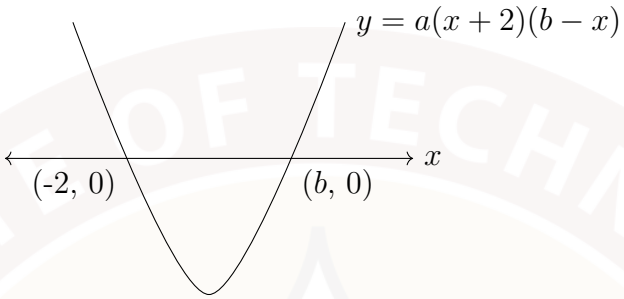
Week 5: Frequently Asked Question
Mathematics for Data Science - 1

Activity Question 1: Q2

Choose the correct option about a with the help of Figure 1.



a $a > 0$

b $a = 5$

c $a = 0$

d $a < 0$

Solution :
We know from week 4 lectures , that a parabola denoted by the equation $f(x) = ax^2 + bx + c$ where $a \neq 0$, will-

- open towards positive y-axis and has minimum value, if $a > 0$. This is called **Upward parabola**.
- open towards negative y-axis and has maximum value if $a < 0$. This is called **Downward parabola**.

Now, equation of the given parabola is $y = a(x + 2)(b - x)$. Converting this equation to a standard form, we get
$y = a(x + 2)(b - x)$
Expanding the terms inside bracket, $y = a(bx + 2b - x^2 - 2x)$
or, $y = ab \times x + 2ab - a \times x^2 - 2ax$
or, $y = (-a) \times x^2 + (ab - 2a) \times x + 2ab$
Hence, the co-efficient of x^2 is $(-a)$ and the given diagram is an **upward parabola**. For constructing an upward parabola with the equation $ax^2 + bx + c$, a should be greater than 0. Therefore in the given problem $y = (-a) \times x^2 + (ab - 2a) \times x + 2ab$, $(-a)$ should be greater than 0. This implies that, $a < 0$. Hence option d is correct.

Activity Question 2: Q3

Which of the following is/are true, if $x^2 - 8x + 13 = 0$ is solved by completing square method.

a 3 should be added on both sides of equation.

b $4 + \sqrt{3}$ is one of the roots.

c $4 - \sqrt{3}$ is one of the roots.

d Equal roots.

Solution:
$x^2 - 8x + 13 = 0$
or, $x^2 - 2 \times 4 \times x + 13 + 3 = 3$ (Adding 3 on both LHS and RHS),
or, $x^2 - 2 \times 4 \times x + 16 = 3$
or, $x^2 - 2 \times 4 \times x + 4^2 = 3$ (Writing $16 = 4^2$),
or, $(x - 4)^2 = 3$
Now, taking the square root on both sides,
or, $(x - 4) = \pm\sqrt{3}$
or, $x = 4 + \sqrt{3}$ and $x = 4 - \sqrt{3}$ Hence, 3 should be added on both sides of equation for solving it and the two roots are $4 + \sqrt{3}$ and $4 - \sqrt{3}$.

Activity Question 3: Q5

Nature of roots of quadratic equation $x^2 - 4 \times x + 7 = 0$ can be determined by which of the following methods?
(a) Completing the square method
(b) Qudratic formula
(c) Graphical method
(d) None of the above methods
Solution:
Completing the square method:
$x^2 - 4 \times x + 7 = 0$
or, $x^2 - 2 \times 2 \times x + 4 + 3 = 0$ (Writing 7 as 4+3),
or, $x^2 - 2 \times 2 \times x + 2^2 = -3$
or, $(x - 2)^2 = -3$
Taking square root on both sides, we get
$(x - 2) = \sqrt{3}$
Hence, $x = 2 \pm \sqrt{-3}$.
As $\sqrt{-3}$ is not a real number, hence there will be no real roots.
Using the Qudratic formula:
The quadratic formula states that, for a quadratic equation $ax^2 + bx + c = 0$ where a, b, c are rational numbers , the roots will be $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Putting the values in given equation, we get,

$x = \frac{-(-4)\pm\sqrt{(-4)^2-4\times1\times7}}{2\times1}$
or, $x = \frac{-(-4)\pm\sqrt{16-28}}{2\times1}$
or,$x = \frac{-(-4)\pm\sqrt{-12}}{2\times1}$
or,$x = \frac{4\pm\sqrt{4\times-3}}{2\times1}$
or,$x = \frac{4\pm2\sqrt{-3}}{2\times1}$ (Taking 4 out of square root)
Dividing numerator and denominator by 2,
$x = 2 \pm \sqrt{-3}$. Hence from here also we understand that there will be no real roots.
Using Graphical method:

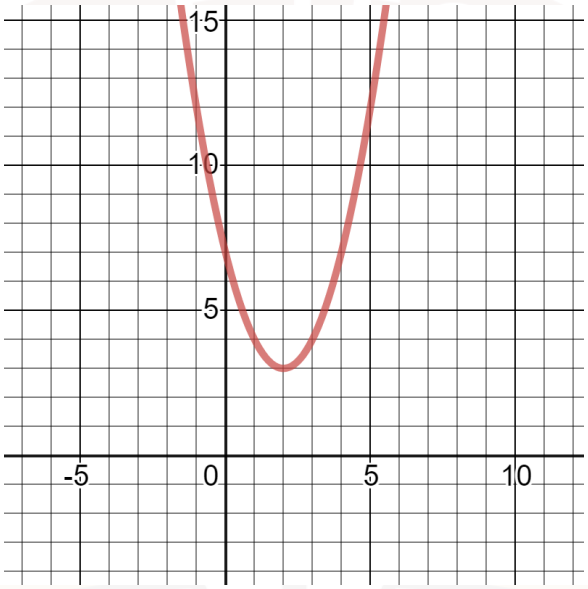


Figure for graphical solution of Activity Question 3: Q5

Using Desmos, we can plot the graph of $x^2 - 4x + 7$. The graph is, as shown in Figure 2. From the graph we can see that no part of the graph touches the real number axis, hence there will be no real roots.
Therefore all 3 methods mentioned in (a),(b),(c) can be utilized to solve this question.