2/51

2301E1013

Candidate's Seat No : 10035

M.C.A. Semester-I Examination

Mathematical Foundations

Time : 3-00 Hours] **January-2023** **[Max. Marks : 50**

Instruction:

- **Write both the Sections in the separate answer book.**
- **Both Sections having equal weightage.**
- **Draw Diagrams wherever necessary.**
- **Make Assumptions wherever necessary.**

SECTION-I

Q-1 Explain the following terms with an appropriate example: [5]

a. Diagonal Matrix.
b. Proper Subset
c. Pendent vertex.
d. Cartesian Product
e. Loop

Q-2 Attempt the following: [8]

a. Find x, y, z and t if $2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$.

b. Describe the following sets in set-builder form.
$A = \{2,4,6,8,10\}$ and $B = \{3,5,7,9, \ldots, 87,89\}$.

c. If $f(x) = \frac{5x+1}{2x+1}$ then find $f'(x)$ at $x = 1$.

d. Find the distance between the points $P(1,-1,3)$ and $Q(2,1,-7)$.

OR

Q-2 Attempt the following: [8]

a. Find $I = \int (x^e + e^x + e^e)\, dx$.

b. Find $\lim_{x \to 0} \frac{x^3 - 3x^2 + x}{4x^3 - 5x^2 + 3x}$

c. Find the adjoint of $A = \begin{bmatrix} 4 & 2 \\ -1 & 3 \end{bmatrix}$.

d. Let $u = (1,2,3)$ and $v = (-2,3,0)$ then find scalar projection of u on v and vector projection of u along v.

Q-3 Attempt the following: [12]

a. Draw the undirected graph G = (V,E) where V = {a,b,c,d,e} and E = $\{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ and its incidence relation given as: e_1=(a,b), e_2=(a,b), e_3=(b,c), e_4=(c,d), e_5=(b,b), e_6=(a,d), e_7=(e,d).

b. For what value of k the system has non-trivial solution x+2y+3z=0, 2x+3y+kz=0, 7x+13y+19z=0.

c. If $A = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 \\ 3 & -4 \\ 5 & 0 \end{bmatrix}$; verify that $(AB)' = B'A'$; where A' and B' denote the transpose matrix of A and B respectively.

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d. Find the inverse of $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ by using elementary row transformations.

OR

Q-3 Attempt the following: [12]

a. Use the Gram-Schmidt process of orthonormalization to construct an orthonormal basis of the subspace generated by (1,1,-1,1), (1,0,0,1), (1,2,0,1).

b. Find $\lim_{x \to 1} \frac{x^4 - 7x^3 + 8x^2 - 3x + 1}{3x^4 - 5x^3 + 6x^2 - 10x + 6}$.

c. Determine which of the following sequences are paths, simple paths, circuit and cycle.

(i) $v_1\ e_1\ v_2\ e_6\ v_4\ e_3\ v_3\ e_2\ v_2$

(ii) $v_5\ e_5\ v_1\ e_8\ v_4\ e_3\ v_3\ e_2\ v_2\ e_6\ v_4\ e_4\ v_5$

d. If U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 3, 5, 7, 9}, B = {1, 5, 6, 8}, C = {1, 4, 6, 7} then verify (i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and (ii) $A \Delta B = B \Delta A$.

SECTION-II

Q-4 Explain the following terms with an appropriate example: [5]

a. One-One function

b. Orthogonal Vectors

c. Undirected Graph

d. Node

e. Pendent Vertex

Q-5 Attempt the following: [10]

a. If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

b. Explain the method of solving equation $f(x) = 0$ by using Bisection method.

OR

Q-5 Attempt the following: [10]

a. Prove the following statement using Venn diagram.

(i) $(A \cup B)' = A' \cap B'$

(ii) $A - (B \cup C) = (A - B) \cap (A - C)$

b. Find $I = \int \left(\frac{x^{24}}{x^{25}}\right) dx$ and (ii) Find $f''(x)$ if $f(x) = 5 \sin^2 x - 2 \cos^3 x$.

Q-6 Attempt the following: [10]

a. Solve by matrix method and find Rank of a matrix for x-2y+3z=2, 2x-3z=3, x+y+z=0.

b. Use Newton's method to find a root of the equation $x^3 - 3x - 5 = 0$.

OR

Q-6 Attempt the following: [10]

a. If $f(x) = x^5 - 5x^4 + 5x^3 - 1$ then find maximum and minimum value.

b. If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ then show that $A^2 - 4A + 3I = 0$ and hence find A^{-1}.