1. Let $A = \{1, 3, 5\}$. Is the following statement: $3 \in A$. True or false?

1 / 1 point

- ○ False
- ◉ True

Correct
The symbol $\in$ stands for "is an element of" and it
is true that $3$ is an element of $A$. The other two elements of $A$ are $1$ and $5$.

2. Let $E = \{-1, -2, -3\}$. Compute the cardinality $|E|$ of $E$:

1 / 1 point

- ○ $E$
- ◉ $3$
- ○ $0$
- ○ $-3$

Correct
Recall that the cardinality of a set is the number
of elements in it. Since $E$ has three elements (which are $-1$, $-2$, $-3$), the
cardinality of $E$ is $|E| = 3$.

3. Let $A = \{1, 3, 5\}$ and $B = \{3, 5, 10, 11, 14\}$.

1 / 1 point

Which of the following sets is equal
to the intersection $A \cap B$?

- ○ $\{3, 5, 10\}$
- ○ $\{1, 3, 5\}$
- ◉ $\{3, 5\}$
- ○ $\{3\}$

Correct
The intersection of two sets consists
precisely of the elements they share in common. The elements $3$ and $5$ are in
both $A$ and $B$.

1. Which of the following real numbers is ***not*** an integer?

1 / 1 point

- ○ $7$
- ◉ $4.3$
- ○ $0$
- ○ $-3$

**Correct**
$4.3$ is a decimal that is between two consecutive integers ($4$ and $5$).

2. Which of the following is the absolute value $|-7|$ of the number $-7$?

1 / 1 point

- ◉ $7$
- ○ $1$
- ○ $-7$
- ○ $0$

**Correct**
The absolute value of a number $x$ is the distance along the number line from $x$ to $0$. In this case, $-7$ is $7$ units away from $0$, and so $|-7| = 7$.

**3.** Suppose I tell you that $x$ and $y$ are two real numbers which make the statement $x < y$ true. Which pair of numbers ***cannot*** be values for $x$ and $y$?

**1 / 1 point**

- (selected) $x = 5$ and $y = 3.3$
- $x = -1$ and $y = 0$
- $x = -17.3$ and $y = -17.1$
- $x = 1$ and $y = 7.3$

**Correct**
The statement $x < y$ means that $x$ is to the left of $y$ on the real number line. Since $5$ is to the right of $3.3$, these cannot be values for $x$ and $y$.

**4.** Suppose I tell you that $w$ is a real number which makes both of the following statements true: $w > 1$ and $w < 1.2$. Which of the following numbers could be $w$?

**1 / 1 point**

- $w = 11$
- (selected) $w = 1.05$
- $w = 1.2$
- $w = 0$

**Correct**
$1.05 > 1$ is true since $1.05$ is to the right of $1$ on the real number line, and $1.05 < 1.2$ is also true, since $1.05$ is to the left of $1.2$ on the real number line.

5. Suppose that $x$ and $y$ are two real numbers which satisfy $x+3 = 4y + 1$. **Which of the following statements are false?**

1 / 1 point

- (selected) $x = 4y$
- $x + 2 = 4y$
- $x = 4y - 2$
- $2x + 6 = 8y + 2$

Correct
The equation $x = 4y$ cannot be derived from the given equation.

6. Which of the following real numbers is in the open interval $(2, 3)$?

1 / 1 point

- $2$
- (selected) $2.1$
- $1$
- $3$

Correct
Recall that the open interval $(2, 3)$ consists of all
real numbers $x$ which satisfy $2 < x < 3$. Since $2.1 > 2$ and $2.1 < 3$, the number $2.1$ is in this open interval.

7. Which of the following real numbers are in the open ray $(3.1, \infty)$?

1 / 1 point

- ○ $0$
- ○ $3.1$
- ◉ $4.75$
- ○ $-5$

Correct
Recall that $(3.1, \infty) = \{x \in \mathbb{R} \mid x > 3.1\}$. Since $4.75 > 3.1$ is true, $4.75 \in (3.1, \infty)$.

8. Which of the following values for $x$ solves the equation $-3x + 2 = -4$

1 / 1 point

- ○ $x = \frac{2}{3}$
- ○ $x = -2$
- ◉ $x = 2$
- ○ All values of $x$ such that $x \leq 2$

Correct
First we subtract $2$ from both sides of the given equation, to obtain $-3x = -6$. Finally, to isolate $x$ we divide both sides of the equation by $-3$ to obtain $x = 2$.

1. Let $B = \{3, 5, 10, 11, 14\}$. Is the following statement true or false: $3 \notin B$

1 / 1 point

- (•) False
- ( ) True

Correct
The symbol $\notin$ stands for “is not an element of.” Since $3$ is in an element of the set $B$, the given statement is not true.

2. Let $A = \{1, 3, 5\}$ and $B = \{3, 5, 10, 11, 14\}$. Which of the following sets is equal to the union $A \cup B$?

1 / 1 point

- ( ) $\{1, 10, 18\}$
- ( ) $\{3, 5, 10, 11, 14\}$
- (•) $\{1, 3, 5, 10, 11, 14\}$
- ( ) $\{1, 3, 5, 3, 5, 10, 11, 14\}$

Correct
The union of two sets consists precisely
of the elements that are in at least one of the two sets. That is precisely what
is listed here.

**3.** How many real numbers are there between the integers $1$ and $4$?

1 / 1 point

- ○ $2$
- ○ $4$
- ○ None
- ◉ Infinitely many

Correct
There are in fact infinitely many real numbers
between any pair of distinct integers, or indeed any pair of distinct real numbers!

**4.** Suppose I tell you that $x$ and $y$ are two real numbers which make the statement $x \geq y$ true. Which pair of numbers ***cannot*** be values for $x$ and $y$?

1 / 1 point

- ○ $x = 2$ and $y = 1$
- ○ $x = 10$ and $y = 10$
- ◉ $x = -1$ and $y = 0$
- ○ $x = 5$ and $y = 3.3$

Correct
Recall that the statement $x \geq y$ means that $x$ is either equal to $y$ or $x$ is to the right of $y$ on the real number line. Since $-1$ is actually to the left of $0$, these cannot be values for $x$ and $y$.

**5.** Suppose that $z$ and $w$ are two positive numbers with $z < w$. Which of the following inequalities is false?

1 / 1 point

- ○ $-z > -w$
- ○ $w - 7 > z - 7$
- ○ $z + 3 < w + 3$
- ◉ $-5z < -5w$

**Correct**

If we start with $z < w$ and multiply both sides by $-5$, we need to flip the less-than sign, which would give $-5z > -5w$. For an
example, try $z = 1$ and $y = 2$ and see what happens!

**6.** Find the set of all $x$ which solve the inequality $-2x + 5 \leq 7$

1 / 1 point

- ○ $x \geq -6$
- ○ $x \leq -1$
- ○ $x = -1$
- ◉ $x \geq -1$

**Correct**

Subtracting $5$ from both sides of the given inequality
gives $-2x \leq 2$. Then we divide both sides by $-2$, remembering to flip the
inequality sign, and we obtain this answer

7. Which of the following real numbers is not in the closed interval $[2, 3]$

1/1 point

- (selected) $1$
- $2.1$
- $2$
- $3$

**Correct**

Recall that the closed interval $[2, 3]$ consists of all
real numbers $x$ which satisfy $2 \leq x \leq 3$. Since $2 \leq 1$ is false, $1 \notin [2, 3]$

8. Which of the following intervals represents the set of all solutions to:

1/1 point

$-5 \leq x + 2 < 10$?

- $(7, 8)$
- $[-7, 8]$
- (selected) $[-7, 8)$
- $[-5, 10)$

**Correct**

Subtracting $2$ from all sides of the inequalities gives $-7 \leq x < 8$, and the set of all real numbers $x$ which make that true is exactly
the half-open interval $[-7, 8)$.

**9.** Which of the numbers below is equal to the following summation: $\Sigma_{k=2}^{5} 2k$?

1 / 1 point

- ○ $10$
- ◉ $28$
- ○ $14$
- ○ $4$

**Correct**

We compute $\Sigma_{k=2}^{5} 2k = 4 + 6 + 8 + 10 = 28.$

**10.** Suppose we already know that $\Sigma_{k=1}^{20} k = 210$. Which of the numbers below is equal to $\Sigma_{k=1}^{20} 2k$?

1 / 1 point

- ○ $40$
- ○ $210$
- ◉ $420$
- ○ $2$

**Correct**

By applying one of our Sigma notation simplification rules, we can rewrite the summation in question as $2\left(\Sigma_{k=1}^{20} k\right) = 2 \times 210 = 420.$

**11.** Which of the numbers below is equal to the summation $\Sigma_{i=2}^{10} 7$?

1 / 1 point

- ○ $7$
- ○ $48$
- ○ $70$
- ◉ $63$

**Correct**

According to one of our Sigma notation simplification rules, this summation is just equal to $9$ copies of the number $7$ all added together, and so we get $9 \cdot 7 = 63.$

**12.** Which of the following numbers is the variance of the set $Z = \{-2, 4, 7\}$?

1 / 1 point

- ○ $\sqrt{14}$
- ○ $69$
- ○ $42$
- ◉ $14$

**Correct**

To get the variance of a set of numbers, you need to perform four steps:

First compute the mean (which is $3$)

Then calculate all the squared differences between the numbers in the set and this mean (here you get $25, 1, 16$)

Then add all these up (here you get $42$)

Then divide by the number of elements in the set (which is $3$).

Therefore, the variance of $Z$

$$= \frac{1}{3}\left[(-2-3)^2 + (4-3)^2 + (7-3)^2\right]$$

$$= \frac{1}{3}\left[25 + 1 + 16\right] = \frac{42}{3} = 14$$

**13.** Which of the following sets does *not* have zero variance? (hint: don't do any calculation here, just think!)

1 / 1 point

- ◉ $\{2, 5, 9, 13\}$
- ○ $\{5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5\}$
- ○ $\{0, 0, 0, 0, 0, 0, 0\}$
- ○ $\{1, 1, 1, 1\}$

**Correct**

Intuitively, the numbers in this set are spread out.

1. Which of the numbers below is equal to the following summation: $\Sigma_{i=1}^{3} i^2$ ?

1 / 1 point

- ○ $30$
- ◉ $14$
- ○ $1$
- ○ $9$

Correct

We compute $\Sigma_{i=1}^{3} i^2 = 1^2 + 2^3 + 3^2 = 14$

2. Suppose that $A = \Sigma_{k=1}^{100} k^4$ and $B = \Sigma_{j=1}^{100} j^4$

1 / 1 point

Which of the following statements is true?

- ◉ $A = B$
- ○ $B > A$
- ○ There is not enough information to do the problem
- ○ $A > B$

Correct

A = B. Both summations evaluate to the same number, since $k$ and $j$ are just dummy indices.

3. Which of the numbers below is equal to the summation $\Sigma_{i=1}^{10} 7$?

1 / 1 point

- (selected) $70$
- $7$
- $55$
- $0$

Correct
According to one of our Sigma notation simplification rules, this summation is just equal to $10$ copies of the number $7$ all added together, and so we get $10 \times 7 = 70$.

4. Suppose that $X = \Sigma_{i=1}^{5} i^3$ and $Y = \Sigma_{i=1}^{5} i^4$.

1 / 1 point

Which of the following expressions is equal to the summation $\Sigma_{i=1}^{5}(2i^3 + 5i^4)$?

- $X + Y$
- $7$
- $3375$
- (selected) $2X + 5Y$

Correct
To get here, you apply two of our Sigma notation simplification rules $\Sigma_{i=1}^{5} 2i^3 + 5i^4 = 2\left(\Sigma_{i=1}^{5} i^3\right) + 5\left(\Sigma_{i=1}^{5} i^4\right) = 2X + 5Y$.

**5.** Which of the following numbers is the mean $\mu_Z$ of the set $Z = \{-2, 4, 7\}$?

1 / 1 point

- (●) $3$
- ( ) $9$
- ( ) $4$
- ( ) $\frac{13}{3}$

**Correct**
To get the mean of a set of numbers, you need to perform two steps: first add them all up (in this case getting $-2+4+7=9$), and then divide by the number of elements in the set (in this case that number is $3$).

So you should obtain $\mu_Z = \frac{9}{3} = 3$, which you did!

**6.** Suppose the set $X$ has five numbers in it: $X = \{x_1, x_2, x_3, x_4, x_5\}$. Which of the following expression represents the mean of the set $X$?

1 / 1 point

- ( ) $\frac{1}{N}[\sum_{i=1}^{N} x_i]$
- ( ) $\frac{1}{5}[\sum_{i=1}^{5} (x_i - \mu_X)^2]$
- (●) $\frac{1}{5}[\sum_{i=1}^{5} x_i]$
- ( ) $\sum_{i=1}^{5} x_i$

**Correct**
To obtain the mean of a set of numbers, you first add them all up (which is expressed here by the sigma operation inside the square brackets) and then you divide by the number of numbers in the set (which is expressed here by the $\frac{1}{5}$ outside the square brackets).