Mathematics for Data Science-1 Term-2

Graded Assignment Practice Session Week - 5

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Polynomials

- Polynomial refers to "Poly = many" + "nomial = terms". Therefore, it says "many terms".
  For example $p(x) = ax^3 + bx^2 + cx + d$ *has four terms.*
- Polynomial is a special case of functions where exponents of any variable always be non-negative integers.
- In simple words "A polynomial can have constants, variables and exponents, but never **division by a variable**."
  For example $3x^{-2}$ *and* $\frac{2x}{5x+1}$ *are not polynomials.*

Sample question

Which one of the following is not a polynomial?

a. $3x^5 - 2x^2 + 3$

b. $3x^5 - 2x^3 + 3x$

c. $3x^5 - 2x^3 + \frac{x}{3}$

d. $3x^5 - 2x^3 + \frac{3}{x}$

Degree of Polynomials

- The degree of a polynomial with only one variable is the largest exponent of that variable.
  For example, the degree of $p(x) = ax^3 + bx^2 + cx + d$ *is 3.*

Degree of Polynomials

- Addition of two or more polynomials always provide a polynomial and the degree of the resulting polynomial is the degree of polynomial having maximum degree until unless the term having largest exponent of the variable gets canceled.
- Let $p(x) = 5x^3 + 3x + 2$, $q(x) = x^2 + 2x$, and $h(x) = p(x) + q(x)$, then the degree of $h(x)$ would be degree of $p(x)$ i.e., 3.
- But if $p(x) = 5x^3 + 3x + 2$, $q(x) = -5x^3 + 2x$, and $h(x) = p(x) + q(x)$, then the degree of $h(x)$ would not be degree of $p(x)$. Now the degree of $h(x)$ would be 1.

Degree of Polynomials

- Subtraction of two or more polynomials always provide a polynomial and the degree of the resulting polynomial is the degree of polynomial having maximum degree until unless the term having largest exponent of the variable gets canceled.
- Let $p(x) = 5x^3 + 3x + 2$, $q(x) = x^2 + 2x$, and $h(x) = p(x) - q(x)$, then the degree of $h(x)$ would be degree of $p(x)$ i.e., 3.
- But if $p(x) = 5x^3 + 3x + 2$, $q(x) = 5x^3 + 2x$, and $h(x) = p(x) - q(x)$, then the degree of $h(x)$ would not be degree of $p(x)$. Now the degree of $h(x)$ would be 1.

Degree of Polynomials

- Multiplication of two or more polynomials always provide a polynomial and the degree of the resulting polynomial is the sum of the degrees of the polynomials.
- Division of two polynomials might or might not provide a polynomial. If it is a polynomial, then the degree of the resulting polynomial is the degree of numerator minus the degree of denominator.

Question 1

Table provides the information regarding some polynomials. Let $h(x) = \frac{P(x)Q(x) - R(x)S(x) + S(x)P^2(x)}{P(x)Q(x)}$, where $h(x)$ is also a polynomial.

Polynomial	Degree	Condition
$P(x)$	m	$m > 0$
$Q(x)$	n	$m > 2n > 0$
$R(x)$	k	$k = m - n$
$S(x)$	t	$t = 2n$

Question 1

If $m = 4$ and $n = 1$, then choose the correct option.

a. $h(x)$ would always be positive.

b. $h(x)$ would always be a constant.

c. $h(x)$ would always be negative.

d. None of these.i

Solution

- Degree of $P(x)Q(x)$ would be $m+n$.
- Degree of $R(x)S(x)$ would be $k+t = m-n+2n = m+n$.
- Degree of $S(x)P^2(x)$ would be $t+m+m = 2n+2m$.
- Therefore the degree of numerator would be $max\{m+n, 2(m+n)\} = 2(m+n)$.

Solution

- Similarly the degree of denominator would be $m + n$.
- Then the degree of $h(x)$ would be
  $2(m + n) - (m + n) = m + n = 5$

Answer: As $h(x)$ is a polynomial of degree five, it can neither be always positive nor always negative.

Question 2

A manufacturing company produces three types of products A, B, and C from one raw material in a single continuous process. This process generates total solid wastes (W) (in kg) as $W(r) = -0.0002r^3 + 0.2r^2 + 2r$, where r is the amount of raw material used in kg. If instead, the company uses three different batch-processes (one batch process for one product) to produce the above products, then the amount of waste generated because of products A, B, and C are given as $W_A = -0.00001r^4 + 0.015r^3$, $W_B = -0.005r^3 + 0.05r^2$ and $W_C = 0.05r^2$ respectively.

Question 2

Let the company wastes Rs. 5,000 in waste treatment when it uses the single continuous process by consuming 100 kg of raw material. If instead of continuous process the company uses the three-batch-processes, then how much amount (in Rs.) will the company have to pay for waste treatment ?

a. 5000

b. 20000

c. 25000

d. 1500

Solution

- To find the amount waste in waste treatment through batch process, we need to first find the amount of solid waste generated by this process. It will be

$$W_b(r) = W_A(r) + W_B(r) + W_C(r)$$

$$W_b(r) = -0.00001r^4 + 0.015r^3 - 0.005r^3 + 0.05r^2 + 0.05r^2$$

$$W_b(r) = \mathbf{-0.00001r^4 + 0.01r^3 + 0.1r^2}$$

Solution

- Let solid waste generated by the batch process is n times of the solid waste generated by continuous process. Then

$$n = \frac{W_b(r)}{W_c(r)}$$

$$n = \frac{0.00001r^4 + 0.01r^3 + 0.1r^2}{-0.0002r^3 + 0.2r^2 + 2r}$$

$$n = 0.05r$$

Solution

- Money used for batch process = n x Money used in continuous process.
- Money used for batch process = $0.05r \times 5000 = 0.05 \times 100 \times 5000 = 25000$

Answer: 25000.

Question 3

Given a polynomial $P(x) = (2x + 5)(1 - 3x)(x^2 + 3x + 1)$, then find the coefficient of x^2.

a. 20

b. -20

c. 40

d. -40

Solution

Question 4

A curious student created a performance profile of his favourite cricketer as $R = -x^5 + 6x^4 - 30x^3 + 80x^2 + 70x + c$, where R is the total cumulative runs scored by the cricketer in x matches. He picked three starting values as shown in table.

No. of matches	Total cumulative score
1	121
2	284
3	362

Question 4

If minimum Sum Squared Error method is used for the curve fitting, then what will be the value of c?

a. 5/3

b. -5/3

c. 5/6

d. -5/6

Solution

x	y	y_{cal}	Error
1	121	$-1+6-30+80+70+c$	$125+c-121$
2	284	$-32+96-240+320+140+c$	$284+c-284$
3	362	$-243+486-810+720+210+c$	$363+c-362$

$$SSE = (c+4)^2 + c^2 + (c+1)^2$$

$$SSE = 3c^2 + 10c + 17$$

Answer: $c = -5/3$.

THANK YOU