1. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a function. Which of the following expressions corresponds to $f'(2)$, the slope of the tangent line to the graph of $f(x)$ at $x = 2$?

1 / 1 point

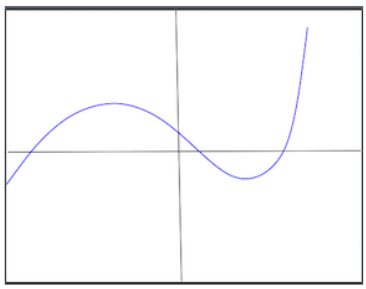
- $f'(2) = 2$
- $f'(2) = mx + b$
- $f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$
- $f'(2) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

Correct

This expression can be obtained from the first screen of our video by plugging in 2 for a.

2. Suppose that $h : \mathbb{R} \to \mathbb{R}$ is a function whose graph is shown as the blue curve in the figure. For how many values of a is $h'(a) = 0$?

1 / 1 point



- 3
- Never
- Always
- 2

Correct

$h'(a)$ gives the slope of the tangent line to the graph of h at the point $x = a$.

When $h'(a) = 0$, this means that the tangent line is horizontal.

There are two places (one on each side of the y-axis) where this tangent line is horizontal, so this answer is correct.

1. Re write the number $784 = 2 \times 2 \times 2 \times 2 \times 7 \times 7$ using exponents. 1 / 1 point

- ○ $(16^4)(49^2)$
- ◉ $(2^4)(7^2)$
- ○ $(2 \times 7)^6$
- ○ $(2^6)(7^6)$

Correct
For this type of problem, count the number of times each relevant factor appears in the product. That number is the exponent for that factor.

2. What is $(x^2 - 5)^0$? 1 / 1 point

- ○ $(x^2) - 5$
- ○ -4
- ◉ 1
- ○ (x^2)

Correct
Any real number (except zero) raised to the"zeroith" power $= 1$.

3. Simplify $((x - 5)^2)^{-3}$ 1 / 1 point

- ○ $(x - 5)$
- ○ $(x - 5)^{-1}$
- ○ $(x - 5)^{-5}$
- ◉ $(x - 5)^{-6}$

Correct
By Rule 2, "Power to a Power," multiply the exponents and get:

$$(x - 5)^{(2 \times -3)} = (x - 5)^{-6}$$

By the definition of negative exponents, this is equal to $\dfrac{1}{(x - 5)^6}$

4. Simplify $(\frac{8^2}{8^7})^2$ 1 / 1 point

- (●) 8^{-10}
- ( ) 8^{-4}
- ( ) 8^{-1}
- ( ) 8^{-5}

Correct

We can first simplify what is inside the parenthesis to 8^{-5} using the Division and Negative Powers Rule.

Then apply division and negative powers-- the result is the same. $\frac{8^4}{8^{14}} = 8^{-10}$

5. $\log 35 = \log 7 + \log x$ 1 / 1 point

Solve for x

- ( ) 4
- (●) 5
- ( ) 28
- ( ) 7

Correct

$\log(x) = \log 35 - \log 7$

$\log(x) = \log\ (\frac{35}{7})$

By the Quotient Rule $\log x = \log 5$

6. $\log_2(x^2+5x+7)=0$ 1 / 1 point

Solve for x

- ○ $x=2$
- ○ $x=2$ or $x=3$
- ○ $x=3$
- ◉ $x=-2$ or $x=-3$

Correct

We use the property that $b^{\log_b a}=a$

Use both sides as exponent for 2.

$2^{\log_2 x^2+5x+7}=2^0$

$x^2+5x+7=1$

$x^2+5x+6=0$

$(x+3)(x+2)=0$

$x=-3$ OR

$x=-2$

7. Simplify $\log_2 72-\log_2 9$ 1 / 1 point

- ◉ 3
- ○ $\log_2 4$
- ○ $\log_2 63$
- ○ 4

Correct

By the quotient rule, this is $\log_2 \frac{72}{9}=\log_2 2^3=3$

8. Simplify $\log_3 9 - \log_3 3 + \log_3 5$ **1 / 1 point**

- ○ 15
- ◉ $\log_3 15$
- ○ $\log_3 8$
- ○ 8

Correct

By the Quotient and Product Rules, this is $\log_3 \dfrac{9 \times 5}{3} = \log_3 15$

9. Simplify $\log_2(3^8 \times 5^7)$ **1 / 1 point**

- ○ $(5 \times \log_2 3) + (8 \times \log_2 5)$
- ◉ $(8 \times \log_2 3) + (7 \times \log_2 5)$
- ○ $56 \times \log_2 15$
- ○ $15 \times \log_2 56$

Correct

We first apply the Product Rule to convert to the sum: $\log_2(3^8) + \log_2(5^7)$. Then apply the power and root rule.

10. If $\log_{10} y = 100$, what is $\log_2 y = ?$ **1 / 1 point**

- ○ 500
- ◉ 332.19
- ○ 301.03
- ○ 20

Correct

Use the change of base formula, $\log_a b = \dfrac{\log_x b}{\log_x a}$

Where the "old" base is x and the "new" base is a.

So $\dfrac{100}{\log_{10}(2)} = \dfrac{100}{0.30103} = 332.19$

11. A tree is growing taller at a continuous rate. In the past 12 years it has grown from 3 meters to 15 meters. What is its rate of growth per year? **1 / 1 point**

- (●) 13.41%
- ( ) 11.41%
- ( ) 12.41%
- ( ) 10.41%

Correct

$$\frac{\ln \frac{15}{3}}{12} = 0.1341$$

12. Bacteria can reproduce exponentially if not constrained. Assume a colony grows at a continually compounded rate of 400% per day. How many days before a colony with initial mass of $6.25 \text{ X } 10^{-10}$ grams weights 1000 Kilograms? **1 / 1 point**

- (●) 8.75 days
- ( ) 87.5 days
- ( ) 875 days
- ( ) 0.875 days

Correct

$$6.25 \times 10^{-10} \times e^{4t} = 10^6$$

$$4t = \ln\left(\frac{10^6}{(6.25 \times 10^{-10})}\right) = 35.00878$$

$$t = \ln \frac{10^6}{6.25 \times 10^{-10}} = 8.752195$$

1. Convert $\frac{1}{49}$ to exponential form, using 7 as the factor. **1 / 1 point**

- ○ 49^{-1}
- ◉ 7^{-2}
- ○ $\frac{7}{7^3}$
- ○ (7^2)

Correct
The rule for a factor to a Negative exponent is to divide by the same factor to a positive exponent with the same absolute value.

2. A light-year (the distance light travels in a vacuum in one year) is $9,460$ trillion meters. Express in scientific notation. **1 / 1 point**

- ○ 0.946×10^{16}
- ○ 9.46×10^{15} kilometers
- ◉ 9.46×10^{15} meters.
- ○ 9460×10^{12} meters

Correct
$9,460$ is (9.4×10^3) meters and one trillion meters is 10^{12} meters. $(9.4 \times 10^3)(10^{12})$ = 9.4×10^{15}. A kilometer is 1000 meters.

3. Simplify $(x^8)(y^3)(x^{-10})(y^{-2})$ **1 / 1 point**

- ○ $(x)(y^{-2})$
- ○ $(x^{-80})(y^{-6})$
- ○ $(x^2)(y)$
- ◉ $(x^{-2})(y)$

Correct
By the Division and Negative Powers Rule, this is $(x^{(8-10)})(y^{(3-2)})$

4. Simplify $[(x^4)(y^{-6})]^{-1}$ **1 / 1 point**

- (selected) $(x^{-4})(y^6)$
- $\frac{(x^{-}4)}{(y^6)}$
- $\frac{(x^4)}{(y^{-6})}$
- $(x^3)(y^{-7})$

Correct
By the Power to a Power Rule, each of the exponents is multiplied by (-1)

5. Solve for x: **1 / 1 point**

$\log_2(39x) - \log_2(x-5) = 4$

- ○ $\frac{39}{23}$
- ○ $\frac{80}{38}$
- ◉ $\frac{-80}{23}$
- ○ $\frac{23}{80}$

Correct

$\log_2 \frac{39x}{(x-5)} = 4$ by the Quotient Rule.

Since both sides are equal, we can use them as exponents in an equation.

$$2^{\log_2 \frac{39x}{(x-5)}} = 2^4$$

$$\frac{39x}{(x-5)} = 16$$

$$39x = 16 \times (x-5)$$

$$39x = 16x - 80$$

$$23x = -80$$

$$x = \frac{-80}{23}$$

6. Simplify this expression:

1 / 1 point

$$(x^{\frac{1}{2}})^{\frac{-3}{2}}$$

- ○ x^{-1}
- ○ $x^{\frac{4}{3}}$
- ○ $x^{\frac{1}{3}}$
- ◉ $x^{\frac{-3}{4}}$

Correct

We use the Power to a Power Rule -- multiply exponents:

$$x^{\frac{1}{2}\times\frac{-3}{2}} = x^{\frac{-3}{4}}$$

7. Simplify $\log_2 8 - \log_2 4 - (\log_3 4.5 + \log_3 2)$ 1 / 1 point

- ○ 2
- ○ 0
- ◉ -1
- ○ 1

Correct
This is equivalent to:

$$\log_2(\tfrac{8}{4}) - \log_3(4.5 \times 2) = 1 - 2 = -1$$

8. If $\log_3 19 = 2.680$, what is $\log_9 19$? 1 / 1 point

- ○ 0.4347
- ○ 0.8934
- ◉ 1.304
- ○ 5.216

Correct
To convert from $\log_3$ to $\log_9$, divide by $\log_3 9$. Which is equal to 2, so the answer is 1.34

9. If $\log_{10} b = 1.8$ and $log_a b = 2.5752$, what is a? 1 / 1 point

- 3
- 4
- 5 (selected)
- 6

Correct

To solve for a in the formula;

$$\log_a b = \frac{\log_x b}{\log_x a}$$

$\log_a b = 2.5752$ and $\log_{10} b = 1.8$

Therefore, $\log_{10} a$ must equal to $\frac{1.8}{2.5752} = 0.69897$

Treating both sides of equation $\log_{10} a = 0.69897$ as exponents of 10 gives $a = 10^{0.69897} = 5$

10. An investment of $1,600$ is worth $7,400$ after 8.5 years. What is the continuously compounded rate of return of this investment? 1 / 1 point

- 18.02% (selected)
- 17.01%
- 20.01
- 19.01%

Correct

$$\frac{\ln \frac{7400}{1600}}{8.5} = 0.18017$$

11. A pearl grows in an oyster at a continuously compounded rate of $.24$ per year. If a 25-year old pearl weighs 1 gram, what did it weigh when it began to form?

1 / 1 point

- (●) 0.002478
- ( ) 0.02478
- ( ) 0.2478
- ( ) 0.0002478

Correct

$$e^{(0.24\times 25)} = \frac{1}{x}$$

$$x = \frac{1}{(e^{0.24\times 25})}$$

$$x = \frac{1}{403.4288}$$

$$x = 0.002478$$

12. $\log_2 z = 6.754$. What is $\log_{10}(z)$? 1 / 1 point

- 0.82956
- 0.49185
- 1.3508
- (selected) 2.03316

Correct

$$\frac{\log_2 z}{\log_2 10} =$$

$$(\log_{10} z) \times (\log_2 10) = 3.321928$$

Therefore, $\log_{10} z = \dfrac{6.754}{3.321928} = 2.03316$

13. Suppose that $g : \mathbb{R} \to \mathbb{R}$ is a function, and that $g(1) = 10$. Suppose that $g'(a)$ is negative for every single value of a.Which of the following could possibly be $g(1.5)$? 1 / 1 point

- $g(1.5) = 10.1$
- $g(1.5) = 11$
- (selected) $g(1.5) = 9.7$
- $g(1.5) = 103.4$

Correct

Since the slope of the tangent line to the graph of g is negative everywhere on the graph, we know that g is *decreasing* function! And therefore we must have $g(1.5) < g(1)$. That is the case here, so this value is at least possible.