Week - 2
Assignment Solutions
Mathematics for Data Science - 1, March '21

1 Multiple Choice Questions (MCQ):

1. You are running a car rental company. The company charges ₹500 per day and ₹8 per kilo-metre. A customer returns a car with the odometer readings (shows the distance travelled since the time of renting the car) of 300 kilo-metre and the bill amounts to ₹5900. How many days was this car rented? (Ans: c)

   ◯ 5
   ◯ 6
   ◯ 7
   ◯ 8

Q1. Soln: Company charges ₹500 per day and ₹8 per km.
if no. of days a car was rented be 'x' and total km travelled be 'y', then the total cost can be represented as
cost = ₹500x + ₹8y — (1) ⇒ eqn. of a straight line
Here, total cost = ₹5900
total distance covered = 300 km
∴ eqn (1) becomes 5900 = 500x + 8 × 300
⇒ 500x = 5900 − 2400 = 3500
∴ x = $\frac{3500}{500}$ = 7 days

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2. You are climbing a ladder which is slanted at an angle of 45 degrees (measured in the anticlockwise direction) with respect to the ground. The ladder, leaning against a wall, is at a vertical distance of 1 metre from the ground. If you are at a location which cuts the ladder in the ratio 2 : 1 from the top to bottom, what are the coordinates of your location? Assume origin $(0, 0)$ to be at the intersection of the ladder and the ground. (Ans: b)

   - ◯ $(1/2, 1/2)$
   - ◯ $(1/3, 1/3)$
   - ◯ $(2/3, 2/3)$
   - ◯ $(1/3, 2/3)$

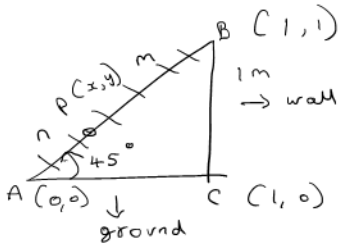
Solution:

Consider a ladder AB leaning on a wall BC.

AB makes 45° (in the anticlockwise direction) with the ground AC

$$\tan 45 = 1 = \frac{BC}{AC}$$

$$\Rightarrow AC = BC = 1\text{ m}$$



Coordinates of A are $(0, 0)$

Coordinates of C are $(1, 0)$

Coordinates of B are $(1, 1)$

Your location is at P with the coordinates (x, y)

Let (x_A, y_A), (x_B, y_B) represent the coordinates of the points A and B respectively. Then,

$$\frac{m}{n} = \frac{2}{1} = \frac{x - x_B}{x_A - x} = \frac{y - y_B}{y_A - y}$$

since $x_A = 0$, $x_B = 1$, $y_A = 0$, $y_B = 1$

$$\frac{x - 1}{0 - x} = 2 \Rightarrow x - 1 = -2x, \quad 3x = 1 \quad \text{or} \quad x = \frac{1}{3}$$

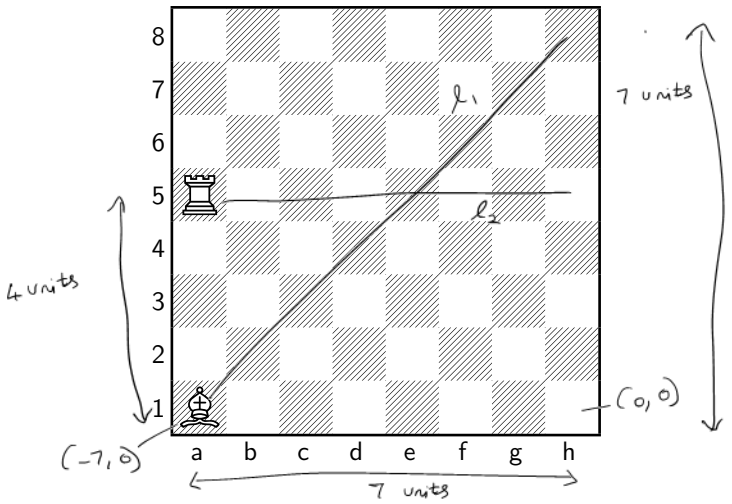
and, $\frac{y - 1}{0 - y} = 2 \Rightarrow y - 1 = -2y \quad \text{or} \quad y = \frac{1}{3}$

$\therefore$ Coordinates of the point P (your location on the ladder) are $\left(\frac{1}{3}, \frac{1}{3}\right)$.

Read the following passage for answering Q3, Q4, Q5

In a chessboard, the Bishop (shown in the cell '$a1$') can move diagonally along the same coloured squares and a Rook (shown in the cell '$a5$') can move only in the horizontal or in vertical directions. There is no restriction on the maximum number of cells the Bishop and the Rook can move in the chessboard. Both of them can never move out of the board.

Assume each cell in the board is a point on the coordinate plane such that the bottom right corner cell '$h1$' is the origin (0,0). Consider the right direction to be +ve X-axis and upward direction to be +ve Y-axis. Each cell consists of unit distance in both axes, i.e., co-ordinate of '$g1$' to be $(-1, 0)$, co-ordinate of '$h2$' to be $(0, 1)$ etc.



3. If B and R represent the initial coordinates of Bishop and Rook respectively, then which of the following is correct? (Ans: b)

B = a1 = (-7, 0)
R = a5 = (-7, 4)

- ◯ $B = (-8, 1), R = (-8, 5)$
- ◯ $B = (-7, 0), R = (-7, 4)$
- ◯ $B = (-5, 1), R = (-5, 5)$
- ◯ $B = (-6, 1), R = (-6, 4)$

4. The Bishop is initially located in the cell '$a1$' and moves to the cell '$h8$' following a straight line 'l_1'. l_1 can be represented as: (Ans: a)

- ◯ $y = x + 7$
- ◯ $y = 2x - 5$

- ◯ $y = 3x + 1$
- ◯ $y = x + 8$

5. The Rook moves from the cell '$a5$' to the cell '$h5$' along a straight line 'l_2'. l_2 can be represented as: (Ans: a)
   - ◯ $y = 4$
   - ◯ $y = 5$
   - ◯ $y = x + 4$
   - ◯ $y + x = -3$

Q 4. Solution: Bishop B moves from a1 to h8

h8 = (0, 7), a1 = (−7, 0)

slope of the line l_1 joining a1 and h8

$= \frac{0-7}{-7-0} = 1$

equation of l_1 : $y = mx + c$

$m = 1$,

substituting coordinates of h8 in the equation,

$7 = 0 + c \Rightarrow c = 7$

∴ equation of l_1 is $y = x + 7$. So, option (a) is correct

Q 5 Solution: R moves from a5 to h5

Coordinates of h5 are (0, 4) and as are (−7, 4)

slope of the line $l_2 = \frac{4-4}{-7-0} = 0$

equation of the line l_2 : $y = mx + c$

$m = 0$

$y = c$

substituting coordinates of h5 in the equation,

$y = 4 = c$

∴ $c = 4$

Thus, l_2 equation is $y = 4$. So, option (a) is correct

6. Three points $P(-4,4), Q(3,-3)$ and $R(g,h)$ are collinear. Identify the coordinates of the point R from the following: (Ans: c)

- ◯ $(-1,-1)$
- ◯ $(2,2)$
- ◯ $(1,-1)$
- ◯ $(1,1)$

Let $P = (x_P, y_P) = (-4, 4)$, $Q = (x_Q, y_Q) = (3, -3)$

and $R = (x_R, y_R) = (g, h)$

P, Q, R all lie on the same line.

slope of the line $= \frac{y_Q - y_P}{x_Q - x_P} = \frac{y_R - y_Q}{x_R - x_Q}$

i.e. $\frac{-3-4}{3-(-4)} = \frac{h-(-3)}{g-3}$

$\frac{-7}{7} = \frac{h+3}{g-3} = -1 \Rightarrow h + 3 = -g + 3$

$h = -g$

Also, slope $= \frac{y_Q - y_P}{x_Q - x_P} = \frac{y_R - y_P}{x_R - x_P}$

i.e. $-1 = \frac{h-4}{g-(-4)} \Rightarrow h - 4 = -g - 4$

$h = -g$

From the given options, only the point $(1,-1)$ satisfies the condition $h = -g$. Hence, option (c) is correct.

2 Multiple Select Questions (MSQ):

7. A carpenter has a call out fee (basic charges) of ₹200 and also charges ₹80 per hour. Which of the following are true? (Ans: a,c,d)

   ◯ If y is the total cost in (₹) and x is the total number of working hours, then the equation of the total cost is represented by $y = 80x + 200$.

   ◯ Following the same notations of y, x, equation of the total cost is represented by $y = 200x + 80$.

   ◯ The total charges, if the carpenter has worked for 4 hours, would be ₹520.

   ◯ If the carpenter charged ₹350 for fixing a L-stand and changing door locks, then the number of working hours would be approximately one hour and 53 minutes.

Basic charges = ₹200, x = no. of working hours, y = total cost

$y = ₹200 + ₹80x$

∴ option (a) is correct and (b) is wrong

option (c): no. of working hours = 4 = x

total cost $y = 200 + 80x = 200 + 80 \times 4$

$= 200 + 320 = ₹520$

So, option (c) is correct.

option (d): Total charges = ₹350

$= y = 200 + 80x$

$\Rightarrow 80x = 350 - 200 = 150$

$x = \frac{150}{80} = \frac{80}{80} + \frac{70}{80}$ hours

$= 1 \text{ hour} + \frac{70}{280} \times 60$ minutes

$= 1 \text{ hour} + 52.5$ minutes

Hence, option (d) is correct.

8. A swimming pool leaks at a slow rate of 0.5 gallons per hour. If the pool holds 325 gallons of water when it is full, then which of the following statements are false? (Ans: b,d)

- ◯ 301 gallons of water would remain in the pool after two full days.
- ◯ 324 gallons of water would remain in the pool after two full days.
- ◯ Quantity of water remaining in the pool (y in gallons) after x number of hours can be computed by the equation: $y = 325 - 0.5x$.
- ◯ The pool would be empty on the 25^{th} day since the beginning of the leak.

leak rate = 0.5 gallons per hour

Full capacity of the pool = 325 gallons

So, amount of water after 'x' no. of hours since the leak

$= y = 325 - 0.5x$

$\therefore$ option (c) is correct

After 2 full days, $x = 2 \times 24 = 48$ hours

$y = 325 - 0.5 \times 48 = 325 - 24 = 301$ gallons

$\therefore$ option (a) is correct and option (b) is false

pool would be empty when $y = 0 = 325 - 0.5x$

$\Rightarrow x = \frac{325}{0.5} = 325 \times 2$

$= 650$ hours

$= \frac{650}{24}$ days

$= \frac{648}{24} + \frac{2}{24}$

$= 27 + \frac{2}{24}$ days

i.e. on the 28th day pool would get empty

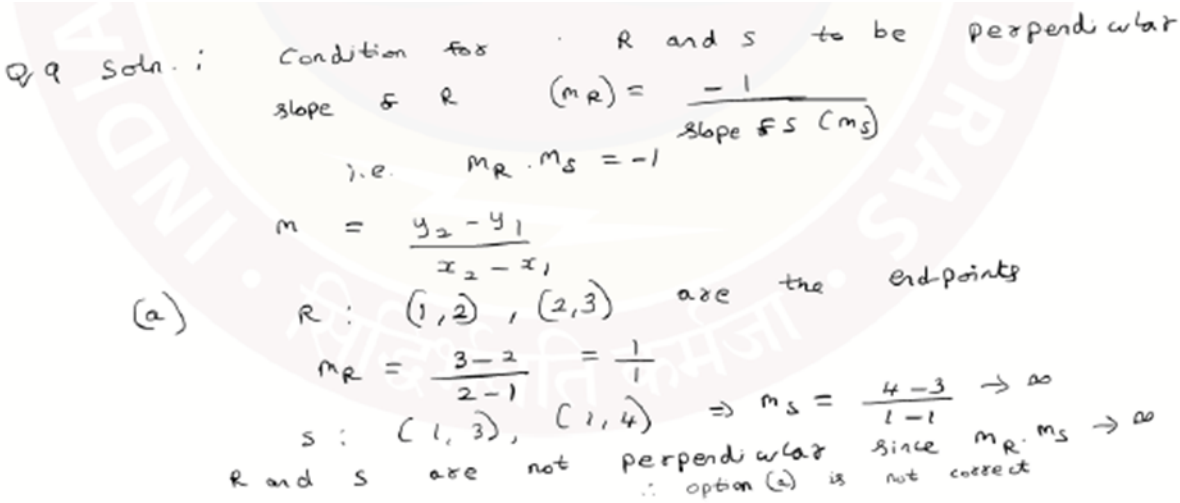
$\therefore$ option (d) is false

9. The skeleton of an aircraft wing comprises of the ribs and spars. Note that ribs and spars are arranged perpendicular to each other during the manufacturing of a wing. We have eight structural parts with the two end-points of each of them known. Identify which of the following pairs of structures (indicated with their end-points) can potentially be chosen for the ribs and spars combination. R and S represent structures designed for the ribs and spars respectively (Ans b,c)

   - ◯ R: $(1,2),(2,3)$, S: $(1,3),(1,4)$
   - ◯ R: $(0.5,1),(1.5,3)$, S: $(2.5,1),(0.5,2)$
   - ◯ R: $(2.5,6),(3.5,7)$, S: $(4.5,8),(3.5,9)$
   - ◯ R: $(3,2),(5,5)$, S: $(2,3),(4,4)$

3 Numerical Answer Type (NAT):

10. You are keen to know the area of the plot, which you want to buy, in the outskirts of Chennai city. Interestingly, the plan of the plot indicates that it is of a triangular boundary. The corners of the plot are marked by the points A $(1,3)$, B $(-5,7)$ and C $(8,12)$ with respect to some fixed coordinate system. Compute the area of the plot in square units. (Ans: 41)



(b) R : (0.5, 1) , (1.5, 3)

$m_R = \frac{3-1}{1.5-0.5} = \frac{2}{1} = 2$

S : (2.5, 1) , (0.5, 2)

$m_S = \frac{2-1}{0.5-2.5} = \frac{1}{-2} = -\frac{1}{2}$

$m_R \times m_S = 2 \times (-\frac{1}{2}) = -1$

∴ these 2 structures can be used as a rib-spar pair ∴ option (b) is correct

(c) ✓ R : (2.5, 6), (3.5, 7)

$m_R = \frac{7-6}{3.5-2.5} = \frac{1}{1} = 1$

S : (4.5, 8), (3.5, 9)

$m_S = \frac{9-8}{3.5-4.5} = \frac{1}{-1} = -1$

$m_R \times m_S = 1 \times (-1) = -1$

∴ these 2 structures can be used as a rib-spar pair ∴ option (c) is correct

(d) R : (3, 2), (5, 5)

$m_R = \frac{5-2}{5-3} = \frac{3}{2}$

S : (2, 3) , (4, 4)

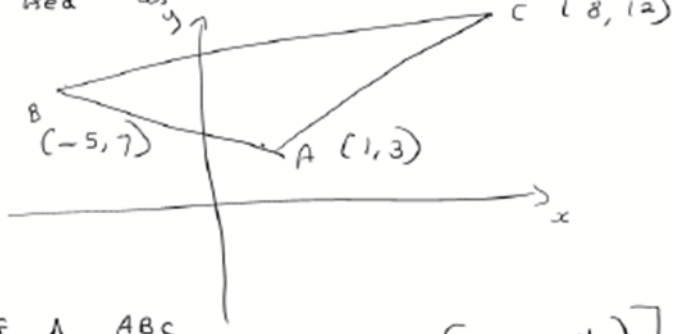
$m_S = \frac{4-3}{4-2} = \frac{1}{2}$

$m_R \times m_S = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4} \neq -1$

∴ these 2 structures are not perpendicular and cannot be used as a rib-spar combination

Q 10. Soln. :

Let (x_A, y_A) indicate coordinates of point A and similarly for points B and C respectively



Area of the plot = Area of Δ ABC

$= \frac{1}{2}\left|\left[x_A (y_B - y_C) + x_B (y_C - y_A) + x_C (y_A - y_B)\right]\right|$

Note that $x_A = 1$, $y_A = 3$, $x_B = -5$, $y_B = 7$, $x_C = 8$, $y_C = 12$

$$\Rightarrow \text{Area} = \frac{1}{2}\left|\left[1(7-12) + (-5)(12-3) + 8(3-7)\right]\right|$$

$$= \frac{1}{2}\left|\left[-5 + (-5)\times 9 + 8 \times (-4)\right]\right|$$

$$= \frac{1}{2}\left|-5 - 45 - 32\right| = \frac{1}{2}\left|-82\right|$$

$$= 41 \text{ Square units}$$

Ans : 41