Lab08 Probability and Distributions

**Handout date:** Wednesday, October 21, 2020

**Due date:** Friday, October 30, 2020 by midnight at eLearning’s Lab08Submit link

*This lab counts 4 % toward your total grade*

**Objectives:** In this lab you   
[a] practice probability theory;  
[b] evaluating event probabilities from underlying theoretical distributions.

**Format of answer:** Your answers (graphs and verbal description) should be handed in as ***hard-copy*** in ***one*** document. Add a running title into the header of the document with the following information: ***your name***, ***Lab08*** and ***page numbers***. Label each answer properly starting with its task number. Maintain the sequence of questions. For your answers, use ***Word's Equation Editor***. Format any code and computer output properly before inserting it into the document with your answer. -code and text output need to be in a ***monospaced*** font (i.e., fixed-pitch font) such as Courier New so proper spacing and alignments are preserved. Excessive, but irrelevant, output will lead to a deduction of your accumulated points.

# Task 1: Combinatorics (0.4 points)

Give for a set with the 6 elements the ***number of distinct sample outcomes*** given a sample size 4 elements. Show the *professionally typeset* equations that you applied to derive each answer.

(a) Sampling without replacement and distinguishing the order.

(b) Sampling without replacement and irrespectively of the order.

(c) Sampling with replacement and distinguishing the order.

(d) Sampling with replacement and irrespectively of the order.

# Task 2: Joint, marginal and conditional probabilities (1.0 point)

A speed radar identifies speeders and it also captured the license plates, which helped to distinguish between cars own by local residents, non-residents and commercial vehicles. In a given morning in total 250 cars passed by the speed radar.

(a) Complete table by filling in the missing joint and marginal frequencies. Insert the missing **counts in red**. (0.2 points)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Speed\Plates** | **Resident** | **Not Resident** | **Commercial** | **Sum** |
| **Tolerable** |  | **90** |  |  |
| **Excessive** |  | **30** | **15** | **50** |
| **Sum** | **100** |  |  | **250** |

(b) Calculate the ***joint probabilities*** and the ***marginal probabilities*** for the observed table. Report these in a properly formatted table rounded to three decimal places. (0.2 points)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Speed\Plates** | **Resident** | **Not Resident** | **Commercial** | **Sum** |
| **Tolerable** |  |  |  |  |
| **Excessive** |  |  |  |  |
| **Sum** |  |  |  |  |

(c) Calculate the joint probabilities of the cells ***assuming independence*** between Origin and Speed. Report these in a properly formatted table rounded to three decimal places. (0.2 points)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Speed\Plates** | **Resident** | **Not Resident** | **Commercial** | **Sum** |
| **Tolerable** |  |  |  |  |
| **Excessive** |  |  |  |  |
| **Sum** |  |  |  |  |

(d) Calculate the ***conditional column probabilities*** for the speed conditional on who drives the vehicle (local resident, not a resident or commercial driver). Report these in a properly formatted table rounded to three decimal places. (0.2 points)

|  |  |  |  |
| --- | --- | --- | --- |
| **Speed\Plates** | **Resident** | **Not Resident** | **Commercial** |
| **Tolerable** |  |  |  |
| **Excessive** |  |  |  |
| **Sum** |  |  |  |

(e) Interpret the conditional probabilities. Do the “*data tell a story*” about the driving habits of the local residents, drivers from outside the neighborhood and commercial drivers? (0.2 points)

# Task 3: Application of Probability Rules (0.4 points)

A group of 3 colleagues fly jointly before Covid-19 times to a conference. Their “cheapskate” boss only funds no-frill tickets, for which the seat assignment is done randomly by the airline. Their plane has no-frill rows of 3 seats each (window, middle, and aisle).

On both flights (to the conference and back) all 3 colleagues end up in the middle seat. Typeset your calculations with the equation editor.

(a) What is the probability that the group of colleagues is jointly unlucky on just one of the flights to end up on a middle seat? (0.1 points)

(b) What is the probability that the group of colleagues is jointly unlucky on both flights ending up on the middle seats? (0.1 points)

Probabilities based on sets: Let the following relationship between the sets and hold:

(c) Is the probability of or larger? Justify your answer (0.1 points)

(d) Does the inequality hold? Justify your answer (0.1 points)

# Task 4: Bayesian Theorem (1 point)

A real estate agent has a client with a specific preference of two school districts. The client is only a short period in town, so that the real estate agent can only show homes in one school district. School district has 18 available houses whereas school district has 12 available houses.

(a) Give the ***prior probabilities*** for each school district. Which school district shall the real estate agent take her clients to maximize the likelihood of finding a new home for the client? Make use of standard probability notation and show your equation. (0.2 points)  
  
While talking to the client the agent learns that the client prefers a home with mature trees on the property. This information is not part of regular real estate listings. However, from previous visits to districts and the agent has seen more trees in district . She *subjectively* thinks that a home site in district is treed with probability whereas for school district it is .

(b) Calculate the total probability . Make use of standard probability notation and show your equation. (0.4 points)

(c) Calculate the posteriori probabilities and . Make use of standard probability notation and show your equation. Will the real estate revise her choice of neighborhood in which she will show her client's homes? (0.4 points)

*Professionally typeset* the equations.

# Task 5: The binomial and Poisson distributions (0.8 points)

Chapter 14 “Probability Distributions” in Lander shows you how to calculate probabilities of events for the binomial distribution and Poisson distribution.

(a) Calculate the probabilities of for a the *binomial distribution* with and : (0.4 points)  
[i]

[ii]

[iii]

[iv]

(b) Calculate the probabilities of for a *Poisson distribution* with (0.2 points)  
[i]

[ii]

(c) What are the expectation and variance for the binomial in tasks 5 (a) and the Poisson distribution in task 5(b)? *Professionally typeset* the used equations. (0.2 points)

# Task 6: Normal Distribution (0.4 point)

(a) ***Manually*** draw the density-function and distribution-function of a continuous uniformly distributed random variable into the figures below. (0.2 points)

|  |  |
| --- | --- |
| Figure 1: Density Function | Figure 2: Distribution Function |

(b) Manually calculate the following probabilities for : (0.2 points)

(i)

(ii)