Mathematics for Data Science - 1
Practice Assignment Solution
Week 11

1 MULTIPLE CHOICE QUESTIONS:

1. Consider the following weighted graph in Figure PA-12.1.

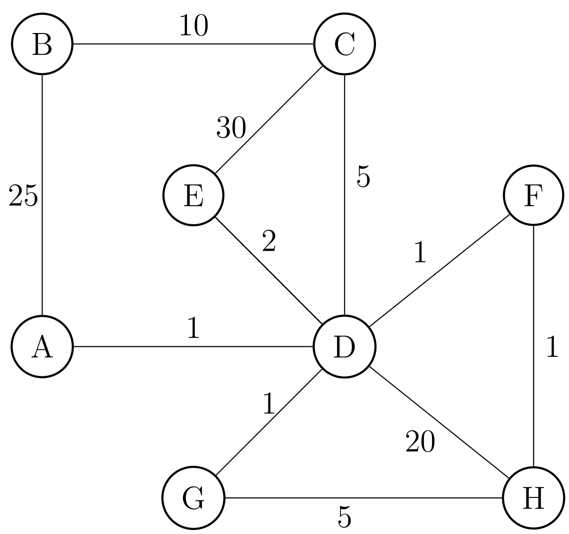


Figure PA-12.1

The shortest weighted path from H to B is [Ans: d]

(a) $H \rightarrow G \rightarrow D \rightarrow A \rightarrow B$

(b) $H \rightarrow D \rightarrow C \rightarrow B$

(c) $H \rightarrow F \rightarrow D \rightarrow E \rightarrow C \rightarrow B$

(d) $H \rightarrow F \rightarrow D \rightarrow C \rightarrow B$

Answer

As the edge weights are positive we can use Dijkstra's algorithm

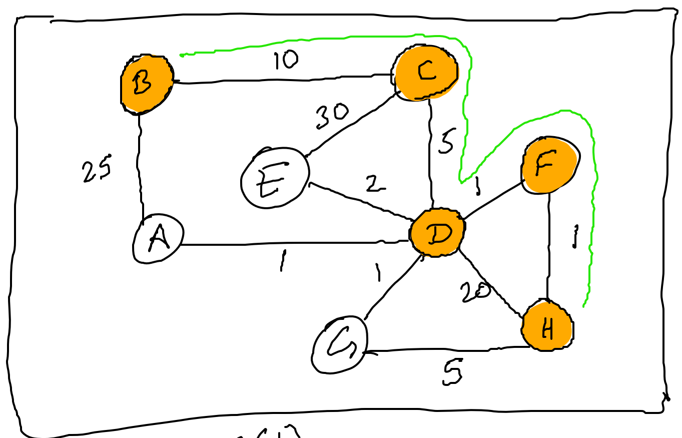


Figure (1)

Clearly from Figure (1) the shortest path to reach vertex B from vertex H is $\boxed{H \rightarrow F \rightarrow D \rightarrow C \rightarrow B}$ as in this path the total weight is minimum which is 17.

2. Suppose Dijkstra's algorithm is run on the graph below (Figure PA-12.2), starting at node A. In what order do the shortest distances to the other vertices get finalized?

[**Ans: a**]

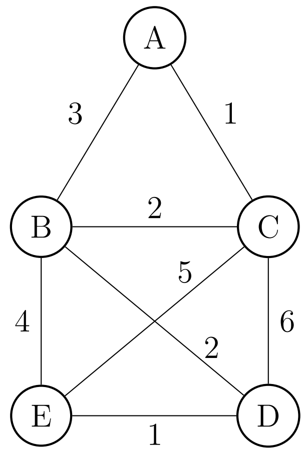


Figure PA-12.2

(a) A, C, B, D, E
(b) A, C, B, E, D
(c) A, C, D, B, E
(d) A, C, D, E, B

Answer

Using Dijkstra's algorithm the shortest distances to reach other vertices are shown in Figure(2)

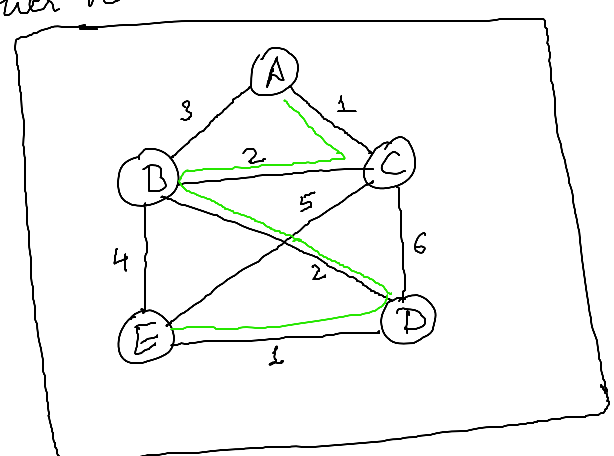


Figure (2)

Notice that from source vertex A to vertex C the distance is shortest as compared with the distance from vertex A to vertex B.

The same logic is applied for every other vertices and we get the shortest distance from vertex A to other vertices as follows $\boxed{A \rightarrow C \rightarrow B \rightarrow D \rightarrow E}$

3. If we perform Floyd-Warshall algorithm for the graph shown below (Figure PA-12.3),

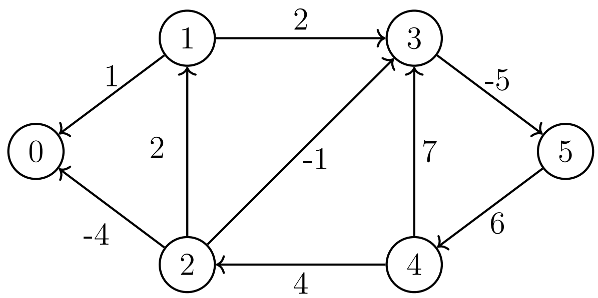


Figure PA-12.3

then which of the following matrices represents SP^4? [Ans: c]

(a)

SP^4	0	1	2	3	4	5
0	∞	∞	∞	∞	∞	∞
1	1	∞	∞	2	∞	∞
2	-4	2	∞	-1	∞	∞
3	∞	∞	$-\infty$	∞	∞	-5
4	∞	∞	4	7	∞	∞
5	∞	∞	∞	∞	6	∞

(b)

SP^4	0	1	2	3	4	5
0	∞	∞	∞	∞	∞	∞
1	1	∞	∞	2	∞	∞
2	-4	2	∞	-1	∞	∞
3	∞	∞	∞	∞	∞	-5
4	0	6	4	3	∞	∞
5	∞	∞	∞	∞	6	∞

(c)

SP^4	0	1	2	3	4	5
0	∞	∞	∞	∞	∞	∞
1	1	∞	∞	2	∞	-3
2	-4	2	∞	-1	∞	-6
3	∞	∞	∞	∞	∞	-5
4	0	6	4	3	∞	-2
5	∞	∞	∞	∞	6	∞

(d)

SP^4	0	1	2	3	4	5
0	∞	∞	∞	∞	∞	∞
1	1	∞	∞	2	∞	-3
2	∞	2	∞	-1	∞	-6
3	∞	∞	∞	∞	∞	-5
4	0	6	4	3	∞	∞
5	∞	∞	∞	∞	6	∞

Answer:

- To find: SP^4 adjacency matrix
- Approach: Find SP^0, SP^1, SP^2, SP^3 and then find SP^4
- Floyd-Warshall Algorithm

Let $SP^k[i,j]$ be the length of the shortest path from i to j via vertices $\{0, 1, \dots k-1\}$.

- Note:- $SP^0[i,j] = W[i,j]$; where $W[i,j]$ is weight of an edge from i to j.

For SP^1 find shortest path via vertex $\{0\}$

For SP^2 " " vertices $\{0,1\}$

For SP^3 " " " $\{0,1,2\}$

SP^0	0	1	2	3	4	5
0	∞	∞	∞	∞	∞	∞
1	1	∞	∞	2	∞	∞
2	-4	2	∞	-1	∞	∞
3	∞	∞	∞	∞	∞	-5
4	∞	∞	4	7	∞	∞
5	∞	∞	∞	∞	6	∞

SP^1	0	1	2	3	4	5
0	∞	∞	∞	∞	∞	∞
1	1	∞	∞	2	∞	∞
2	-4	2	∞	-1	∞	∞
3	∞	∞	∞	∞	∞	-5
4	∞	∞	4	7	∞	∞
5	∞	∞	∞	∞	6	∞

SP^2	0	1	2	3	4	5
0	∞	∞	∞	∞	∞	∞
1	1	∞	∞	2	∞	∞
2	-4	2	∞	-1	∞	∞
3	∞	∞	∞	∞	∞	-5
4	∞	∞	4	7	∞	∞
5	∞	∞	∞	∞	6	∞

SP^3	0	1	2	3	4	5
0	∞	∞	∞	∞	∞	∞
1	1	∞	∞	2	∞	∞
2	-4	2	∞	-1	∞	∞
3	∞	∞	∞	∞	∞	-5
4	0	6	4	3	∞	∞
5	∞	∞	∞	∞	6	∞

SP^4	0	1	2	3	4	5
0	∞	∞	∞	∞	∞	∞
1	1	∞	∞	2	∞	-3
2	-4	2	∞	-1	∞	-6
3	∞	∞	∞	∞	∞	-5
4	0	6	4	3	∞	-2
5	∞	∞	∞	∞	6	∞

2 MULTIPLE SELECT QUESTIONS:

Using the graph below answer the following questions [Question 4 and 5]

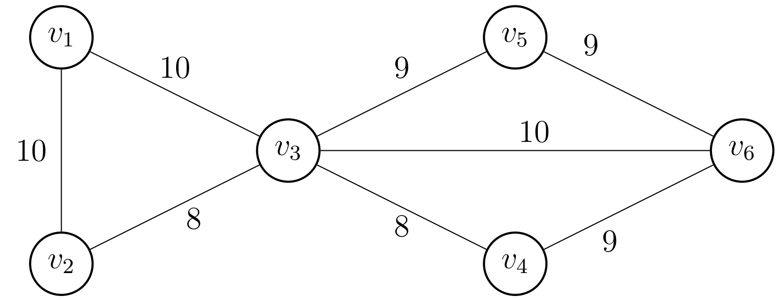
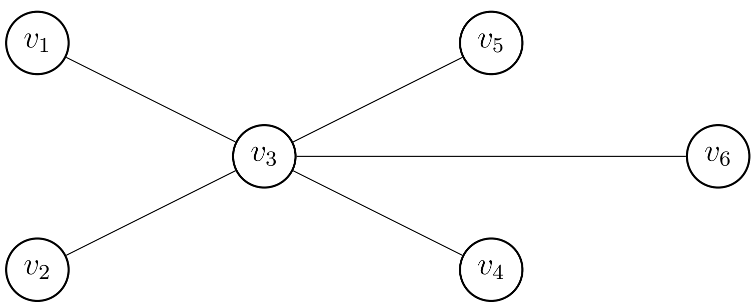


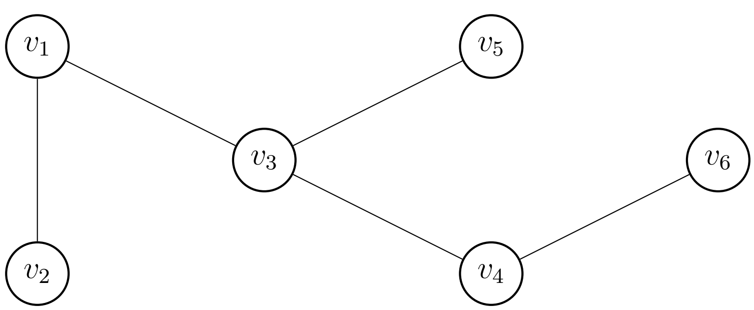
Figure PA-12.4

4. Which of the following could be the minimum cost spanning tree computed by running Prim's algorithm on the graph in Figure PA-12.4? [Ans: c,d]

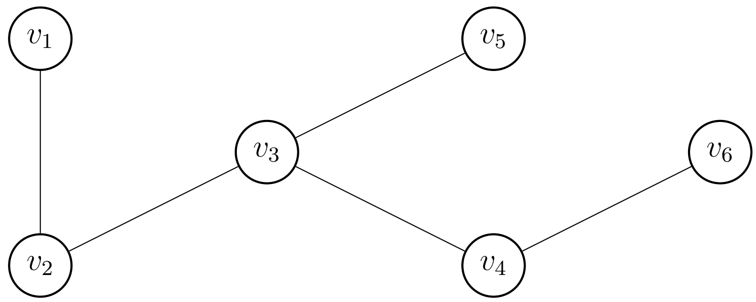
(a)



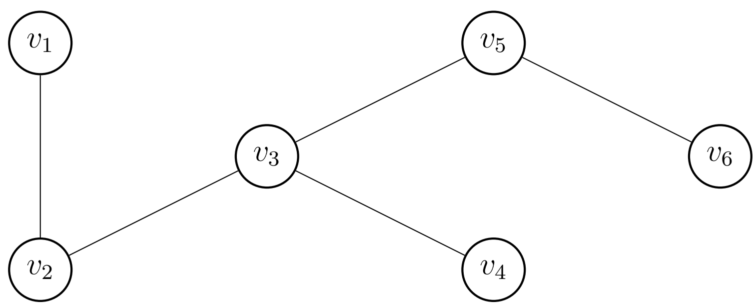
(b)



(c)



(d)



Answer:

Prim's algorithm:

Step1: Select the edge with minimum cost (edge weight)

For Eg: $V_2 \xrightarrow{8} V_3$

Step2: Now check for all the edges adjacent to (V_1, V_2). Select the one which has the lowest weight and include it in the tree. For eg: $V_2 \xrightarrow{8} V_3 \xrightarrow{8} V_4$ (Make sure no cycles are formed)

Step3: Repeat step 2 by adding one more edge {adjacent to (V_2, V_3) or (V_3, V_4)} which has the lowest weight edge

Eg:

or $V_2 \xrightarrow{8} V_3 \xrightarrow{8} V_4 \xrightarrow{9} V_6$

Step4:- Repeat the above step to cover all the edges to obtain MCST.

Note:- In the given graph (Figure PA-12.4) there are many edges which have same weights and thus we can have multiple MCST.

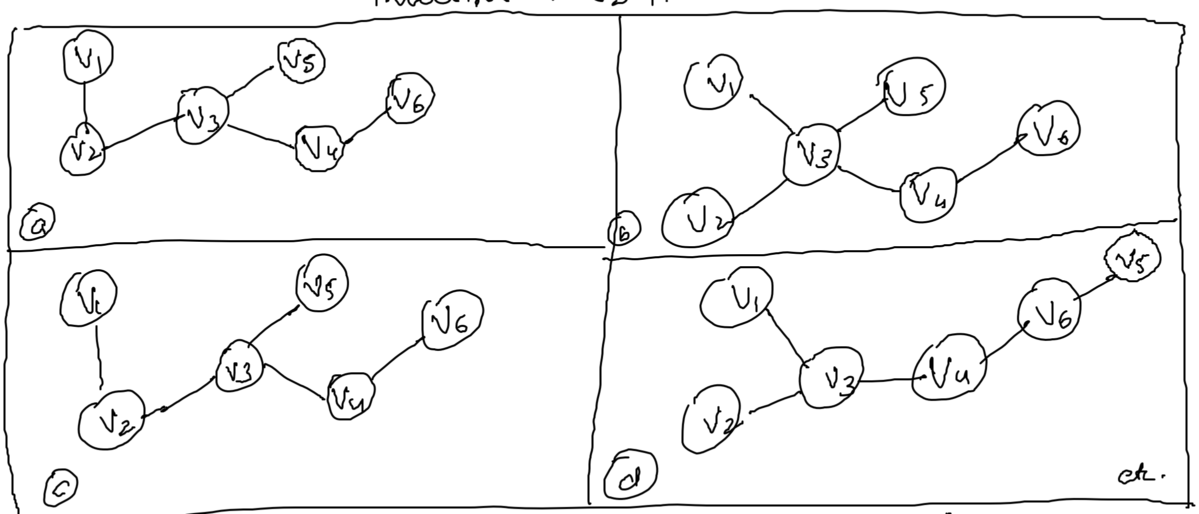
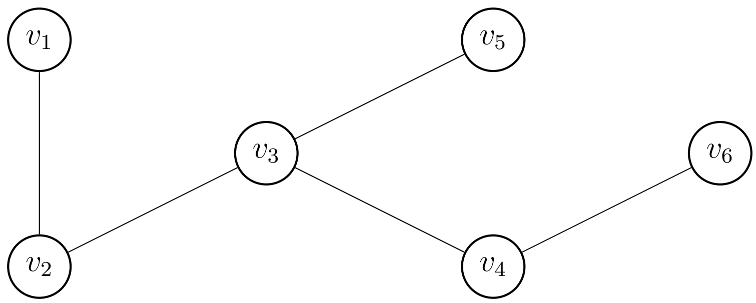


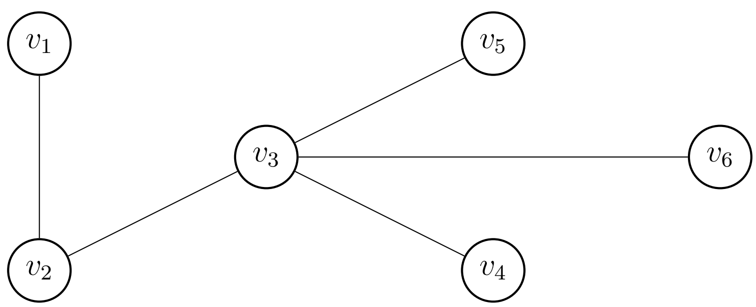
Figure (3): MCST using Prim's algorithm.

5. Which of the following could be the minimum cost spanning tree computed by running Kruskal's algorithm on the graph in Figure PA-12.4? [Ans: a,d]

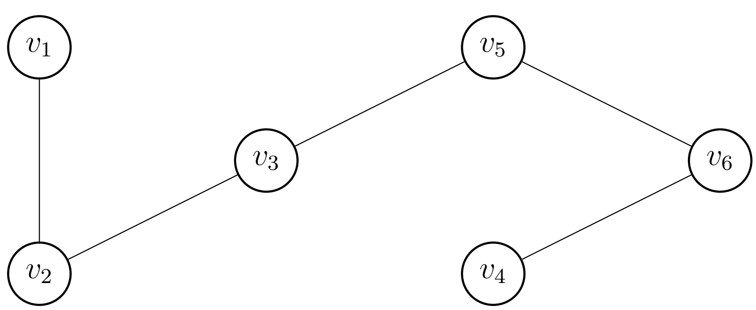
(a)



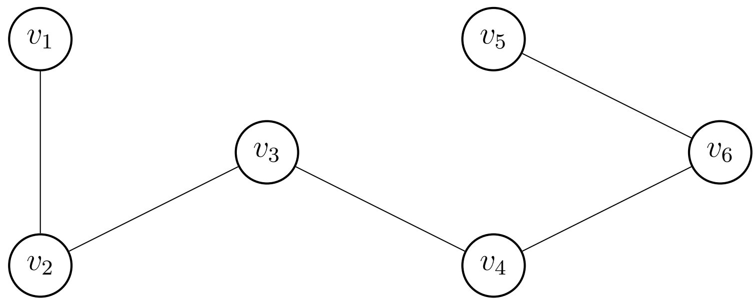
(b)



(c)



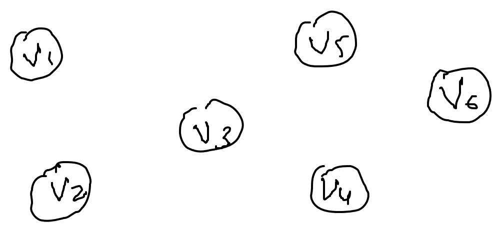
(d)



Answer:- Note:- In each step make sure cycle is not created.

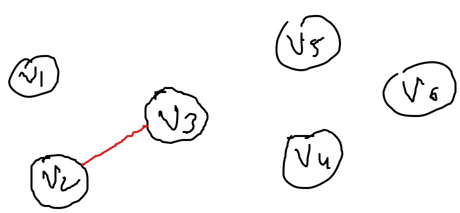
Step 1: For the graph given in (Figure PA-12.4) break them into n components (here $n=6$)

Eg:



Step 2: Connect 2 components in ascending order of cost.

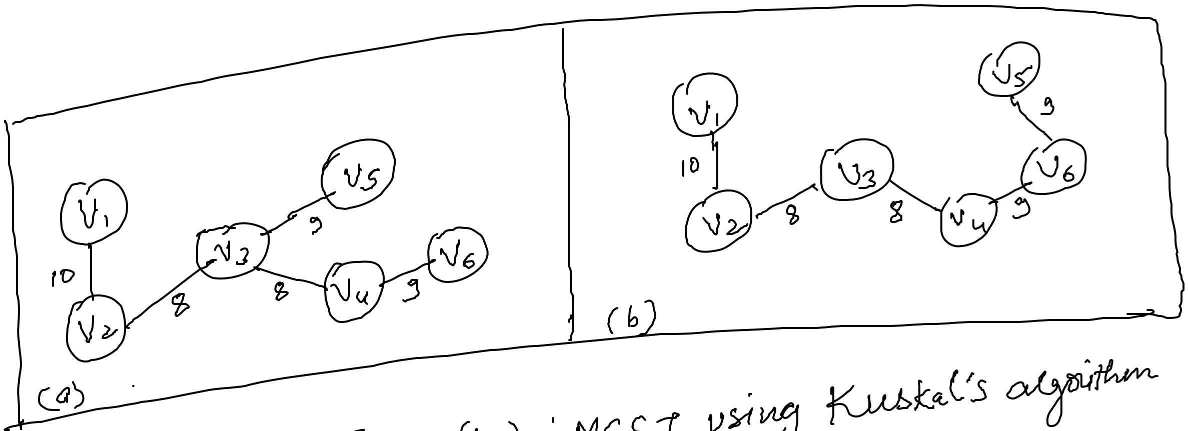
Eg:



Note: We can connect either V_2—V_3 or V_3—V_4 as they have same edge weights. In above Eg. we are connecting vertices V_2 and vertex V_3

Step 3: Repeat step 2 until all the edges are connected.

see Figure (4).



Figure(4) : MCST using Kuskal's algorithm

Note:- In the given graph (Figure PA-12.4) there are many edges which has same weights and thus we will have multiple MCST. Some of them are shown in Figure(4)

6. While using Bellman-Ford Algorithm for the graph shown below (Figure PA-12.5), let $D(v)$ be the shortest distance of vertex v from the source vertex 4 after 7 iterations.

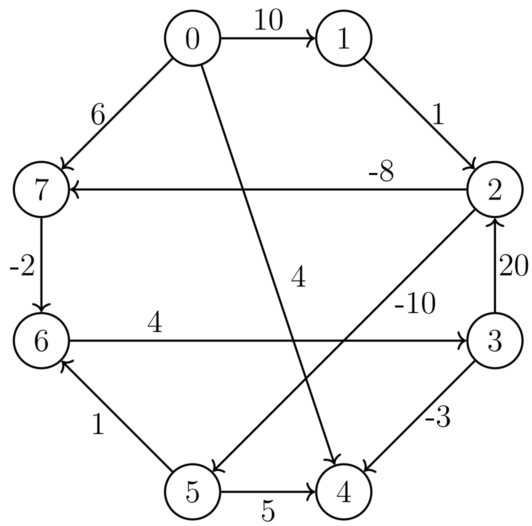


Figure PA-12.5

If the direction of edges in the graph are reversed, then which of the following is (are) CORRECT? [Ans: a,d]

(a) $D(0) = 2$

(b) $D(2) = 9$

(c) Bellman-Ford is not applicable for the new graph.

(d) $D(v)$ is negative for some vertex v.

Answer:-

Consider the new graph shown in Figure(10) after changing the directions of the edges of the graph given in Figure PA-12.5

Now, using Bellman-Ford algorithm we get adjacency matrix as shown in the Table 1. Note, after 7 iterations we get the values of D(v) as shown in last column.

Clearly D(0) = 2 ; D(2) = -9 ; D(v) is negative for some vertex v.

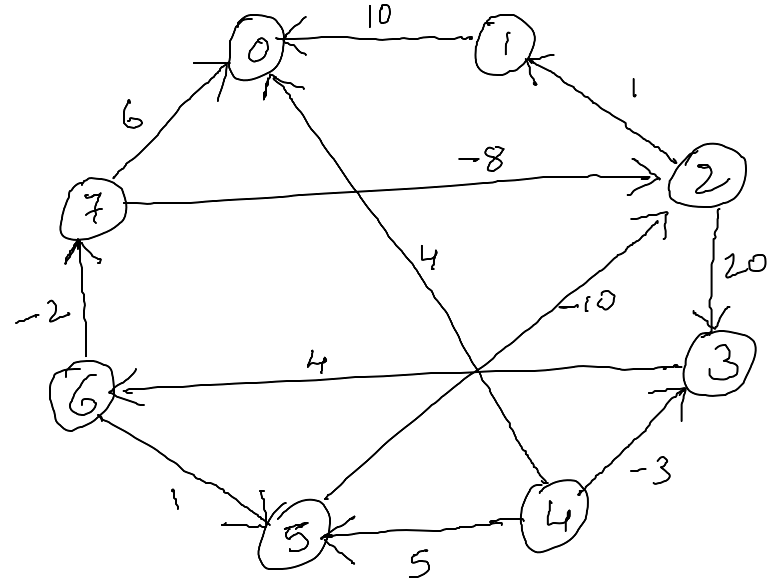


Figure (10)

v	D(v)							
0	∞	4	4	4	4	4	2	2
1	∞	∞	∞	-4	-4	-8	-8	-8
2	∞	∞	-5	-5	-9	-9	-9	-9
3	∞	-3	-3	-3	-3	-3	-3	-3
4	0	0	0	0	0	0	0	0
5	∞	5	5	2	2	2	2	2
6	∞	∞	1	1	1	1	1	1
7	∞	∞	∞	-1	-1	-1	-1	-1

Table 1

Bellman Ford is applicable for this new graph because there is no negative weight cycle.

7. Which of the following options are correct? [Ans:c, d]

(a) Let G be a weighted graph and in which the weights of all the edges are different. If we run a shortest path algorithm on G, then we will get a unique shortest path from the starting vertex to every other vertex.

(b) Suppose $G = (V, E)$ is a weighted graph, where $V = \{v_1, v_2, \ldots, v_n\}$. Let P be a shortest path from v_i to v_j $(i \neq j)$. If we increase the weight of each edge in the graph by one, then P will still be the shortest path from v_i to v_j.

(c) A graph G can have more than one spanning tree.

(d) Suppose $G = (V, E)$ is a weighted graph and the weights of all the edges are positive. Let P be a shortest path from $a \in V$ to $b \in V$. If we double the weight of every edge in the graph G, then the shortest path remains same but the total weight of path changes.

Answer:-

option (a)

Let G be weighted graph as shown in Figure (7)

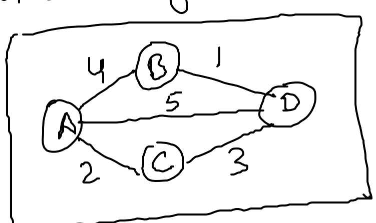


Figure (7)

Note: All the edge weights are different.

There are more than one shortest path to reach the vertex D from the starting vertex A. Thus, the statement as in option (a) is wrong.

Option (b)

Consider the graph shown in Figure (8)

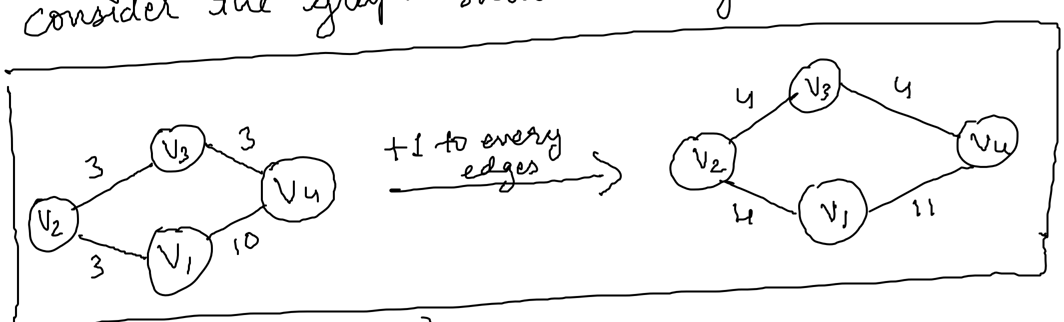


Figure (8)

Initially, the shortest path, to reach vertex V_4 from V_1 was $V_1 \rightarrow V_2 \rightarrow V_3 \rightarrow V_4$ (total cost is 9). If we increase the weights of each edge by one, (see Figure(8)) then the shortest path from V_1 to V_4 becomes $V_1 \rightarrow V_4$. So we are getting a new shortest path after the increament so option (b) is wrong.

<u>option(c)</u>

A simplest example could be a graph G, when all the edges have same weights.

<u>option (d)</u>

If we multiply all edge weights by 2, then the shortest path doesn't change. Because weights of all paths from a to b gets multiplied by 2. Here the number of edges in a path doesn't matter.

8. Which of the following options are correct? [Ans: a,d]

(a) Dijkstra's algorithm works for graphs having no negative weight edge.

(b) Floyd–Warshall algorithm works for graphs with negative weight cycles.

(c) Dijkstra's algorithm works on any graph without negative weight cycles.

(d) Shortest path problem is not applicable for a graph with a negative weight cycle.

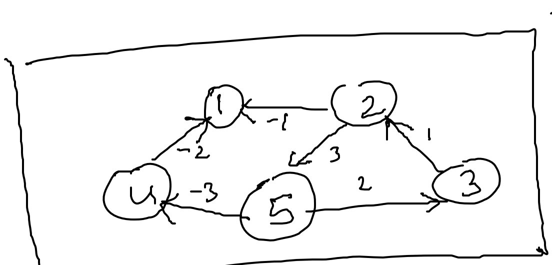
Answer

(a) Dijkstra's algorithm work for graph with non-negative edges, is the basic condition for this algorithm. So option (a) is correct.

(b) Floyd-Warshall algorithm doesn't works for graphs with negative weight cycles. So, option (b) is wrong.

(c) This option is incorrect. For Dijkstra's algorithm to work, the edge weights must be non-negative. A graph can have negative edges even though there are no negative weight cycles.

For example: Figure (6)



vertices 2, 3 and 5 forms a non-negative cycle but there are edges b/w vertices 2 to 1, 4 to 1 and 5 to 4 with negative weight edges.

Figure (6)

(d) Shortest path problem is applicable for non-negative weight cycle. So, option (d) is correct.

3 NUMERICAL ANSWER TYPE:

9. What is the weight of a minimum cost spanning tree of the graph given below (Figure PA-12.6)?
   [ans: 38]

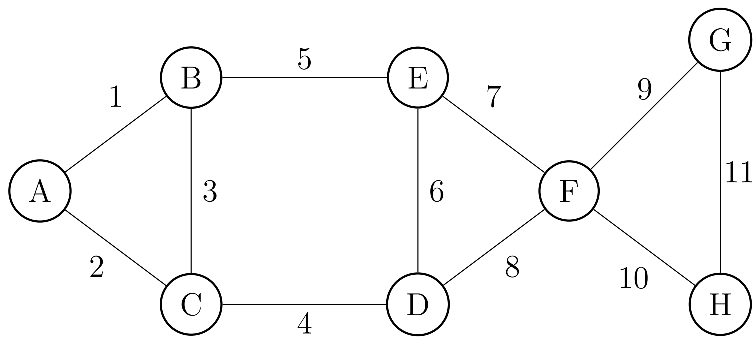


Figure PA-12.6

Answer:-

Using Prim's algorithm, MCST of graph shown in Figure PA-12.6 is (see Figure (5))

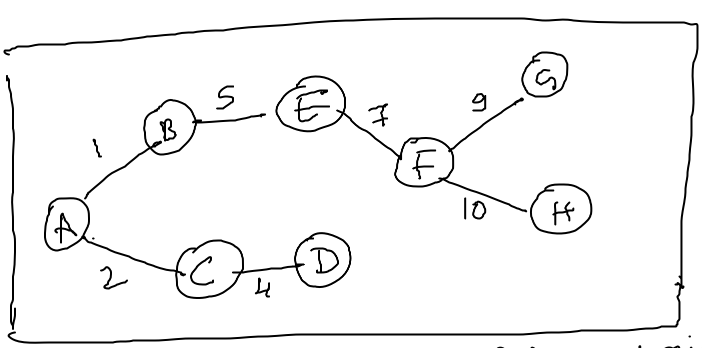


Figure (5): MCST using Prim's algorithm

Weight of MCST will be sum of weights of all the edges in MCST which is $1+2+4+5+7+9+10 = \underline{38}$

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