Week - 2
Practice Assignment Solutions
Straight line - 1
Mathematics for Data Science - 1

NOTE: There are some questions which have functions with discrete valued domains (such as month or year). For simplicity, we treat them as continuous functions.

1 Multiple Choice Questions (MCQ):

1. If R is the set of all points which are 5 units away from the origin and are on the axes then R is:

   ○ $R = \{(5, 5), (-5, 5), (-5, -5), (5, -5)\}$
   ○ $R = \{(5, 0),\ (5, -5),\ (5, 5),\ (-5, 0)\}$
   ○ $R = \{(5, 0),\ (0, 5),\ (5, 5),\ (0, -5)\}$
   ○ $\boldsymbol{R = \{(5, 0),\ (0, 5),\ (-5, 0),\ (0, -5)\}}$
   ○ $R = \{(5, 0),\ (0, 5),\ (-5, 0),\ (-5, 5)\}$
   ○ There is no such set.

   Solution:

   The points on the $x-$axis are represented by $(\pm a,\ 0)$, and on the $y-$axis are represented by $(0,\ \pm b)$, where a and b are the distances of the points $(\pm a,\ 0)$ and $(0,\ \pm b)$, respectively, from the origin. Therefore, the points (5, 0), (0, 5), (-5, 0), (0, -5) lie on the axes and are 5 units away from the origin. See Figure PS-2.1 for reference.

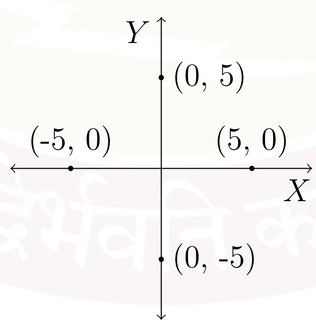


Figure PS-2.1

2. A point P divides the line segment MN such that $MP : PN = 2 : 1$. The coordinates of M and N are $(-2, 2)$ and $(1, -1)$ respectively. What will be the slope of the line passing through P and the point $(1, 1)$?

   - ◯ $\frac{4}{3}$
   - ◯ $\mathbf{1}$
   - ◯ Inadequate information.
   - ◯ $-\frac{4}{3}$
   - ◯ $\tan(\frac{4}{3})$
   - ◯ None of the above.

Solution:

By the sectional formula, the coordinates of a point (x, y) that divides a line segment defined by two points $(x_1, y_1), (x_2, y_2)$ in the ratio $m : n$ is given by

$$x = \frac{m \times x_2 + n \times x_1}{m + n}$$

$$y = \frac{m \times y_2 + n \times y_1}{m + n}$$

Since point P divides the line segment formed by the points $M(-2, 2)$ and $N(1, -1)$ in the ratio 2:1, we obtain the coordinates of point P denoted by, say (x_p, y_p), using the sectional formula as follows.

$$x_p = \frac{2 \times 1 + 1 \times (-2)}{2 + 1} = 0$$

$$y_p = \frac{2 \times (-1) + 1 \times 2}{2 + 1} = 0$$

Hence point $P = (0, 0)$ denotes the origin as shown in Figure PS-2.2

Now, we compute the slope of the line passing through P and (1,1) as,

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 0}{1 - 0} = 1$$

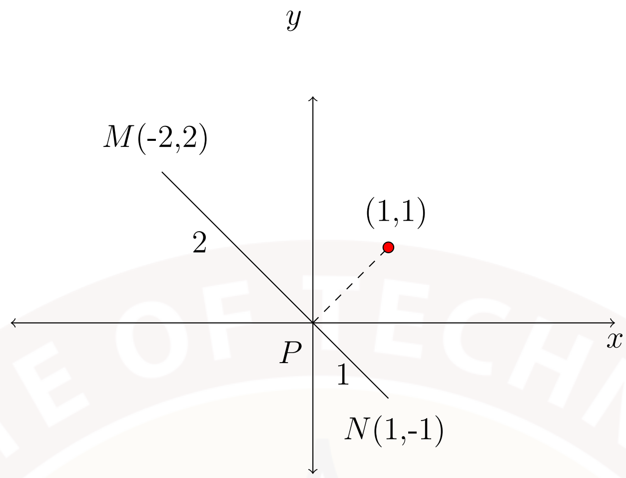


Figure PS-2.2

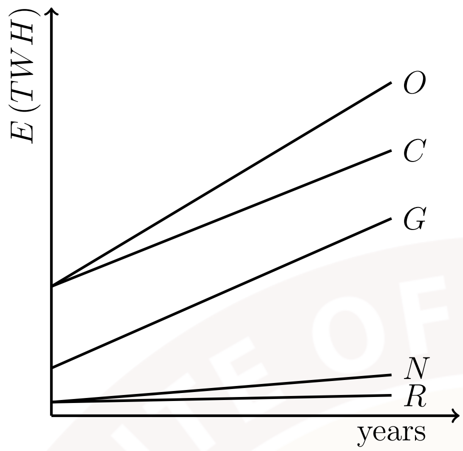
Use the following information to solve questions 3 and 4.

Table PS-2.1 shows the different types of energies consumed (approximate values) in years 1965 and 2015 across the world

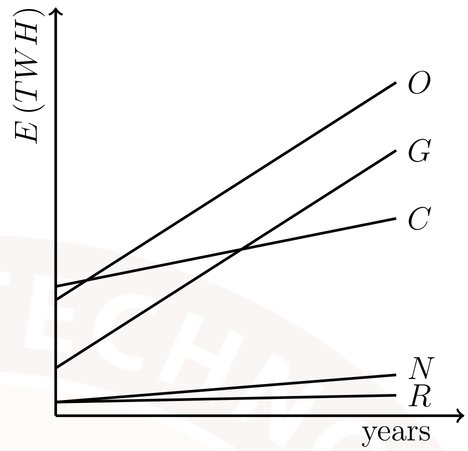
Energy type	Approximate energy used (TWH)	
	1965	2015
Oil (O)	19000	49000
Coal (C)	17000	38000
Gas (G)	7000	29000
Nuclear (N)	2000	6000
Renewable (R)	2000	3000

Table PS-2.1

3. A student assumes a linear relationship between energy consumed (E) and the number of years after 1965. Choose the option which best represents the linear relationships assumed by the student (from 1965 to 2015). [Ans: Option C]



A.



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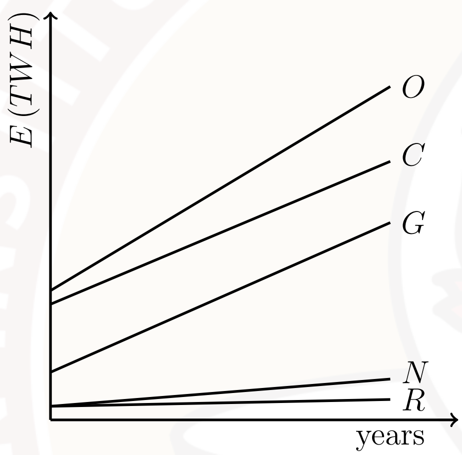
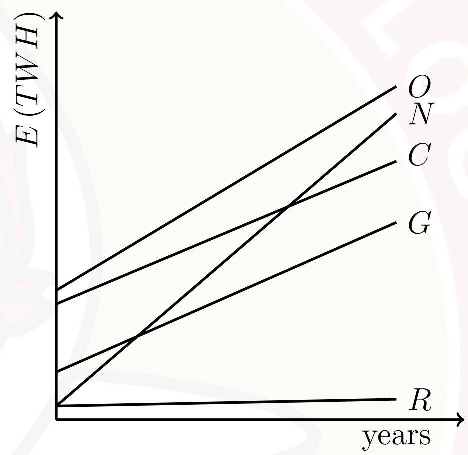


C.



D.

Solution:

Let x-axis and y-axis represent the years and the energy consumption respectively. The energy consumption in 2015 is in the order $O > C > G > N > R$, which is represented correctly in options (A) and (C). However, option (A) shows the energy consumption of O and C being same in the year 1965, which is not true. Hence, option (A) is not correct. Therefore, the correct answer is option (C).

4. The student estimated the energy consumption in 2025 and created Table PS-2.2. Choose the correct option.

Energy type	Approximate energy used (TWH)		
	1965	2015	2025
Oil (O)	19000	49000	o
Coal (C)	17000	38000	c
Gas (G)	7000	29000	g
Nuclear (N)	2000	6000	n
Renewable (R)	2000	3000	r

Table PS-2.2

- ◯ $o = 64000$
- ◯ $c = 48500$
- ◯ $g = 38500$
- ◯ $n = 8000$
- ◯ $r = 3500$
- ◯ **None of the above**.

Solution:

As earlier, let x-axis and y-axis represent the years and the energy consumption respectively. Using the data provided for two years, we can find the equation of the line in two-point form. Equation for the energy type *oil* (O) will be

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\implies y - 19000 = \frac{49000 - 19000}{2015 - 1965}(x - 1965)$$

On solving the above equation with $x = 2025$,

$$\implies y - 19000 = \frac{49000 - 19000}{2015 - 1965}(2025 - 1965)$$

$$y = 55000$$

Equation for the energy type *coal* (C) will be

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\implies y - 17000 = \frac{38000 - 17000}{2015 - 1965}(x - 1965)$$

On solving the above equation with $x = 2025$:

$$\implies y - 17000 = \frac{38000 - 17000}{2015 - 1965}(2025 - 1965)$$

$$y = 42200$$

Equation for the energy type *gas* (G) will be

$$(y - y_1) \ = \ \frac{y_2 \ - \ y_1}{x_2 \ - \ x_1} \ (x \ - \ x_1)$$

$$\implies y - 7000 = \frac{29000 \ - \ 7000}{2015 \ - \ 1965} \ (x - 1965)$$

On solving the above equation with $x = 2025$,

$$\implies y - 7000 = \frac{29000 \ - \ 7000}{2015 \ - \ 1965} \ (2025 - 1965)$$

$$y = 33400$$

Equation for the energy type *nuclear* (N) will be

$$(y - y_1) \ = \ \frac{y_2 \ - \ y_1}{x_2 \ - \ x_1} \ (x \ - \ x_1)$$

$$\implies y - 2000 = \frac{6000 \ - \ 2000}{2015 \ - \ 1965} \ (x - 1965)$$

On solving the above equation with $x = 2025$,

$$\implies y - 2000 = \frac{6000 \ - \ 2000}{2015 \ - \ 1965} \ (2025 - 1965)$$

$$y = 6800$$

Equation for the energy type *renewable* (R) will be:

$$(y - y_1) \ = \ \frac{y_2 \ - \ y_1}{x_2 \ - \ x_1} \ (x \ - \ x_1)$$

$$\implies y - 2000 = \frac{3000 \ - \ 2000}{2015 \ - \ 1965} \ (x - 1965)$$

On solving the above equation with $x = 2025$:

$$\implies y - 2000 = \frac{3000 \ - \ 2000}{2015 \ - \ 1965} \ (2025 - 1965)$$

$$y = 3200$$

Thus, none of the options given is correct.

2 Multiple Select Questions (MSQ):

1. The elements of a relation R are shown as points in the graph in Figure P-2.3. Choose the set of correct options:

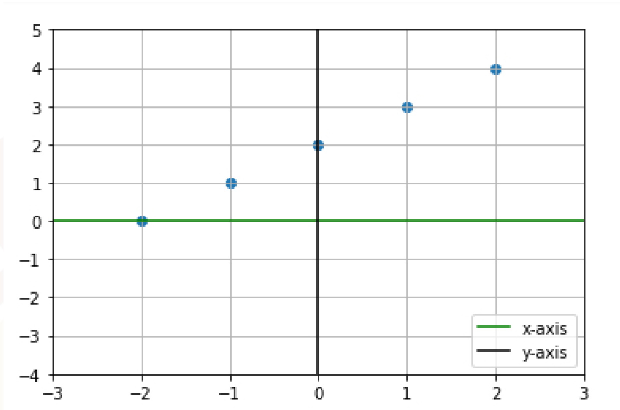


Figure PS-2.3

- ◯ **$\boldsymbol{R}$ can be represented as $\boldsymbol{R = \{(-2,0), (-1,1), (0,2), (1,3), (2,4)\}}$.**
- ◯ We can write R as $R = \{(a,b)|(a,b) \in X \times Y, b = a+2\}$, where X is the set of all values on the $x - axis$, and Y is the set of all values on the $y - axis$.
- ◯ R cannot be a function because it is a finite set.
- ◯ R is a symmetric relation.
- ◯ **$\boldsymbol{R}$ is a function because it has only one output for one input.**
- ◯ If R is a function then it is a bijective function on $X \times Y$, where X is the set of all values on the $x - axis$, and Y is the set of all values on the $y - axis$.
- ◯ **We can write $\boldsymbol{R}$ as $\boldsymbol{R = \{(a,b)|(a,b) \in X \times Y, b = a+2\}}$, where $\boldsymbol{X = \{-2,-1,0,1,2\}}$ and $\boldsymbol{Y = \{0,1,2,3,4\}}$.**

Solution:

- Option (a) is correct since the coordinates of the points in the Figure P-2.3 are as is defined by the function.
- Option (b) is incorrect. We can write R as $\{R = (a,b) \mid (a,b)\ in\ X \times Y,\ b = a+2\}$, where $X = \{-2,-1,0,1,2\}$ and $Y = \{0,1,2,3,4\}$. Here R is a finite set so we can not write for all values of x-axis or y-axis.
- Option (c) is incorrect since R can be a function of a finite set.
- Option (d) is incorrect since R is not a symmetric relation. For example, corresponding to the element (-2, 0), there is no element (0, -2) in R.

- Option (e) is correct since for every value of X there is single corresponding value in Y.
- Option (f) is incorrect since R as a function is not defined for all values on the x-axis, and Y is not the set of all values on the y-axis, whereas $X = \{-2, -1, 0, 1, 2\}$ and $Y = \{0, 1, 2, 3, 4\}$.
- Option (g) is correct, and explained in accordance with definition of function.

2. Find the values of a for which the triangle ΔABC is an isosceles triangle, where A, B, and C have the coordinates $(-1,1)$, $(1,3)$, and $(3,a)$ respectively.

   - ◯ **If $\boldsymbol{AB = BC}$, then $\boldsymbol{a = 1}$.**
   - ◯ If $AB = BC$, then $a = -1$ or -5.
   - ◯ **If $\boldsymbol{BC = CA}$, then $\boldsymbol{a = -1}$.**
   - ◯ If $BC = CA$, then $a = 1$.

Solution:

As we know, for an isosceles triangle two of its sides are equal. According to the question the vertices of C is $(3,a)$ therefore, depending on the value of a we can have length of $AB = BC$ or $BC = CA$

Since the vertices of triangle are given, we can find the length of each side using distance formula.

Value of a when length of $AB = BC$:

Length of any side of triangle is given by

$$\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$\implies \sqrt{(3-1)^2 + (1-(-1))^2} \;=\; \sqrt{(a-3)^2 + (3-1)^2}$$

$$\implies \sqrt{8} \;=\; \sqrt{4 + (a-3)^2}$$

Squaring them on both sides, we have

$$\implies (a-3)^2 \;=\; 4 \implies a - 3 \;=\; \pm 2$$

Therefore,

$$a \;=\; 5,\; 1$$

But, if $a = 5$ then the three points will be co-linear therefore,

$$\boldsymbol{a = 1}$$

Value of a when length of $BC = CA$:

$$\sqrt{(a-3)^2 + (3-1)^2} \;=\; \sqrt{(a-1)^2 + (3-(-1))^2}$$

$$\implies \sqrt{4 + (a-3)^2} \;=\; \sqrt{16 + (a-1)^2}$$

Squaring on both sides of the equation, we get

$$\implies 4 + (a-3)^2 \;=\; 16 + (a-1)^2 \implies (a-3)^2 - (a-1)^2 \;=\; 12$$

$$\implies (2a-4)(-2) \;=\; \implies 12 \implies a = -1$$

Therefore,

$$\boldsymbol{a \;=\; -1}$$

3. A plane begins to land when it is at a height of 1500 metre above the ground. It follows a straight line path and lands at a point which is at a horizontal distance of 2700 metre away. There are two towers which are at horizontal distances of 900 metre and 1800 metre away in the same direction as the landing point. Choose the correct option(s) regarding the plane's trajectory and safe landing.

   - ○ The trajectory of the path could be $\frac{y}{27} + \frac{x}{15} = 100$ if $x - axis$ and $y - axis$ are horizontal and vertical respectively.
   - ○ The maximum safe height of the towers are 1000 metre and 1500 metre respectively.
   - ○ **The trajectory of the path could be $\boldsymbol{\frac{y}{15} + \frac{x}{27} = 100}$ if $\boldsymbol{x - axis}$ and $\boldsymbol{y - axis}$ are horizontal and vertical respectively.**
   - ○ The maximum safe height of the towers are 1500 metre and 500 metre respectively.
   - ○ **The maximum safe height of the towers are 1000 metre and 500 metre respectively.**
   - ○ None of the above.

Solution:

Let us consider the height of plane from ground as $y-$axis and horizontal distance on ground as $x-$axis as shown in Figure PS-2.4

Then, the point P(0,1500) represents the position of the airplane when it began its descent and point Q(2700,0) represents the point where the plane landed.

The two towers which are 900m and 1800m away from the $y-$axis are represented by A and B respectively.

The equation of a straight line path traced by plane from $P(0, 1500)$ to $Q(2700, 0)$ can be obtained using the intercept-form.

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\implies \frac{x}{2700} + \frac{y}{1500} = 1$$

On rearranging:

$$\frac{y}{15} + \frac{x}{27} = 100$$

Now, to check the maximum safe height of towers:

For tower A at $X-$coordinate $= 900m$, the maximum safe height will be:

$$\frac{y}{15} + \frac{900}{27} = 100$$

$$\implies y = 1000m$$

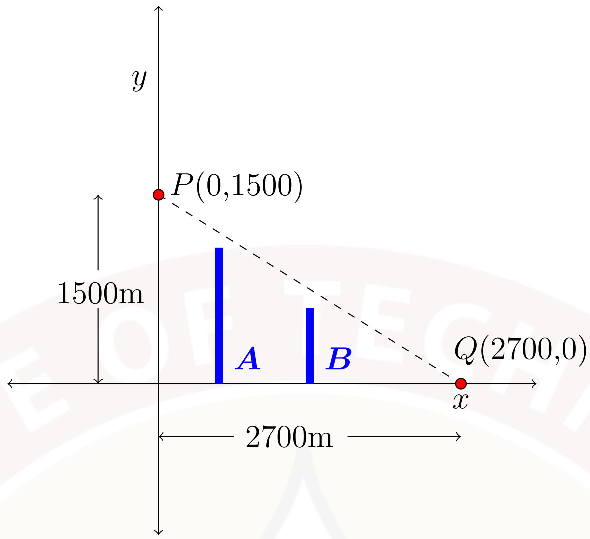


Figure PS-2.4

For tower B at $X-$coordinate $= 1800m$, the maximum safe height will be:

$$\frac{y}{15} + \frac{1800}{27} = 100$$

$$\implies y = 500m$$

3 Numerical Answer Type (NAT):

Use the following information to solve the question 1-2.
The coordinates of points A, B, C and E are shown in the figure PS-2.5 below. Points D and F are the midpoints of lines BC and AD respectively. Using the data given and Figure PS-2.5, answer the questions below.

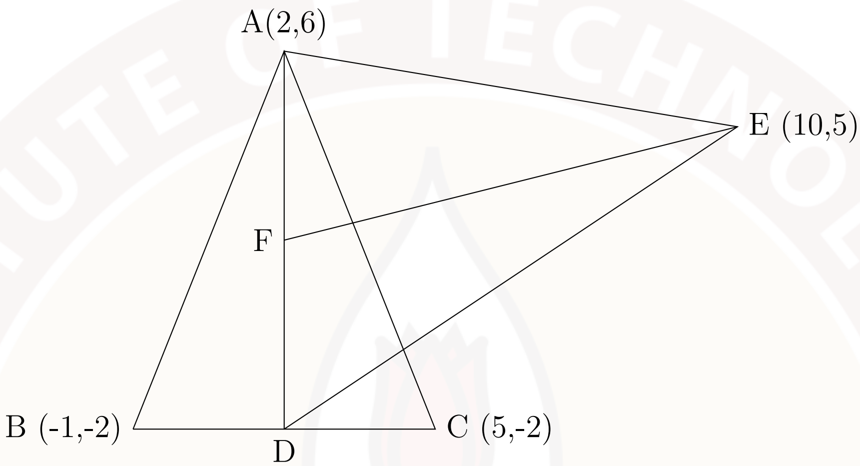


Figure PS-2.5

1. Find the area of triangle ADE. [Ans: 32]

Solution:

By the sectional formula, the coordinates of a point (x, y) that divides a line segment defined by two points $(x_1, y_1), (x_2, y_2)$ in the ratio $m : n$ is given by

$$x = \frac{m \times x_2 + n \times x_1}{m + n}$$

$$y = \frac{m \times y_2 + n \times y_1}{m + n}$$

Since point D is the midpoint of the line segment BC formed by the points $B(-1, -2)$ and $C(5, -2)$ so they are in the ratio 1:1. Thus, we can obtain the coordinates of the point D denoted by, say (x_d, y_d), using the sectional formula as follows.

$$x_d = \frac{1 \times 5 + 1 \times (-1)}{1 + 1} = 2$$

$$y_d = \frac{1 \times (-2) + 1 \times (-2)}{1 + 1} = -2$$

Therefore,

$$\implies D(2, -2)$$

Now, area of triangle ADE with vertices A(2, 6), D(2, -2) and E(10, 5) can be obtained as:

$$= \frac{1}{2}| \ x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \ |$$

$$= \frac{1}{2}| \ 2(-2-5) + 2(5-6) + 10(6-(-2)) \ |$$

$$= 32$$

2. Let the slope of a line FG be 2 and the coordinate of the point G be (a, 9). Then, what is the value of a? [Ans: 5.5]

Solution:

As seen earlier, the point F is the midpoint of the line segment AD formed by the points $A(2, 6)$ and $D(2, -2)$ so they are in the ratio 1:1. Thus we can obtain the coordinates of the point F denoted by, say (x_f, y_f), using the sectional formula as follows.

$$x_f = \frac{1 \times 2 + 1 \times 2}{1+1} = 2$$

$$y_f = \frac{1 \times (-2) + 1 \times 6}{1+1} = 2$$

Therefore,

$$\implies F(2, 2)$$

Now, the slope of FG will be $= \frac{9 \ - \ 2}{a \ - \ 2} = 2$
On solving the above equation, we get $\boldsymbol{a \ = \ 5.5}$

3. Leo rents a motorcycle for 2 days. Hence, the rental company provides the motorcycle at Rs. 500 per day with 100 km free per day. The additional charges after 100 km are Rs. 2 per km Leo drives the motorcycle for a total of 500 km. How much (Rs.) will he have to pay to the rental company? [Ans: 1600]

Solution:

Leo has rented a motorcycle for 2 days, thus he has to pay Rs. 1,000 for free 200 km ride. Thereafter, he has to pay Rs. 2 per km. for rest of 300km, which accounts for Rs. 600. Thus, in total he has to pay Rs. 1,600.