Maths Week 2: Problem Solving Tips
Mathematics for Data Science - 1

1 Coordinate geometry

A **Cartesian coordinate system** is a coordinate system which specifies each point in a plane by a set of numerical coordinates.
These coordinates are the signed distances to the point from two fixed perpendicular oriented lines, measured in the same unit of length. Each reference line is called an **axis** (plural axes) of the system, and the point where two axes meet is called **origin**, whose coordinate is (0, 0).

Quadrant	Abscissa (X-axis)	Ordinate (Y-axis)
I	+ve	+ve
II	-ve	+ve
III	-ve	-ve
IV	+ve	-ve

- **Distance between any two points** (x_1,y_1) and (x_2,y_2) in cartesian plane (X-Y plane) is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

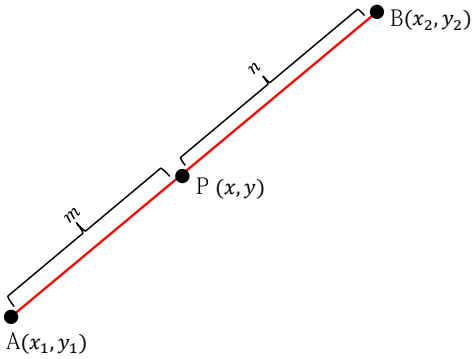
- **Area of a triangle** : Area of a triangle (Δ) formed by 3 points, (x_1,y_1) , (x_2,y_2) and (x_3,y_3) in X-Y plane is

$$\Delta = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

- **Section formula** :
If a point P (x, y) **internally** cuts the line segment AB, which connects two points A (x_1,y_1) and B (x_2,y_2) in a ratio $m : n$, then the value of x and y will be

$$x = \frac{mx_2 + nx_1}{m + n}$$

$$y = \frac{my_2 + ny_1}{m + n}$$

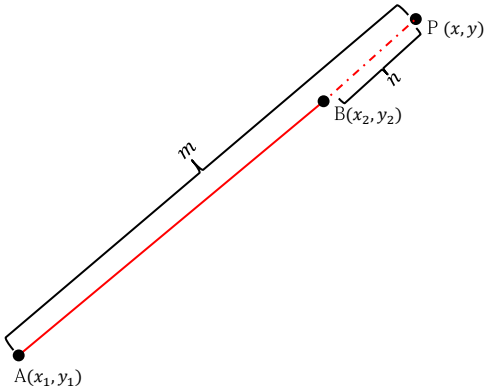


Internal section in the ratio $m : n$

If a point P (x, y) **externally** cuts the line segment AB, which connects two points A (x_1,y_1) and B (x_2,y_2) in a ratio $m : n$, then the value of x and y will be

$$x = \frac{mx_2 - nx_1}{m - n}$$

$$y = \frac{my_2 - ny_1}{m - n}$$



External section in the ratio $m : n$

2 Straight line

- **Equation of a straight line** : In two dimensions, the equation of straight line parallel to the Y-axis is given by $x = a$ for some real number a, where the straight line intersects

the X-axis at the point $(a, 0)$. In general the equation of the straight lines is often given in the slope-intercept form:

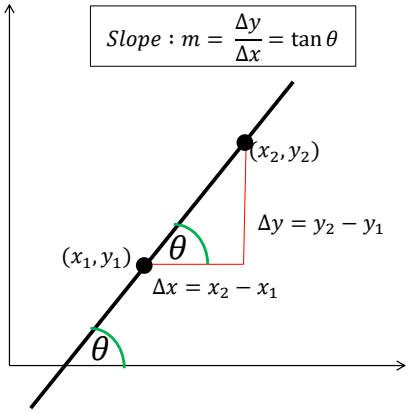
$$y = mx + c$$

where:
m is the slope or gradient of the line.
c is the y-intercept of the line.
x is the independent variable of the function $y = f(x)$.

- **Slope** : Slope of a line (denoted by m) describes both direction and steepness of a line. Numerically, slope

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = tan\theta,$$

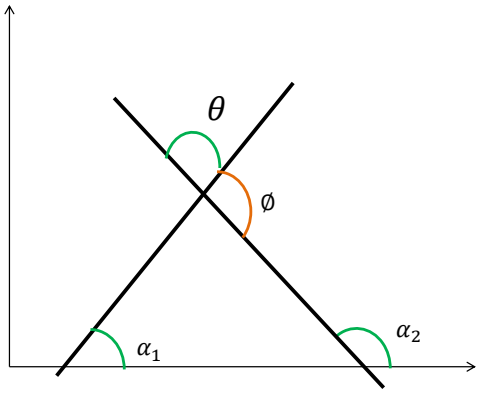
where θ is the inclination of the line with respect to the X-axis.



- Two non-vertical lines are parallel only if their slopes are equal, i.e. $m_1 = m_2$.
- Two non-vertical lines l_1 (with slope m_1) and l_2 (with slope m_2) are perpendicular if and only if $m_1 m_2 = -1$
- Three points will be colinear if the area enclosed by the triangle formed by those three points is 0.
- **Two-point form of straight line** connecting two points (x_1, y_1) and (x_2, y_2):

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

- **Angle between two lines** : Let l_1 and l_2 be two lines with slopes m_1 and m_2 and inclinations α_1 and α_2 respectively (let us assume, $\alpha_2 > \alpha_1$).



If l_1 and l_2 intersect each other forming adjacent angles ϕ and θ, then

$$tan\theta = \frac{m_2 - m_1}{1 + m_2 m_1}$$

$$tan\phi = \frac{m_1 - m_2}{1 + m_1 m_2}$$