Sample Answer Lab01: Evaluation of Centroids and Directional Data

Handed out: Wednesday, January 22, 2020

Return date: Wednesday, February 05, 2020, at the beginning of the class.

Task 1: Cluster identification [2 points]

Open the file **`ClusteredPoints.dbf`** to identify bivariate normal distributed clusters is this dataset with the function **`Mclust( )`**.

1.1 Estimate the optimal number of clusters in this dataset [0.5 points]

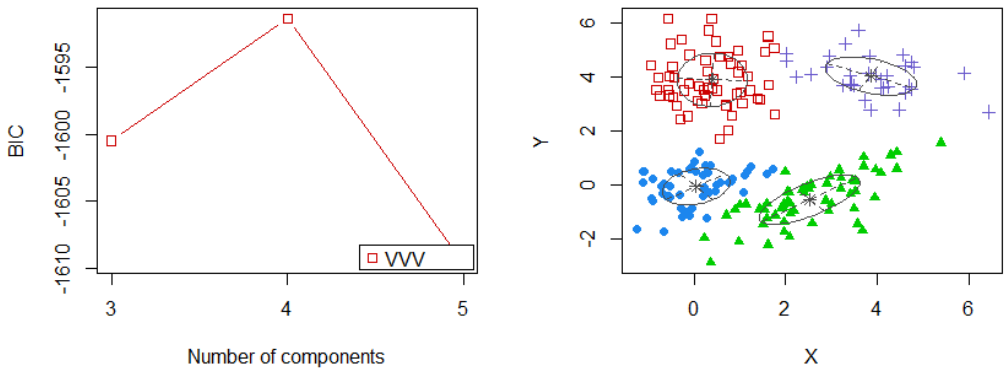
```
library(foreign)

library(mclust)

ClusterP <- read.dbf('./ClusteredPoints.dbf')

p.Mclust <- Mclust(ClusterP, G=c(3,4,5), modelNames="VVV")

summary(p.Mclust)
```

```
    ----------------------------------------------------
    Gaussian finite mixture model fitted by EM algorithm
    ----------------------------------------------------

    Mclust VVV (ellipsoidal, varying volume, shape, and
    orientation) model with 4 components:

     log.likelihood   n df       BIC       ICL
          -734.7653 200 23 -1591.392 -1608.453

    Clustering table:
     1  2  3  4
    50 58 59 33
```

The estimated optimal number of clusters is 4.

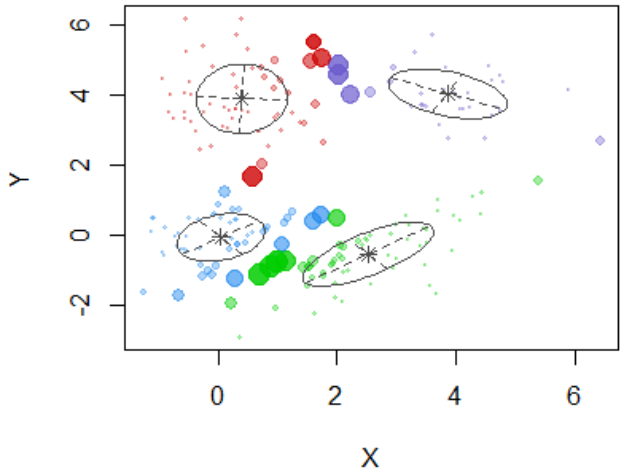
1.2 Generate a plot of the estimated classification [0.5 points]

```
plot(p.Mclust, what = "classification")
plot(p.Mclust,what = "BIC")
```



1.3 Generate a plot of the estimated uncertainty of assigning a point to a particular cluster and interpret the plot. [0. 5 points]

```
plot(p.Mclust, what = "uncertainty")
```



1.4 Report in a table the estimate number of points in a cluster and the cluster centroids. [0.5 points]

```
df1 <- data.frame(p.Mclust$parameters$mean)
colnames(df1) <- table(p.Mclust$classification)
```

Cluster No.	1	2	3	4
No. of points	50p	58	59	33
Cluster centroids	X 0.049 Y -0.046	X 0.399 Y 3.913	X 2.529 Y -0.528	X 3.868 Y 4.047

Task 2: Calculation of City Centroids/Medians [2 points]

Open the map in the file **MyStudyArea.zip**.

Task 2.1: Calculate the simple city centroids for each region. [0.25 points]

```
Shape <- rgdal::readOGR(dsn="./MyStudyArea/MyFourRegions.shp",integer64="warn.loss")
City <- rgdal::readOGR(dsn="./MyStudyArea/MyPopPlaces.shp",integer64="warn.loss")
CityDf <- as.data.frame(City)
MeanX <- tapply(CityDf[,"coords.x1"],CityDf[,"REGIONID"],mean,simplify=F)
MeanY <- tapply(CityDf[,"coords.x2"],CityDf[,"REGIONID"],mean,simplify=F)
(cbind(MeanX,MeanY))
```

```
  MeanX          MeanY
1 -0.001546545  0.001479636
2 -0.001352091  0.001024545
3 -0.0009622    0.0004012
4 -0.0005575714 0.001822714
```

Task 2.2: Calculate the population weighted city centroids for each region. [0.25 points]

```
regionSet <- unique(CityDf[,"REGIONID"])
xy_ma <- matrix(NA,length(regionSet),2)
```

```
for(i in 1:length(regionSet)){
  idxReg <- regionSet[i]
  City.id <- which(CityDf[,"REGIONID"] == idxReg)
  x.w.mean <- weighted.mean(CityDf[City.id,4],CityDf[City.id,"POPULATION"],na.rm=TRUE)
  y.w.mean <- weighted.mean(CityDf[City.id,5],CityDf[City.id,"POPULATION"],na.rm=TRUE)
  xy_ma[i,] <-c(x.w.mean,y.w.mean)

}
xy_ma
```

```
              [x]         [y]
[1,] -0.0014238401 0.0014111868
[2,] -0.0008487914 0.0015543882
[3,] -0.0014386750 0.0011918997
[4,] -0.0011261227 0.0004243523
```

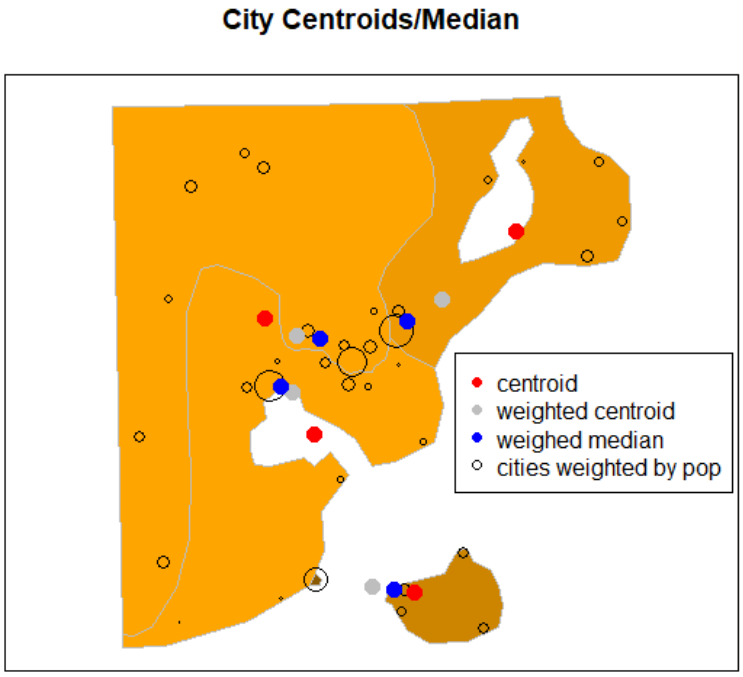
Task 2.3: Calculate the population weighted Euclidian city medians for each region. [0.25 points]

```
tol <- 0.0001
maxIter <- 80
nofPoints <- 10
xy_eu <- matrix(NA,length(regionSet),2)
for(i in 1:length(regionSet)){
  iIter <- 0
  City.id <- which(CityDf[,"REGIONID"] == regionSet[i])
  w <- CityDf[City.id,"POPULATION"]
  x <- CityDf[City.id,4]
  y <- CityDf[City.id,5]
  xOld <- xy_ma[i,1]
  yOld <- xy_ma[i,2]
  repeat{
    d <- sqrt((x-xOld)^2+(y-yOld)^2)                          # distance equation in step
1
    xNew <- sum(w*x/d)/sum(w/d)                               # update equation in step 2
    yNew <- sum(w*y/d)/sum(w/d)
    xyDiff <- sqrt((xNew-xOld)^2+(yNew-yOld)^2)               # calculate tolerance score
    xOld <- xNew
    yOld <- yNew
    iIter <- iIter + 1
    print(xyDiff)
    if (xyDiff < tol | iIter > maxIter) break
```

```
    Sys.sleep(0.5)
  } #end::repeat
  xy_eu[i,] <- c(xNew,yNew)
}
xy_eu
```

```
             [x]          [y]
[1,] -0.001328753 0.0013968477
[2,] -0.001002505 0.0014595419
[3,] -0.001486036 0.0012146427
[4,] -0.001151801 0.0004233249
```

Task 2.4: Map all three calculate centroids/medians jointly into a map with a proper map legend. Discuss which centroids or the median is most appropriate to show the center of the population distribution in a region. [0.5 points]



Weighted median is the most appropriate to show the center of population distribution within a region. The left two regions have concave shapes, the centroids are outside of the region, the region on the right has a hole inside, the centroid is also outside of the area. Since population distributions within a region are highly positively skewed, the weighted centroids are greatly impacted by the skewness, for example, the weighted centroid is outside of the bottom region. The island region has a small bridge head on the mainland. Therefore, the weighted centroid is pulled outside the study area.

The weighted median points are all within their region, and they best summarized the centers of population distributions within each region.

Task 2.5: The -script **`EuclidianMedian.r`** (see Lecture01) can be substantially optimized. [0.75 points]

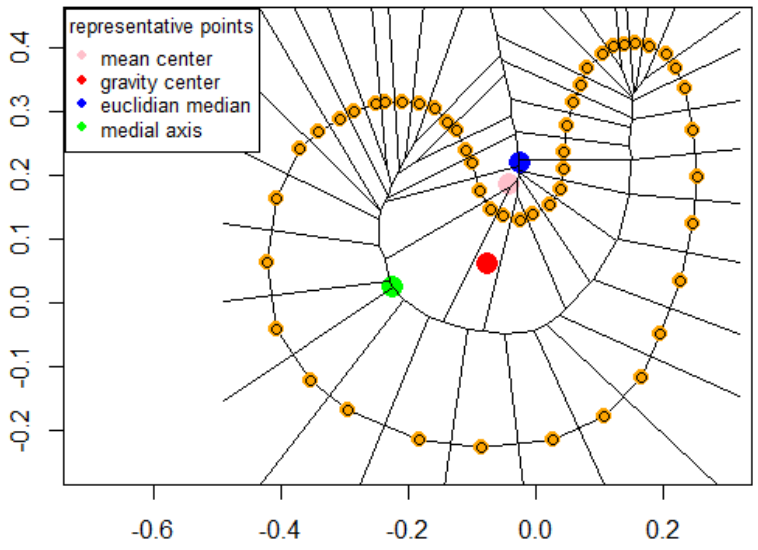
Use the user-defined function **`euclidMedian( )`** in the script in combination with the optimization function **`nlm( )`** to find the population weighted Euclidian Median. Show that section of your script which uses this optimization approach.

```
euclidMedian <- function(xy){
  ## Function that evaluates the cost at any point XY given
  ## the global vector argments: x, y, and w
  cost <- sum(w * sqrt( (x-xy[1])^2 + (y-xy[2])^2 ))
  return(cost)
} #end::euclidMedian

Med.nlm<- nlm(euclidMedian, c=(5,5))

cat("x-Median = ",round(nlmOpti$estimate[1],3) ,"\n",
    "y-Median = ",round(nlmOpti$estimate[2],3) ,"\n",
    "Min. Cost= ",nlmOpti$minimum,"\n")
```

Task 3: Polygon Centers [2 points]

Open the shape file **`ShapeKindey.shp`** which outlines the boundary of a polygon. You will identify several more or less representative central points for this polygon. In tasks 3.1 to 3.4 plot these points into the polygon and make also sure to highlight the boundary shape points. All distinct points should be in one plot with a legend. No need to your code.

Task 3.1: Add the ***arithmetic mean center*** to the map of your polygon. Does this point reasonably well represent the center of the polygon? [0.5 points]



Note: the orange points depict the shape points of the polygon. These are used to evaluate the different polygon centers. Each shape point has an equal weight.

Task 3.2: Add the ***center of gravity*** to the map of the polygon. Does this point reasonably well represent the center of the polygon? [0.5 points]

Outside of the polygon.

Task 3.3: Add the ***Euclidian median*** to the map of the polygon. Does this point reasonably well represent the center of the polygon? [0.5 points]

Inside the polygon but pulled toward the region of high density shape points. This point is calculated by default for shape-files. It can be extracted from a polygon with command **`coordinates( )`** .

Task 3.4: Add the ***medial axis*** point to the map of the polygon. Does this point reasonably well represent the center of the polygon? [0.5 points]

Inside the polygon and furthest away from the boundary edge.

Task 4: Directional Data Analysis [3 points]

The file **`CarCrashDirection.dbf`** holds information about the time and direction a car was travelling, when an accident occurred. An insurance analyst wants to know if, dependent on the time of day, accidents happen at predominate travelling directions.

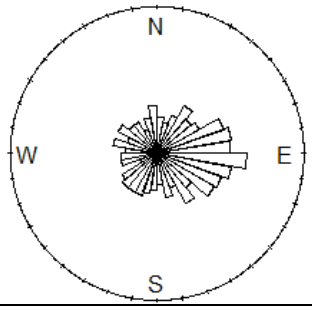
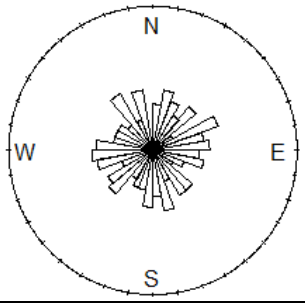
The analyst breaks the time of the crash down into a factor with four categories (a) morning rush hour, (b) day time, (c) evening rush hour, and (d) night time.

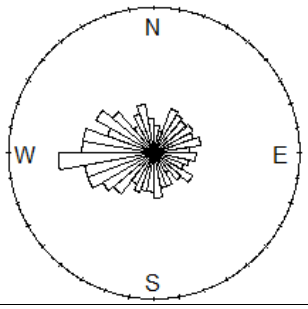
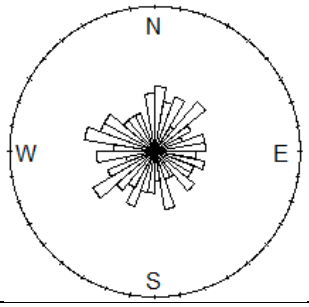
It is known that irrespectively of the time of day an equal number of cars is travelling in any direction, that is, the overall travel direction of all trips is uniformly distributed at any time.

Task 4.1: Why is it important for the analysis of the accident directions that the overall travel direction of all trips at any time is uniformly distributed? [1 point]

If the general traffic at any time would have a directional bias and assuming that the traffic accidents are proportional to the overall traffic, then we would expect to see coincidentally more accidents where more traffic takes place. On the other hand, if the general traffic follows a uniform directional pattern than any excessive direction of accidents points at a directional accident bias. Accident directions are a subset of the overall travel directions.

Task 4.2: Generate 4 rose diagrams for accident directions at each time of day and calculate the average direction and mean resultant length ρ. Interpret the plots. [1 point]

Car Crash at Morning Rush Hours	Car Crash at Day
	
N:259, average: 95.27 Rho: 0.360, *p*-val: 0	N:155, average: 18.90 Rho: 0.018, p-val: 0.949

Car Crash at Evening Rush Hours	Car Crash at Night
	
N:341, average: -87.59 Rho: 0.324, p-val: 0	N:99, average: -61.86 Rho: 0.101, p-val: 0.367

The four plots indicate that the car accidents in the day and night follow an uniform distribution, however during the morning rush hours, we can observe more car accidents in the eastern direction and during the evening rush hours, more accidents can be observed in the western direction.

The ρ-values for the day and night plots are close to 0, which also suggest an approximately uniform distribution. For the two plots on the right side, their ρ-values are 0.360 and 0.324 respectively, which means their distributions are less likely to be uniform. Moreover, their direction means suggest that morning rush hours have more accidents on the eastern direction and evening rush hours have more accidents on the western direction. This may be related to the blinding effects of the sun being low on the horizon.

Task 4.3: Perform Rayleigh tests for each time period to test whether the accidents at a given time period occur uniformly distributed or at a predominate direction. Draw your conclusions with regards to your visual interpretation in task 4.2. [1 point]

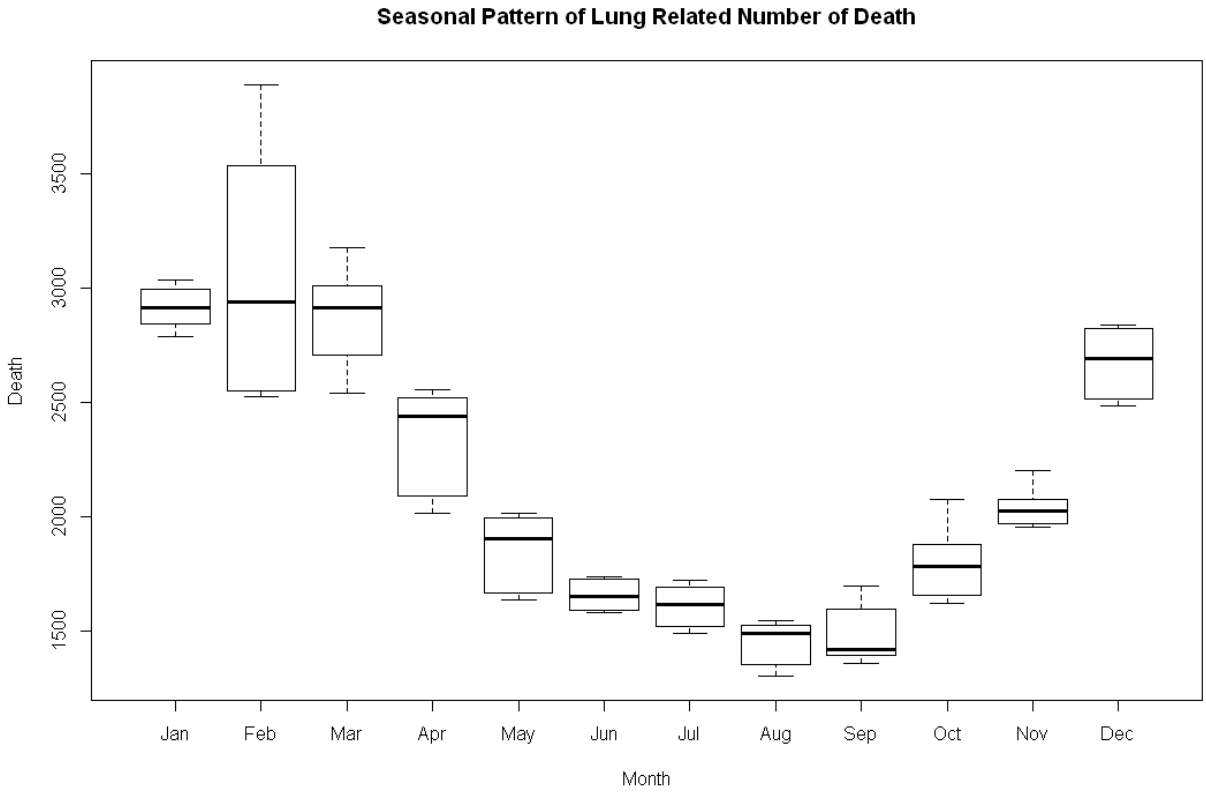
See p-values in above plot. The morning and evening accident distributions have a significant directional bias, excessive crashes in the morning for vehicles driving into the rising easterly sun and in the evening for vehicles driving into the westerly setting sun.

Task 5: Regression with a Circular Response Variables [1 point]

The workspace **`LungDf.RData`** contains data of lung disease related death counts by month over seven years.

Task 5.1: Discuss the seasonal pattern of the lung disease related death counts using a boxplot stratified by the months. [0.25 points]

```
boxplot(lungDeath~monthFac, data=lungDf,
        main="Seasonal Pattern of Lung Related Number of Death",
        xlab="Month", ylab="Death")
```



Lung death follows a seasonal pattern with the highest frequency of death in February and the lowest numbers in the summer months.

Task 5.2: Test with linear regression whether there is a *trend* in the lung disease related death counts over time [0.25 points]

```
summary(lm(lungDeath~monthNum,data=lungDf))
> ## Test of time trend
> summary(lm(lungDeath~t, data=lungDf))

Call:
lm(formula = lungDeath ~ t, data = lungDf)

Residuals:
   Min     1Q Median     3Q    Max
-861.6 -506.4 -146.6  480.9 1708.4

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 2273.642    145.459  15.631   <2e-16 ***
t             -3.502      3.463  -1.011    0.315
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 610.7 on 70 degrees of freedom
Multiple R-squared:  0.0144,  Adjusted R-squared:  0.0003176
F-statistic: 1.023 on 1 and 70 DF,  p-value: 0.3154
```

The number of lung death does not have a trend over time.

Task 5.3: Estimate with linear regression the *amplitude* and of the seasonal behavior of the lung disease related death counts. [0.5 points]

```
> ## Estimate seasonal model
> lung.lm <- lm(lungDeath~t+xCos+xSin, data=lungDf)
> summary(lung.lm)

Call:
lm(formula = lungDeath ~ t + xCos + xSin, data = lungDf)

Residuals:
    Min      1Q  Median      3Q     Max
-456.04 -126.48  -11.36  129.56  945.12

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 2193.959     55.716  39.378   <2e-16 ***
t             -1.319      1.329  -0.992    0.325
xCos         476.199     38.754  12.288   <2e-16 ***
xSin         632.901     39.047  16.209   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 232.4 on 68 degrees of freedom
Multiple R-squared:  0.8614,  Adjusted R-squared:  0.8553
F-statistic: 140.8 on 3 and 68 DF,  p-value: < 2.2e-16

>
> ## Calculate phaseShift and amplidute. Input has to be period length (i.e.,
1/lambda)
> round(ampPhase(lung.lm, "xCos", "xSin", 12), 2)
             Estimate     SE  2.5 % 97.5 %
Amplitude:     792.04  38.86 715.88 868.21
Phase Shift:     1.77   0.09   1.58   1.95
```

The phase shift indicates that the number of death peak around February (second month of the year) and that the winter number of death is around 800 cases higher than the average of 2200 cases, whereas the summer number of cases is around 800 cases lower than the average.