# Markov Chains

## Definition

A Markov chain or Markov process is a stochastic model describing a sequence of possible events (Also known as state by convention) in which **the probability of each event depends only on the state attained in the previous event.**

In mathematical denotation, it is written as

This assumption makes a lot of problems in the real world easier.

Another rule is the sum of arrows from one state must be 1 since it represents probability.

### Transient state and recurrent state

Assume we have the following transition relations. Since there is no edge (arrow / probability) from other states to state 0, we called state 0 as **Transient state. Which means the probability of revisiting 0 is less than 1.**

In the contrast, states 1 and 2 are known as **recurrent state, which means the probability of revisiting is 1.**

A picture containing text, electronics, device, vector graphics

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In this situation, we claimed this Markov chain is **reducible**, it can be divided into two **irreducible** situations (**all states are recurrent**).

Diagram, schematic

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The most famous reducible Markov chain is **Gambler’s ruin,** which can be divided into three irreducible one, as follows

Diagram, schematic

Description automatically generated

## Example

### Find the stationary state of a Markov chain.

Imagine we have the following relationships among three states. We could denote those relationships by a **transition matrix A** (0 means no edge between two vertices)

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Diagram

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If we have the pizza now, denote as the vector , use the vector to multiply the transition matrix, we could get the probability of next day

If is the eigenvector of the transition matrix A, **we get by the constrain .** After solving those equations, we get (The number of eigenvectors is usually more than 1)

So, this is the stationary probability of the given situation.

# Chapman-Kolmogorov Theorem

## Design purpose

Assume we have the following Markov chain, **what is the probability of reaching the state from state after exactly steps.**

A screenshot of a video game

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Before dig into the general situation, we restrict the situation to “**probability of reaching state 2 from state 0 after 2 steps**”, it equals to

It equals to

So, **the chapman-Kolmogorov theorem** says, the probability from state to after steps, denote as

## Proof

# Hidden Markov Models (HMM)