Lab11: Hypothesis Testing Concepts & One-Sample Tests

**Handout date:** Wednesday, November 13, 2019

**Due date:** Wednesday, November 20, 2019 at the beginning of class

*This lab counts 4 % toward your total grade*

### Note: Study for this lab Chapter 10 “Power Analysis” in Kabacoff “R in Action. Data Analysis and Graphics with R”. Available online at UTD’s Library.

# Task 1: Hypothesis Testing from a Binomial Distribution (2points)

A developer takes 225 independent random soil samples in a 150 acres plot, which is zoned for a new residential development. From previous studies in the broader neighborhood around the building site it is known that 39% of the homesteads need subsequent foundation repairs because they are built on shifting clay soil. This could have been avoided by laying more expensive foundations in order to avoid subsequent foundation work (welcome to the DFW real estate market).

The builder is interested whether it worthwhile to immediately build the more expensive foundations for all homes in the new development, or just to set some money aside for warrantee work to fix approximately 39% of the foundations (home owners good luck here that the builder is still in business when problems emerge).

Out of the 225 surface samples taken 125 indicate shifting clay soil. The developer is willing to take a 5% risk of making a wrong decision, which is spending more money on reinforced all foundations even though it is not necessary. On the other hand, if a substantial number of foundations fail, he will make a loss and by exhausting the set-a-side warrantee fund.

[a] Verbally explain whether the given scenario leads to a *one*- or a *two*-sided hypothesis test and what the *null* and the *alternative* hypothesis are. (0.3 points)

[b] Formulate and in a statistical notation. (0.1 points)

[c] Based on the sample observations perform an exact test of the null hypothesis using the binomial distribution. Try to exhaust the error probability as closely as possible. Properly interpret the test outcome in terms of its *prob*-value. Execute the test using -code and show your code. (0.4 points)

[d] Perform the same test using the normal approximation for the binomial distribution. Properly interpret the test outcome in terms of its *prob*-value. Execute the test using -code and show your code. (0.4 points)

[e] Why do both tests differ slightly in their conclusions? Which test would you prefer? (0.2 points)

[f] Plot the -error, i.e., , of the exact test assuming the true population probability ranges from , i.e., **pi.H1 <- seq(0.35, 0.90, by=0.01)**. Show your code and interpret the graph. (0.4 points)

[g] Why is the -error at equal to ? At which hypothetical value of the estimated sample proportion do you think the beta error becomes acceptable. Justify. (0.2 points)

Hints: The -functions **qbinom( )** and **pbinom( )** allow you to perform the exact test (see Chapter 9 script **ExactBinomialTest.r**) and the function **pnorm( )** will give you the error probability of the approximate test. The function **binom::binom.power( )** calculates the power.

# Task 2: Testing for Correlation (1 point)

In Lab 5 Task 8 you visualized the correlation pattern of 3 datasets. Continue with the three data-frames **cc1**, **cc2** and **cc3** in workspace **Part1Data.Rdata** to test whether pairs of variables are significantly correlated. See the function **cor.test()** for the significance test. For all tests assume that an error probability of and use independence, i.e., , as benchmark.   
Interpret the *prob*-values and the calculated confidence intervals. Draw your conclusions about the stated hypotheses. Make sure that you properly account for the one- and two-sided test scenarios.

[a] Test the hypotheses against for the variable pair in **cc1**. Interpret the output. (0.2 points)

[b] Test the hypotheses against for the variable pair in **cc2**. Interpret the output. (0.2 points)

[c] Test the hypotheses against for the variable pair in **cc3**. Interpret the output. (0.2 points)

[d] Use the function **pwr::pwr.r.test( )** to evaluate for and the required sample sizes to achieve a power of 0.7, 0.8 and 0.9. Explain how the power and sample size relate to each other. You may want to think in terms of the test statistic distributions assuming the null and the alternative hypotheses are true. (0.2 points)

[e] Use the function **pwr::pwr.r.test( )** to evaluate for a sample size of and the power for given error probabilities of 0.1, 0.05 and 0.01. Explain how the power and the error probability relate to each other. You may want to think in terms of the test statistic distributions assuming the null and the alternative hypotheses are true. (0.2 points)

# Task 3: One-Sample Testing for a Given Expected Value (1 point)

A stream has been monitored weekly for several years. Its total dissolved solids in the stream is 60 parts per million. Overall, the distribution of the dissolved solids appears to be normal distributed without systematic variation.

Following recent changes in the land-use within the catchment area new weekly samples were taken for 36 weeks. These samples indicate that the dissolved solids have increased on average to 68 parts per million with a week to week standard deviation of 16 parts per million.

[a] Perform the six steps of a classical hypothesis testing using a one-sided hypothesis against with a given error probability of . (0.3 points)

[b] Calculate the prob-value of the observed sample. (0.1 points)

[c] Use the **pwr::pwr.t.test( )** function to fill in the missing values into the table below. (0.2 points)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Scenario: | Effect Size: | Sample Size: | -error: | Power: |
| 1 | 0.25 | 36 | 0.05 | **???** |
| 2 | 0.25 | 36 | 0.01 | **???** |
| 3 | **???** | 36 | 0.01 | 0.5 |
| 4 | **???** | 144 | 0.01 | 0.5 |
| 5 | 0.50 | **???** | 0.01 | 0.99 |

[d] Address the following questions in a sentence or two (0.4 points)

Why is the power shrinking when the -error shrinks?

Why is the power shrinking for a smaller effect size?

Why is the power increasing of a larger sample size?

Can power analysis be used to determine a required sample size?