Mathematics for Data Science - 1
Graded Assignment
Week 9

1 MULTIPLE CHOICE QUESTIONS:

1. The maximum number of non-zero entries in an adjacency matrix of a simple graph having n vertices can be [option: d]

   (a) n^2

   (b) $\frac{n(n-1)}{2}$

   (c) $\frac{n(n-1)}{4}$

   (d) $n(n-1)$

   Solution:
   Number of non zero entry means number of ones in the adjacency matrix which is equal to the sum of the degrees of all vertices. In a graph a vertex can have maximum $n-1$ degree. In at most case if each vertex has the degree $n-1$, then the sum of degrees of all vertex will be $(n-1)+(n-1)+.....n\ times$ which means $n \times (n-1)$.

2. We have a graph G with 6 vertices. We write down the degrees of all vertices in G in descending order. Which of the following is a possible listing of the degrees? [option: c]

   (a) 6,5,4,3,2,1

   (b) 5,5,2,2,1,1

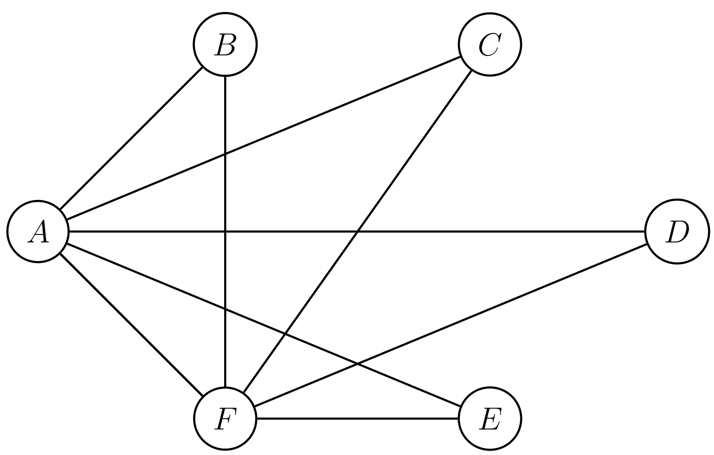
   (c) 5,3,3,2,2,1

   (d) 2,1,1,1,1,1

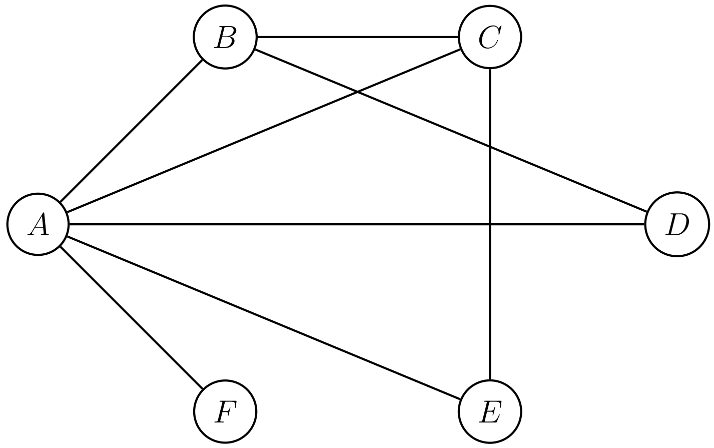
Solution:
Step 1:
Any vertex in a graph can have maximum degree (n-1).
Here n-1 = 6-1 = 5, therefore we do not need to check option 1.

Step 2:
Sum of degree of all vertices always be even therefore option 4 can not be correct option.

Step 3:
If two vertices in a graph have (n-1) degree it means there will be no vertex with degree 1 (shown in figure). Vertices A and F have degree 5.



Step 4:
Option 3 satisfies all the possible conditions therefore the correct option is option 3.


A
B
C
D
E
F

3. We are trying to find the correct path in a maze. We start at the entrance. At some points, we have to choose a direction to explore. If we reach a dead end, we come back to the most recent intersection where we still have an unexplored direction to investigate. What is a good data structure to keep track of the intersections we have visited? [option: b]

   (a) List
   (b) Stack
   (c) Queue
   (d) Array

   Solution:
   This is a recursive exploration of the maze, so intermediate stages should be stored on a stack.

4. Below table shows the adjacency list w.r.t outgoing edges of a directed graph G.

1	{2,4}
2	{3,5,6}
3	{7}
4	{3,5,6}
5	{6,7}
6	{1}
7	{1,2,6}

Table 1: adjacency list w.r.t forward edges

Which of the following tables shows the adjacency list w.r.t incoming edges of the graph G? [option: c]

1	{6,7}
2	{1,6}
3	{2,4}
4	{1}
5	{2,7}
6	{2,4,5,7}
7	{3,5}

(a)

1	{6,7}
2	{1,7}
3	{2,4}
4	{1,5}
5	{2,4}
6	{2,4,7}
7	{3,5}

(b)

1	{6,7}
2	{1,7}
3	{2,4}
4	{1}
5	{2,4}
6	{2,4,5,7}
7	{3,5}

(c)

1	{6,7}
2	{1,4}
3	{2,7}
4	{1,5}
5	{2,4}
6	{2,4,7}
7	{3,5}

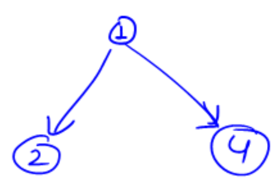
(d)

Solution

If outgoing edges are represented as

1	{2,4}

⟹



There is one edge coming from 1 to 2 and one from 1 to 4. And if we create an adjacency list for incoming edges we will find as:

2	{1}
4	{1}

If we apply the same approch we will get the list as shown in option c.

'1' will have incoming edge from 6 and 7.

'2' will have " " " 1 and 7

'3' " " " " " 2 and 4

'4' " " " " " 1.

'5' " " " " " 2 and 4.

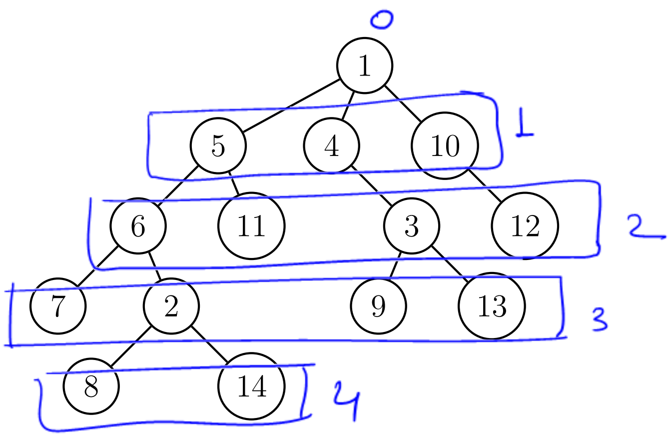
'6' " " " " " 2, 4, 5, and 7.

'7' " " " " " 3 and 5

Therefore:

1	{6, 7}
2	{1, 7}
3	{2, 4}
4	{1}
5	{2, 4}
6	{2, 4, 5, 7}
7	{3, 5}

5. Suppose we obtain the following BFS tree rooted at node 1 for an undirected graph with vertices (1,2,3,4,5,.....14).



Which of the following cannot be an edge in the original graph? [option: A]

(a) (8,11)
(b) (3,10)
(c) (4,5)
(d) (6,9)

option (a) (8, 11) $\Rightarrow$ level 2 and 4 ie level i and $i+2$. There fore it can not be an edge in original graph.

option (b) (3, 10) $\Rightarrow$ $\Rightarrow$ i and $i+1$

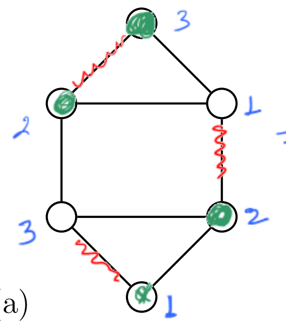
level 2 level 1. So could be connected

option (c) (4, 5) $\Rightarrow$ $\Rightarrow$ i and i

level 1 level 1 So could be connected.

2 MUTIPLE SELECT QUESTIONS:

6. Which of the following graphs satisfies the below properties:
   1. $|VC(G)| = 3$, where $VC(G)$ is the minimum vertex cover of a graph G.
   2. $|PM(G)| = 3$, where $PM(G)$ is the perfect matching of a graph G.
   3. The graph is a 3-colouring. [option: c,d]

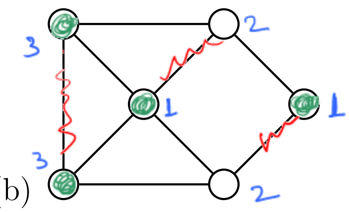


(a)

⇒ 3 colouring

⇒ 3 perfect matching

⇒ minimum vertex cover = 4

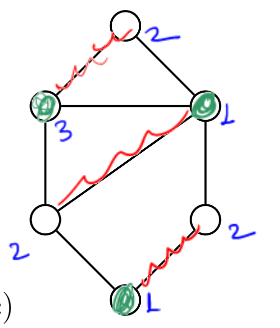


(b)

⇒ 3 colouring

⇒ 3 perfect matching

⇒ minimum vertex cover = 4

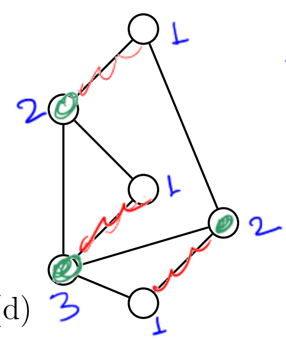


(c)

⇒ 3 colouring

⇒ 3 perfect matching

⇒ minimum vertex = 3



(d)

⇒ 3 colouring

⇒ 3 perfect matching

⇒ minimum vertex cover = 3

7. Which of the following statements is(are) true? [option: a,b]

(a) BFS can be used to identify the vertex which is at the farthest distance from v in any graph, in terms of number of edges, where vertex v is the starting vertex.

(b) BFS and DFS identifies all the vertices reachable from the starting vertex v.

(c) BFS cannot be used to check for cycles in the graph while DFS can be used to check for cycles in the graph.

(d) DFS can be used to identify the shortest distance from starting vertex v to every other vertex in the graph, in terms of number of edges.

Solution:

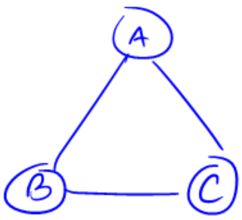
(a) BFS provides a tree level wise which means it shows how many minimum edges will be required to reach from source to a vertex. The levels can be used to identify the vertex which is at the farthest distance from v in any graph, in terms of number of edges, where vertex v is the starting vertex.

(b) BFS and DFS both show the connectivity of vertices from a source vertex which means we can use this to find the reach-ability too. Therefore, option b is correct.

(c) BFS and DFS removes the cycle from original graph so it can not be used to find the cycles.

(d) DFS is based on the vertex where to start the search from. Therefore it can not be used to find the shortest distance.

For example:



The shortest distance or number of edges required to reach from A to C is one.

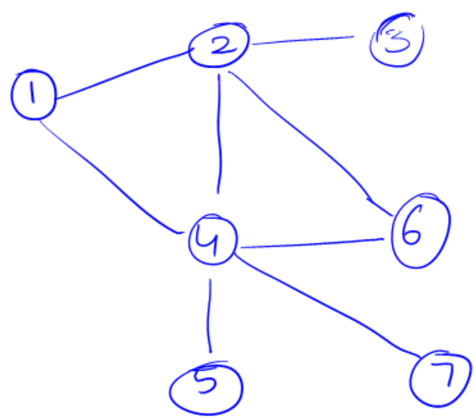
But if we use DFS then we can find two ways:

2 edges.

1 edge.

3 NUMERICAL ANSWER TYPE:

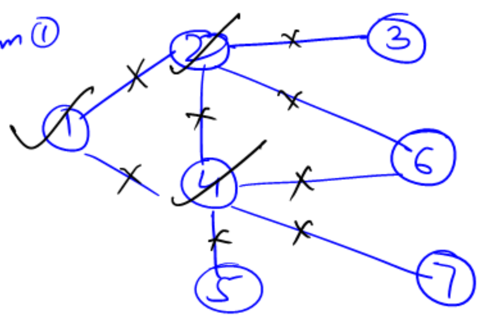
8. If $A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array}\!\!\begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$ represents adjacency matrix of a graph G, then the cardinality of the maximum independent set of the graph G is [Answer: 5]



To find the cardinality of maximum independent set we will first find the minimum vertex cover.

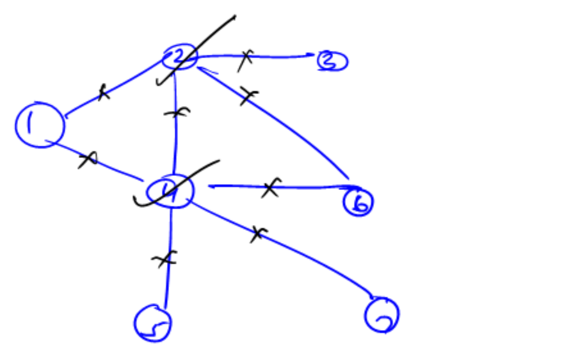
approach 1.

Start from ①



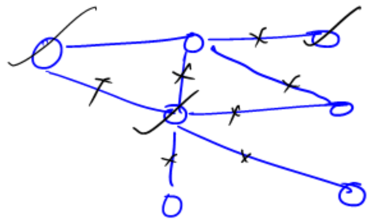
minimal vertex cover = 3.

approach 2 - Starting from ②



minimal vertex cover = 2

Approach 3: starting from (4)



minimal vertex cover = 3

Therefore,

we can not get vertex cover less than 2.

Therefore the minimum vertex cover is 2 using the approach (2)

Cardinality of maximum independent set = 7 - 2

minimum vertex cover

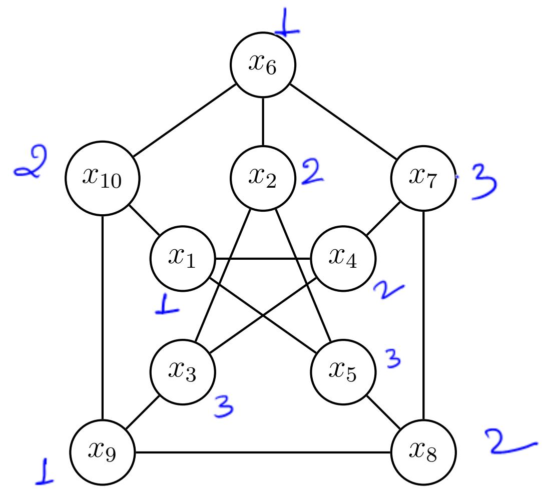
Number of total vertices

= 5

Independent (maximum) set = { 1, 3, 5, 6, 7 }

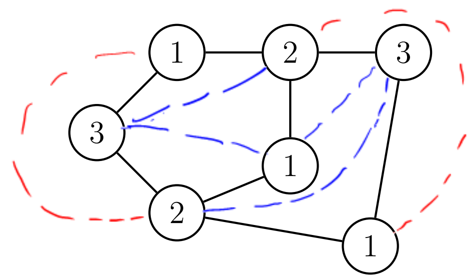
Ans.

9. A company manufactures 10 chemicals $x_1, x_2, x_3, ...x_{10}$. Certain pairs of these chemicals are incompatible and would cause explosions if brought into contact with each other. Below graph shows the incompatibility of the chemicals, each vertex represents the chemical and each edge between a pair of chemicals represents that those two chemicals are incompatible. As a precautionary measure the company wishes to partition its warehouse into compartments, and store incompatible chemicals in different compartments. What is the least number of compartments into which the warehouse should be partitioned?
[Answer: 3]



Minimum number of campartment can be found using the colouring of Graph i.e. $\underline{3}$ Ans.

10. An incomplete undirected graph is given below and the numbering on each vertex denotes the colouring of the graph('1' denotes color 1, '2' denotes color 2, and '3' denotes color 3). Find the number of maximum edges that can be added to the given graph such that the colouring is retained and the graph is planar.
NOTE: Planar graph is a graph that can be drawn on the plane in such a way that its edges intersect only at their endpoints. [Answer: 6]



Total 6 edges.