Lecture01

Tuesday, November 17, 2020 8:55 PM

Module 1 objectives, assignments, and supplementary materials

Learning Objectives

Upon successful completion of Module 1, you will be able to:

• Understand the different definitions of probability.

• Understand the meaning of conditional probability.

• Compute conditional probabilities using Bayes' theorem.

• Understand probability distributions, including probability density functions.

• Compute probabilities of outcomes for basic distributions.

Assignments

This module has four required quizzes and an honors quiz. A score of 75% is required to pass. Quizzes can be attempted up to four times in an eight-hour period.

Additional Materials

In addition to regular lectures and quizzes, this module includes the following materials.

Lesson 1: Probability

- Background reading: This reviews the rules of probability, odds, and expectation. (3 paradigms)
    - Classical framework (outcomes have equal probability)
    - Frequentist framework (probability are related to frequency)
    - Bayesian framework (personal perspective/subjective)

• Discussion prompt: Read what your peers have to say about the prompt and share your ideas on the discussion board.

Lesson 2: Bayes’ theorem

• Supplementary reading: This optional reading extends Bayes’ theorem beyond just two possibilities. It will also be helpful for the Lesson 2 quiz.

Lesson 3: Review of distributions

• Supplementary reading: This reading provides a vital reference for future lessons. It includes a review of indicator functions; expectation and variance; important probability distributions that will be used throughout the course (some not included in the lesson videos); the central limit theorem; and the continuous version of Bayes’ theorem.

Bayes' theorem

Tuesday, November 17, 2020 9:34 PM

Lesson 2.1 Conditional probability

$$\mathrm{P(A|B)} = \frac{P(A \cap B)}{P(B)}$$

Independence: $\mathrm{P(A|B)} = \mathrm{P(A)}$, $\mathrm{P(A} \cap B) = \mathrm{P(A)P(B)}$

Lesson 2.2 Bayes' theorem

$$\mathrm{P(A|B)} = \frac{P(B|A)P(A)}{P(B)} = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$

Distribution

Tuesday, November 17, 2020 10:04 PM

Lesson 3.1 Bernoulli and binomial distributions

Bernoulli : $X \sim B(p), P(X=1)=p, P(X=0)=1-p$

$f(X=x|p)=f(x|p)=p^x(1-p)^{1-x}$

$E(X)=p, Var(X)=p(1-p)$

Question

Which of the following functions is equivalent to the one below?

$$f(x)=\begin{cases} x & \text{if } x \le 0 \\ 2x & \text{if } x > 0. \end{cases}$$

- (selected) $f(x)=x+xI_{\{x>0\}}(x)$
- $f(x)=(xI_{\{x\le 0\}}(x)) \times (2xI_{\{x>0\}}(x))$
- $f(x)=2xI_{\{x\le 0 \cup x>0\}}(x)$
- $f(x)=xI_{\{x\le 0\}}(x)-2xI_{\{x>0\}}(x)$

Binomial

$X \sim Bin(n,p)$

$P(X=x|p)=f(x|p)=C_n^x p^x(1-p)^{n-x}$

E(X)=np, Var(X)=np(1-p)

Lesson 3.2 Uniform distribution

$X \sim U[0,1], f(x) = \begin{cases} 1 & if\ x \in [0,1] \\ 0 & otherwise \end{cases}$

We could also notate it as $I_{\{0 \leq x \leq 1\}}(x)$

If $X \sim [\theta_1, \theta_2], f(x|\theta_1, \theta_2) = \frac{1}{\theta_2 - \theta_1}$

Also, could be notated as $I_{\{\theta_1 \leq x \leq \theta_2\}}$

Lesson 3.3 Exponential and normal distributions

Exponential distribution

$X \sim Exp(\lambda), f(x|\lambda) = \lambda e^{-\lambda x}\ for\ x \geq 0$

$E(X) = \frac{1}{\lambda}, Var(X) = \frac{1}{\lambda^2}$

Normal distribution

$X \sim N(\mu, \sigma^2), f(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\{\frac{-(x-\mu)^2}{2\sigma^2}\}$

$E(X) = \mu, Var(X) = \sigma^2$