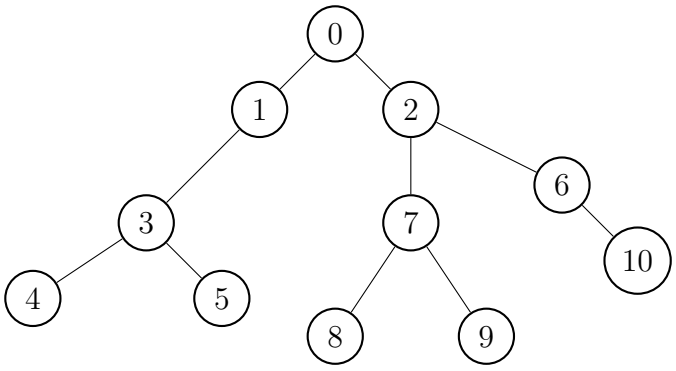
Mathematics for Data Science - 1
Practice Assignment
Week 9

1 MULTIPLE CHOICE QUESTIONS:

1. Suppose we obtain the following DFS tree rooted at node 0 for an undirected graph with vertices $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.



Which of the following cannot be an edge in the original graph? [Ans: c]

(a) $(1, 4)$
(b) $(0, 4)$
(c) $(7, 10)$
(d) $(2, 9)$

Soln. DFS tree for an undirected graph can have edges only in the same branch because if there is an edge between two vertices in G which are in different branches of DFS tree, then the neighbours of vertex 'u' must be visited in DFS in order to remove it from the stack.

clearly from Figure 1, we have five branches (b1, b2, b3, b4, b5)

✓ Option (a) :- Vertices (1, 4) ∈ branch(b1)

✓ Option (b) :- Vertices (0, 4) ∈ branch(b1)

✗ Option (c) :- Vertex (7) ∈ branch(b3)
& vertex (10) ∈ branch(b5) which cannot form a possible edge in original unrooted DFS tree.

✓ Option (d) :- Vertex (2, 9) ∈ branch(b4)

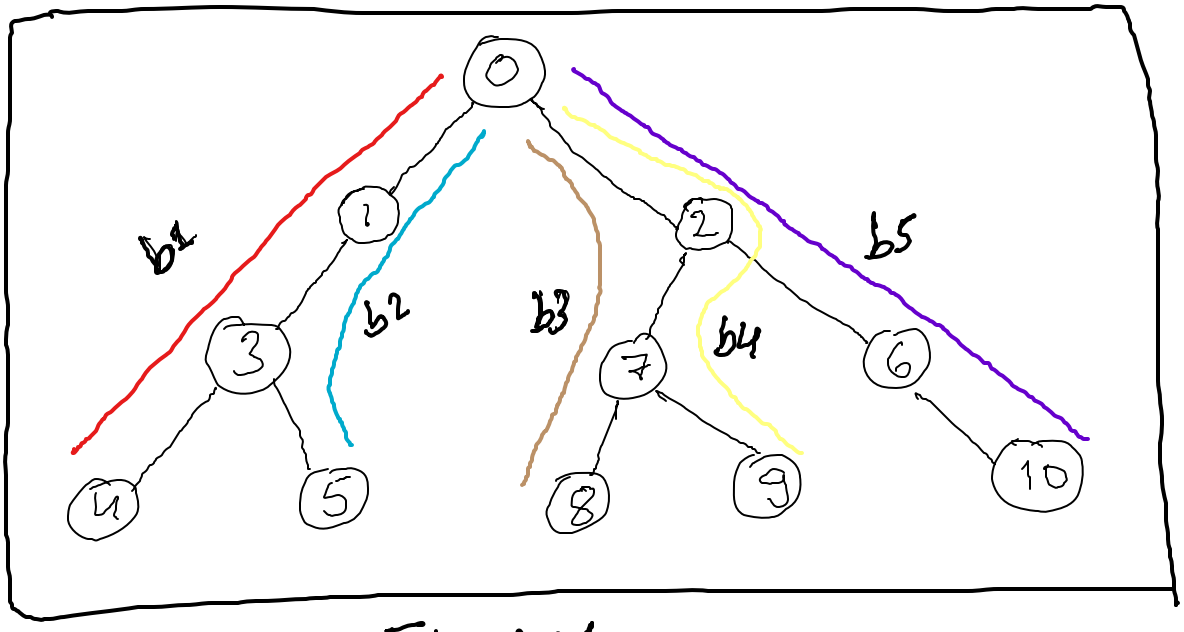
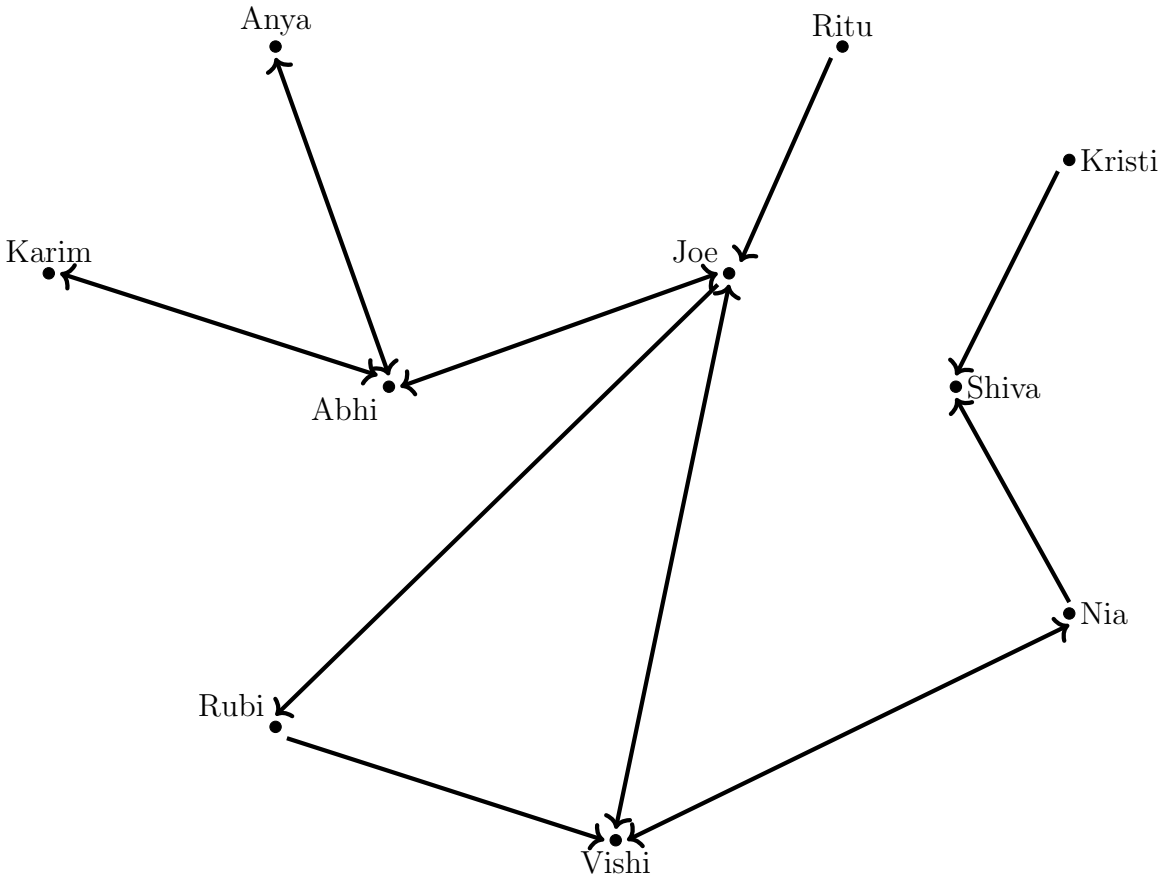


Figure 1

Use the following information for the questions 2-4:
Ten friends in a college decided to have a night party at the home of one of them. Unfortunately at D-day the government closes many of the routs of the city. The below graph shows the location of their homes and the open routs as the graph $G = (V, E)$, where V is the set of nodes and E is the edges representing the open routs.



2. The possible place for the party is. [Ans: a]

   (a) Shiva's house.
   (b) Abhi's house.
   (c) Joe's house.
   (d) Vishi's house.

Soln:-

Given that $G = (V, E)$ where V is set of nodes representing a person's house & E is the edge representing the open routes. Let us consider the reachability of each node

Note: Vertex (v) is reachable from vertex (u) if there is a path from u to v, where $u \& v \in V$

(i) Kristi's & Ritu's house are not reachable by anyone

(ii) Anya's, Karim's, Abhi's, Joe's, Rubi's, Vishi's and Nia's house are not reachable by Kristi and Shiva

(iii) Shiva's house is reachable by everyone so the best possible place for the party is Shiva's house as this node is reachable from every other nodes.

3. Let $V_1 = \{\text{Kristi, Shiva, Nia}\}$ and $E_1 = E \cap (V_1 \times V_1)$, that is, E_1 is the subset of edges of G with both end points in V_1. Choose the correct option. [Ans: d]

   (a) $G_1 = (V_1, E_1)$ is an undirected graph.

   (b) $G_1 = (V_1, E_1)$ is a cyclic graph.

   (c) $G_1 = (V_1, E_1)$ will not be a graph.

   (d) $G_1 = (V_1, E_1)$ is a directed graph.

Soln

Given that: $V_1 = \{\text{Kristi, Shiva, Nia}\}$

$V_1 \times V_1 = \{(\text{Kristi, Shiva}), (\text{Shiva, Kristi}), (\text{Kristi, Nia}), (\text{Nia, Kristi}), (\text{Shiva, Nia}), (\text{Nia, Kristi})\}$

Note: $V_1 \times V_1$ represents the interconnected path in V_1

$E = \{(\text{Kristi, Shiva}), (\text{Nia, Shiva}), \ldots\}$

$E_1 = E \cap (V_1 \times V_1) = \{(\text{Kristi, Shiva}), (\text{Nia, Shiva})\}$

So, clearly $G_1 = (V_1, E_1)$ is a directed graph as E_1 is directed. Figure 1-1 shows the subgraph G_1.

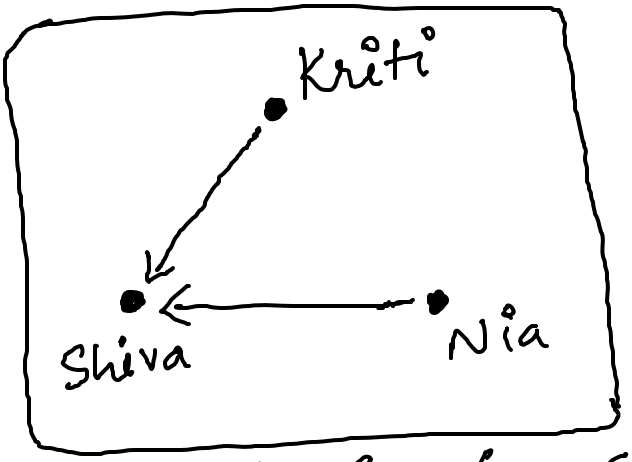


Figure 1-1: $G_1 = (V_1, E_1)$

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4. If Joe wants to have the party on his home, then at most how many members can join the party. [Ans: d]

   (a) 5.

   (b) 6.

   (c) 7.

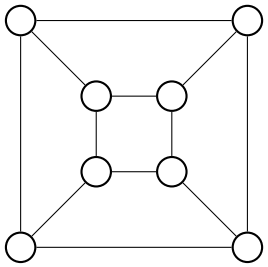
   (d) 8.

Sol:

As seen earlier Joe's home was not reachable by Kristi and Shiva. & there are 10 members altogether. Thus 8 members in total will join the party

2 MUTIPLE SELECT QUESTIONS:

5. Suppose G be a graph (shown in the below figure). Let V be the set of vertices of G, V_i be the maximum independent set and V_c be the minimum vertex cover. Which of the followings is(are) true? [Ans: a,d]



(a) Cardinality of V_i is 4.

(b) Cardinality of V_c is 3.

(c) Cardinality of V_i is 5.

(d) Cardinality of V_c is 4.

Soln Vertex cover:

In a graph G, vertex cover is the set of vertices that includes at least one end point of every edge of the graph.

So, minimum vertex cover (V_c) is a vertex cover having smallest possible number of vertices.

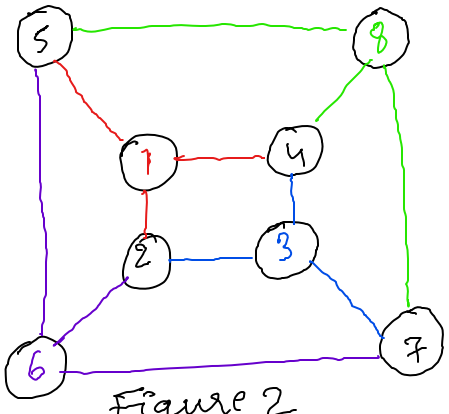


Figure 2

From Figure 2, one of the possible minimum vertex cover $(V_c) = \{1, 3, 6, 8\}$

Thus, cardinality of $(V_c) = 4$

Independent set:

Given a graph $G = (V, E)$, where V is vertex & E is edges, $F \subseteq V$ is an independent set if there are no edges between vertices in F.

Maximum independent set (V_i) is said to be maximal if no vertex of 'G' can be added to F

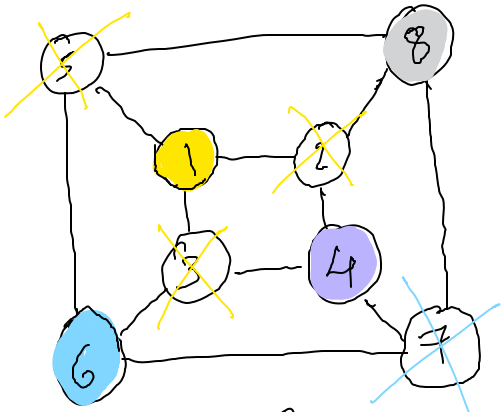
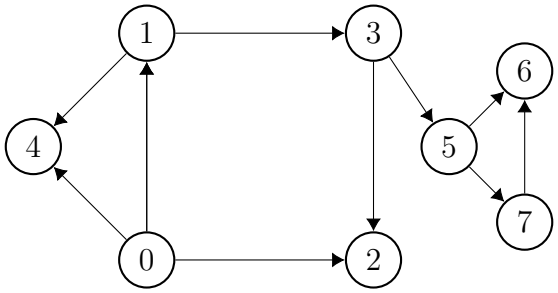


Figure 3

From Figure 3, one of the possible values of $V_i = \{1, 4, 6, 8\}$

Thus, cardinality of $V_i = 4$

6. Consider the graph given below.



Suppose we perform BFS/DFS so that when we visit a vertex, we explore its unvisited neighbours in a random order. Which of the following options are correct? [a,c,d]

(a) If we perform Breadth First Search at node 0, then one of the possible order in which the nodes will be visited is 01423567.

(b) If we perform Depth First Search at node 0, then one of the possible order in which the nodes will be visited is 04123576

(c) If we perform Breadth First Search at node 0, then one of the possible order in which the nodes will be visited is 01423576.

(d) If we perform Depth First Search at node 0, then one of the possible order in which the nodes will be visited is 04132567.

Soln

BFS tree from node 0, for the given graph could be as follows

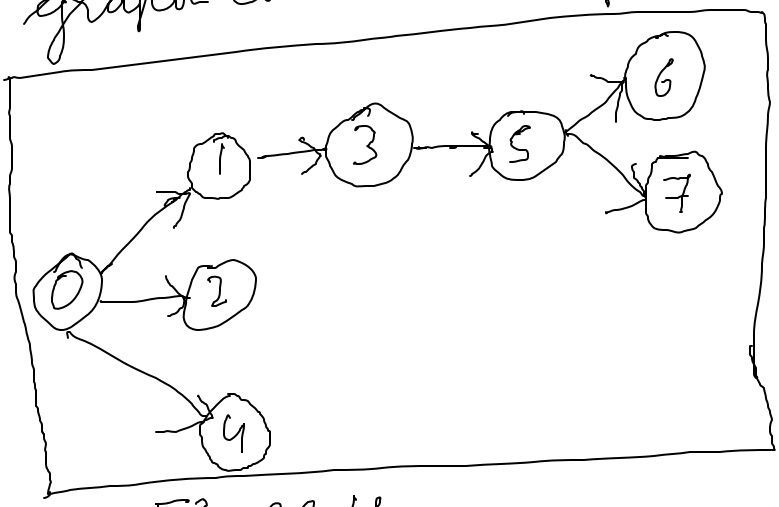


Figure 4

Explore 0, visit 1, 4, 2
Explore 1, visit 3
Explore 4
Explore 2
Explore 3, visit 5
Explore 5, visit 6, 7
Explore 6
Explore 7

0 1 4 2 3 5 6 7

Explore 0, visit 1, 4, 2
Explore 1, visit 3
Explore 4,
Explore 2
Explore 3, visit 5
Explore 5, visit 7, 6
Explore 7
Explore 6

0 4 1 2 3 5 7 6

These are the two possible orders however more orders are possible too.

DFS tree from the node 0, for the given graph could be as follows.

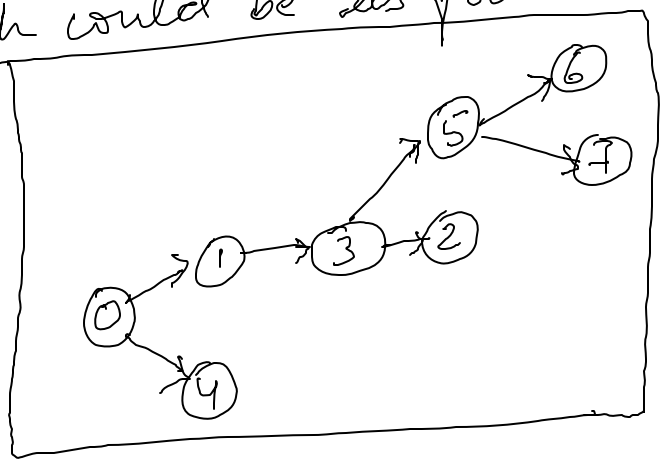


Figure 5

one of the possible order is 0 4 1 3 2 5 6 7.

Thus option (a), (c), (d) are right options

7. Which of the followings options are correct? [Ans: a,b,c,d]

(a) In Depth First search of a directed graph only back edges generate cycles.

(b) If the maximum independent set of a graph G contains only 1 element, then the graph must be connected.

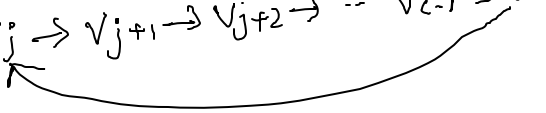
(c) If we add an edge to a tree T, then the resulting graph becomes cyclic.

(d) In a connected graph G having n vertices, at least two vertices have same degree.

Soln

(a) In a DFS, the vertices $v_0, v_1, v_2, \ldots v_{n-1}$ are connected by outward edge.

Suppose there is a backward edge (v_i, v_j) where $j < i$ for some $i, j \in \{0, 1, \ldots n-1\}$ then $v_j \to v_{j+1} \to v_{j+2} \to \ldots v_{i-1} \to v_i$ is a cycle.



(b) Consider a graph G, which is disconnected, then atleast the graph has 2 components of connected graph. Then, the maximum independent set of a graph G contains more than 2 elements in independent set.

Thus, for a graph with 1 element in maximum independent set must be a connected graph.

(c) Suppose in a tree there exists a path from vertex i to vertex j, for all $i, j \in V$

Now if we add an edge 'x' connecting vertex i to vertex j then there exists a path from vertex i to vertex j as shown below in Figure 6 which forms cycle

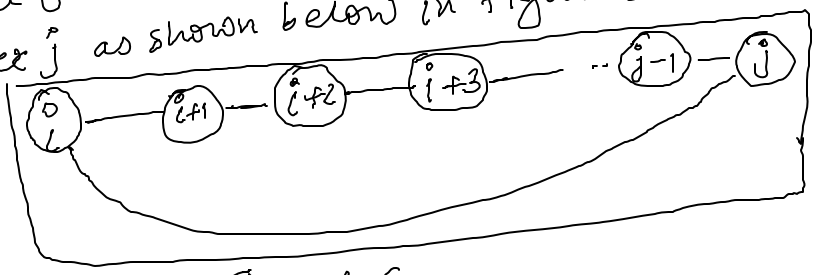
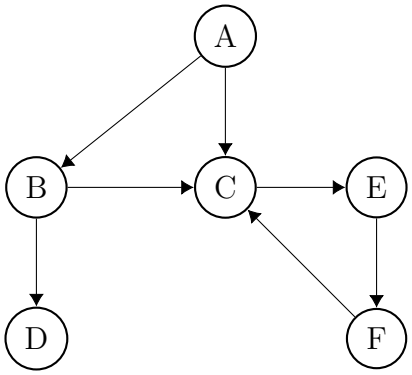


Figure 6

Thus if we add an edge to a tree T, then the resulting graph becomes cyclic.

(d) Let $v_1, v_2, v_3 \ldots v_n$ be the vertices and let v_1 has degree 1, v_2 has degree 2, v_3 has degree 3, ... v_{n-1} has degree $n-1$. Now the degree of vertex v_n should choose from $\{1, 2, 3, \ldots n-1\}$ which is one of the degree of the above vertices i.e., if the degree of the vertex v_n is 'k', then v_k and v_n has the same degree 'k'.

8. Consider the following directed graph.



Suppose DFS of this graph is performed from node A, such that when we visit a vertex, we explore its unvisited neighbours in alphabetical order.

Which of the following options are correct? [Ans: a,d]

(a) AC is a forward edge.

(b) CE is a backward edge.

(c) BD is a forward edge.

(d) FC is a backward edge.

Soln DFS tree of the given graph when performed from node A is shown in Figure 7

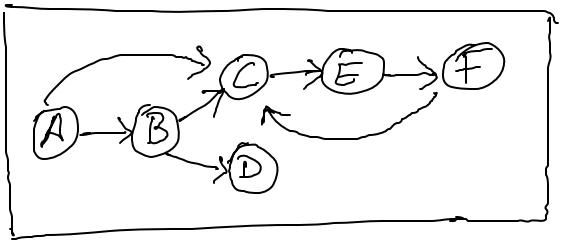


Figure 7

The only forward edge is (A)→(C) } Refer Figure
and only backward edge is (F)→(C) }
Rest all edges are the normal edge of DFS tree.

Therefore, AC forms forward edge & FC forms backward edge respectively.

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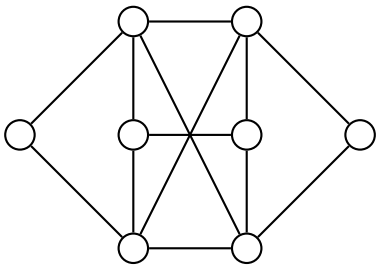
3 NUMERICAL ANSWER TYPE:

9. The cardinality of the maximum independent set of the graph given below is [ans: 4]



Soln

One of the possible way to find the maximum independent set is shown in Figure 8.

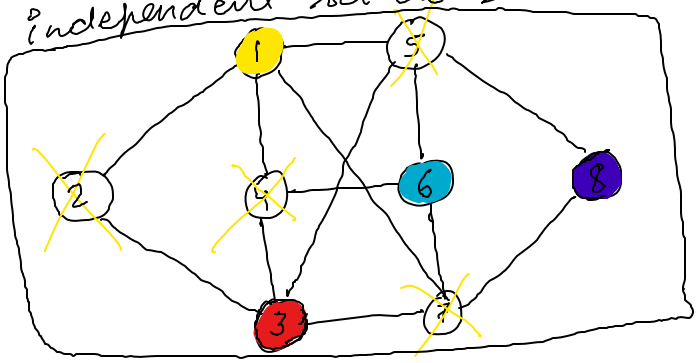
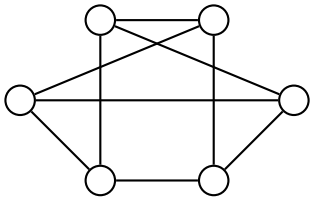


Figure 8

$V_i = \{1, 3, 6, 8\}$

Cardinality of $V_i = 4$

10. The minimum colouring of the below graph is [Answer: 2]



Soln We know that, graph $G = (V, E)$, set of colors C coloring is a function $c : V \to C$ such that $(u, v) \in E \Rightarrow c(u) \neq c(v)$

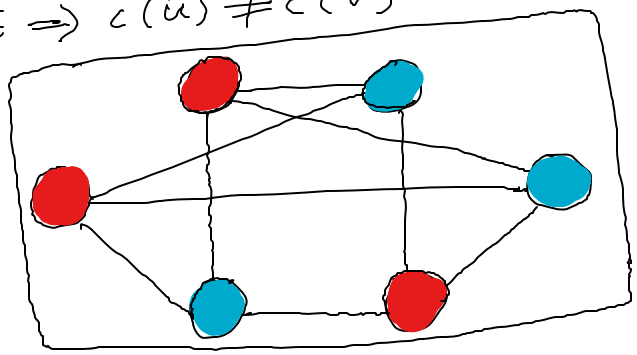


Figure 9

In simplest form, it is a way of coloring the vertices of a graph such that no two adjacent vertices are of the same color

From Figure 9 the minimum coloring required is 2.