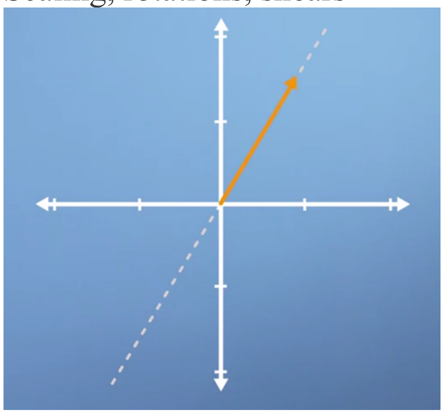
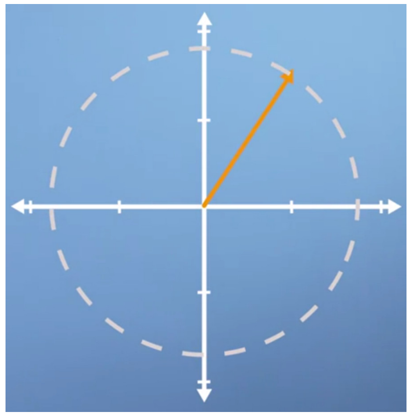
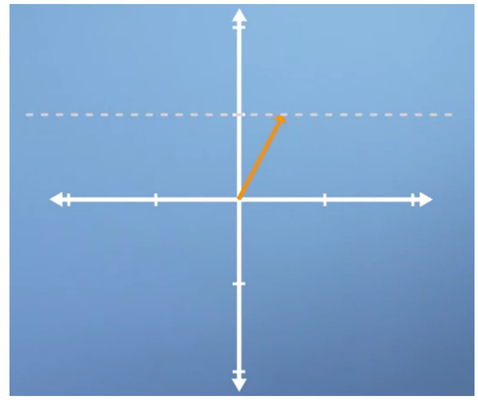
Week5 Eigen Factor

Sunday, November 1, 2020 9:10 PM

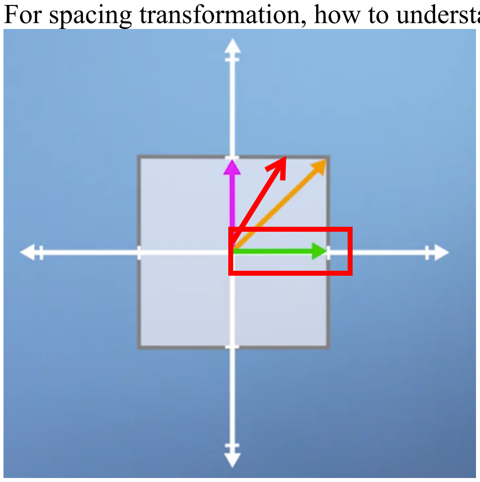
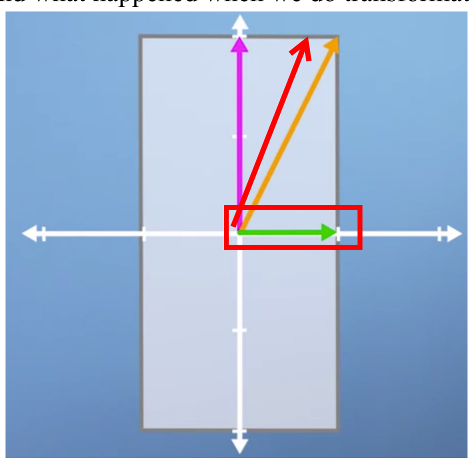
- **What are eigenvalues and eigenvectors?**

  Space operation including

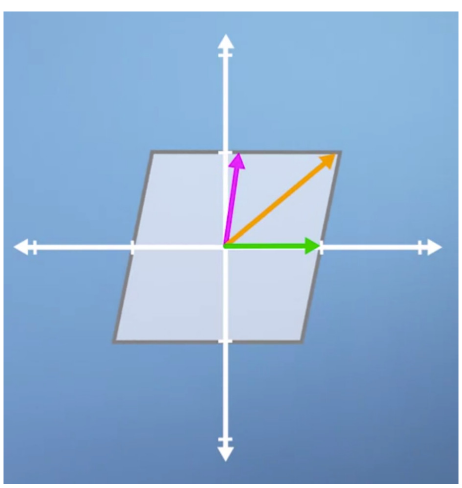
  1. Scaling, rotations, shears

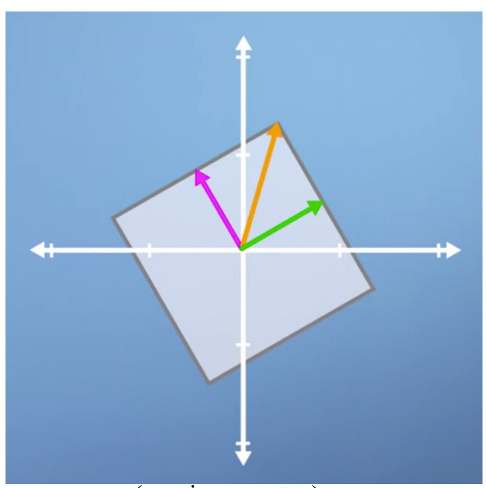
For spacing transformation, how to understand what happened when we do transformation? We use a square help us to visualize it.

For this situation, the horizontal vector do not be changed after transformation, it is special for characterizing this square, which is why we refer to them as eigenvectors. (Since the horizontal value do not be changed, so it's eigen value is 1, the vertical one is 2)



(only 1 eigen vector)

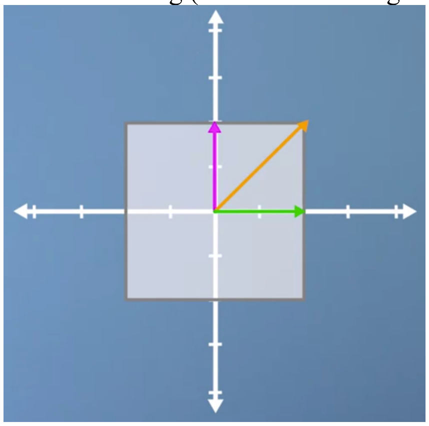


(no eigen vector)

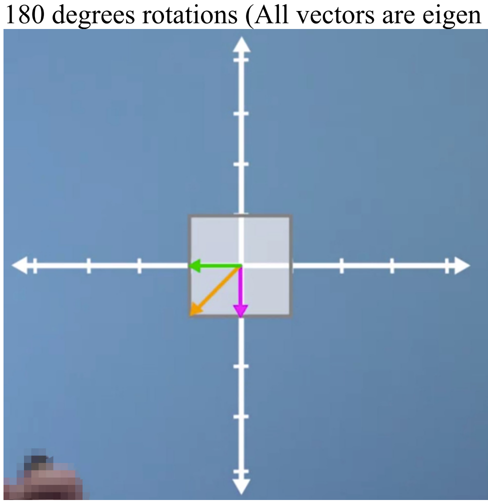
Once the vector direction is not be changed, then it could be the eigen vector.

- **Special eigen-cases**

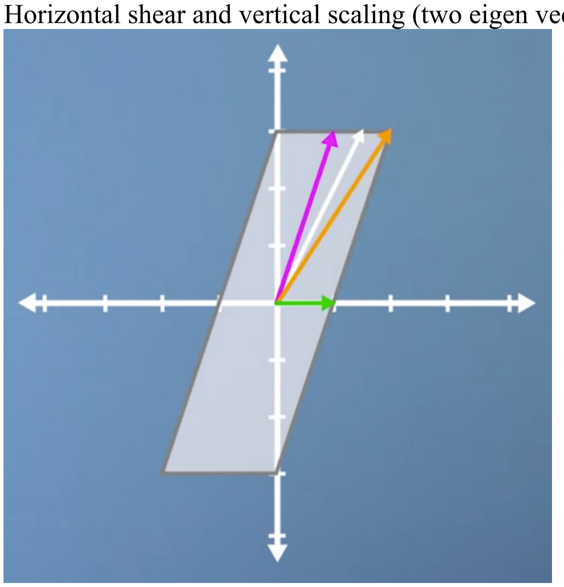
Uniform scaling (All vectors are eigen vectors).



180 degrees rotations (All vectors are eigen vectors).



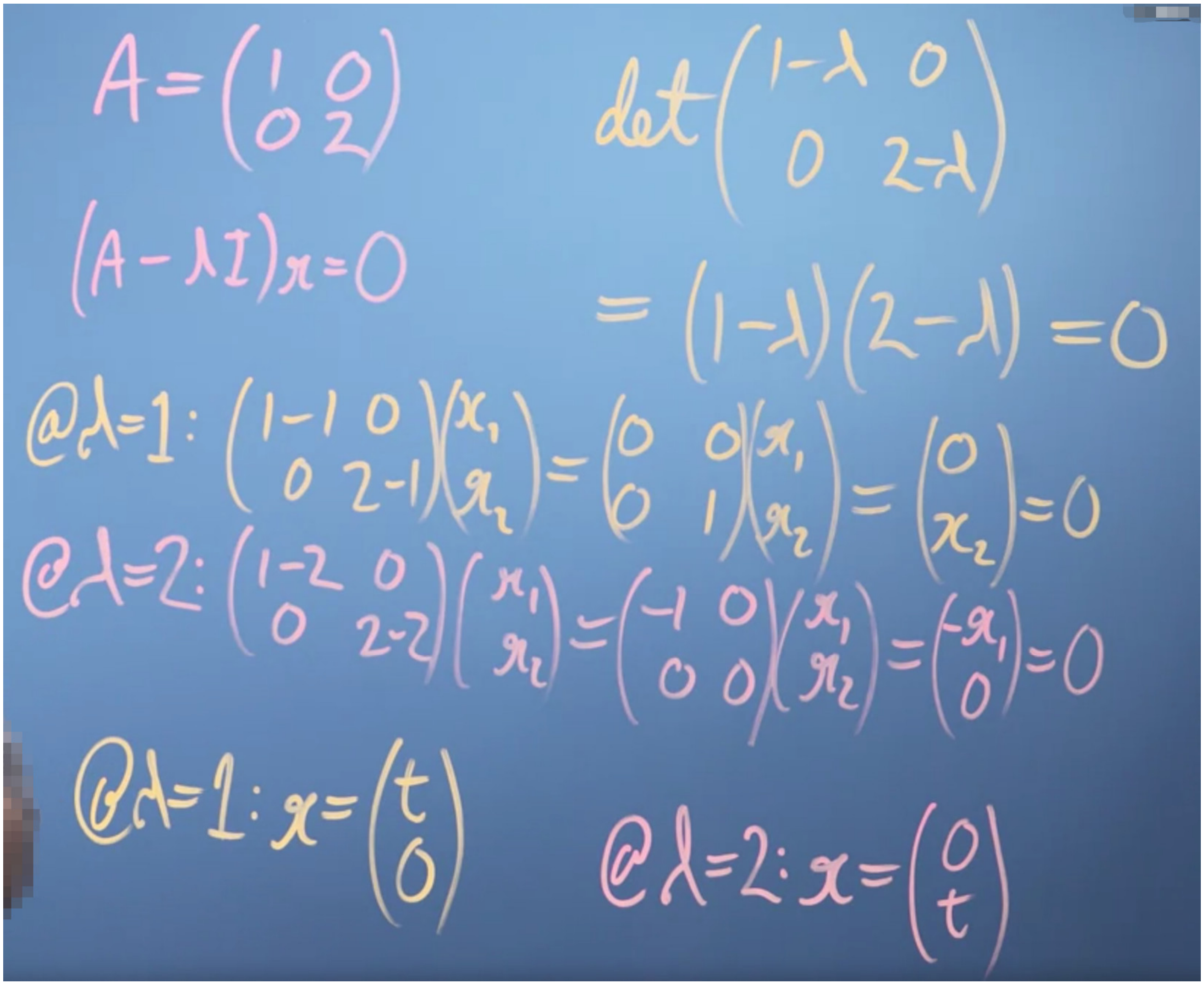
Horizontal shear and vertical scaling (two eigen vector: including the white one)



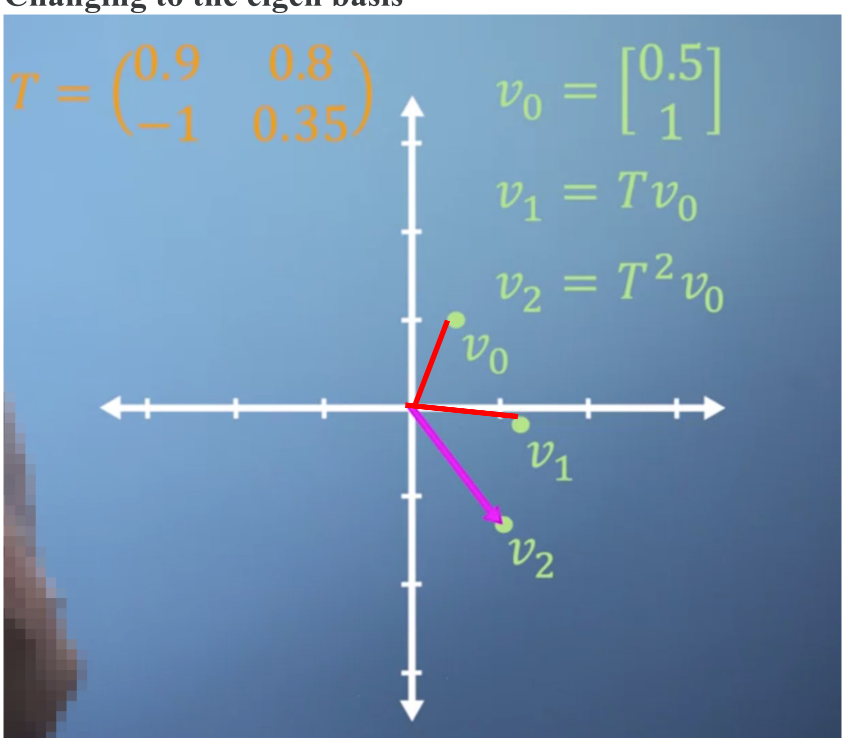
- **Calculating eigenvectors**

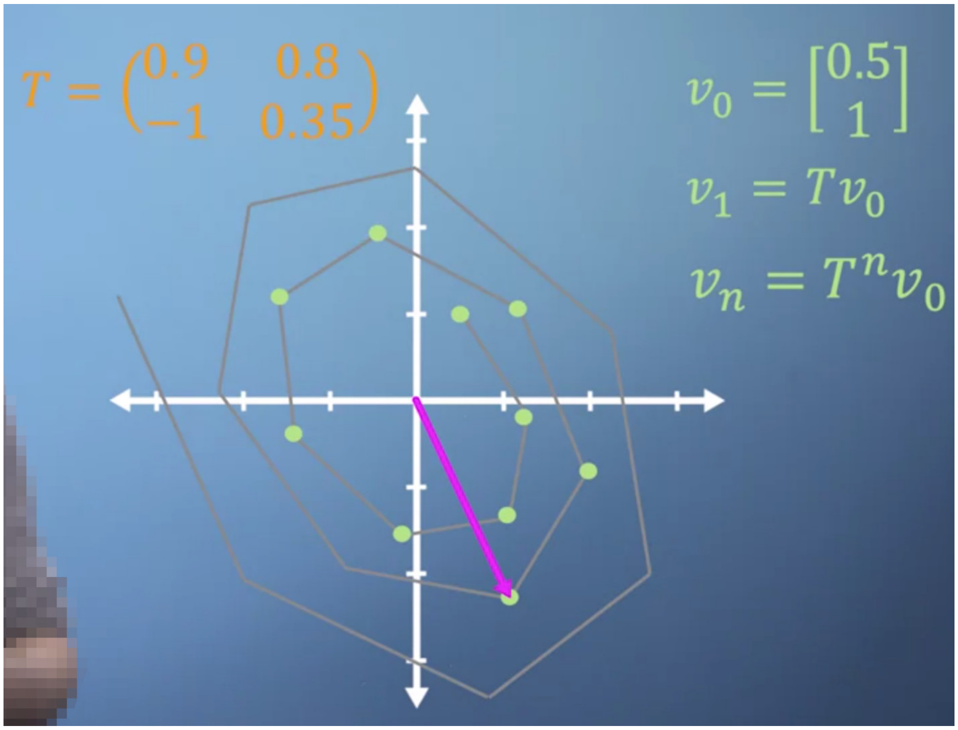
Since eigenvectors does not change the direction, So if we notate it as x. Then we have Matrix $A * x = \lambda x$. Which λ is just a scaling number. A is n dimensional matrix (n*n), so we could have $(A - \lambda I)x = 0$. So eigher $A - \lambda I = 0$ or $x = 0$. We do not care about $x = 0$ Since it is a trivial solution. So, we just need to find $A - \lambda I = 0$.

Suggest A is a 2*2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then we have $\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) = 0, \Rightarrow \lambda^2 - (a + d)\lambda + ad - bc = 0$

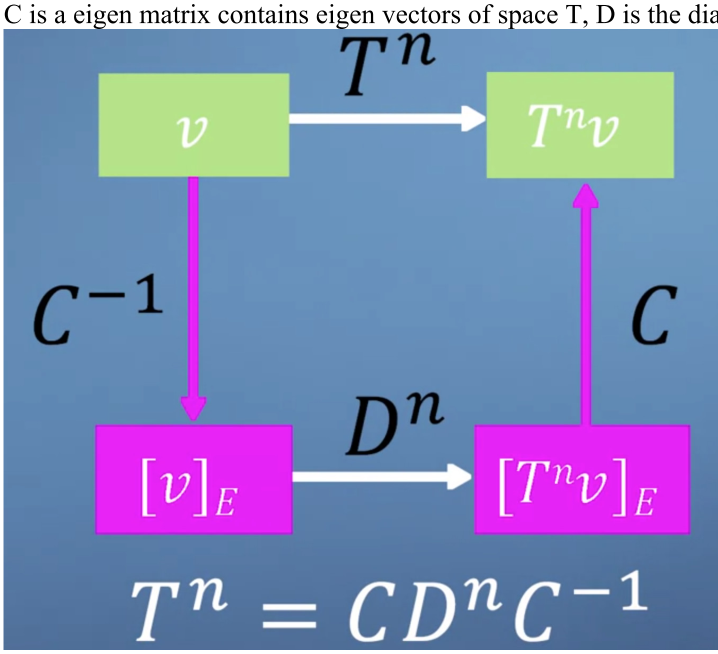


- **Changing to the eigen basis**





C is a eigen matrix contains eigen vectors of space T, D is the diagonal matrix, calculated by $= \mathrm{C}^{-1}\mathrm{TC}$



- **Eigen basis example**

  Suggest we have transforming matrix $\mathrm{T} = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$, we could get when $\lambda = 1, \mathrm{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \lambda = 2: \mathrm{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

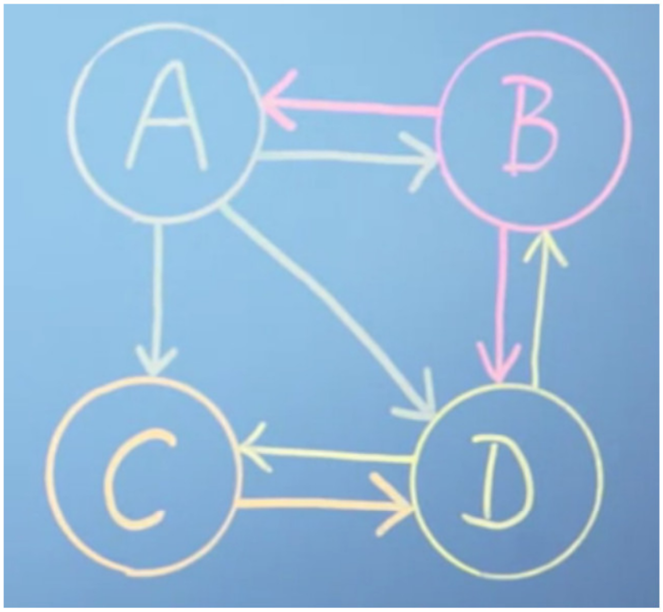
  And $\mathrm{T}^2 = \begin{pmatrix} 1 & 3 \\ 0 & 4 \end{pmatrix}$, since we already have two eigen vectors, so we could calcuate it by using eigen matrix $\mathrm{C} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$,

  $\mathrm{T}^2 = \mathrm{C} * \mathrm{D}^2 * \mathrm{C}^{-1} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}^2 \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 0 & 4 \end{pmatrix}$

- **Introduction to PageRank**

  PageRank algorithm used by google, each website are considered as a node, and their relationships(links) could be drawn as a network. Use the concept of Procrastinating Pat, conceptualize the network as a matrix (the sum of each column/vector = 1)

Complete the calculation before the lecturer: what should the link vectors $\mathbf{l}_{\mathrm{B}}$ and $\mathbf{l}_{\mathrm{C}}$ look like?

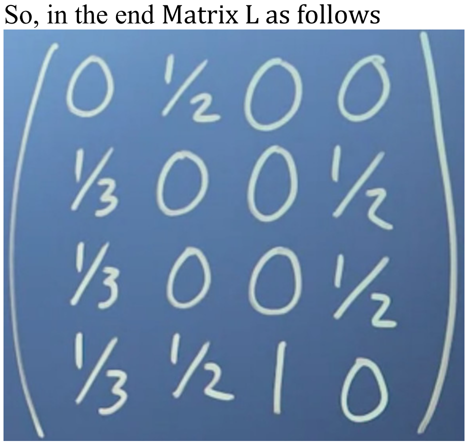


- $\mathbf{l}_{\mathrm{B}} = (\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}})$ and $\mathbf{l}_{\mathrm{C}} = (0, 0, 0, 1)$
- $\mathbf{l}_{\mathrm{B}} = (\frac{1}{2}, 0, 0, \frac{1}{2})$ and $\mathbf{l}_{\mathrm{C}} = (0, 0, 0, 1)$

Correct
Well done!

- $\mathbf{l}_{\mathrm{B}} = (\frac{1}{3}, 0, 0, \frac{1}{3})$ and $\mathbf{l}_{\mathrm{C}} = (0, 0, 0, \frac{1}{3})$
- $\mathbf{l}_{\mathrm{B}} = (1, 0, 0, 1)$ and $\mathbf{l}_{\mathrm{C}} = (0, 0, 0, 1)$

So, in the end Matrix L as follows



We assume all webpage are outwarded, so the rank A should be $r_A = \sum_{j=1}^{n} L_{A,j} r_j$, for more general, $r^{i+1} = L * r^i$, eventually, r would be the eigen vector of matrix L

After iteration, we get the following results.

	A	B	C	D
rank =	0.1250	0.2083	0.2083	0.4583
	0.1354	0.2118	0.2118	0.4410
	0.1084	0.2611	0.2611	0.3694
	0.1084	0.2544	0.2544	0.3791
	0.1249	0.2311	0.2311	0.4129
	0.1227	0.2350	0.2350	0.4072
	0.1186	0.2425	0.2425	0.3963
	0.1194	0.2412	0.2412	0.3983
	0.1203	0.2395	0.2395	0.4007
	0.1201	0.2398	0.2398	0.4003
	0.1200	0.2401	0.2401	0.3999
	0.1200	0.2400	0.2400	0.4000
	0.1200	0.2400	0.2400	0.4000
	0.1200	0.2400	0.2400	0.4000
	0.1200	0.2400	0.2400	0.4000
	0.1200	0.2400	0.2400	0.4000
	0.1200	0.2400	0.2400	0.4000
	0.1200	0.2400	0.2400	0.4000

Two advantages for using power method to calculate eigen vector : 1. gives you eigen value =1

Recently, for considering about both stability and efficiency, scholars provide a compromise way $r^{i+1} = d(L * r^{i}) + \frac{1-d}{n}$. d is probability user input a web address instead of click links from the current page.