Week - 3
Practice problems
Straight line
Mathematics for Data Science - 1

1 Multiple Choice Questions (MCQ):

1. A vehicle is travelling on a straight line path and it passes through the points $A(-4, 2)$, $B(-1, 3)$, and $C(2, \mu)$. The value of μ is:
   - ○ 2
   - ○ **4**
   - ○ -2
   - ○ 10

   Solution:

   Since the vehicle is travelling on a straight line path and passes through the points A, B, and C, it follows that A, B, and C are collinear. Hence the slope of the straight line path joining A and B will be equal to the slope of the straight line path joining B and C. Using the slope formula for two points, we have

$$\frac{3-2}{-1+4} = \frac{\mu - 3}{2+1}$$

$$\implies \boldsymbol{\mu = 4}.$$

2. Suppose two boats are starting their journey from the ferry ghat A (considered as the origin), one towards ferry ghat B along the straight line $y = -2x$ and the other towards the ferry ghat C along a straight line perpendicular to the path followed by B. The river is 1 km wide uniformly and parallel to the X-axis. Suppose Rahul wants to go to the exact opposite point of A along the river.

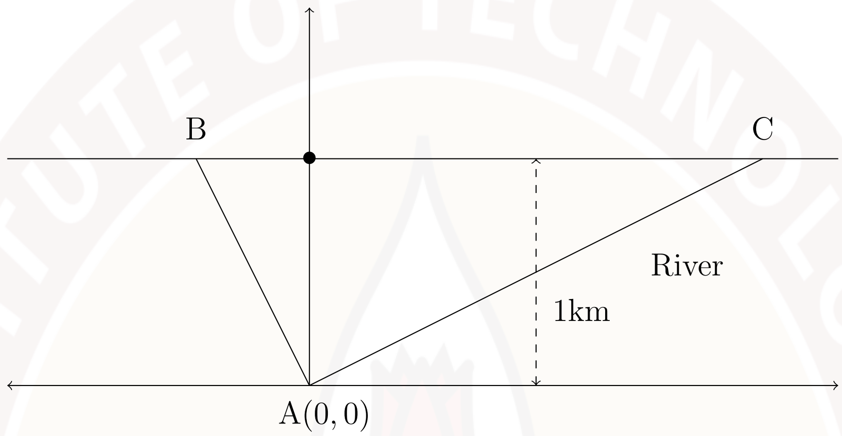

Figure PS-3.1

Then, answer the following questions.

(a) How much total distance does Rahul have to travel to reach his destination if he takes the boat towards ferry ghat B?

- ○ $\sqrt{5}$
- ○ $\sqrt{5}+2$
- ○ $\frac{\sqrt{5}}{2}$
- ○ $\frac{\sqrt{5}+1}{2}$

Solution:

See the Figure PS-3.2 for reference:

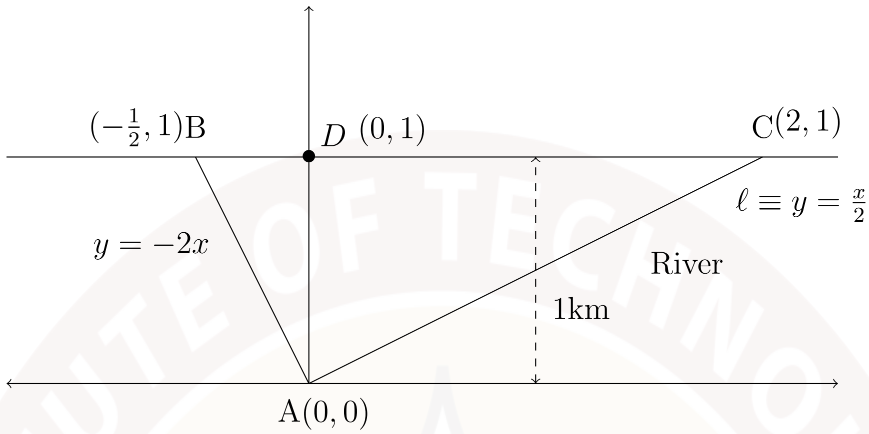

Figure PS-3.2

Since the point A is assumed to be the origin, the side of the river from which Rahul is starting his journey is considered to be the X-axis. The path towards Rahul's destination, which is perpendicular to the X-axis, is hence the Y-axis. Let D be Rahul's destination, which is 1 km away from the point A and is on the opposite side of the river. It follows that the point D is $(0, 1)$.

Hence, the equation of the line representing the opposite side of the river is $y = 1$. Solution of the equations $y = 1$ and $y = -2x$ gives the location of ferry ghat B which is the point $(-\frac{1}{2}, 1)$.

Using the distance formula between two points, the distance between ferry ghat A and ferry ghat B is given by

$$\sqrt{(-\frac{1}{2} - 0)^2 + (1 - 0)^2} = \frac{\sqrt{5}}{2}\text{units}$$

Similarly, the distance between ferry ghat B and the point D is $\frac{1}{2}$ units.

Hence, the total distance that Rahul has to travel to reach his destination D if he takes the boat toward ferry ghat B is given by

$$\frac{\sqrt{5}}{2} + \frac{1}{2} = \frac{\sqrt{5} + 1}{2} \text{ units}$$

(b) How much total distance does Rahul have to travel to reach his destination if he takes the boat towards ferry ghat C?

- ◯ $\sqrt{5}$
- ◯ $\boldsymbol{\sqrt{5} + 2}$
- ◯ $\frac{\sqrt{5}}{2}$
- ◯ $\frac{\sqrt{5}+1}{2}$

Solution: Let ℓ denote the path towards ferry ghat C from A. The equation of path ℓ will be $y = mx$ since it passes through the origin. Since ℓ is perpendicular to the line $y = -2x$, which has a slope $m_1 = -2$, it follows that

$$m = -\frac{1}{m_1} = \frac{1}{2}$$

$\implies$ the equation of ℓ is $y = \frac{x}{2}$.

Solution of the equations $y = \frac{x}{2}$ and $y = 1$ gives the location of ferry ghat C which is (2,1).

Using the distance formula between two points, the distance between ferry ghat A and ferry ghat C is

$$\sqrt{(2-0)^2 + (1-0)^2} = \sqrt{5} \text{ units}$$

Similarly, the distance between ferry ghat C and the destination point D is 2 units.

Hence, the total distance that Rahul has to travel to reach his destination D if he takes the boat towards ferry ghat C is $\sqrt{5} + 2$ units.

3. Suppose a bird is flying along the straight line $4x - 5y = 20$ on the plane formed by the path of the flying bird and the line of eye point view of a person who shoots an arrow which passes through the origin and the point $(10, 8)$. What is the point on the co-ordinate plane where the arrow hits the bird?

   - ◯ $(20, 12)$
   - ◯ $(25, 16)$
   - ◯ **The arrow will miss the bird.**
   - ◯ Inadequate information.

   Solution:
   Using the two point form of a line, the equation of the path of arrow passing through the origin and the point (10,8) is

$$(y - 0) = \frac{8 - 0}{10 - 0}(x - 0) \implies 8x - 10y = 0$$

   The slope intercept form of the above line is given by

$$y = \frac{8}{10}x$$

   From the above line, we obtain the slope as

$$m_1 = \frac{8}{10} = \frac{4}{5}$$

   Similarly, for the path of the bird along the straight line $4x - 5y = 20$, we get the slope

$$m_2 = \frac{4}{5}$$

   Here, $m_1 = m_2$,
   That is, the lines $8x - 10y = 0$ and $4x - 5y = 20$ have the same slope. Therefore, the path of flying bird and the path of the arrow are parallel to each other. Hence, the arrow will miss the bird.

4. We plot the displacement (S) versus time (t) for different velocities as it follows the equation $S = vt$, where v is the velocity. Identify the best possible straight lines in the Figure P-3.2 for the given set of velocities.

Table PS-3.1

v_1	v_2	v_3	v_4
1	-2	0.5	-1

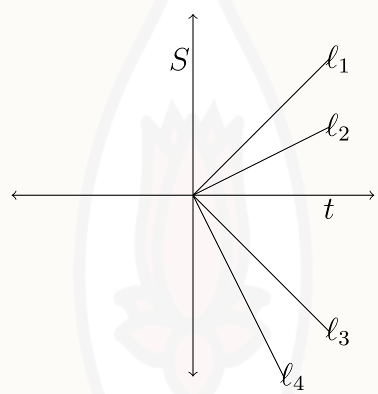

Figure PS-3.3

- ○ $v_1 \to \ell_1$, $v_2 \to \ell_2$, $v_3 \to \ell_3$, and $v_4 \to \ell_4$.
- ○ $v_1 \to \ell_1$, $v_2 \to \ell_4$, $v_3 \to \ell_3$, and $v_4 \to \ell_2$.
- ○ **$\boldsymbol{v_1 \to \ell_1}$,$\boldsymbol{v_2 \to \ell_4}$, $\boldsymbol{v_3 \to \ell_2}$, and $\boldsymbol{v_4 \to \ell_3}$.**
- ○ $v_1 \to \ell_2$, $v_2 \to \ell_4$, $v_3 \to \ell_1$, and $v_4 \to \ell_3$.

Solution:

From Figure PS-3.3, ℓ_1 and ℓ_2 have positive slope and the slope of ℓ_1 is greater than the slope of ℓ_2. Similarly the slopes of ℓ_3 and ℓ_4 are negative and the slope of line ℓ_3 is greater than the slope of line ℓ_4.

Substituting the value of v in equation $s = vt$, we get the equations

$$s = t, s = -2t, s = 0.5t, s = -t$$

By comparing the above equations of lines and the lines in Figure PS-3.3, we conclude that v_1 corresponds to the line ℓ_1, v_2 corresponds to the line ℓ_4, v_3 corresponds to the line ℓ_2, and v_4 corresponds to the line ℓ_3.

2 Multiple Select Questions (MSQ):

5. A constructor is asked to construct a road which is at a distance of $\sqrt{2}$ km from the municipality office and perpendicular to a road which can be defined by the equation of the straight line $x - y = 8$ (considering the municipality office to be the origin). Find out the possible equations of the straight lines to represent the new road to be constructed.

   - ○ $x - y - 2 = 0$
   - ○ $\boldsymbol{x + y + 2 = 0}$
   - ○ $x - y + 2 = 0$
   - ○ $\boldsymbol{x + y - 2 = 0}$

   Solution:

   See the Figure PS-3.4 for reference:

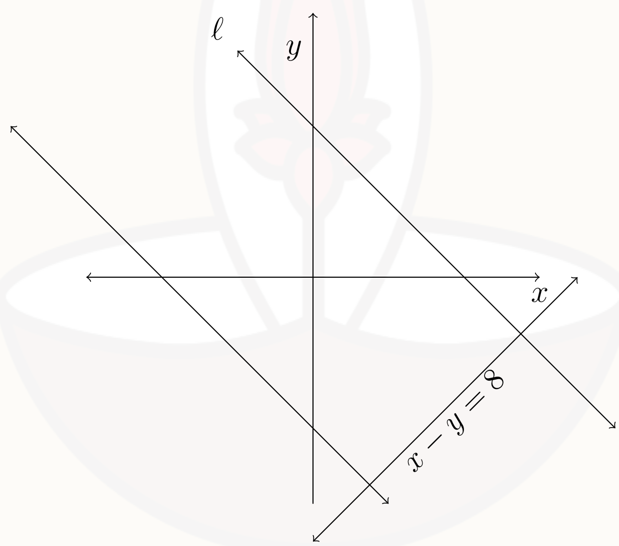

Figure PS-3.4

Let the new road constructed be denoted by ℓ. Given, ℓ is perpendicular to the straight line $x - y = 8$. That is, ℓ is perpendicular to the line $y = x - 8$ whose slope is $m_1 = 1$. Therefore, the slope of ℓ is

$$m_2 = -\frac{1}{m_1} = -1$$

By the slope intercept form, the equation of ℓ is

$$y = m_2\ x\ +\ c$$

$$\implies y = -x + c \text{ , where } c \text{ is a constant}$$

That is, ℓ is the line given by

$$x + y - c = 0$$

It is given that the distance of ℓ from the municipality office is $\sqrt{2}$.
The distance formula of a point (x_1, y_1) from a line $(Ax + By + C = 0)$ is given by $\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$. Substituting $x_1 = 0, y_1 = 0, A = 1, B = 1, C = -c$ in formula, we will get the distance of the point(0,0) from the line ℓ,

$$\frac{|1 \times 0 + 1 \times 0 - c|}{\sqrt{1^2 + 1^2}}$$

which is equal to $\sqrt{2}$. So,

$$\frac{|0 + 0 - c|}{\sqrt{1 + 1}} = \sqrt{2}$$

$$\implies \frac{|c|}{\sqrt{2}} = \sqrt{2}$$

$$\implies |c| = 2$$

$\implies c = +2$ or $c = -2$.
Hence, the equation of the new road ℓ is

$$x + y + 2 = 0$$

or

$$x + y - 2 = 0.$$

6. Suppose there are two roads perpendicular to each other which are both at the same distance from Priya's house (considered as the origin). The meeting point of the two roads is on the x-axis and at a distance of 5 units from Priya's house.
Choose the correct possible equations representing the roads.

   - ○ Inadequate information.
   - ○ $y = \frac{1}{2}x + 5, y = -2x - 5$
   - ○ $\boldsymbol{y = -x - 5, y = x + 5}$
   - ○ $y = 2x - 10, y = -2x - 10$
   - ○ $y = 2x - 5, y = -\frac{1}{2}x - 5$
   - ○ $\boldsymbol{y = -x + 5, y = x - 5}$
   - ○ $x = 5, x = -5$

Solution:
Denote the two roads by ℓ_1 and ℓ_2. The meeting point of ℓ_1 and ℓ_2 are on the X-axis and at a distance of 5 units from Priya's house (origin) i.e x-intercepts of the roads are 5 or -5 and passing through the points (5,0) or (-5,0) respectively.

Case 1: when x-intercept is 5 and passes through (5,0)
Using intercept form of a line on the axes, the equation of line ℓ_1 is

$$\frac{x}{5} + \frac{y}{b} = 1$$

where b is a constant.

That is, ℓ_1 is

$$bx + 5y - 5b = 0 \tag{1}$$

See Figure PS-3.5 for reference.

The slope of the road ℓ_1 is $m_1 = -\frac{b}{5}$.
Since the road ℓ_2 is perpendicular to ℓ_1, the slope of road ℓ_2 is

$$m_2 = -\frac{1}{m_1} = \frac{5}{b}$$

Using the slope intercept form, the equation of the road ℓ_2 is

$$y = \frac{5}{b}x + c \Longrightarrow by - 5x - bc = 0 \text{ where } b \text{ and } c \text{ are constant}$$

The roads ℓ_1 and ℓ_2 are at the same distance from Priya's house (origin).
Using distance formula of a line from a point, we get

$$\frac{|-5b|}{\sqrt{b^2 + 25}} = \frac{|-bc|}{\sqrt{b^2 + 25}} \Longrightarrow |c| = |5| \Longrightarrow c = 5 \quad or - 5$$

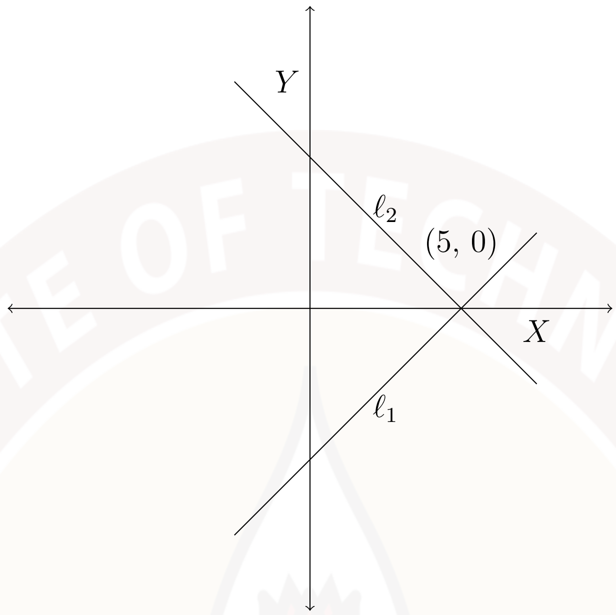

Figure PS-3.5

When $c = 5$, the equation of road ℓ_2 becomes $by - 5x - 5b = 0$. Since ℓ_2 passes through $(5, 0)$, we get $b = -5$.
Therefore, the equation of the road ℓ_2 is $y = -x + 5$.
Substituting $b = -5$ in Equation (1), we will get the equation of the road ℓ_1 as $y = x - 5$.
When $c = -5$, we will get the same equation alternatively.

Case 2: when x-intercept is -5 and passing through (-5,0)
We follow the same process as in Case 1 and we get the equation of the road ℓ_2 as $y = x + 5$ and the equation of the road ℓ_1 as $y = -x - 5$.
See Figure PS-3.6 for reference.

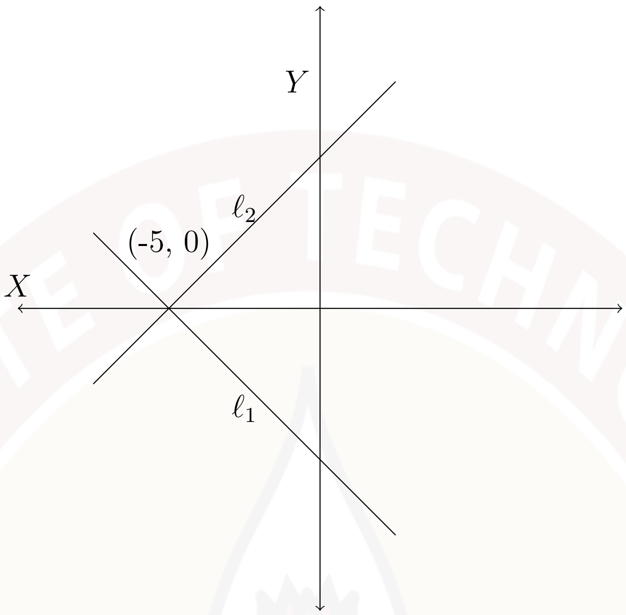

Figure PS-3.6

7. Consider the following two diagrams.

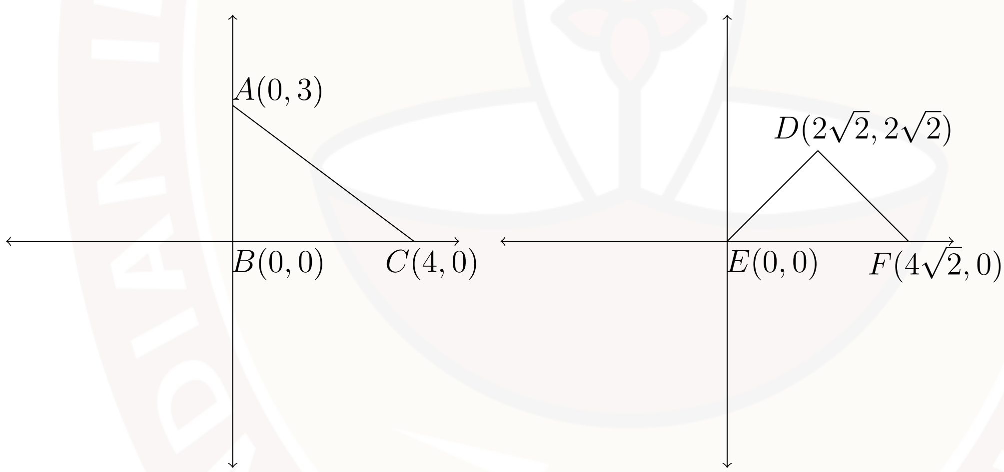

Figure PS-3.7

Which of the following option(s) is(are) true about the triangles ΔABC and ΔDEF given in Figure PS-3.7?

- ◯ Only ΔABC is a right angled triangle while ΔDEF is not.
- ◯ **Both ΔABC and ΔDEF are right angled triangles.**
- ◯ The area of ΔABC is greater than the area of ΔDEF.
- ◯ Both the triangles have the same area.

◯ **The area of ΔDEF is 8 sq.unit.**

Solution:

In Figure PS-3.7, vertices A and C are on $Y-$axis and $X-$ axis respectively and the vertex B is at the origin itself.
Therefore, $\triangle ABC$ is a right angle triangle.
The distance formula between two points $(x_1, y_1), (x_2, y_2)$ is given by

$$\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

Using the above formula, in $\triangle DEF$, the length of side DE is

$$\sqrt{(2\sqrt{2} - 0)^2 + (2\sqrt{2} - 0)^2} = \sqrt{(2\sqrt{2})^2 + (2\sqrt{2})^2} = 4$$

Similarly, the length of side DF is

$$\sqrt{(4\sqrt{2} - 2\sqrt{2})^2 + (0 - 2\sqrt{2})^2} = 4$$

The length of side EF is $4\sqrt{2}$. We have

$$DE^2 + DF^2 = 16 + 16 = 32 = (4\sqrt{2})^2 = EF^2$$

Hence, by the Pythagoras theorem, $\triangle DEF$ is also a right angled triangle.
Area of the right angled $\triangle ABC = \frac{1}{2} \times 4 \times 3 = 6$ sq. unit.
Area of the right angled $\triangle DEF = \frac{1}{2} \times 4 \times 4 = 8$ sq. unit.

8. Let the diagonals of a quadrilateral with one vertex at $(0,0)$ bisect each other perpendicularly at the point $(1,2)$. Further, let one of the diagonals be on the straight line $y = 2x$. Then, which of the following is (are) correct statements?

- ◯ **The diagonally opposite vertex of $(0,0)$ is $(2,4)$.**
- ◯ The other diagonal is on the straight line $y = -\frac{1}{2}x$.
- ◯ **The other diagonal is on the straight line $y = -\frac{1}{2}x + \frac{5}{2}$.**
- ◯ The diagonally opposite vertex of $(0,0)$ is $(\frac{3}{2}, 3)$.

Solution:

Figure PS-3.8 shows a sketch of the quadrilateral.

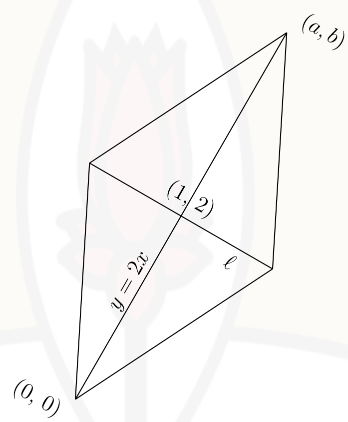

Figure PS-3.8

The diagonal $y = 2x$ has slope $m_1 = 2$.
Let the other diagonal, perpendicular to the line $y = 2x$, be on the line ℓ.
So, the slope of the line ℓ is

$$m_2 = -\frac{1}{m_1} = -\frac{1}{2}$$

From the slope intercept form, the equation of the line ℓ is $y = -\frac{x}{2} + c$, where c is a constant.
Since both the diagonals intersect at the point (1,2) and one diagonal is on line ℓ, the point (1,2) belongs to ℓ and hence $c = \frac{5}{2}$.
Hence, the equation of the line ℓ is $\boldsymbol{y = -\frac{1}{2}x + \frac{5}{2}}$.

Let the opposite vertex of $(0,0)$ be (a,b).
Since the point (1, 2) is the bisection point of the both diagonals, it follows that the point $(1,2)$ is mid-point of the line segment joining the points $(0,0)$ and (a,b).
Using the section formula of a line segment,

$$\frac{a}{2} = 1 \implies a = 2$$

$$\frac{b}{2} = 2 \implies b = 4$$

Hence the diagonally opposite vertex of $(0, 0)$ is $(2, 4)$.

9. A woman is reported missing in a locality. The police department finds a human femur bone during their investigation. They estimate the height H of a female adult (in cm) using the relationship $H = 1.8f + 70$, where f is the length (in cm) of the femur bone. The length of the femur found is 35 cm, and the missing woman is known to be 130 cm tall. In the particular locality, maximum height of a female is 195 cm and the minimum length of a female femur bone is 15 cm. Based on the given data answer the following questions.

   (a) Choose the set of correct options.

   - ◯ If an error of 1 cm is allowed, bone could belong to missing female.
   - ◯ **If an error of 3 cm is allowed, bone could belong to missing female.**
   - ◯ If the height as a function of femur length is known to be accurate, the range of the function is [70, 195].
   - ◯ **If the height as a function of femur length is known to be accurate, the range of the function is [97, 195].**
   - ◯ **If the height as a function of femur length is known to be accurate, the domain of the function is $[15, \frac{625}{9}]$.**

   Solution:

   The relationship between height of a woman H and the length of her femur bone f is given by

$$H = 1.8f + 70. \tag{2}$$

   Since the length of the femur bone found during the investigation is 35 cm, we have

$$H = 1.8 \times 35 + 70 = 133 \text{ cm}$$

   The height of missing woman is known to be 130cm. Since $133 - 130 = 3 \leq 3$ and by our assumption, an error of 3 cm is allowed, it is possible that the femur bone found during the investigation belongs to the missing woman.

   Given that the maximum height of a female in that location is 195 cm.
   Substituting $H = 195$ in Equation (2) , we get the maximum length of female femur bone in that location i.e maximum $f = \frac{625}{9}$ cm.
   Since the minimum length of of femur bone known in that location is 15 cm and if height as a function of femur length is known to be accurate then the domain of the function is $[15, \frac{625}{9}]$.

   Given that the minimum length of the female femur bone in that location is 15 cm.
   The minimum height of a female in that location is $H = 1.8 \times 15 + 70 = 97$ cm.
   Since the maximum height of a female in that location is 195 cm, the range of the height function is $[97, 195]$

(b) A new detective agency came up with a relationship $H = mf + 70$, where H is the height of a male adult (in cm) and f is the length (in cm) of the femur bone. They have used the following sample set given below in the Table P-3.2 , such that the sum squared error is minimum.

height(H) (in cm)	150	160	170	180
length of femur bone(f) (in cm)	40	42	48	56

Table PS-3.2

Choose the correct option (only one option is correct).

- ◯ $m = 1$
- ◯ $m = 1.5$
- ◯ $\boldsymbol{m = 2}$
- ◯ $m = 2.5$

Solution:

From Table PS-3.3, we can see that the minimum SSE is for $m = 2$.

H (in cm)	f (in cm)	$(H - mf - 70)^2$			
		$m = 1$	$m = 2$	$m = 1.5$	$m = 2.5$
150	40	1600	0	400	400
160	42	2304	36	729	225
170	48	2704	16	784	400
180	56	2916	4	676	900
SSE		$\sum = 9524$	$\sum = 56$	$\sum = 2589$	$\sum = 1925$

Table PS-3.3

3 Numerical Answer Type (NAT):

10. What will be the slopes of the straight lines perpendicular to the following lines?
    a) $2x + 5y - 9 = 0$

    Answer:2.5
    Solution:
    Using the slope intercept form, the slope of the line $2x + 5y - 9 = 0$ is $m_1 = -\frac{2}{5}$.
    Let the slope of the perpendicular line be m_2. Then

$$m_1.m_2 = -1$$

$$\implies m_2 = \frac{5}{2} = 2.5$$

    b) $-5x + 25y + 28 = 0$

    Answer:-5
    Solution:
    Using the slope intercept form, the slope of the line $-5x + 25y + 28 = 0$ is $m_1 = \frac{1}{5}$.
    Let the slope of the perpendicular line be m_2, then

$$m_1.m_2 = -1$$

$$\implies m_2 = -5.$$