1. If $x =$ "It is raining," what is $\sim(\sim x)$? 1 / 1 point

- ○ "It is not raining"
- ○ "It is never raining"
- ◉ "It is raining"
- ○ "It is always raining"

Correct
The second negation cancels out the first one.

Similarly $\sim(\sim(\sim x)) = \sim x$

2. If the statement "I am 25 years old" is assigned probability 0, what probability is assigned to the statement "I am not 25 years old"? 1 / 1 point

- ◉ 1
- ○ -1
- ○ 0
- ○ Unknown

Correct
It is always the case that $p(x) + p(\sim x) = 1.$

3. If I assign to the statement x = "it will rain today" a probability of $p(x) = 0.35$, what probability must I assign to the statement "it will not rain today?" 1 / 1 point

- ○ $.5$
- ○ $.35$
- ◉ $.65$
- ○ 0

Correct
$p(x) + p(\sim x) = 1$

4. Is the following collection of statements a probability distribution?

1 / 1 point

1. I own a Toyota pickup truck

2. I do not own a Toyota pickup truck

3. I own a non-Toyota pickup truck

4. I do not own a non-Toyota pickup truck

- (●) No
- ( ) Yes

Correct
The statements are not *exclusive*:1 and 4 could both be true, 2 and 3 could both be true, 2 and 4 could both be true, and even (1) and (3) could both be true (if I owned more than one pickup truck).

5. I don't know what it means to be "ingenuous." What probability would I assign to the statement, "I am ingenuous OR I am not ingenuous"?

1 / 1 point

- (●) 1
- ( ) .5
- ( ) -1
- ( ) 0

Correct
It is always the case, regardless of the content of the statement x, that $p(x \text{ or } \sim x) = 1$

6. A friend of mine circumscribes a circle inside a square, so that the diameter of the circle and the edge of the square are the same length. He asks me to close my eyes and pick a point at random inside the square. He says the probability that my point will also be inside the circle is $\frac{\pi}{4}$

1 / 1 point

Is this correct?

- (●) Yes
- ( ) No

Correct

Probabilities can be any real number between 0 and 1. They do not need to be rational numbers – a numerator that is a transcendental number like Pi is acceptable.

Note that the correct probability does not depend on the length r of the circle's radius. For a circle with any radius r to be circumscribed inside a square, the square must have sides each of length 2r. The area of the circle is Pi*r^2 and the area of the square is (2r)^2 = 4*r^2 = The probability of landing in a circle of area Pi*r^2 when it is known that one is in the area of the square is equal to the ratio of the area of the circle to the area of the square in which it is circumscribed, or Pi*r^2/4*r^2, which equals Pi/4.

7. The probability of drawing a straight flush (including a Royal Flush) in a five-card poker hand is 0.0000153908

1 / 1 point

What is the probability of **not** drawing a straight flush?

- (●) $.9999846092$
- ( ) $.9996582672$
- ( ) $.9967253809$
- ( ) $.9999745688$

Correct

$p(\sim x) = 1 - p(x)$

8. What is the probability that a fair, six-sided die will come up with a prime number? (Recall that prime numbers are positive integers other than 1 that are divisible only by themselves and 1)

1 / 1 point

- (selected) $\frac{1}{2}$
- $\frac{1}{3}$
- $\frac{1}{6}$
- $\frac{2}{3}$

Correct
The faces with 2, 3 and 5 satisfy the condition – which makes 3 relevant outcomes out of the "universe" of 6 outcomes = $\frac{3}{6} = \frac{1}{2}$

9. The joint probability p (the die will come up 5, the next card will be a heart) Is equal to the joint probability:

1 / 1 point

- (selected) p (the next card will be a heart, the die will come up 5)
- p (the next card will **not** come up 5, the next card will be a heart)
- p (the next card will be a heart, the die will **not** come up 5)
- p (the die will **not** come up 5, the next card will **not** be a heart)

Correct
In joint probabilities, the order does not change the probability: $p(A, B) = p(B, A)$

1. I am given the following 3 joint probabilities:

p(I am leaving work early, there is a football game that I want to watch this afternoon) = $.1$

p(I am leaving work early, there is not a football game that I want to watch this afternoon) = $.05$

p(I am not leaving work early, there is not a football game that I want to watch this afternoon) = $.65$

What is the probability that there is a football game that I want to watch this afternoon?

- ○ .35
- ○ .2
- ◉ .3
- ○ .1

Correct

Getting the answer is a two-step process. First, recall that the sum of probabilities for a probability distribution must sum to 1. So the “missing” joint distribution

p(I am not leaving work early, there is a football game I want to watch this afternoon) must be $1–(0.1 + 0.05 + 0.65) = 0.2$

By the sum rule, the marginal probability p(there is a football game that I want to watch this afternoon) = the sum of the joint probabilities

P(I am leaving work early, there is a football game that I want to watch this afternoon) + P(I am not leaving work early, there is a football game I want to watch this afternoon) = $.1 + .2 = .3$

2. The
Joint probability of my summiting Mt. Baker in the next two years AND publishing a best-selling book in the next two years is $.05$. If
the probability of my publishing a best-selling book in the next two years is 10%, and the probability of my summiting Mt. Baker in the next two years is 30%, are these two events dependent or independent?

1 / 1 point

- ○ Independent
- ◉ Dependent

Correct
We know this because the joint distribution of 5% does not equal the product distribution of $(0.1) \times (0.3) = 3\%$. If I summit Mt. Baker, I am more likely to publish a best-selling book, and vice versa.

3. The
Joint probability of my summiting Mt. Baker in the next two years AND my
publishing a best-selling book in the next two years is $.05$.

If
the probability of my publishing a best-selling book in the next two years is 10%, and the probability of my summiting Mt. Baker in the next two years is 30%, what is the probability that (sadly) in the next two years I will neither
summit Mt. Baker nor publish a best-selling book?

1 / 1 point

- ○ .25
- ○ .9
- ◉ .65
- ○ .95

Correct
Set A = I will summit Mt. Baker in the next two years

Set B = I will publish a best-selling book in the next two years.

Since $p(A) = 0.3$ and $p(A, B) = 0.05$, by the SUM RULE we know that $p(A, \sim B) = (0.3 - 0.05) = 0.25$

Since $p(B) = 0.1, p(\sim B) = 0.9$

Since $p(\sim B) = 0.9$ and $p(A, \sim B) = 0.25$ and again by the SUM RULE, $p(\sim A, \sim B) = 0.9 - 0.25 = .65$

4. I

have two coins. One is fair, and has a probability of coming up heads of .5. The second is bent, and has a probability of coming up heads of $.75$. If I toss each coin once, what is the probability that *at least* one of the coins will come up heads?

1/1 point

- ○ .375
- ○ .625
- ○ 1.0
- ◉ .875

Correct

We apply the rule p(A or B or both)

= 1 - (p(~A)p(~B))

= 1 - ((1- .5)(1-.75))

= 1 - .125

=.875

5. What is $\frac{11!}{9!}$?

1/1 point

- ○ $4,435,200$
- ○ $554,400$
- ◉ 110
- ○ $110,000$

Correct

$$\frac{11!}{9!} = 11 \times 10 = 110$$

6. What is the probability that, in six throws of a die, there will be exactly one each of "1" "2" "3" "4" "5" and "6" ?

1 / 1 point

- (selected) .01543210
- .00187220
- .01176210
- .01432110

Correct

There are $6! = 720$ permutations where each face occurs exactly once.

There are $6 \times 6 \times 6 \times 6 \times 6 \times 6 = 46656$ total permutations of 6 throws.

The probability is therefore $\frac{720}{46656} = 0.01543210$

7. On 1 day in 1000, there is a fire and the fire alarm rings.

1 / 1 point

On 1 day in 100, there is no fire and the fire alarm rings (false alarm)

On 1 day in $10,000$, there is a fire and the fire alarm does not ring (defective alarm).

On $9,889$ days out of $10,000$, there is no fire and the fire alarm does not ring.

If the fire alarm rings, what is the (conditional) probability that there is a fire?

Written $p(\text{there is a fire} \mid \text{fire alarm rings})$

- (●) 9.09%
- ( ) 1.12%
- ( ) 1.1%
- ( ) 90.9%

Correct

10 days out of every $10,000$ there is fire and the fire alarm rings.

100 days out of every $10,000$ there is no fire and the fire alarm rings.

110 days out of every $10,000$ the fire alarm rings.

The probability that there is a fire, given that the fire alarm rings, is $\frac{10}{110} = 9.09\%$

8. On 1 day in 1000, there is a fire and the fire alarm rings.

1 / 1 point

On 1 day in 100, there is no fire and the fire alarm rings (false alarm)

On 1 day in $10,000$, there is a fire and the fire alarm does not ring (defective alarm).

On $9,889$ days out of $10,000$, there is no fire and the fire alarm does not ring.

If the fire alarm does not ring, what is the (conditional) probability that there is a fire?

p(there is a fire | fire alarm does not ring)

- ○ $.01000\%$
- ◉ 0.01011%
- ○ $.10011\%$
- ○ 1.0001%

Correct

On $(1+9,889)$ = $9,890$ days out of every $10,000$ the fire alarm does not ring.

On 1 of those $10,000$ days there is a fire.

$$\frac{1}{9890} = 0.01011\%$$

2. If I flip a fair coin, with heads and tails, ten times in a row, what is the probability that I will get exactly six heads?

1 / 1 point

- ○ 0.021
- ○ 0.187
- ◉ 0.2051
- ○ 0.305

Correct

By Binomial Theorem, equals

$$\binom{10}{6}\left(0.5^{10}\right)$$

$$= \left(\frac{10!}{4! \times 6!}\right)\left(\frac{1}{1024}\right)$$
$$= 0.2051$$

3. If a coin is bent so that it has a 40% probability of coming up heads, what is the probability of getting *exactly* 6 heads in 10 throws?

1 / 1 point

- ○ 0.0974
- ○ 0.1045
- ◉ 0.1115
- ○ 0.1219

Correct

$\binom{10}{6} \times 0.4^6 \times 0.6^4 = 0.1115$

4. A bent coin has 40% probability of coming up heads on each independent toss. If I toss the coin ten times, what is the probability that I get at least 8 heads?

1 / 1 point

- ○ 0.0132
- ○ 0.0312
- ◉ 0.0123
- ○ 0.0213

Correct

The answer is the sum of three binomial probabilities:

$$((\binom{10}{8}) \times (0.4^8) \times (.6^2)) + ((\binom{10}{9}) \times (0.4^9) \times (0.6^1)) +$$

$$((\binom{10}{10})) \times (0.4^{10}) \times (0.6^0))$$

5. Suppose I have a bent coin with a 60% probability of coming up heads. I throw the coin ten times and it comes up heads 8 times.

1 / 1 point

What is the value of the “likelihood” term in Bayes’ Theorem -- the conditional probability of the data given the parameter.

- ◉ 0.120932
- ○ 0.043945
- ○ 0.168835
- ○ 0.122885

Correct

Bayesian “likelihood” --- the p(observed data | parameter) is

p(8 of 10 heads | coin has p = .6 of coming up heads)

$$\binom{10}{8} \times (0.6^8) \times (0.4^2) = 0.120932$$

6. We have the following information about a new medical test for diagnosing cancer.

1 / 1 point

Before any data are observed, we know that 5% of the population to be tested actually have Cancer.

Of those tested who do have cancer, 90% of them get an accurate test result of “Positive” for cancer. The other 10% get a false test result of “Negative” for Cancer.

Of the people who do not have cancer, 90% of them get an accurate test result of “Negative” for cancer. The other 10% get a false test result of “Positive” for cancer.

What is the conditional probability that I have Cancer, if I get a “Positive” test result for Cancer?

**Formulas in the feedback section are very long, and do not fit within the standard viewing window. Therefore, the font is a bit smaller and the word "positive test" has been abbreviated as PT.

- ○ 4.5%
- ◉ 32.1% probability that I have cancer
- ○ 67.9%
- ○ 9.5%

7. We have the following information about a new medical test for diagnosing cancer.

1 / 1 point

Before any data are observed, we know that 8% of the population to be tested actually have Cancer.

Of those tested who do have cancer, 90% of them get an accurate test result of "Positive'' for cancer.

The other 10% get a false test result of "Negative'' for Cancer.

Of the people who do not have cancer, 95% of them get an accurate test result of "Negative'' for cancer.

The other 5% get a false test result of "Positive'' for cancer.

What is the conditional probability that I have cancer, if I get a "Negative'' test result for Cancer?

- ○ $.80\%$
- ○ 88.2%
- ◉ 0.9%
- ○ 99.1%

8. An urn contains 50 marbles – 40 blue and 10 white. After 50 draws, exactly 40 blue and 10 white are observed.

1 / 1 point

You are not told whether the draw was done “with replacement” or “without replacement.”

What is the probability that the draw was done with replacement?

- ○ 1
- ◉ 12.27%
- ○ 13.98%
- ○ 87.73%

9. According to Department of Customs Enforcement Research: 99% of people crossing into the United States are not smugglers.

1 / 1 point

The majority of all Smugglers at the border (65%) appear nervous and sweaty.

Only 8% of innocent people at the border appear nervous and sweaty.

If someone at the border appears nervous and sweaty, what is the probability that they are a Smuggler?

- ○ 92.42%
- ◉ 7.58%
- ○ 7.92%
- ○ 8.57%