Week - 4
Tutorial
Quadratic functions
Mathematics for Data Science - 1

Syllabus Covered:

- Quadratic functions (Vertex, axis of symmetry, minima, and maxima).
- Slope of quadratic function
- Solution of quadratic equation using graph (Zeroes of quadratic functions)

1. (a) Find the minimum value of y where $y = x^2 + x + 2$.
   (b) Find the x-intercept of the given curve $y = x^2 + x + 2$.
   (c) Find out the length of the line segment on the straight line passing through the y-intercept of the given curve and the point $(-2, 4)$.

2. Find the value of k for which the curve $x = y^2 - 6y + k$ touches X-axis exactly at one point.

3. A train goes along a path $x = y^2 - 6y + 8$. There are two stops on the line $x = 0$ and Arav's home is at the origin. How much minimum distance will Arav have to cover in order to catch the train?

4. On the basis of some measured data of a vehicle, a student fitted a curve for the vehicle's speed (in kmph) x and its fuel economy (mileage in kmpl) $f(x)$ as $40f(x) = 88x - x^2 + 1200$. According to his fit what is the maximum economy that can be obtained by the vehicle and what should the speed be for the same?

5. The production rate (R) of a material in a factory depends on two factors f_1 and f_2 as $R = f_1 f_2$. Factor f_1 and f_2 are the functions of purity of the raw material x as $f_1(x) = ax + b$ and $f_2 = -cx + d$. Find the purity of material for which the production is maximum where a, b, c, and d are positive.

6. Consider the function$f_1 = -x^2 + 8x + 6$. Two points P and Q are on the resulting parabola such that they are two units away from the axis of symmetry. If V represents the vertex of the curve, answer the following.
   (a) If the triangle PVQ is rotated around its axis of symmetry then what is the curved surface area of the resulting cone? Given that the curved surface area of a cone is πrl, where r is the radius of the base and l is the slant height of a cone?

(b) Consider another curve representing function f_2 such that $f_2 = (x-4)^2$. Now let A be the set of all points inside the region bounded by these curves (including the curves), what is the range of $y - coordinate$ of the points in A ?

7. Let a curve C represent the relation $y^2 = 4ax$. Is y a function of x?

8. An advertiser is analysing the growth of likes for their new ad on YouTube. She analyzed that the increase in likes in a given second is equal to $4t_{av}$ where t_{av} is midpoint of the time interval. For example, the increase in likes from 3 seconds to 4 seconds is equal to 4×3.5. Answer the following questions.
   (a) If the total likes follow the path as $l(t) = at^2 + bt + c$ then what is the value of b?
   (b) Find the total likes at 60 seconds.
   (c) If the domain of the function l is $[k, \text{inf})$, what will be the value of k?