Week - 3
Practice Assignment-2 Solution
Straight line - 2
Mathematics for Data Science - 1

1 Multiple Choice Questions (MCQ)

1. In a triangle made by the points of the intersection of the two axes and the line $bx+ay = ab$, d is the height of the perpendicular from the origin to the opposite side. Then $\frac{1}{a^2}+\frac{1}{b^2}$ is equal to

   - ○ $\sqrt{2}d^2$
   - ○ $\frac{\sqrt{2}}{d^2}$
   - ○ $\boldsymbol{\frac{1}{d^2}}$
   - ○ d^2

   Solution:
   The intercept form of the equation of the line $bx + ay = ab$, where $a > 0, b > 0$ (without loss of generality) is given by $\frac{x}{a} + \frac{y}{b} = 1$.
   Clearly x-intercept $= a$ and y-intercept $= b$.

   See Figure AS-3.1 for reference:

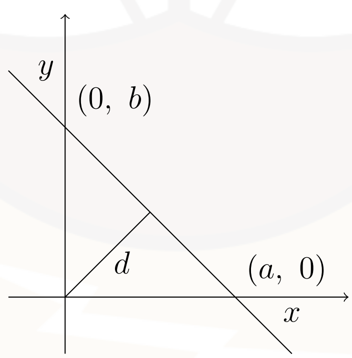

Figure AS-3.1

   Using perpendicular distance formula, we have
   $$d = \frac{|\ Ax_1 + By_1 + C\ |}{\sqrt{A^2 + B^2}}$$
   Here, $x_1 = 0,\quad y_1 = 0,\quad A = b,\ B = a,\quad$ and $C = \ -ab$
   Therefore,
   $$d = \frac{|0\ +\ 0\ - ab|}{\sqrt{a^2 + b^2}}$$

Squaring both sides:

$$d^2 = \frac{a^2b^2}{a^2 + b^2}$$

$$\implies \frac{1}{d^2} = \frac{a^2 + b^2}{a^2b^2}$$

$$\implies \boldsymbol{\frac{1}{d^2} = \frac{1}{a^2} + \frac{1}{b^2}}$$

2. An elephant standing at the junction of two straight roads represented by the equations $x-y+2=0$ and $y-1=0$ wants to reach another road whose equation is $x-y-3=0$. If the elephant can move in any direction and wants to cover the shortest distance to its destination road, then the equation of the path that the elephant should follow is:

   - ◯ $\boldsymbol{x+y=0}$
   - ◯ $-x+y=0$
   - ◯ $x-y+1=0$
   - ◯ $2x-y=0$

Solution:

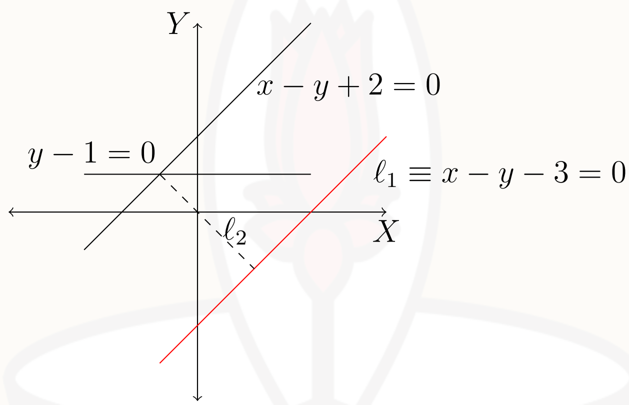

Figure AS-3.2

The junction of the two roads is the intersection point of the lines $x-y+2=0$ and $y=1$. That is, the solution of the equations $x-y+2=0$ and $y=1$ is the point $(-1,1)$.
Let ℓ_1 denote the road that the elephant wants to reach. Then $\ell_1 \equiv x-y-3=0$.
Let the shortest route from the junction to the road ℓ_1 be denoted by ℓ_2. Then, ℓ_2 will be a road that is perpendicular to ℓ_1.
The slope of ℓ_1 is $m_1=1$.
If the slope of ℓ_2 is m_2, then $m_1m_2=-1 \implies m_2=-1$.
Using the slope intercept form, the equation of ℓ_2 is $y=-x+c$. Since ℓ_2 passes through the point (-1,1), it will satisfy ℓ_2. Therefore,

$$1=-(-1)+c \implies c=0$$

Hence, the elephant should follow the path

$$y=-x+0 \implies \boldsymbol{\ell_2 \equiv x+y=0}$$

3. Slope of a line which cuts intercepts of equal lengths on the positive sides of the axes is

   - ◯ 0
   - ◯ **-1**
   - ◯ 1
   - ◯ $\sqrt{3}$

Solution:

Let the line cut an intercept of a $(a > 0)$ units on both axes.
So, the intercept form of the line will be

$$\frac{x}{a} + \frac{y}{a} = 1$$

After rearranging the equation, the slope intercept form of the equation is $y = -x + a$.
Clearly, the slope is -1.

4. Distance between the lines $15x + 9y + 14 = 0$ and $10x + 6y - 14 = 0$ is

- ◯ $\boldsymbol{\frac{35}{3\sqrt{34}}}$
- ◯ $\frac{35}{2\sqrt{34}}$
- ◯ $\frac{5}{3\sqrt{34}}$
- ◯ $\frac{35}{\sqrt{34}}$

Solution:

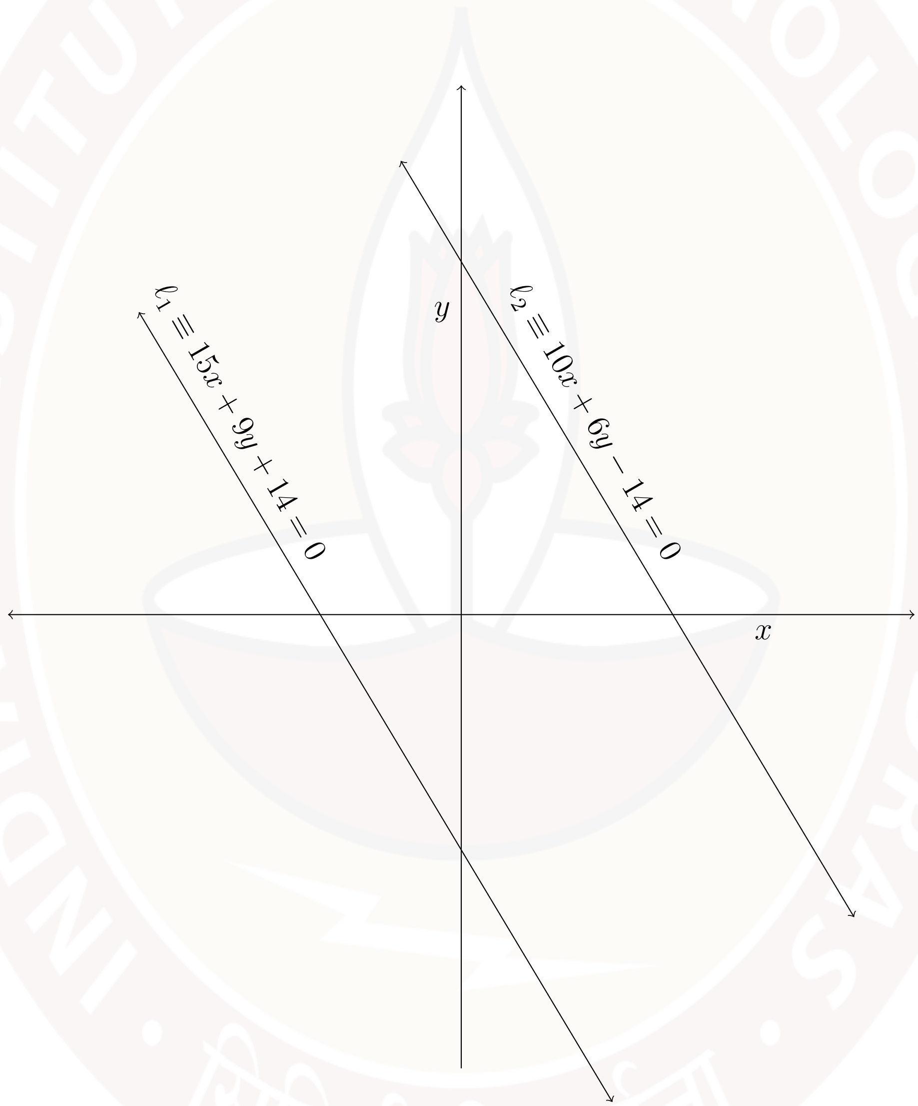

Figure AS-3.3

Let ℓ_1 be the line $15x + 9y + 14 = 0$. Let m_1 be the slope of ℓ_1. Then $m_1 = -\frac{5}{3}$. Let ℓ_2 be the line $10x + 6y - 14 = 0$. Let m_2 be the slope of ℓ_2. Then $m_2 = -\frac{5}{3}$.
This shows that the lines ℓ_1 and ℓ_2 are parallel.

ℓ_1 and ℓ_2 represent straight lines.
Divide both sides of the equations of ℓ_1 and ℓ_2 by 3 and 2 respectively.
Then, ℓ_1 becomes $5x + 3y + \frac{14}{3} = 0$ and ℓ_2 becomes $5x + 3y - 7 = 0$.
Using the distance formula between two parallel lines, the distance d between the lines is

$$\begin{aligned} d &= \frac{|\frac{14}{3} - (-7)|}{\sqrt{5^2 + 3^2}} \\ &= \frac{35}{3\sqrt{34}} units \end{aligned}$$

5. The equation of the straight road which is equidistant from two parallel straight roads represented by $6x + 4y - 5 = 0$ and $3x + 2y + 4 = 0$ is

   ◯ $12x - 8y + 3 = 0$

   ◯ $8x - 12y - 3 = 0$

   ◯ $\mathbf{12x + 8y + 3 = 0}$

   ◯ $8x + 12y + 3 = 0$

Solution:

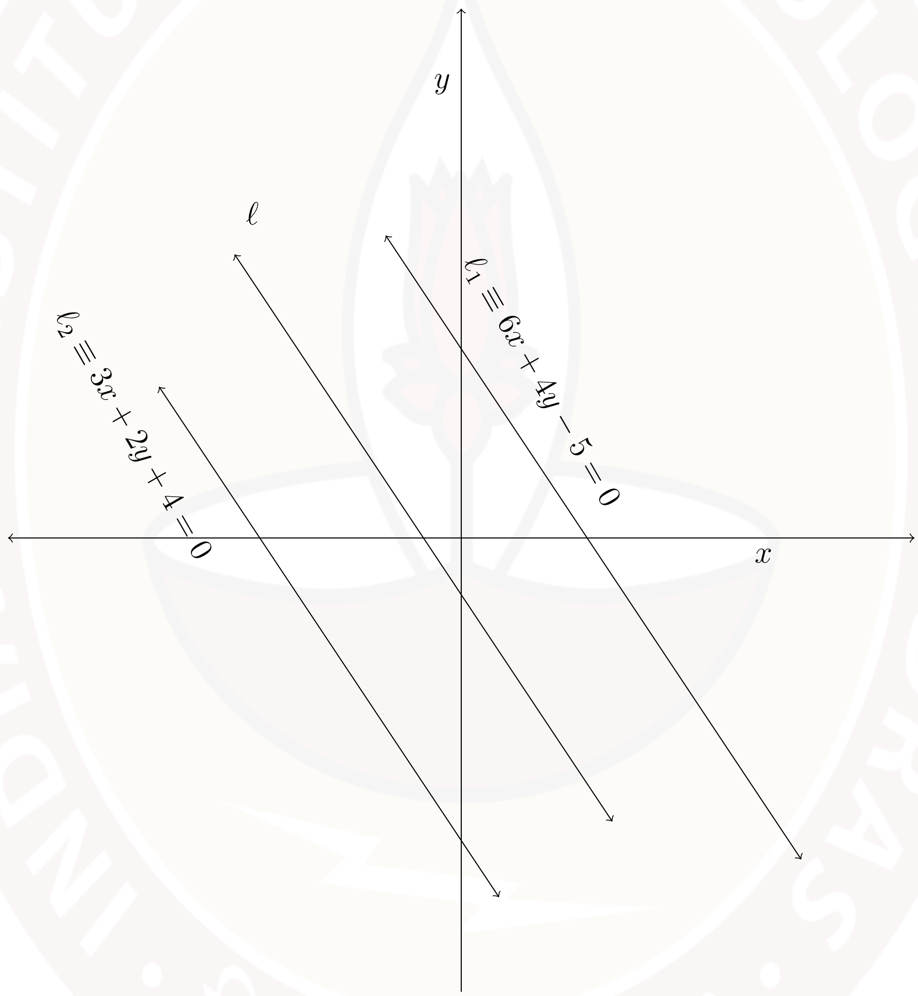

Figure AS-3.4

Let ℓ_1 and ℓ_2 represent the roads

$$\ell_1 \equiv 6x + 4y - 5 = 0, \text{ with slope } m_1 = -\tfrac{3}{2}$$
$$\text{and } \ell_2 \equiv 3x + 2y + 4 = 0, \text{ with slope } m_2 = -\tfrac{3}{2}.$$

Multiplying the both sides of equation of ℓ_2 by 2, we get $\quad \ell_2 \equiv 6x + 4y + 8 = 0$.

Let ℓ be the equation of the required road. Using the slope intercept form, the equation ℓ (with same slope) will be:

$$\ell \equiv 6x + 4y + c = 0$$

If ℓ is equidistant from ℓ_1 and ℓ_2, then

$$\frac{|\ c - (-5)\ |}{\sqrt{6^2 + 4^2}} = \frac{|\ c - 8\ |}{\sqrt{6^2 + 4^2}} \Longrightarrow |c + 5| = |c - 8|$$

Case 1:

$$c + 5 = c - 8 \Longrightarrow 5 = -8 \text{ which is not possible.}$$

Case 2:

$$c + 5 = -(c - 8) \Longrightarrow c = \tfrac{3}{2}.$$

Therefore,

$$\ell \equiv 6x + 4y + \frac{3}{2} = 0$$

or

$$\boldsymbol{\ell \equiv 12x + 8y + 3 = 0}$$

Hence the equation of the required road is $\ell \equiv \mathbf{12}\boldsymbol{x} + \mathbf{8}\boldsymbol{y} + \mathbf{3} = \mathbf{0}$.

6. A line perpendicular to the line segment joining the points $A\,(1,0)$ and $B\,(2,3)$, divides it at C in the ratio of $1:3$. Then the equation of the line is

   - ◯ $2x + 6y - 9 = 0$
   - ◯ $\mathbf{2x + 6y - 7 = 0}$
   - ◯ $2x - 6y - 9 = 0$
   - ◯ $2x - 6y + 7 = 0$

Solution:

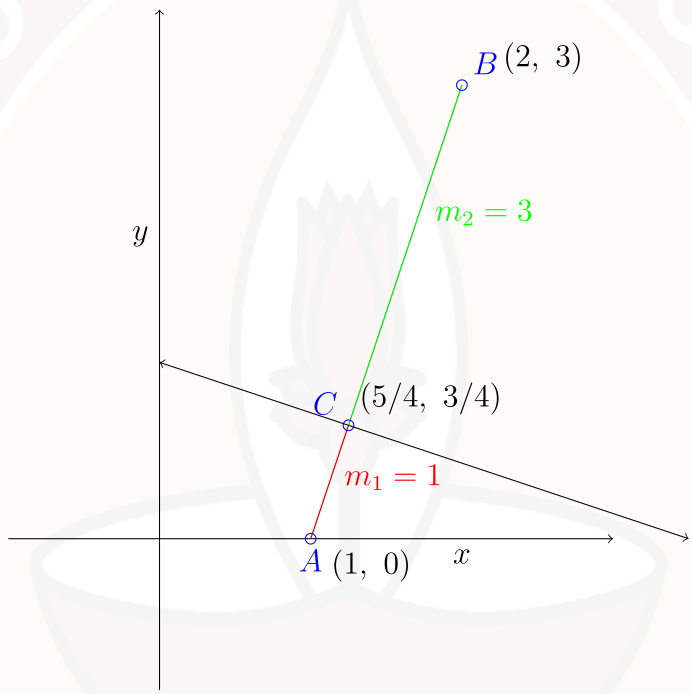

Figure AS-3.5

Since the perpendicular line divides the line segment joining the points A and B at C in the ratio of $1:3$, (see Figure AS-3.5) the coordinate of C is

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

where $x_1 = 1, y_1 = 0, x_2 = 2, y_2 = 3, m_1 = 1$ and $m_2 = 3$ (using the section formula).
Hence coordinate of C is $(\frac{5}{4}, \frac{3}{4})$.
Using the slope formula, the slope of the line joining the points A and B is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-0}{2-1} = 3$$

Therefore, the slope of the perpendicular line which is perpendicular to the line segment

joining the points A and B and passing through the point C (say ℓ) is $m' = -\frac{1}{m} = -\frac{1}{3}$.
From the slope intercept form, the equation of the line ℓ is $x + 3y = d$, where d is a constant.
Since the line ℓ passes through point C, C satisfies the equation of ℓ. We get $d = \frac{7}{2}$.
Hence the equation of the required line ℓ is $\boldsymbol{2x + 6y - 7 = 0}$.

7. An experiment is performed in an ideal condition to find out the velocity of a particle obeying the equation $S = vt$ where the distance travelled (S) inside a labeled glass tube by a particle with an uniform velocity (v) is measured with respect to time taken(t). The data collected by repeating the experiment five times is shown in the table below. If we use the method to minimize the squared sum error, what is the most likely velocity

S (in cm)	3	9	13	14	17
t (in sec)	2	4	6	8	10

Table AS-3.1

of the particle among the given options?

- ◯ 2.5
- ◯ 1.5
- ◯ **2**
- ◯ 1

Solution:

Consider Table AS-3.2.

S (in cm)	t (in cm)	$(S-vt)^2$			
		$v = 2.5$	$v = 1.5$	$v = 2$	$v = 1$
3	2	4	0	1	1
9	4	1	9	1	25
13	6	4	16	1	49
14	8	36	4	4	36
17	10	64	4	9	49
SSE		$\sum = 109$	$\sum = 33$	$\sum = 16$	$\sum = 160$

Table AS-3.2

From the table, it is clear that $v = 2$ gives the minimum Sum Squared Error. Hence **most likely velocity of the particle is 2.**

Second Method:

Observe Figure AS-3.6, which shows the lines represented by different slopes (v values) and the points mentioned in Table AS-3.1.

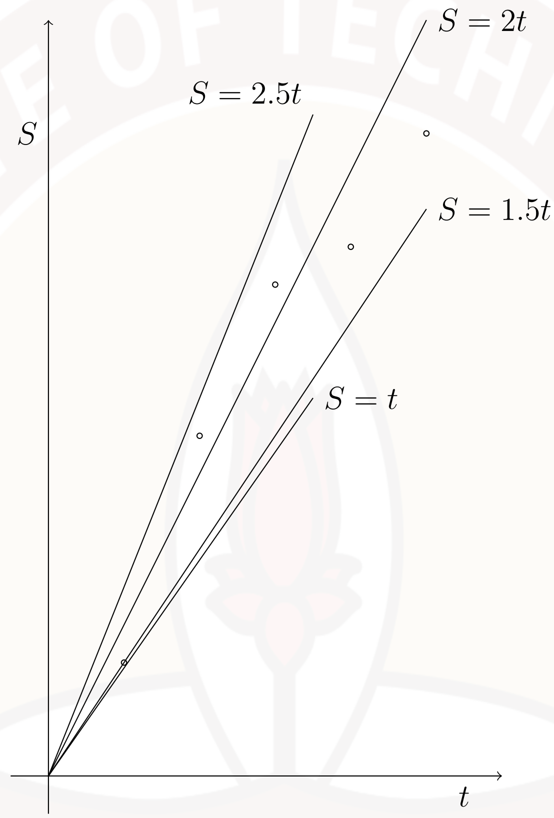

Figure AS-3.6

From Figure AS-3.6 also, it is clear that the most likely velocity of the particle is 2.

2 Multiple Select Questions (MSQ)

1. The equations of lines with slope 3 and the length of the perpendicular from the origin equal to $\sqrt{10}$ are

   - ◯ $x = y + 10$
   - ◯ $y = 2x - 5$
   - ◯ $\boldsymbol{y = 3x + 10}$
   - ◯ $x = 3y - 10$
   - ◯ $\boldsymbol{y = 3x - 10}$
   - ◯ $x = 3y + 10$

   Solution:

   Given, the slope of the line is 3 and the distance from the origin is $\sqrt{10}$.
   From the slope intercept form, the equation of the line is $y = 3x + c$, where c is a constant.
   The general form of the line is $-3x + y - c = 0$.

   Using perpendicular distance formula, the distance of the line from the origin is

   $$d = \frac{|\ 0\ +\ 0\ -\ c\ \ |}{\sqrt{(-3)^2 + 1}} = \frac{|-c|}{\sqrt{10}}$$

   Therefore, we get

   $$\frac{|c|}{\sqrt{10}} = \sqrt{10}$$

   After solving the above equation, we get $|c| = 10 \Rightarrow \quad c = +10\ \ or\ c = -10$.
   Hence, the equation of the line is $\boldsymbol{-3x + y + 10 = 0\ or\ -3x + y - 10 = 0}$.

2. Ankit is located at $(3,3)$. He called Ajay to ask his location. Ajay describes the path he had taken from home (located at the origin) as: " I walked three units towards East and then nine units towards North. And I repeated the same pattern thrice." Now Ankit wants a direct path to reach Ajay, then choose the correct options.

   - ◯ He should follow $3x - y = 0$.
   - ◯ **He should follow $4x - y - 9 = 0$.**
   - ◯ **Ankit will have to walk a distance of $6\sqrt{17}$ units.**
   - ◯ Ankit will have to walk a distance of $3\sqrt{17}$ units.
   - ◯ Ajay has walked a distance of $9\sqrt{10}$ units from his home.
   - ◯ **Ajay has walked a distance of 36 units from his home.**

Solution:

Figure AS-3.7 shows the location of Ankit and the path travelled by Ajay.

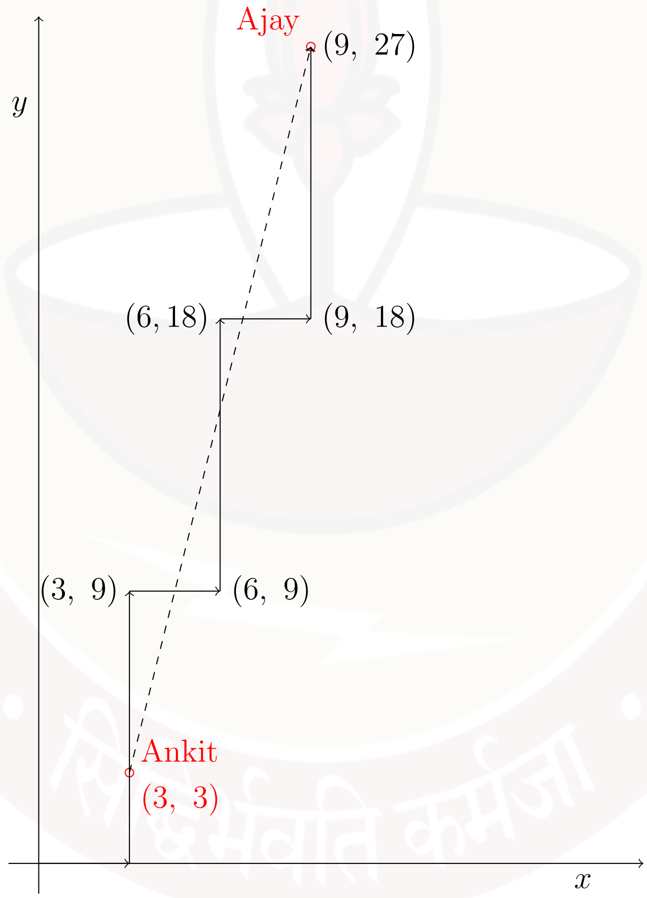

Figure AS-3.7

From Figure AS-3.7, the location of Ajay is (9,27).
Ankit will have to follow the path which connects their locations directly.
Using two point form of a line, the equation of the line passing through the points (3,3) and (9,27) is

$$4x - y - 9 = 0.$$

The distance which Ankit will have to walk will be the distance between their locations (3,3) and (9,27).
Using distance formula, the distance between the points (3,3) and (9,27) is

$$d = \sqrt{(9-3)^2 \ + \ (27-3)^2} = 6\sqrt{17} \ units$$

Also, the travelled distance by Ajay $= 3+9+3+9+3+9 = 36$ units.

3. To determine the gas constant R, two students A and B perform an experiment based on the ideal gas equation given as $Pv = RT$. Both use the same gaseous sample having $v = 16.6\ m^3/mol$ and reported the approximate value of R as $8.3\ J/(K\triangle mol)$ using the minimisation of sum squared error. The data collected by both the students are reported below. Choose the correct option:

$P(T)(Pa)$	137	139	142	142	141
$T(K)$	274	276	278	280	282

Table AS-3.3: Data collected by student A.

$P(T)(Pa)$	137	141	142	148	145
$T(K)$	276	280	284	288	290

Table AS-3.4: Data collected by student B.

- ◯ **$\boldsymbol{A}$ has better fit than $\boldsymbol{B}$.**
- ◯ B has better fit than A.
- ◯ A and B both have same fit.
- ◯ SSE calculated by B is 14.
- ◯ **SSE calculated by $\boldsymbol{A}$ is 14.**
- ◯ SSE calculated by both A and B is 14.

Solution:

Given, the ideal gas equation

$$Pv = RT$$
$$\implies P = \frac{R}{v}T$$
$$\implies P = \frac{8.3}{16.6}T$$
$$\implies P = \frac{1}{2}T$$

Now, from Table AS-3.5 and Table AS-3.6, we can say that $\boldsymbol{A}$ is a better fit than $\boldsymbol{B}$ and SSE calculated by $\boldsymbol{A}$ is 14.

$T(K)$	$P(T)(Pa)$	$(P - \frac{1}{2}T)^2$
274	137	0
276	139	1
278	142	9
280	142	4
282	141	0
		$\sum = 14$

Table AS-3.5: SSE calculated by $\boldsymbol{A}$

$T(K)$	$P(T)(Pa)$	$(P - \frac{1}{2}T)^2$
276	137	1
280	141	1
284	142	0
288	148	16
290	145	0
		$\sum = 18$

Table AS-3.6: SSE calculated by $\boldsymbol{B}$

3 Numerical Answer Type (NAT)

1. Find the values of r for which the line $(r+5)x - (r-1)y + r^2 + 4r - 5 = 0$ is
   (a) parallel to the x-axis [Ans: -5]
   (b) parallel to the line $2x - y + 4 = 0$, [Ans: 7]
   (c) parallel to the y-axis. [Ans: 1]
   (d) perpendicular to the line $x - y + 32 = 0$ [Ans: -2]

Solution:

Using the slope intercept form of the line $\ell \equiv (r+5)x - (r-1)y + r^2 + 4r - 5 = 0$, the slope of the line is

$$m_1 = \frac{r+5}{r-1}$$

a.
Since line ℓ is parallel to the X-axis and the slope of the X-axis is $m_2 = 0$,

$$m_1 = m_2 \implies \frac{r+5}{r-1} = 0 \implies r = -5.$$

Hence $r = -5$.

b.
Since line ℓ is parallel to the line $2x - y + 4 = 0$ and the slope of the line $2x - y + 4 = 0$

is $m_2 = 2$, we have

$$m_1 = m_2 \implies \frac{r+5}{r-1} = 2 \implies r = 7.$$

Hence $r = 7$.

c.

Since line ℓ is parallel to the Y-axis ($x = 0$), using parallel line condition, we have

$$a_1 b_2 = a_2 b_1$$

where $a_1 = 1, b_1 = 0, a_2 = r + 5, b_2 = -(r - 1) = 1 - r$,

$$\implies 1 - r = 0$$

Hence $r = 1$.

d.

Since line ℓ is perpendicular to the line $x - y + 32 = 0$ which has slope $m_2 = 1$, we have

$$m_1 \times m_2 = -1 \implies \frac{r+5}{r-1} = -1 \implies r = -2.$$

Hence $r = -2$.

2. A, B and C have coordinates $(2, 9), (10, -7)$ and $(6, p)$ respectively. Line AB is perpendicular to line BC. Then the value of p is
[Ans: -9]
Solution:
From the two point form of a line, the equation of the line passing through the points A and B is

$$2x + y = 13$$

The slope of line $2x + y = 13$ is $m_1 = -2$.
Again from the two point form of a line, the equation of the line passing through points the B and C is

$$y = -\frac{(p+7)}{4}x + \frac{10(p+7)}{4} - 7$$

The slope of this line is,

$$m_2 = \frac{-(p+7)}{4}$$

Since lines AB and BC are perpendicular to each other, we have

$$m_1 \times m_2 = -1$$

$$\implies \frac{2(p+7)}{4} = -1 \implies p = -9$$

Hence $p = -9$.