Modulus & inner product

Length of vector (also called size)

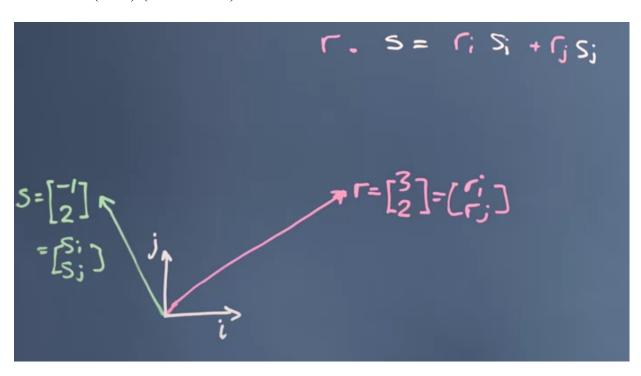
$$r * r = |r|^2$$

The dot product of a vector, also called inner scalar or projection product (just a number)

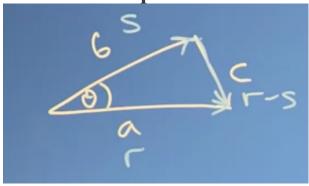
r * s = s * r (commutative)

r * (s + t) = r * s + r * t (distributive)

r * as = a (r * s) (associative)



Cosine & dot product



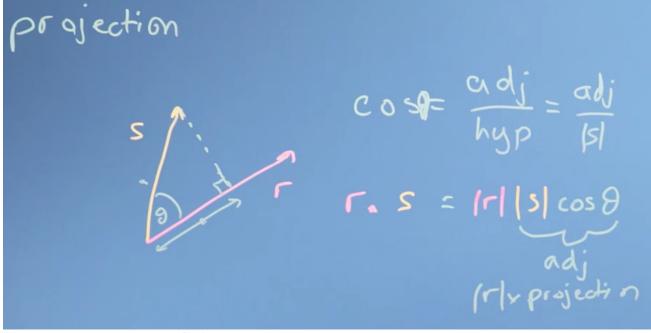
Cosine rule

$$c^{2} = a^{2} + b^{2} - 2ab * \cos \theta$$
Based on this rule, so
$$|r - s|^{2} = |r|^{2} + |s|^{2} - 2|r||s| \cos \theta$$

$$|r|^{2} + |s|^{2} - 2r * s = |r|^{2} + |s|^{2} - 2|r||s| \cos \theta$$
So, $r * s = |r||s| \cos \theta$

$$\cos \theta = \frac{r * s}{|r||s|}$$

Projection



Scalar Projection
$$\frac{r*s}{|r|} = |s| \cos \theta$$

Vector Projection $r * \frac{r*s}{|r||r|} = \frac{r*s}{r*r} * r$

Changing basis

Vectors we used to define a space called basis vectors

Basis, vector space, and linear independence

Basis is a set of n vectors that:

- (i) are not linear combinations of each other (linearly independent)
- (ii) span the space
- The space is then n-dimensional

The first rule suggests that, if we have 3 basis vectors b_1 , b_2 , b_3 , we could not find any numbers to satisify that: $b_3 = \alpha * b_1 + \beta * b_2$, it called linear independent among basis vectors