# Norm of vector

## Definition and purpose

In mathematics, a **norm** is a function from a real or complex vector space to the nonnegative real numbers that behaves in certain ways like the distance from the origin:

* it commutes with scaling
* obeys a form of the triangle inequality
* and is zero only at the origin

For example, if we want to measure two vectors/points from the 2-D coordinate system, we may want to use the Euclidean Distance (also called norm), which is written as

A picture containing line chart

Description automatically generated

To generalize, we change 2 into a variable, so the equation becomes

/

If we set , we get the Manhattan distance

/ Manhattan distance

## Usage

It can be utilized as a constraint. For example, if we consider the 2-D space, with , it looks like

Chart, line chart

Description automatically generated

For , it looks like

Chart

Description automatically generated

For

Chart

Description automatically generated

This property has been widely used in multiple theorems. Recall the definition of the Mean Squared Error

For Ridge Regression Constraint

Chart, schematic, box and whisker chart

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For Lasso Regression Constraint

Diagram, shape

Description automatically generated with medium confidence

# Norm of Matrix

## Definition and purpose

The norm of a matrix is **a real number** which is a **measure of the magnitude of the matrix**.

Recall that we use the norm of vectors to compare the “length” of vectors. We also want to come out with a metric to measure the “size” of matrices. E.g.

Because matrices are considered as linear transformations, so we define the norm of matrices as, with any given vector , it satisfies

with constraint

Recall that,

Chart

Description automatically generated

So, the means, after the transformation, the largest change of of vectors which lies on this circle

The norm of matrices shares the following properties:

## Calculation

### The norm

The of a square matrix is the maximum of the absolute column sums

Put simply, we **sum the absolute values down each column** and then take the **biggest answer**.

e.g.

Then

### The infinity norm

The infinity-norm of a square matrix is the **maximum of the absolute row sums**.

*e.g.*

Then

### The norm

The Euclidean norm of a square matrix is the **square root of the sum of all the squares of the elements.**

*e.g.*

Then