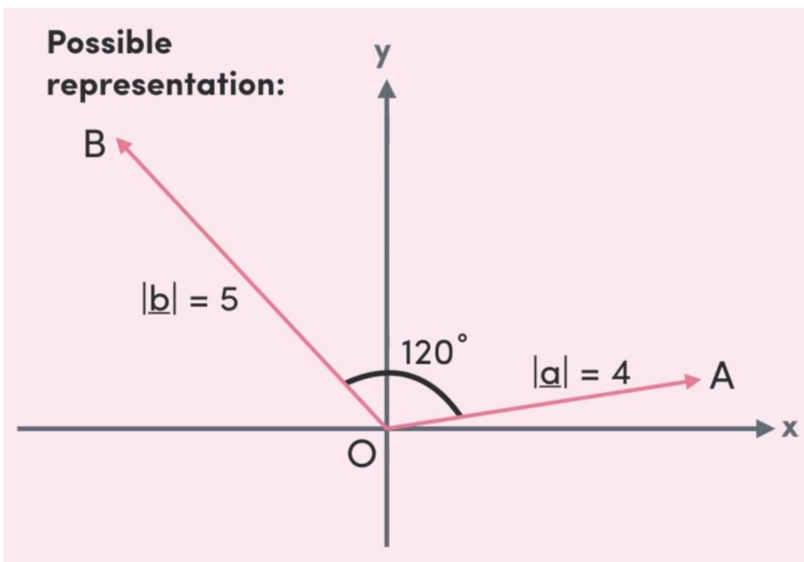
$$|a|^2 = x_1^2 + \ y_1^2$$

$$|b|^2 = x_2^2 + \ y_2^2$$

$$2x_1x_2 + 2y_1y_2 = 2|a||b|cos\theta$$

$$x_1x_2 + y_1y_2 = |a||b|\cos\theta$$

## Geometrical interpretation




$|b|\cos\theta$ means we project vector b to the line of vector A, then calculate the product of them