Week3-Stationarity-Quiz

2020年12月6日 19:42

1. For a weakly stationary process, which of the following are true? **1 / 1 point**

- [x] The mean function is constant.

  Correct

  Yes! Everywhere we look on the process, the mean is the same.

- [x] The variance function is constant.

  Correct

  Yes! Everywhere we look on the process, the variance is the same.

- [ ] The autocovariance is constant.

2. A random walk is an example of a weakly stationary process. **1 / 1 point**

- ○ Yes.
- ◉ No.

  Correct

  That's right! We don't have constancy of variance (and may not have constancy of mean).

3. A moving average is an example of a weakly stationary process. **1 / 1 point**

- ◉ Yes.
- ○ No.

  Correct

  You bet! the mean is constant (equal to zero) and the autocovariance depends just upon lag spacing.

4. Suppose you have the MA(2) process:

1 / 1 point

$$X_t = Z_t + .5\, Z_{t-1} + .5\, Z_{t-2}, \quad \sigma^2 = 1$$

How many terms in the ACF are nonzero?

- ○ There are no nonzero terms.
- ○ Exactly 2.
- ◉ Exactly 3.
- ○ An infinite number.

Correct

Yes! Using our formulas, we obtain 3 nonzero terms:

$$\gamma(k) = \sum_{i=0}^{2-k} \beta_i\ \beta_{i+k}$$

$$\rho(k) = \frac{\gamma(k)}{\gamma(0)}$$

5. Let's think about our MA2 process from the last question.

1 / 1 point

$$X_t = Z_t + \ .5\, Z_{t-1} + \ .5\, Z_{t-2}, \quad \sigma^2 = 1$$

What is the autocovariance at lag zero? That is, calculate $\gamma(0)$.

1.5

Correct

Great job! We perform the following calculation.

$$\gamma(0) = \sum_{i=0}^{2} \beta_i \ \beta_i = \beta_0\beta_0 + \beta_1\beta_1 + \beta_2\beta_2 = 1^2 + .5^2 + .5^2 = 1.5$$

6. Again, consider the MA2 example.

1 / 1 point

$$X_t = Z_t + .5\, Z_{t-1} + .5\, Z_{t-2}, \quad \sigma^2 = 1$$

calculate the autocorrelation function at lag 2.

0.333

✓ **Correct**

Great work. We calculate as

$$\gamma(2) = \sum_{i=0}^{0} \beta_i \ \beta_{i+2} = 1 \cdot .5 = .5$$

$$\rho(2) = \frac{\gamma(2)}{\gamma(0)} = \frac{0.5}{1.5} = \frac{1}{3}$$

7. Run the following code to simulate our MA(2) process as shown above. Be sure to replace XX's with the appropriate coefficients.

1 / 1 point

```
set.seed=1
(acf(arima.sim(n=1000, model=list(ma=c(0.5,0.5)))))
```

Run

Reset

From your graph or the function output, estimate $\rho(1)$.

- ○ 1
- ◉ 0.531
- ○ 0.338

✓ **Correct**

Terrific! That is $\rho(1)$.

Week3-Series, Backward Shift Operator, Invertibility and Duality - Quiz

2020年12月6日 19:48

1. Determine if the geometric series is convergent or divergent, and find the sum of the series if it is convergent.

1 / 1 point

$-3+\frac{3}{2}-\frac{3}{4}+\frac{3}{8}-\ldots$

- ○ It is divergent.
- ◉ It is convergent, and the sum is -2.
- ○ It is convergent, and the sum is $\frac{1}{2}$.

Correct

Correct! It is convergent since $r=-\frac{1}{2}$, and $|-\frac{1}{2}|<1$, and the sum is $\frac{a}{1-r}=\frac{-3}{1+\frac{1}{2}}=-2$.

2. Express the rational function as a geometric series: $\frac{4}{1+x}$

1 / 1 point

- [x] $4-4x+4x^2-4x^3+4x^4-\ldots$

Correct

Correct! We know $\frac{a}{1-r}=a+ar+ar^2+ar^3+\ldots$. In this case, $a=4$ and $r=-x$.

- [x] $4(1-x+x^2-x^3+\ldots)$

Correct

Correct! We know $\frac{a}{1-r}=a+ar+ar^2+ar^3+\ldots$. In this case, $a=4$ and $r=-x$.

- [x] $4\sum_{n=1}^{n=\infty}(-1)^{n-1}x^{n-1}$

Correct

Correct! We know $\frac{a}{1-r}=a+ar+ar^2+ar^3+\ldots$. In this case, $a=4$ and $r=-x$.

3. Express the following model by utilizing Backward shift operator.

1 / 1 point

$X_t = 0.5X_{t-1} + Z_t + 0.7Z_{t-1}$

- [x] $(1 - 0.5B)X_t = Z_t + 0.7Z_{t-1}$

> ✓ **Correct**
> Correct! We write $X_{t-1} = BX_t$. We can continue using B to write $Z_{t-1} = BZ_t$.

- [ ] $(1 + 0.5B)X_t = (1 - 0.7B)Z_t$
- [x] $(1 - 0.5B)X_t = (1 + 0.7B)Z_t$

> ✓ **Correct**
> Correct! We write $X_{t-1} = BX_t$ and $Z_{t-1} = BZ_t$.

4. We write the model $X_t = X_{t-1} + 2X_{t-2} + Z_t$ as $\phi(B)X_t = Z_t$. What is $\phi(B)$?

1 / 1 point

- [ ] $\phi(B) = (1 - B)(1 + 2B)$.
- [x] $\phi(B) = (1 + B)(1 - 2B)$.

> ✓ **Correct**
> Correct! $1 - B - 2B^2 = (1 + B)(1 - 2B)$.

- [x] $\phi(B) = 1 - B - 2B^2$.

> ✓ **Correct**
> Correct! We obtain this since $X_t - X_{t-1} - 2X_{t-2} = Z_t$.

5. Is the following process invertible?

1 / 1 point

$X_t = Z_t + 3Z_{t-1}$

- ○ It is an invertible process since the coefficient 3 is larger than 1.
- ◉ It is not an invertible process.

> ✓ **Correct**
> Correct! The root of the polynomial $1 + 3B$, $-\frac{1}{3}$, is not outside of a unit circle.

6. For what values of the θ, the process $X_t = Z_t - \theta Z_{t-1} - 6\theta^2 Z_{t-2}$ is an invertible process.

1 / 1 point

- (selected) $|\theta| < \frac{1}{3}$
- $|\theta| > \frac{1}{3}$
- $|\theta| < \frac{1}{2}$

Correct

Correct! Since $X_t = (1 - 3\theta B)(1 + 2\theta B)Z_t$, we need $|\theta| < \frac{1}{3}$ and $|\theta| < \frac{1}{2}$. But that means $|\theta| < \frac{1}{3}$.

7. Is the AR(2) process $X_t = X_{t-1} + 2X_{t-2} + Z_t$ stationary?

1 / 1 point

- (selected) It is not a stationary process.
- It is a stationary process.

Correct

Correct! The roots of the AR polynomial are -1 and $\frac{1}{2}$, none of which are outside of the unit circle.

8. Find all possible values of β so that the AR(2) process $X_t = 2\beta X_{t-1} - \beta^2 X_{t-2} + Z_t$ is stationary.

1 / 1 point

- ○ $|\beta| > 1$
- ◉ $|\beta| < 1$
- ○ $|\beta| = 1$

✓ **Correct**

Correct! AR polynomial, $(1 - \beta B)^2$, has root $B = 1/\beta$. This root is outside of the unit circle if $|\beta| < 1$.

9. Determine if the process is stationary, invertible or both: $X_t = 0.5X_{t-1} + Z_t + 4Z_{t-1}$

1 / 1 point

- ○ Neither stationary nor invertible.
- ◉ Stationary but not invertible.
- ○ Invertible but not stationary.
- ○ Stationary and invertible.

✓ **Correct**

Correct!

AR polynomial has root of 2 (outside of the unit circle) $\rightarrow$ stationary

MA polynomial has root of -0.25 (inside of the unit circle) $\rightarrow$ not invertible

10. Find all values of β and θ such that duality exists for the following process, i.e., it is stationary an invertible: $X_t = \beta^2 X_{t-1} + Z_t + 8\theta^3 Z_{t-1}$.

1 / 1 point

- ◉ $|\beta| < 1$ and $|\theta| < \frac{1}{2}$
- ○ $|\beta| > 1$ and $|\theta| > \frac{1}{2}$
- ○ $|\beta| < 1$ and $|\theta| > \frac{1}{2}$

✓ **Correct**

Correct! Roots of the polynomials are $\frac{1}{\beta^2}$ and $-\frac{1}{8\theta^3}$. Thus we want them to be outside of the unit circle.

Week3-AR(p)-and-the ACF-Quiz

2021年1月1日 11:11

1. Think about a first order autoregressive process with phi=1. Is this process stationary?

1 / 1 point

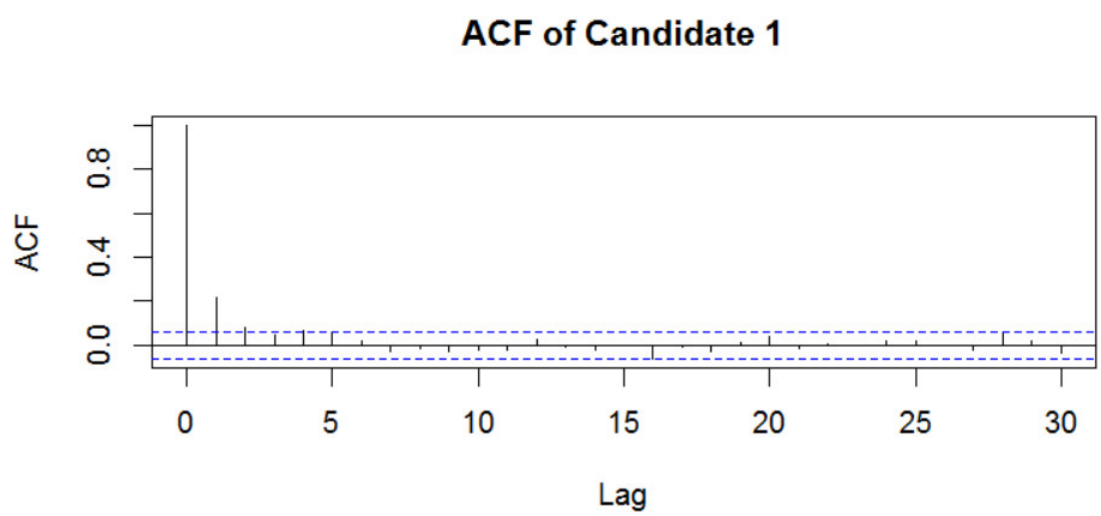
- ○ Yes.
- ◉ No.

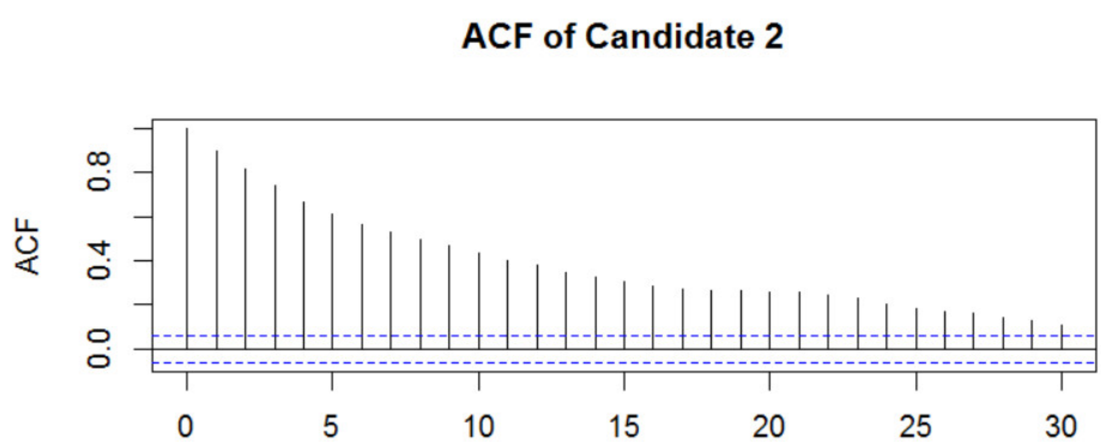
Correct

Right! An AR(1) process is only stationary when -1 < phi < 1. In our case, we just push past this region and so we are not stationary. In fact, this is a random walk.

2. We have two candidate ACFs for an AR(1) process. Which of them corresponds to phi=.2?

1 / 1 point





- (selected) Candidate 1
- Candidate 2

Correct

Good! We have a rapid decay. Recall that rho(k) = phi^k so when phi=.2 our ACF drops off rapidly.

Week3-Difference-equations-and-Yule-Walker-equations

2021年1月1日 11:15

1. The following difference equation is given: $a_n = 4a_{n-1} - 3a_{n-2}$. What is the auxiliary or the characteristic equation? **1 / 1 point**

- (●) $\lambda^2 - 4\lambda + 3 = 0$
- ( ) $\lambda^2 + 4\lambda - 3 = 0$
- ( ) $a_n - 4a_{n-1} + 3a_{n-2} = 0$

✓ **Correct**
Correct! $a_n = \lambda^n$ will give us the above equation.

2. Solve the difference equation $a_n = 4a_{n-1} - 3a_{n-2}$. **1 / 1 point**

- [x] $a_n = c_1 + c_2 3^n$

✓ **Correct**
Correct! We take linear combination of $1^n = 1$ and 3^n.

- [ ] $a_n = 1 + 3^n$
- [x] $a_n = c_1 3^n + c_2$

✓ **Correct**
Correct! We take linear combination of 3^n and $1^n = 1$.

3. Solve the difference equation $a_n = 4a_{n-1} - 3a_{n-2}$ with initial data $a_0 = 2$ and $a_1 = -2$. **1 / 1 point**

- (●) $a_n = 4 - 2(3^n)$
- ( ) $a_n = 6 - 4(2^n)$

✓ **Correct**
Correct! For sanity check, we see that $a_0 = 4 - 2 = 2$ and $a_1 = 4 - 2(3) = -2$.

4. Stationary AR(1) process is given: $X_t = 0.4X_{t-1} + Z_t$. Find the Yule-Walker equations.

1 / 1 point

- ○ $\rho(k) = 0.4\rho(k-1)$ for all $k \in \mathbb{Z}$.
- ○ $(1 - 0.4B)X_t = Z_t$
- ◉ $\rho(k) = 0.4\rho(k-1)$ when $k \geq 1$.

✓ **Correct**

Correct!

AR(1) process gives us Yule-Walker equations starting at lag 1.

5. Find the solution of the Yule-Walker equations of the process $X_t = 0.4X_{t-1} + Z_t$.

1 / 1 point

- ☐ $\rho(k) = c0.4^k$ for $k \geq 1$.
- ☑ $\rho(k) = 0.4^k$ for $k \geq 0$.

✓ **Correct**

Correct! We get a geometric sequence that is decreasing to zero.

- ☑ $\rho(k) = 0.4^k$ for $k \geq 0$, and $\rho(k) = \rho(-k)$ for $k \in \mathbb{Z}^-$.

✓ **Correct**

Correct! $\rho(k)$ is an even function.

6. Find the Yule-Walker equations and general solutions of them that govern autocorrelation coefficients of the AR(3) process

1 / 1 point

$X_t = \frac{1}{2}X_{t-1} + \frac{1}{9}X_{t-2} - \frac{1}{18}X_{t-3} + Z_t$

- ○ $\rho(k) = \frac{1}{2}\rho(k-1) + \frac{1}{9}\rho(k-2) - \frac{1}{18}\rho(k-3)$

  $\rho(k) = c_1(\frac{1}{2})^k + c_2(\frac{1}{9})^k + c_3(-\frac{1}{18})^k$

- ○ $18\rho(k) = 9\rho(k-1) + 2\rho(k-2) - \rho(k-3)$

  $\rho(k) = c_1(2)^k + c_2(9)^k + c_3(-18)^k$

- ◉ $\rho(k) = \frac{1}{2}\rho(k-1) + \frac{1}{9}\rho(k-2) - \frac{1}{18}\rho(k-3)$

  $\rho(k) = c_1(\frac{1}{2})^k + c_2(\frac{1}{3})^k + c_3(-\frac{1}{3})^k$

✓ **Correct**

Correct!

Characteristic equation can be written as $18\lambda^3 - 9\lambda^2 - 2\lambda + 1 = 0$. Factorization $(2\lambda - 1)(3\lambda - 1)(3\lambda + 1) = 0$ gives us the roots $\frac{1}{2}$, $\frac{1}{3}$ and $-\frac{1}{3}$.