Week - 6

Tutorial

Graphs of polynomials

Mathematics for Data Science - 1

1. Figure T-7.3 shows the graph of polynomial $p(x)$.

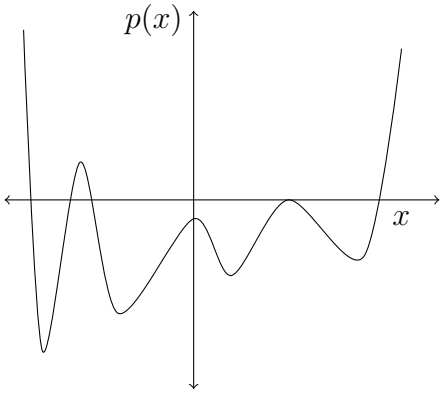

Figure T-7.1

Based on the graph, comment on the following statements.

(a) Number of turning points.

(b) Number of roots.

(c) Minimum possible degree of the polynomial based on the roots.

(d) Minimum possible degree of the polynomial based on turning points.

(e) Minimum degree of the polynomial.

(f) The end behavior and the coefficient of highest degree term.

2. Suppose a newly laid road follows the path $P(x) = (x^4 - 5x^3 + 6x^2 + 4x - 8)(x^2 - 15x + 50)$ from $x = -5$ to $x = 20$ and a railway track is laid along the $X-$axis.

    1. How many level crossings are there (level crossing is an intersection where a railway track crosses a road)?
    2. How many turning points are there on the road?

3. Saraswathi bought an 8 gram gold chain for $Rs.40,000$ on 1st November 2020. After 10 months, she sold the chain and bought a new 10 grams gold chain by paying extra $Rs.10,000$. Suppose the rate of the gold (in thousands) per gram, denoted by $G(t)$, is a polynomial function of the time t. That is, $G(t) = 0.07t^3 - 1.4t^2 + 7t + 5$.(consider $t = 0$ to be the time when Saraswathi bought her first gold chain and t denotes the number of months after she bought her first gold chain). Let G(t) be the polynomial of the rate for both used and new gold.

    1. What is the rate of the gold per gram when she sold her first gold chain?
    2. If she has sold the first gold chain after 6 months. How much extra would she have paid for buying the 10 grams gold chain?

4. A skydiver jumps out of a plane travelling at 3000 m above sea level. When she was about 500m above the sea level she opens her parachute. She dives into the sea and reaches 30 m deep in the sea. She then swims and reaches the sea coast from there she takes a helicopter and reaches her home as shown in the figure.
   Note: The given figure is a rough diagram and answers should be based on the figure.

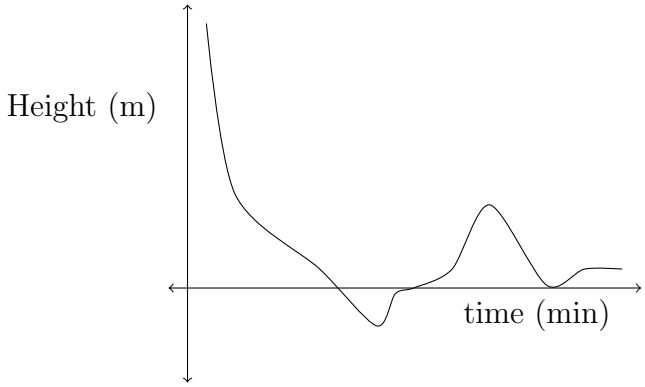

Figure T-7.2

- ◯ Range of the curve so formed is [-30m, 3000m].
- ◯ The domain of the curve will be the time taken for the entire journey.
- ◯ Number of turning points are 5
- ◯ The degree of the polynomial formed by the curve will be at least 6.

5. Electrocardiogram refers to the recording of electrical changes that occur in the heart during a cardiac cycle. It may be abbreviated as ECG or EKG. The electric signal produced by the heart muscle are shown in the figure below.

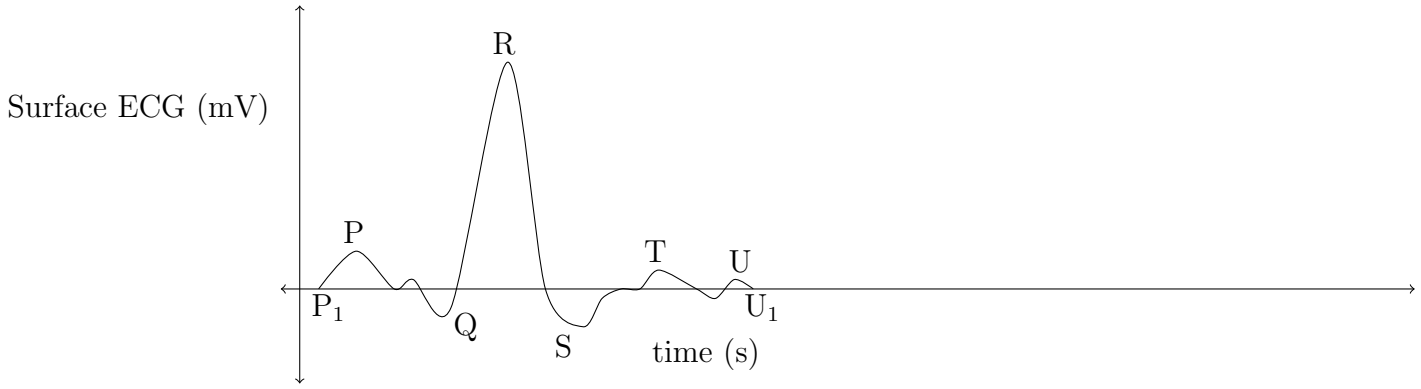

Figure T-7.3

(a) Identify the number of turning points and also the minimum sum of multiplicities of the polynomial so formed by ECG?

(b) Person usually dies when there is no electrical activity i.e, flat lined surface ECG as shown in figure after U_1, what polynomial will it be called for the domain after U_1?

6. A company's profit varies according to the months. The profit (in thousands) for year 2018 is represented by polynomial as $p(x) = 5 + 150x - 46.7x^2 + 5.44x^3 - 0.211x^4$, where x represents the month number starting from January as $x = 1$. The company declares the month as a golden month if the profit is more than or equal to 150 thousand. Find out how many months the company enjoyed the golden month in the year 2018.
Hint:
$-145 + 150x - 46.7x^2 + 5.44x^3 - 0.211x^4 = -a(x - 1.7)(x - 4.117)(x - 8.776)(x - 11.189)$, $a > 0$

Use the following information for questions 7 and 8.

Given that $p(x) = (x^2 + kx + 4)(x - 5)(x - 3)$, and K is the set of values of k.

7. Choose the correct option if $p(x)$ always have four real roots.

   A. $K = \{z | z \in (-\infty, -4] \cup [4, \infty)\}$

   B. $K = \{z | z \in (-\infty, -4) \cap (4, \infty)\}$

   C. $K = \{z | z \in (-\infty, -5.8) \cup (-5.8, -\frac{52}{12}) \cup (-\frac{52}{12}, -4) \cup (4, \infty)\}$

   D. None of the above.

8. Choose the correct option if $p(x)$ always have four distinct real roots.

   A. $K = \{z | z \in (-\infty, -4) \cup (4, \infty)\}$

   B. $K = \{z | z \in (-\infty, -4) \cap (4, \infty)\}$

   C. $K = \{z | z \in (-\infty, -5.8) \cup (-5.8, -\frac{52}{12}) \cup (-\frac{52}{12}, -4) \cup (4, \infty)\}$

   D. None of the above.

9. Let the demand of a particular product for a company be $d(x)$ and the production of the product be $p(x)$ for 12 months, where x is the number of months after January (for January, $x = 0$). Given that $d(x) - p(x) = a(x^2 + 1)(x - 2)(x - 5.8)(x - 11.6)$, $a > 0$, then find out in which months should company reduce the production after march.