Week4 Matrices as objects that map one vector onto another; all the types of matrices

Tuesday, October 27, 2020 6:22 PM

- **Einstein summation convention and the symmetry of the dot product**

Set Matrix A $= \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}$, and B $= \begin{bmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{nn} \end{bmatrix}$. If we mutiply A by B.

For example, $(\mathrm{ab})_{23} = \mathrm{a}_{21}\mathrm{b}_{13} + \mathrm{a}_{22}\mathrm{b}_{23} + \cdots + \mathrm{a}_{2\mathrm{n}}\mathrm{b}_{\mathrm{n}3}$. Einstein notates it as $(\mathrm{ab})_{\mathrm{ik}} = \sum_j a_{ij}b_{jk} = a_{ij}b_{jk}$ (It's very nice for programming--3 loops)

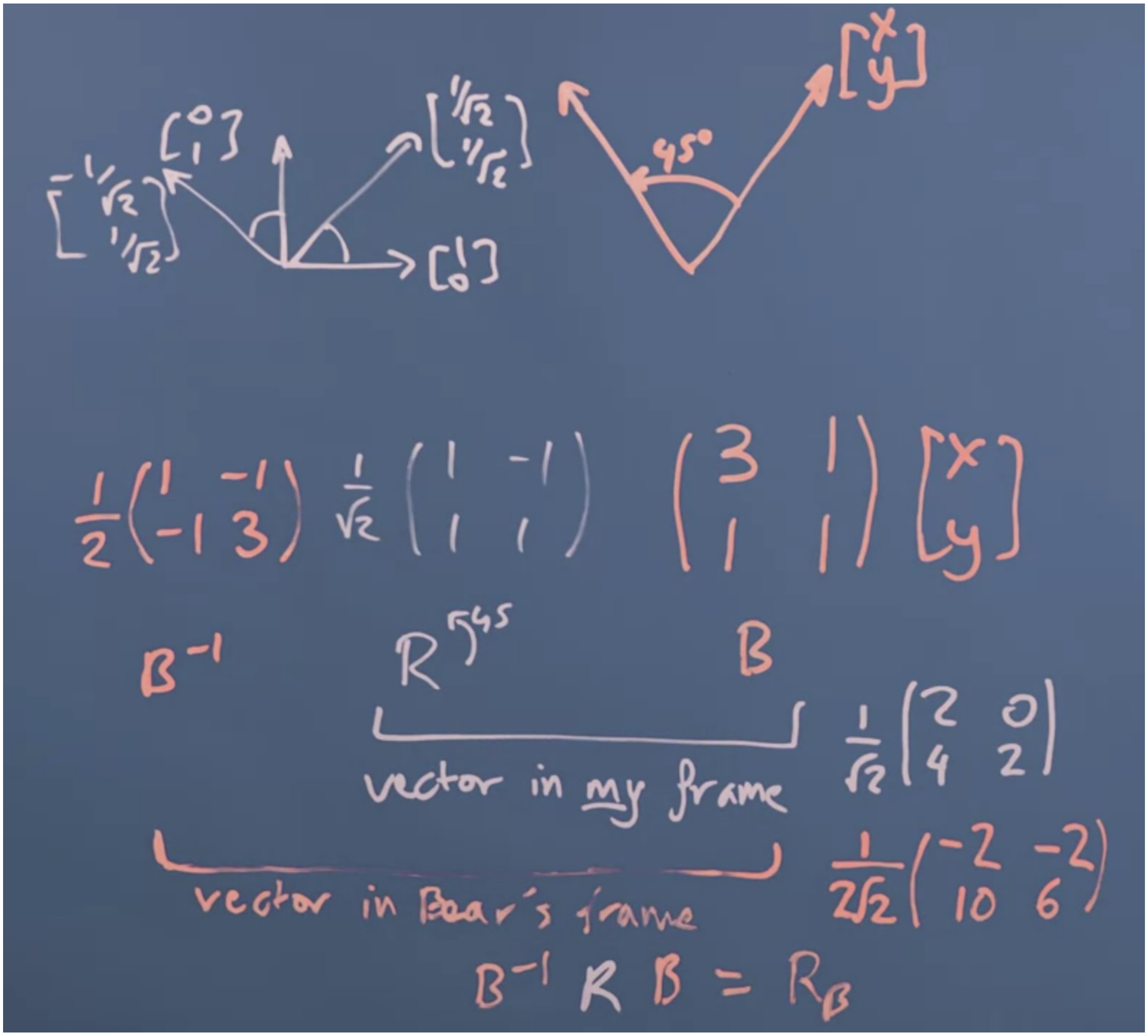
- **Matrices changing basis**

Suggest we have equation as follow:

$\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \frac{3}{2} \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$. We could treate the first matrix as two baisis vector, $\begin{bmatrix} \frac{3}{2} \\ \frac{1}{2} \end{bmatrix}$ is the vector before transforming, and $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$ is the vector after transforming with $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ basis.

As learned before, the inverse matrix for $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is $|A|^{-1} * \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$. So transforming vector under $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ to $\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$, we just need to mutiply $\frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$

- **Doing a transformation(rotation) in a changed basis**



If we under the normal coordinate system $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$, then $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}$ stands for 45° rotation on Counterclockwise. So if we multiply $\frac{1}{2}\begin{bmatrix}1 & -1\\-1 & 3\end{bmatrix}$ and $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}$, means we do the rotation then transform it to $\begin{bmatrix}3 & 1\\1 & 1\end{bmatrix}$ basis.

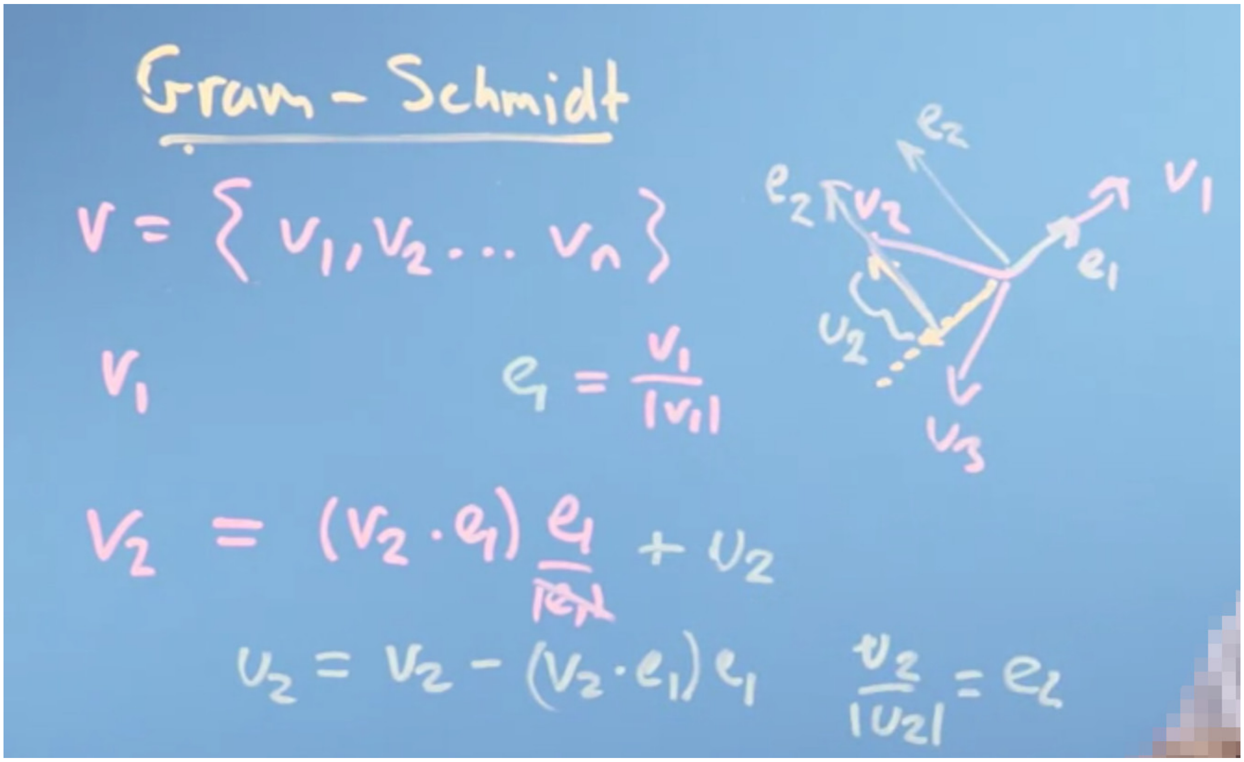
- **Orthogonal matrices**

  Transpose matrix : Set A = $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$, $A^T = \begin{bmatrix}1 & 3\\2 & 4\end{bmatrix}$

  Orthonormal basis set: Set A = $[(a_1)(a_2)\ldots(a_n)]$. If $a_i * a_j = 0$ for $i \neq j$, and $a_i * a_j = 1$ for $i = j$. Then, $A^T * A = I, A^T = A^{-1}$.

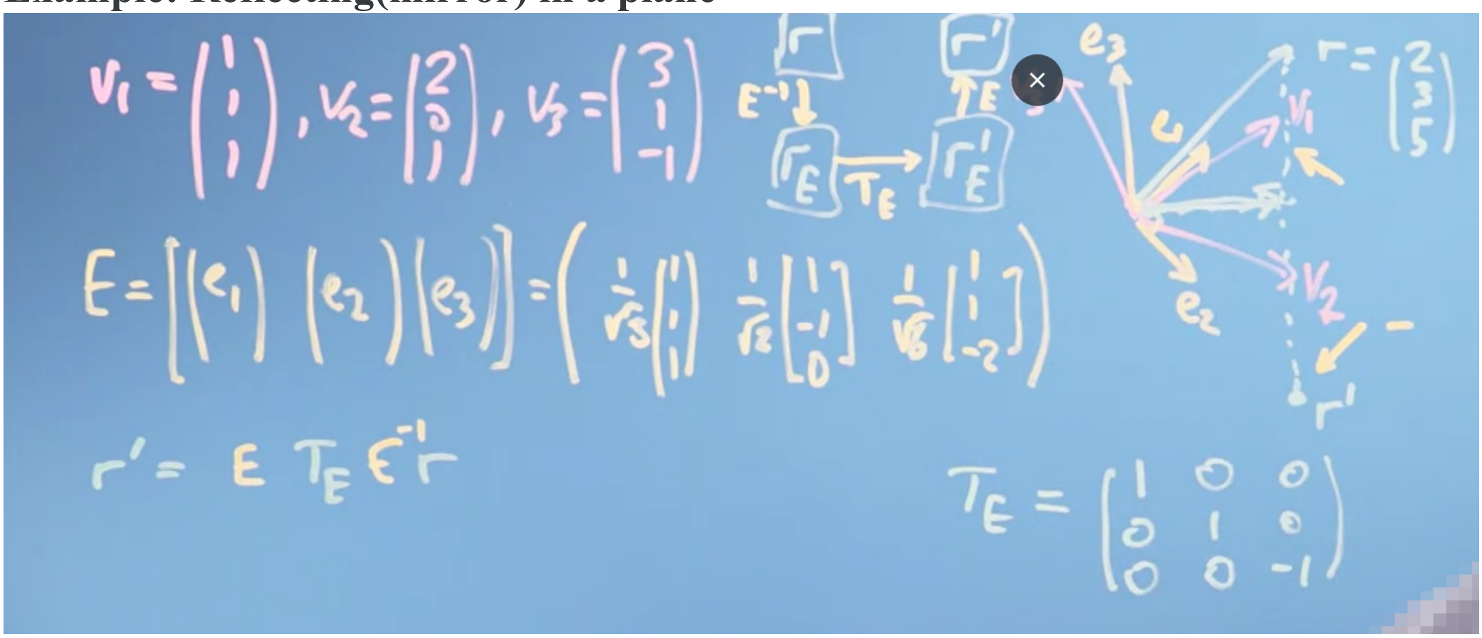
  For orthogonal matrix, the projection just dot product (Prove it), and determinant = either 1 or -1.

- **The Gram–Schmidt process**
  - How to construct an orthonormal basis:



$U_3 = V_3 - (V_3 * e_1)e_1 - (V_3 * e_2)e_2$

- **Example: Reflecting(mirror) in a plane**



This is, we have space matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix}$, and $r = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$. We want to reflect(mirror) r based on z value in A space. For doing this, we transfor A matix to a orthonormal matrix E, and then $r' = E * T_E * E^{-1} * r$