Assignment-2

Modi Akshay
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(1)
(a) $V = \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}$

$$R_x(\pi/2) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\pi/2 & -\sin\pi/2 & 0 \\ 0 & \sin\pi/2 & \cos\pi/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T(t_x, t_y, t_z) = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y(\pi/2) = \begin{bmatrix} \cos\pi/2 & 0 & \sin\pi/2 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\pi/2 & 0 & \cos\pi/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M = T(-1,3,2) \cdot R_x(-\pi/2)\, R_y(\pi/2)$$

$$= \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(ii) Let new coordinates be V'

then $$V' = MV = \begin{bmatrix} 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -3 \\ 1 \end{bmatrix}$$

$$V' = \begin{bmatrix} 0 & 1 & -3 \end{bmatrix}^T$$

new origin O', $O' = MO$

$$= \begin{bmatrix} 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

$$O' = \begin{bmatrix} -1 & 3 & 2 \end{bmatrix}$$

(iii) $$n = \frac{1}{2\sin\theta}\begin{bmatrix} R_{32} - R_{23} \\ R_{13} - R_{31} \\ R_{21} - R_{12} \end{bmatrix}$$

Also, $\theta = \cos^{-1}\left(\frac{tr(R) - 1}{2}\right)$

$tr(R) = 0 + 0 + 0 = 0$

$$\theta = \cos^{-1}\left(\frac{0-1}{2}\right) = \cos^{-1}(0.5) = 2\pi/3$$

$$n = \frac{1}{2\sin(2\pi/3)}\begin{bmatrix} -1-0 \\ 1-0 \\ -1-0 \end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$$

$$\hat{n} = \frac{1}{\sqrt{3}}(-\hat{i} + \hat{j} - \hat{k})$$

(iv) Rodrigues' formula states

$$\vec{x}' = \vec{x} + (\sin\theta)\hat{n} \times \vec{x} + (1-\cos\theta)\hat{n} \times (\hat{n} \times \vec{x})$$

$$N = \frac{1}{\sqrt{3}}\begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix} \qquad N = \begin{bmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0 \end{bmatrix}$$

$N\vec{x} = \hat{n} \times \hat{x}$, $\quad \vec{x}' = \vec{x} + (\sin\theta)N\vec{x} + (1-\cos\theta)N^2\vec{x}$

$$R' = I + (\sin\theta)N + (1-\cos\theta)N^2$$

$$N^2 = \frac{1}{3}\begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -2 & -1 & 1 \\ -1 & -2 & -1 \\ 1 & -1 & -2 \end{bmatrix}$$

$\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$, $\cos\frac{2\pi}{3} = -\frac{1}{2}$,

$$R' = I + \frac{\sqrt{3}}{2}N + \left(+\frac{1}{2} + 1\right)N^2 = I + \frac{\sqrt{3}}{2}N + \frac{3}{2}N^2$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} \times \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix} + \frac{3}{2} \times \frac{1}{3} \times \begin{bmatrix} -2 & -1 & 1 \\ -1 & -2 & -1 \\ 1 & -1 & -2 \end{bmatrix}$$

$$R' = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \quad \& \quad R = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

$R' = R$, both are same

2A: let $\vec{v}$ be rotated along the $\hat{u}$ by angle θ.

$\vec{v} = \hat{u}(\hat{u}\cdot\vec{v}) - \hat{u}(\hat{u}\times\vec{v})$

rotation vector,

$R\vec{x} = \hat{u}(\hat{u}\cdot\vec{v}) + \sin\theta(\hat{u}\times\vec{v}) - \cos\theta(\hat{u}\times(\hat{u}\times\vec{v}))$

$\hat{u}\times\hat{u}\times\vec{v} = (\hat{u}\cdot\vec{v})\hat{u} - (\hat{u}\cdot\hat{u})\vec{v}$

$= (\hat{u}\cdot\vec{v})\hat{u} - \vec{v}$

$R\vec{v} = \hat{u}(\hat{u}\cdot\vec{v}) + \sin\theta(\hat{u}\times\vec{v})$

$- \cos\theta((\hat{u}\cdot\vec{v})\hat{u} - \vec{v})$

$= \cos\theta\,\vec{v} + \sin\theta(\hat{u}\times\vec{v}) + (1-\cos\theta)(\hat{u}\cdot\vec{v})\hat{u}$

$= \cos\theta.\vec{v} + \sin\theta(\hat{u}\times\vec{v}) + (1-\cos\theta)(\hat{u}\cdot\vec{v})\vec{u}$

By replacing $\vec{v}$ with $\vec{x}$

$Rx = \cos\theta x + \sin\theta(u\times x) + (1-\cos\theta)(u^T x)u$

3A: $\vec{x} = K[R \mid t]\, x$

camera $C_1 \rightarrow K_1$

camera $C_2 \rightarrow K_2$

C_1 used in world coordinate frame

$$x_1 = K_1 [I \mid 0] X \quad - (1)$$

$C_2 \rightarrow$ got by rota of C_1

$$x_2 = K_2 (R \mid 0) X \quad - (2)$$

$$(2) \Rightarrow X = [K_2 [R \mid 0])^{-1} x_2 \quad - (3)$$

$$(1) \Rightarrow x = (K_1 [I \mid 0])^{-1} x_1 \quad - (4)$$

$$(3)\&(4) \Rightarrow (K_1 [I \mid 0])^{-1} x_1 = (K_2 [R \mid 0])^{-1} x_2$$

$$(K_1 [I \mid 0])^{-1} x_1 = [R \mid 0]^{-1} K_2^{-1} x_2$$

$$x_1 = K_1 \underset{3\times4}{[I \mid 0]} \underset{4\times3}{[R \mid 0]^{-1}} K_2^{-1} x_2$$

$$\Rightarrow [I \mid 0][R \mid 0]^{-1} = R^{-1}$$

$$x_1 = K_1 R^{-1} K_2^{-1} x_2$$

$$x_1 = (K_1 R^{-1} K_2^{-1}) x_2$$

$$= H_{3\times3}$$

$$\therefore x_1 = H x_2$$

5A:-

ii) we have l_normals & c_normals and offset values of lidar & camera of n pairs.

$$l\_n = [l\_n_1 \; l\_n_2 \; l\_n_3 \; --- \; l\_n_n] \quad \text{size } n \times 3$$

$$c\_n = [c\_n_1 \; c\_n_2 \; c\_n_3 \; ---- \; c\_n_n] \quad n \times 3$$

$$offset\_c = [\alpha_{c_1} \; \alpha_{c_2} \; \alpha_{c_3} \; --- \; \alpha_{cn}] \quad n \times 1$$

$$offset\_l = [\alpha_{l_1} \; \alpha_{l_2} \; \alpha_{l_3} \; ---- \; \alpha_{ln}] \quad n \times 1$$

Translation ~~matrix~~ vector calculated as,

$$T = (c\_n^T c\_n)^{-1} c\_n^T (offsect\_c - offsect\_l)$$

& Rotation matrix is calculated by decomposing through SVD, where

$$l\_n @ c\_n^T = USV^T$$

$$\& \quad R = VU^T$$

By using these formula, we can calculate V & R