# Pearson Correlation and Spearman Correlation test

# Pearson Correlation test

## Definition and purpose

Measures the strength and direction of a linear association between two variables. It defines as

## Geometric Interpretation

Assume we have two regression line, with angle

Diagram

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# Spearman Correlation test

## Definition and purpose

Measures the strength and direction of a **monotonic association** (whether linear or not) between **two ranked variables (or ordinal data)**

For a sample of size n, the n raw scores are converted to ranks , and is computed as

**If the trend line between two datasets goes up and down, the spearman correlation is not suitable for this situation.**

## Assumption

1. Random Sample
2. **A monotonic association exists**

Chart, scatter chart

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1. Variables are at least ordinal

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1. Data contains paired samples
2. Independence of observations

## Determining significance

### Permutation Test

Use the premutation test (rely on resampling (bootstrapping) the original data assuming the null hypothesis, based on the resampled data to conclude how likely the original data to occur under the null hypothesis)

## Fisher Transformation (Fisher Z-transformation)

The fisher transformation can be used to test hypotheses about the value of the population correlation coefficient between X and Y.

### Definition

And standard error equals to

Or directly use this equation

To get the standard z score (std error = 1)

### Examples

Suppose we have the null hypothesis assuming , our sample size is 12, what is the p-value (risk of error) for

After the Fisher Z-transformation, we get a normal distribution with with standard error .

For the alternative hypothesis, we get the Z score after the transforming

Histogram

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# Kendall rank correlation coefficient

### Definition

Kendall’s rank correlation is a statistic used to measure the ordinal association (**degree of concordance**) between two measured quantities. Both **Kendall’s**  and **Spearman’s**  can be formulated as special cases of **General correlation coefficient.**

The greater the number of “**inversions**”, the smaller the coefficient will be. Range [-1,1]

**Here we talk about , not (can handle the tied correlations)**

is the number of concordant pairs, is the number of discordance pairs.

**Concordant pairs:** The number of observed ranks below a particular rank which are larger than that particular rank.

**Discordant pairs:** The number of observed ranks below a particular rank which are smaller in value than that particular rank.

### Determining significance

### Example

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Master | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| Student | 2 | 1 | 4 | 3 | 6 | 5 | 8 | 7 | 10 | 9 | 12 | 11 |
| C | 10 | 10 | 8 | 8 | 6 | 6 | 4 | 4 | 2 | 2 | 0 |  |
| D | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |  |

So, it is significance association here.

Another example is

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Master | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| Student2 | 12 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 1 |
| C | 0 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |  |
| D | 11 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  |

# Kendall’s vs Spearman’s

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Master | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|  | Student | 2 | 1 | 4 | 3 | 6 | 5 | 8 | 7 | 10 | 9 | 12 | 11 |
|  | C | 10 | 10 | 8 | 8 | 6 | 6 | 4 | 4 | 2 | 2 | 0 |  |
| D | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |  |
|  |  | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|  | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Master | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|  | Student | 12 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 1 |
|  | C | 0 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |  |
| D | 11 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  |
|  |  | 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 11 |
|  | 121 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 121 |

A close-up of a document

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Text

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