Week3 Matrices, vectors, and solving simultaneous equation problems

Monday, October 26, 2020 1:48 PM

- **How matrices transform space**

If we have equations like $2a + 3b = 8; 10a + b = 13$; we could get matrix mutiply vector like

$$\begin{pmatrix} 2 & 3 \\ 10 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{bmatrix} 8 \\ 13 \end{bmatrix}.$$

From this matrix multiplication, we could assume matrix A transform vector r to r', so we could get the following properties

$A * r = r'; A * (n * r) = n * r'; A(r + s) = Ar + As$

For example, $\begin{pmatrix} a \\ b \end{pmatrix} = \left(a * \begin{bmatrix} 1 \\ 0 \end{bmatrix} + b * \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$, So

$$\begin{pmatrix} 2 & 3 \\ 10 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{bmatrix} 2 & 3 \\ 10 & 1 \end{bmatrix}\left(a * \begin{bmatrix} 1 \\ 0 \end{bmatrix} + b * \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = a\left(\begin{bmatrix} 2 & 3 \\ 10 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) + b\left(\begin{bmatrix} 2 & 3 \\ 10 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$$
$$= a\begin{bmatrix} 2 \\ 10 \end{bmatrix} + b\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

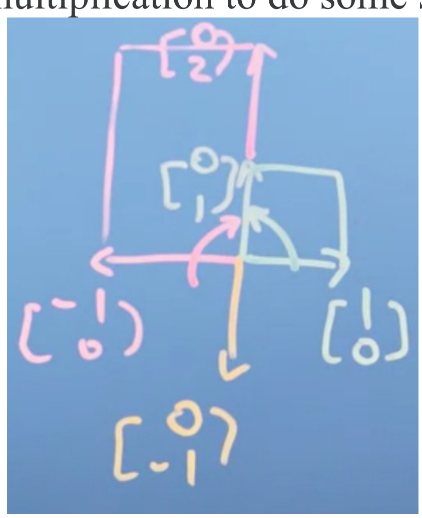
That's why we could assume that: matrix multiply vector is just transform vector to a different basis space($\begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ to $\begin{bmatrix} 2 \\ 10 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$)

- **Types of matrix transformation**
  - Identity matrix (do not change the space basis)

    $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$
  - Inversion matrix

    $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ -y \end{bmatrix}$

We could use matrix multiplication to do some space transformation,



like
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ is relacing the x, y coordinates, so it is a mirror transforming based on $x = y$, $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ is a mirror transformation based on $y = -x$. $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ flips the x value, $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ flips the y value

$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ is a general expression for a rotation in 2D

- **Composition or combination of matrix transformations**

We could applied a complex space transformation by multiplying multiple matrix, Assume we have $A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$.
So $A_2 * A_1 = [\begin{matrix} 0 & -1 \\ -1 & 0 \end{matrix}]$means after performing A_1transformation, we applied A_2transformation. So, it also indicates that $A_1 * A_2 \neq A_2 * A_1$(not commutative)
But $A_3 * (A_2 * A_1) = (A_3 * A_2) * A_1$ (Associative)

- **Gaussian elimination**

If we have the following matrix equation:
$\begin{pmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 2 \end{pmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 15 \\ 21 \\ 13 \end{bmatrix}$, $(notate\ first\ matrix\ as\ A, first\ vector\ as\ r, second\ as\ s)$
For getting vector r, we could mutiply A^{-1} on both side, then we get:
$A^{-1}A * r = A^{-1}s$, then we could get vecotr r

Another way is take first row off from both second and third row, then we have
$\begin{pmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{pmatrix} 15 \\ 6 \\ 2 \end{pmatrix}$. The first matrix now called triangluar matrix . And this called Echelon form. This process called elimination.

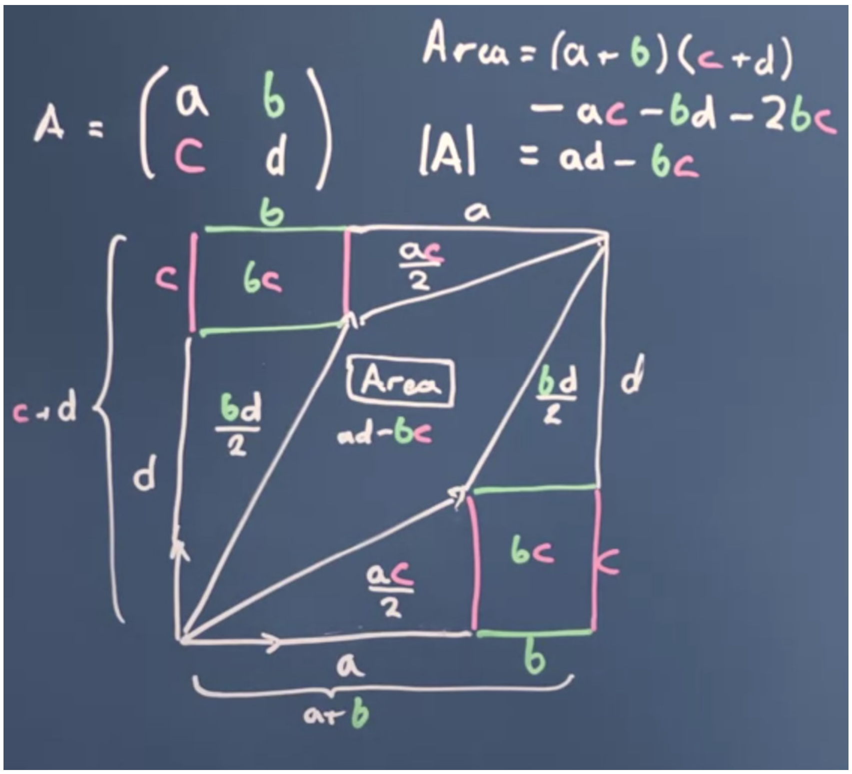
- **From Gaussian elimination to finding the inverse matrix**

Set $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 2 \end{pmatrix}$, and $B = A^{-1}$, $A * B = I$, Set $B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$
By using the elimination process we talk about before, we could transfer matrix A to

identity matrix, the equation would be changed as:
$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} * \mathrm{B} = \begin{pmatrix}0&-1&2\\-2&1&1\\1&0&-1\end{pmatrix}$. Then we get the inverse matrix of A.

- **Determinants and inverses**

  If we treat matrix as a function to scale space (e.g. $\begin{pmatrix}2&3\\10&1\end{pmatrix}$ change the basis vector to $\begin{pmatrix}2\\10\end{pmatrix}\begin{pmatrix}3\\1\end{pmatrix}$), then determinants is a function to calculate the volume (area for 2D) of the polygon after transformation (that's why determinats noted as |A|, just like mode)



For 2D matrix $A = \begin{pmatrix}a&b\\c&d\end{pmatrix}$, we have a therom that

$$\frac{1}{ad-bc} * A * \begin{pmatrix}d&-b\\-c&a\end{pmatrix} = \begin{pmatrix}1&0\\0&1\end{pmatrix}$$

If not all vectors in matrix are linear independent, then we could not find the inverse of this matrix.