Fundamental of single variable calculus

1. Find
$$\lim_{x \to 1} \frac{x^4 - 7x^3 + 8x^2 - 3x + 1}{3x^4 - 5x^3 + 6x^2 - 10x + 6}$$
.

2. If
$$f(x) = x^4 - 5x^2 + 12x - 3$$
 then find $f'(x)$.

3. If
$$f(x) = 1 + x + x^2 + x^3 + \dots + x^{99} + x^{100}$$
 then find $f'(-1)$.

4. If
$$f(x) = \frac{3x+2}{2x+1}$$
 then find $f'(x)$ at $x = 1$.

5. If
$$f(x) = 3x^5 + 4x^4$$
 then find $f'''(x)$.

6. If
$$f(x) = 5 \sin^2 x - 2 \cos^3 x$$
 then find $f''(x)$.

7. Find
$$I = \int (x^3 + 5x - 3) dx$$
.

8. Find
$$I = \int (x^e + e^x + e^e) dx$$
.

9. Find
$$I = \int (8x^3 + 3x^2 + 6x) dx$$

10. Find
$$I = \int \left(\frac{x^{24}}{x^{25}}\right) dx$$
.

Linear Equation

- 11. By using elementary row operations, find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$.
- 12. By using elementary row operations, find the inverse of the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 6 \end{bmatrix}$.
- 13. By using elementary row operations, find the inverse of the matrix A =

$$\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

- 14. Solve the system of equation by matrix method 5x+7y+2=0, 4x+6y+3=0.
- 15. Solve the system of equation by matrix method x+2y+3z=4, x+4y+9z=6, x+y+z=3.
- 16. For what value of k the system has non-trivial solution x+2y+3z=0, 2x+3y+kz=0, 7x+13y+19z=0.
- 17.Reduce the following matrices into row-reduced echelon form and find their ranks.

$$(i) \qquad \begin{bmatrix} 1 & 3 & 4 & 5 \\ 3 & 9 & 12 & 9 \\ 1 & 3 & 4 & 1 \end{bmatrix}.$$

(ii)
$$\begin{bmatrix} 6 & 5 & 1 & 3 \\ 0 & 3 & 2 & 2 \\ -6 & -5 & -1 & 4 \end{bmatrix}$$

Graph and Tree

- 18.Define with example: Node, Edge, Directed Graph, Multi Graph, Null Graph, Degree of a vertex, Pendent vertex.
- 19.Define with example: Incident vertex, Loop, Parallel edges, Directed Edges, Undirected edges and distinct edges.
- 20.Draw the undirected graph G = (V,E) where $V = \{a,b,c,d,e\}$ and $E = \{e_1,e_2,e_3,e_4,e_5,e_6,e_7\}$ and its incidence relation given as: $e_1 = (a,b)$, $e_2 = (a,b)$, $e_3 = (b,c)$, $e_4 = (c,d)$, $e_5 = (b,b)$, $e_6 = (a,d)$, $e_7 = (e,d)$.
- 21.Draw the directed graph G = (V,E) where $V = \{a,b,c,d,e,f,g\}$ and $E = \{e_1,e_2,e_3,e_4,e_5,e_6,e_7,e_8\}$ and its incidence relation given as: $e_1 = (b,a)$, $e_2 = (d,a)$, $e_3 = (b,c)$, $e_4 = (d,c)$, $e_5 = (c,f)$, $e_6 = (f,f)$, $e_7 = (e,c)$, $e_8 = (c,e)$.
- 22. Draw the undirected graph G corresponding to adjacency matrix A =

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}.$$