Quiz01

Tuesday, November 17, 2020 9:29 PM

Lesson 1

LATEST SUBMISSION GRADE

100%

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1. If you randomly guess on this question, you have a .25 probability of being correct. Which probabilistic paradigm from Lesson 1 does this argument best demonstrate?

1 / 1 point

- (●) Classical
- ( ) Frequentist
- ( ) Bayesian
- ( ) None of the above

✓ **Correct**

2. On a multiple choice test, you do not know the answer to a question with three alternatives. One of the options, however, contains a keyword which the professor used disproportionately often during lecture. Rather than randomly guessing, you select the option containing the keyword, supposing you have a better than 1/3 chance of being correct.

1 / 1 point

Which probabilistic paradigm from Lesson 1 does this argument best demonstrate?

- ○ Classical
- ○ Frequentist
- ◉ Bayesian

✓ **Correct**

3. On average, one in three students at your school participates in extracurricular activities. You conclude that the probability that a randomly selected student from your school participates is 1/3.

1 / 1 point

Which probabilistic paradigm from Lesson 1 does this argument best demonstrate?

- ○ Classical
- ◉ Frequentist
- ○ Bayesian

✓ **Correct**

4. **For Questions 4-6, consider the following scenario:**

1 / 1 point

Your friend offers a bet that she can beat you in a game of chess. If you win, she owes you \$5, but if she wins, you owe her \$3.

- Suppose she is 100% confident that she will beat you. What is her expected return for this game? (Report your answer without the \$ symbol.)

3

Correct

This is $3 \cdot (1) - 5 \cdot (0)$. If she is certain she will win, then she expects to receive the \$3.

5. Chess:

1 / 1 point

- Suppose she is only 50% confident that she will beat you (her personal probability of winning is $p = 0.5$). What is her expected return now? (Report your answer without the \$ symbol.)

-1

Correct

This is $3 \cdot (0.5) - 5 \cdot (0.5)$. Clearly, she wouldn't have offered this bet if she was only 50% confident that she would win.

6. Chess:

1 / 1 point

- Now assuming your friend will only agree to fair bets (expected return of \$0), find her personal probability that she will win. Report your answer as a simplified fraction.

Hint: Use the expected return of her proposed bet.

0.625

0.625

Correct

Any value of p (the probability of her winning) lower than 5/8 would result in a negative expected return for your friend. She would not have offered these odds for such a p.

7. **For Questions 7-8, consider the following "Dutch book" scenario:**

1 / 1 point

Suppose your friend offers a pair of bets:

(i) if it rains or is overcast tomorrow, you pay him \$4, otherwise he pays you \$6;

(ii) if it is sunny you pay him \$5, otherwise he pays you \$5.

- Suppose rain, overcast, and sunny are the only events in consideration. If you make both bets simultaneously, this is called a "Dutch book," as you are guaranteed to win money. How much do you win regardless of the outcome? (Report your answer without the \$ symbol.)

1

Correct

-4 + 5 if rain or overcast, 6 - 5 if sunny

8. Dutch book:

1 / 1 point

Apparently your friend doesn't understand the laws of probability. Let's examine the bets he offered.

1. For bet (i) to be fair, his probability that it rains or is overcast must be .6 (you can verify this by calculating his expected return and setting it equal to \$0).
2. For bet (ii) to be fair, his probability that it will be sunny must be .5.

- This results in a "Dutch book" because your friend's probabilities are not coherent. They do not add up to 1. What do they add up to?

1.1

Correct

0.6 + 0.5

Quiz02

Tuesday, November 17, 2020 10:00 PM

1. For Questions 1-4, refer to the following table regarding passengers of the famous Titanic, which tragically sank on its maiden voyage in 1912. The table organizes passenger/crew counts according to the class of their ticket and whether they survived.

1 / 1 point

	Class			
	1st	2nd	3rd	Crew
Survived	203	118	178	212
Did not survive	122	167	528	673

Source: Dawson, Robert J. MacG. (1995), The 'Unusual Episode' Data Revisited. Journal of Statistics Education, 3. https://www.amstat.org/publications/jse/v3n3/datasets.dawson.html

Note: Due to increased research interest in the Titanic following the popular 1997 film, it is known that this data set contains slight inaccuracies.

- If we randomly select a person's name from the complete list of passengers and crew, what is the probability that this person travelled in 1st class? Round your answer to two decimal places.

0.15

Correct

This is (203 + 122) / 2201. This is also P(1st class) = P(1st class and survived) + P(1st class and did not survive).

2. Titanic:

1 / 1 point

- What is the probability that a (randomly selected) person survived? Round your answer to two decimal places.

0.32

Correct

This is (203 + 118 + 178 + 212) / 2201. This is also P(survived) = P(1st class and survived) + P(2nd class and survived) + P(3rd class and survived) + P(crew and survived).

3. Titanic:

1 / 1 point

- What is the probability that a (randomly selected) person survived, given that they were in 1st class? Round your answer to two decimal places.

0.62

Correct
This is 203 / (203 + 122). If we divide the numerator and denominator by 2201, this is also P(survived | 1st class) = P(1st class and survived) / P(1st class).

4. Titanic:

1 / 1 point

- True/False: The events concerning class and survival are statistically independent.

○ True

◉ False

Correct
Note that P(survived | 1st class) is not equal to P(survived). Another way to check is to see if P(survived and 1st class)=P(survived) * P(1st class).

5. **For Questions 5-9, consider the following scenario:**

1 / 1 point

You have three bags, labeled A, B, and C. Bag A contains two red marbles and three blue marbles. Bag B contains five red marbles and one blue marble. Bag C contains three red marbles only.

- If you select from bag B, what is the probability that you will draw a red marble? Express the exact answer as a simplified fraction.

$\frac{5}{6}$

5/6

Correct

6. Marbles: **1 / 1 point**

- If you randomly select one of the three bags with equal probability (so that $P(A) = P(B) = P(C) = 1/3$) and then randomly draw a marble from that bag, what is the probability that the marble will be blue? Round your answer to two decimal places.

Hint: This is the marginal probability $P(\text{blue})$. You can obtain this using the law of total probability (which appears in the denominator in Bayes' theorem). It is $P(\text{blue}) = P(\text{blue} \cap A) + P(\text{blue} \cap B) + P(\text{blue} \cap C)$
$= P(\text{blue} \mid A) \cdot P(A) + P(\text{blue} \mid B) \cdot P(B) + P(\text{blue} \mid C) \cdot P(C)$

0.26

Correct

This is $P(\text{blue}) = P(\text{blue} \cap A) + P(\text{blue} \cap B) + P(\text{blue} \cap C)$
$= P(\text{blue} \mid A) \cdot P(A) + P(\text{blue} \mid B) \cdot P(B) + P(\text{blue} \mid C) \cdot P(C)$
$= 3/5(1/3) + 1/6(1/3) + 0(1/3) = 1/5 + 1/18$

7. Marbles: **1 / 1 point**

- Suppose a bag is randomly selected (again, with equal probability), but you do not know which it is. You randomly draw a marble and observe that it is blue. What is the probability that the bag you selected this marble from is A? That is, find $P(A \mid \text{blue})$. Round your answer to two decimal places.

0.78

Correct

This is $\frac{P(\text{blue}|A) \cdot P(A)}{P(\text{blue})} = \frac{3/5(1/3)}{3/5(1/3)+1/6(1/3)+0(1/3)}$.

8. Marbles: **1 / 1 point**

- Suppose a bag is randomly selected (again, with equal probability), but you do not know which it is. You randomly draw a marble and observe that it is blue. What is the probability that the bag you selected from is C? That is, find $P(C \mid \text{blue})$. Round your answer to two decimal places.

0

Correct

This is $\frac{P(\text{blue}|C) \cdot P(C)}{P(\text{blue})} = \frac{0(1/3)}{3/5(1/3)+1/6(1/3)+0(1/3)}$. This answer is intuitive because there are no blue marbles in bag C.

9. Marbles: **1 / 1 point**

- Suppose a bag is randomly selected (again, with equal probability), but you do not know which it is. You randomly draw a marble and observe that it is red. What is the probability that the bag you selected from is C? That is, find $P(C \mid \text{red})$. Round your answer to two decimal places.

0.45

Correct

This is $\frac{P(\text{red}|C)\cdot P(C)}{P(\text{red})} = \frac{1(1/3)}{2/5(1/3)+5/6(1/3)+1(1/3)}$.

Quiz3.1

Friday, November 20, 2020 11:04 AM

1. When using random variable notation, big X denotes _______.

1 / 1 point

- (●) a random variable
- ( ) a conditional probability
- ( ) distributed as
- ( ) a realization of a random variable
- ( ) the expectation of a random variable
- ( ) approximately equal to

✓ **Correct**

2. When using random variable notation, little x denotes _______.

1 / 1 point

- ( ) a random variable
- ( ) a conditional probability
- ( ) distributed as
- (●) a realization of a random variable
- ( ) the expectation of a random variable
- ( ) approximately equal to

✓ **Correct**

It is a possible value the random variable can take

3. When using random variable notation, X ~ denotes ________. **1 / 1 point**

- ○ a random variable
- ○ a conditional probability
- ◉ distributed as
- ○ a realization of a random variable
- ○ the expectation of a random variable
- ○ approximately equal to

✓ **Correct**

4. What is the value of $f(x) = -5I_{\{x>2\}}(x) + xI_{\{x<-1\}}(x)$ when $x = 3$? **1 / 1 point**

-5

✓ **Correct**

Only the first term is evaluated as non-zero.

5. What is the value of $f(x) = -5I_{\{x>2\}}(x) + xI_{\{x<-1\}}(x)$ when $x = 0$? **1 / 1 point**

0

✓ **Correct**

All indicator functions evaluate to zero.

6. Which of the following scenarios could we appropriately model using a Bernoulli random variable? **1 / 1 point**

- ◉ Predicting whether your hockey team wins its next game (tie counts as a loss)
- ○ Predicting the weight of a typical hockey player
- ○ Predicting the number of wins in a series of three games against a single opponent (ties count as losses)
- ○ Predicting the number of goals scored in a hockey match

✓ **Correct**

Whether they win is a binary outcome which can only take on values $\{0, 1\}$.

7. Calculate the expected value of the following random variable: X takes on values $\{0, 1, 2, 3\}$ with corresponding probabilities $\{0.5, 0.2, 0.2, 0.1\}$. Round your answer to one decimal place.

1 / 1 point

0.9

✓ **Correct**

This is $0(.5) + 1(.2) + 2(.2) + 3(.1)$.

8. Which of the following scenarios could we appropriately model using a binomial random variable (with n > 1)?

1 / 1 point

- (●) Predicting the number of wins in a series of three games against a single opponent (ties count as losses)
- ( ) Predicting whether your hockey team wins its next game (tie counts as a loss)
- ( ) Predicting the weight of a typical hockey player
- ( ) Predicting the number of goals scored in a hockey match

✓ **Correct**

The binomial model assumes a fixed number of independent trials, each with the same probability of success.

9. Suppose $X \sim \text{Binomial}(3, 0.2)$. Calculate $P(X = 0)$. Round your answer to two decimal places.

1 / 1 point

0.51

✓ **Correct**

This is $P(X = 0) = \binom{3}{0} 0.2^0 0.8^3$.

10. Suppose $X \sim \text{Binomial}(3, 0.2)$. Calculate $P(X \leq 2)$. Round your answer to two decimal places.

1 / 1 point

0.99

✓ **Correct**

This is $P(X = 0) + P(X = 1) + P(X = 2)$
$= \binom{3}{0} 0.2^0 0.8^3 + \binom{3}{1} 0.2^1 0.8^2 + \binom{3}{2} 0.2^2 0.8^1$
$= 1 - P(X = 3)$

Quiz4

Friday, November 20, 2020   11:41 AM

1. Which of the following (possibly more than one) must be true if random variable X is continuous with PDF $f(x)$?

1 / 1 point

- [ ] $f(x)$ is a continuous function
- [x] $f(x) \geq 0$ always

Correct

- [ ] $f(x)$ is an increasing function of x
- [ ] $\lim_{x \to \infty} f(x) = \infty$
- [x] $\int_{-\infty}^{\infty} f(x) dx = 1$

Correct

- [ ] $X >= 0$ always

2. If $X \sim \text{Exp}(3)$, what is the value of $P(X > 1/3)$? Round your answer to two decimal places.

1 / 1 point

0.37

Correct

This is $P(X > 1/3) = \int_{1/3}^{\infty} 3e^{-3x} dx$
$= -e^{-3x}|_{1/3}^{\infty}$
$= 0 - (-e^{-3/3}) = e^{-1} = 0.368$

3. Suppose $X \sim \text{Uniform}(0, 2)$ and $Y \sim \text{Uniform}(8, 10)$. What is the value of $E(4X + Y)$?

1 / 1 point

13

Correct

This is $E(4X + Y) = 4E(X) + E(Y) = 4(1) + 9$.

4. **For Questions 4-7, consider the following**:

1 / 1 point

Suppose $X \sim \mathrm{N}(1, 5^2)$ and $Y \sim \mathrm{N}(-2, 3^2)$ and that X and Y are independent. We have $Z = X + Y \sim \mathrm{N}(\mu, \sigma^2)$ because the sum of normal random variables also follows a normal distribution.

- What is the value of μ?

-1

Correct

$\mu = E(Z) = E(X + Y) = E(X) + E(Y) = 1 + (-2)$

5. Adding normals:

1 / 1 point

- What is the value of σ^2?

Hint: If two random variables are independent, the variance of their sum is the sum of their variances.

34

Correct

$\sigma^2 = Var(Z) = Var(X + Y) = Var(X) + Var(Y) = 25 + 9.$

6. Adding normals:

1 / 1 point

If random variables X and Y are not independent, we still have $E(X+Y) = E(X) + E(Y)$, but now $Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y)$ where $Cov(X,Y) = E[(X - E[X])(Y - E[Y])]$ is called the covariance between X and Y.

- A convenient formula for calculating variance was given in the supplementary material: $Var(X) = E[(X - E[X])^2] = E[X^2] - (E[X])^2$. Which of the following is an analogous expression for the covariance of X and Y?

Hint: Expand the terms inside the expectation in the definition of $Cov(X,Y)$ and recall that $E(X)$ and $E(Y)$ are just constants.

- ○ $(E[X^2] - (E[X])^2) \cdot (E[Y^2] - (E[Y])^2)$
- ○ $E[Y^2] - (E[Y])^2$
- ○ $E[X^2] - (E[X])^2 + E[Y^2] - (E[Y])^2$
- ◉ $E(XY) - E(X)E(Y)$

✓ **Correct**

$$
\begin{aligned}
&Cov(X,Y) = E[(X - E[X])(Y - E[Y])] \\
&= E[XY - XE(Y) - E(X)Y + E(X)E(Y)] \\
&= E[XY] - E[XE(Y)] - E[E(X)Y] + E[E(X)E(Y)] \\
&= E[XY] - E(X)E(Y) - E(X)E(Y) + E(X)E(Y)
\end{aligned}
$$

7. Adding normals:

1 / 1 point

- Consider again $X \sim \mathrm{N}(1, 5^2)$ and $Y \sim \mathrm{N}(-2, 3^2)$, but this time X and Y are *not* independent. Then $Z = X + Y$ is still normally distributed with the same mean found in Question 4. What is the variance of Z if $E(XY) = -5$?

Hint: Use the formulas introduced in Question 6.

28

✓ **Correct**

$$
\begin{aligned}
&Var(Z) = Var(X) + Var(Y) + 2Cov(X,Y) = 25 + 9 + 2Cov(X,Y) \\
&= 34 + 2(E[XY] - E[X]E[Y]) \\
&= 34 + 2(-5 - 1(-2)) = 34 - 2(3)
\end{aligned}
$$

8. Free point: 1 / 1 point

1) Use the definition of conditional probability to show that for events A and B, we have $P(A \cap B) = P(B|A)P(A) = P(A|B)P(B)$.

2) Show that the two expressions for independence $P(A|B) = P(A)$ and $P(A \cap B) = P(A)P(B)$ are equivalent.

- [x] Solution (1)

Correct

Write $P(B|A) = \frac{P(A \cap B)}{P(A)}$ and multiply both sides by $P(A)$.

- [x] Solution (2)

Correct

Plug these expressions into those from (1).