# SoftMax and Cross Entropy

Both those two methods serve the classification purpose

## SoftMax

### Why we need it

The softmax function, also known as the normalized exponential function. There are three purpose to apply softmax function

(1) restrict all outputs **within the interval [0,1]**

(2) all outputs will **add up to 1**

(3) new results remain in the **same order as the original (monotonous)**

### Definition

So, it is defended as

## Cross Entropy

### Why we need it

When we evaluate the results for classification, we **find the squared error is too strict**. Usually, we only require the highest value is the right label. For instance, we have a, b, c three possible outputs, if b is the ground truth, we only care about whether the value of b is the largest. If the output of b is 0.6, it does not matter whether the value of a and c is (0.2,0.2) or (0,0.4). But from squared error estimation, the first situation is preferable.

**To overcome the bias from norm, we apply Cross Entropy**

### Definition

#### What is the surprise

The surprise is the metric we use to against probability. For instance, . Then if A was happened, there is not too much surprise for us, since the probability of A is so high. So, in the very beginning, statisticians define surprise as the invert of probability, note as .

However, when , which means the occurrence of A is absolute, if we got A, we would not feel surprised at all. But from the equation defined above, we would get . Therefore, we take the log transformation on it, the equation becomes When , we get . When , we get . The graph of it looks like

Diagram

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In the convention, if we calculate the surprise of two possible outcomes, we use the log base 2, which is

#### Property of the surprise

If we calculate the surprise of two possible outcomes, , then we would get .

But what is the surprise of combination of ? Since the probability of AAB is , So

So, we get the first property: the surprise of a combination equals the sum of surprise of each element, which note as

And the expectation of the surprise equals to

With x is a specific value for Surprise

Now, plug the equation for surprise for x, we get the definition of the entropy

#### Examples

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Therefore, we could use entropy to **measures the difference / similarity in the number** of orange and blue chickens in each area. If we get Entropy = 1, it means we have balanced number of two types of chicken. If we **increase the difference between number of different chickens, we lower the entropy**.

### Cross entropy in Deep learning

Cross entropy is a little different from entropy, it is the expected entropy under the true(observed) distribution P when we use a coding scheme optimized for a predicted distribution Q. It defines as

is the number of output classes

#### Examples

If we have three outputs from a neural network as follows, how we calculate the cross-entropy value? Let’s take Setosa as the example

|  |  |  |
| --- | --- | --- |
| Species | Output from Neural network | Cross entropy |
| Setosa | 0.57 | =-1\*log (0.57) =0.56 |
| Virginica | 0.58 | =-1\*log (0.58) =0.54 |
| Versicolor | 0.52 | =-1\*log (0.52) =0.65 |

The total error of the model is simply summing all cross entropy together,

Application

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As show above, the cross entropy take punishment when model get worst predictions