Week - 2
Solutions for Practice Assignment-2
Straight line - 1
Mathematics for Data Science - 1

1 Multiple Choice Questions (MCQ):

1. A publishing house purchases a printing machine for ₹50,000. At the end of 5 years the value of the machine is supposed to be ₹10,000 only. If the loss in value is assumed to be linear then what is the yearly loss in the value of machine?

   ◯ ₹ 5,000 only.
   ◯ ₹ 6,000 only.
   ◯ ₹ 10,000 only.
   ◯ ₹ 12,000 only.
   ◯ **₹ 8,000 only.**
   ◯ None of the above.

   Solution:
   Let y represents the value of machine and x represents the number of years after purchasing the machine.

$$\text{When, } x = 0 \implies y = 50,000.$$
$$\text{And when, } x = 5 \implies y = 10,000.$$

   Applying equation of the line in two point form:

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$
$$\implies y - 50,000 = \frac{10,000 - 50,000}{5 - 0}(x - 0)$$

   On rearranging the above equation, we have:

$$y = -8000x + 50,000$$

   Then the slope of the line will be the yearly loss in the value of machine i.e. ₹ 8,000.

2. A line is represented by $7y = 56 - 8x$. If mirror image is taken with respect to the $Y - axis$, a new line is formed. What will be the equation of the new line?

   ◯ $7x + 8y = 56$

   ◯ $7x + 8y = -56$

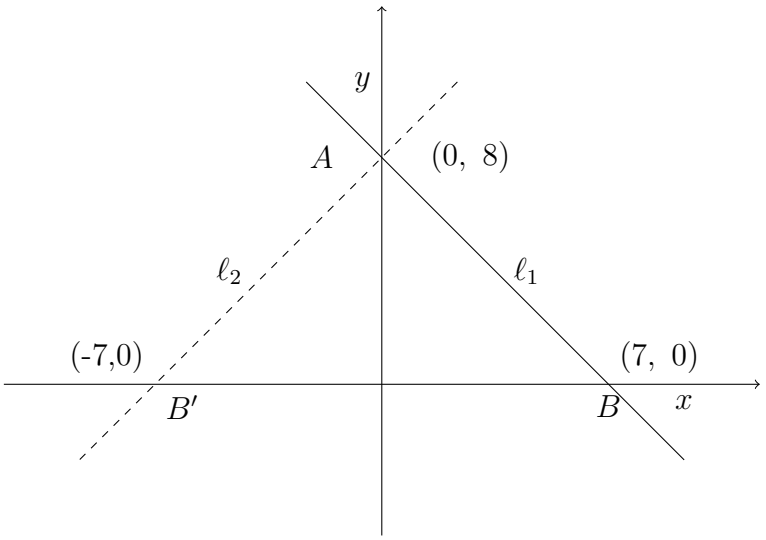
   ◯ $\frac{x}{7} + \frac{y}{8} = 1$

   ◯ $\frac{x}{8} + \frac{y}{7} = 1$

   ◯ $\boldsymbol{y = \frac{8}{7}x + 8}$

   ◯ $y = \frac{7}{8}x - 7$

   Solution:
   Rearranging the equation $7y = 56 - 8x$ in intercept form, we have $\ell_1 \equiv \frac{y}{8} + \frac{x}{7} = 1$.
   Then, the points of intercept are (0,8) and (7,0) as shown in (Figure 1)

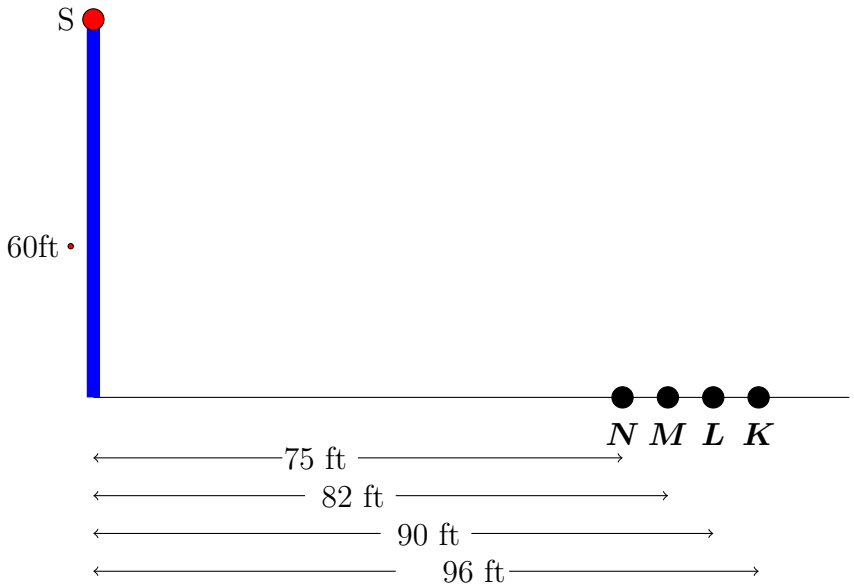


   If we treat $Y-$ axis as mirror then the mirror image line ℓ_1 will be line ℓ_2 and the mirror image of points $A\ (0, 8)$ and $B\ (7, 0)$ will be $A\ (0, 8)$ and $B\ (-7, 0)$ respectively shown in (Figure 1).

   Then the equation of new line passing through A and B' using intercept form $\frac{x}{a} + \frac{y}{b} = 1$ will be $\ell_2 \equiv \frac{y}{8} - \frac{x}{7} = 1$.
   On rearranging:

$$\boldsymbol{y = \frac{8}{7}x + 8}$$

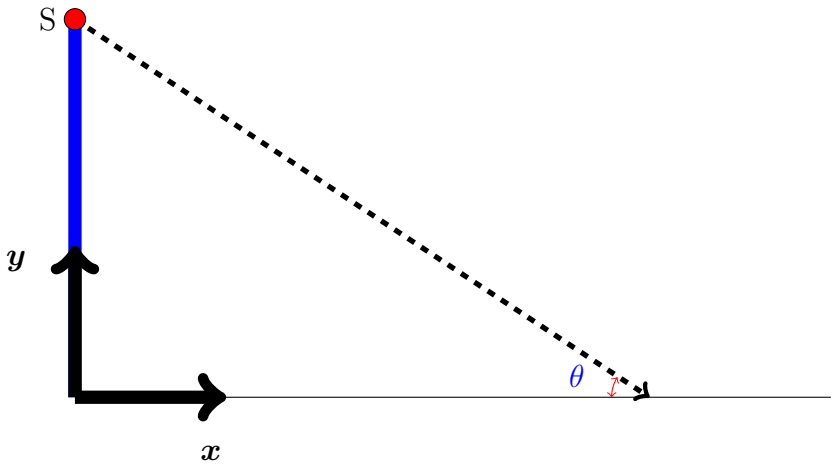
3. A sniper is sitting on top of a tower at a height of 60 ft. There are four workers K, L, M and N standing at a distance of 96 ft, 90 ft, 82 ft, and 75 ft respectively from the base of the tower. The heights of K, L, M, and N are 6 ft, 5.5 ft, 5.7 ft, and 5.2 ft respectively. The sniper misfires a bullet at an angle θ with the horizontal. Since the range covered by the bullet is short, the path of the bullet is assumed to be a straight-line path. If $\tan\theta = \frac{2}{3}$, choose the correct option.



- ◯ All workers are safe.
- ◯ All the workers are safe except K.
- ◯ Only K and N are safe.
- ◯ No one is safe.
- ◯ Only K is safe.
- ◯ **All the workers are safe except M.**

Solution:

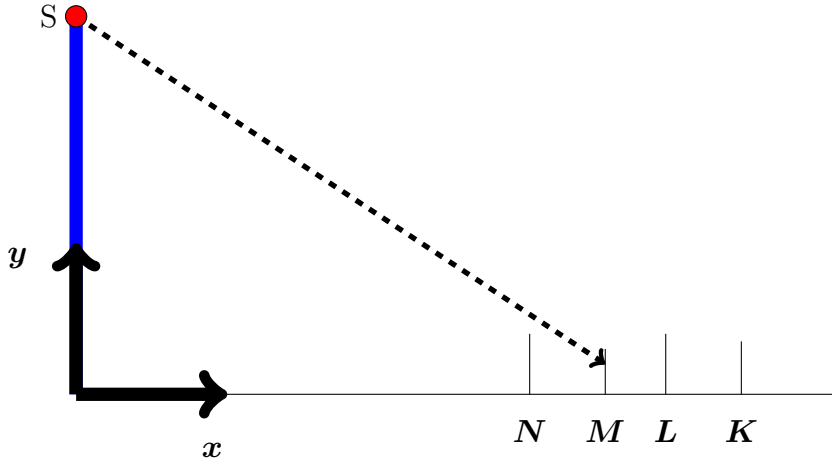
Let us treat height from ground as $Y-$ axis and horizontal distance on ground from tower base as $X-$ axis as shown in Figure.



From figure it is clear that if $x = 0 \longrightarrow y = 60$.
The path of bullet can be written using slope intercept form as $y = mx + c$.
Here $c = 60$ then the path of bullet will be $y = mx + 60$.
Now m will be the slope of line and it is known as $m = tan\ \theta$. According to figure, θ is from -ve $x-$ axis therefore $m = -tan\theta = -\frac{2}{3}$.
There the path of bullet will be $y = -\frac{2}{3}x + 60$.

Worker N is standing at 75 ft away from the tower base which means $x = 75$. Putting this value in the path of bullet $y = -\frac{2}{3} \times 75 + 60 = 10$, where the height of worker is 6 ft. Therefore the bullet will even not touch the worker N.

Similarly for M, $y = 5.333$ which is lower than the height of worker M. It means the bullet will hit worker M and then it will not cross $x = 82$, therefore we do not need to check for others.
Visualization of the above scenario can be scene below:



2 Multiple Select Questions (MSQ):

1. Choose the correct options regarding mirror image of a given point.

   ◯ Point (4,6) is the mirror image of (-4,6) when X-axis is treated as mirror.

   ◯ **Point (-2,8) is the mirror image of (-2,-8) when X-axis is treated as mirror.**

   ◯ Point (-2,8) is the mirror image of (2,8) when X-axis is treated as mirror.

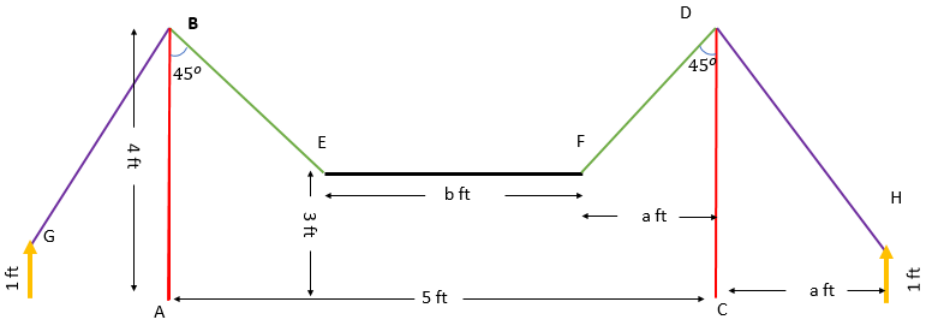
   ◯ **Point (-2,8) is the mirror image of (2,8) when Y-axis is treated as mirror.**

   ◯ Point (4,6) is the mirror image of (-4,-6) when X-axis is treated as mirror.

   ◯ Point (-4,6) is the mirror image of (4,-6) when Y-axis is treated as mirror.

   Solution:
   Only the sign of a ordinate ($Y-$ coordinate) changes when $X-$axis is treated as mirror. Similarly, only the sign of abscissa ($X-$ coordinate) changes when $Y-$axis is treated as mirror. Therefore the options (a), (c), (e), (f) are incorrect.

2. A long horizontal piece of wood EF is suspended above the ground, as shown in the diagram, by two ropes EB and FD which are tied to two bamboo poles AB and CD of equal lengths. The bamboo poles are at equal distances from the edge of wooden piece. The bamboo poles are supported by two different ropes GB and HD respectively as shown in the figure. Choose the correct options.



   ◯ $b = 4$

   ◯ If A is treated as origin then the coordinate of point E will be $(3, 4)$.

   ◯ **If A is treated as origin then the coordinate of point E will be $(1, 3)$.**

   ◯ **If A is treated as origin then the coordinate of point F will be $(4, 3)$.**

   ◯ **The slope of thread DH is -3.**

◯ The slope of thread DH is -$\frac{1}{3}$.

Solution:
Let us consider point A as origin and distance from A on ground as $X-$axis. Therefore, its coordinate is A $(0, 0)$ and $y-$ coordinate of E will be 3 in rectangular coordinate system.
Then, the coordinate of B is (0,4). The slope (m) of the line segment BE is $-tan\ 45 = -1$;
Let coordinate of E be $(x, 3)$, then the slope of BE:

$$m = -1 = \frac{4\ -\ 3}{0-x}$$
$$\longrightarrow x = 1$$

Similarly, coordinates of F is (4,3) and $a = 1$ (Using slope of FD).
Therefore, $b = 4 - 1 = 3$.

Coordinates of D and H are $(5, 4)$ and $(6, 1)$ respectively, using D and H, slope of thread DH can be found:

$$\text{Slope of thread } DH\ =\ \frac{y_2\ -\ y_1}{x_2\ -\ x_1} = \frac{1\ -\ 4}{6\ -\ 5}$$

Slope of thread = -3

3. A line represented by $\frac{y}{3} - \frac{x}{7} = 1$ intersects the $X - axis$ and the $Y - axis$ at points A and B respectively. The mirror image of this line (taken with $X - axis$ as the mirror) intersects the $Y - axis$ at point C. The mirror image of the point A (with $Y - axis$ as the mirror) is denoted by D. Choose the correct options.

   - ◯ **Equation of line segment CD is $\frac{x}{7} - \frac{y}{3} = 1$.**
   - ◯ Equation of line segment CD is $\frac{x}{7} + \frac{y}{3} = 1$.
   - ◯ ***DB* is parallel to *AC*.**
   - ◯ DC is perpendicular to AC.
   - ◯ **The area of the geometry enclosed by above four lines is 42 square units.**
   - ◯ The area of the geometry enclosed by above four lines is 10.5 square units.

   Solution:
   Option (a): Correct
   Let, l_1: $\frac{y}{3} - \frac{x}{7} = 1$ as shown in (Figure 2) represents the line in intercept form of a given.
   Then, the Coordinates of $X-$intercept is A(-7, 0) and $Y-$intercept is B(0,3)
   The mirror image of this line l_1 will be line l_2 as shown in (Figure 2).
   Since, when $X-$axis is treated as mirror only the sign of $Y-$coordinate changes. Therefore, coordinate of C is (0,-3).
   Similarly, coordinate of D is (7,0) as $Y-$axis is mirror, hence, only the sign of $X-$coordinate of point A changes.
   Equation of line segment CD can be obtained using the points C and D in the intercept form i.e. l_2: $\frac{x}{7} - \frac{y}{-3} = 1$

   Option (b): Incorrect
   Because, as seen above the equation of line segment CD is,
   $\frac{x}{7} - \frac{y}{-3} = 1$
   Option (c): Correct
   Slope of BD can be found using B(0,3) and D(7,0)

$$\text{Slope of } BD = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 3}{7 - 0}$$

$$\text{Slope of } BD = -\frac{3}{7}$$

   Slope of AC can be found using A(-7,0) and C(0,-3)

$$\text{Slope of } AC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 0}{0 - (-7)}$$

$$\text{Slope of } AC = -\frac{3}{7}$$

   Since, slpoe of $BD = AC$, therefore are form parallel to each other.
   Option (d): Incorrect

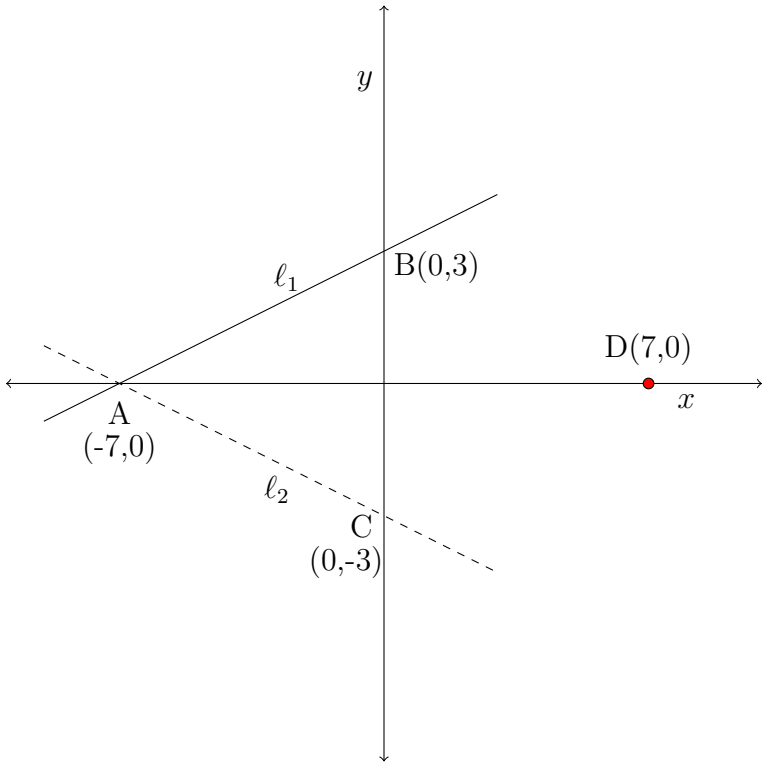


Figure 1: representation of ℓ_1 and ℓ_2 on coordinate plane

Slope of AC (m_1) = -1
Slope of DC (m_2) can be found using the points D(7,0) and C(0,-3)

$$\text{Slope of } DC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 3)}{7 - 0}$$

$$\text{Slope of } DC\ (m_2) = -\frac{3}{7}$$

For a line to be perpendicular to each other,

$$m_1 \times m_2 = -1$$
$$-1 \times (-\tfrac{3}{7}) \neq -1$$

Therefore, line segment DC and AC are not perpendicular to each other.
Option (e): Correct

The area of ABC = area of $DBC = a$ (geometric similarity)
The area formed by geometry $ABDC$ = Area of triangle ABC + area of triangle DBC $= a + a = 2a$
Area of a triangle ABC can be obtained using the vertices A(-7,0), B(0,3) and C(0,-3) is:

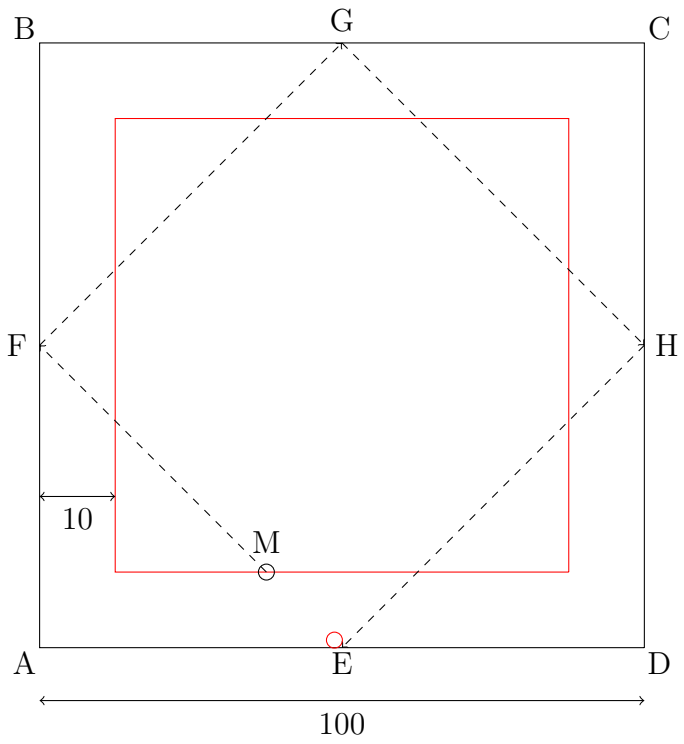
$$a = \tfrac{1}{2}|\ x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\ |$$

$$a = \tfrac{1}{2}|\ -7(3 - (-3)) + 0(-3 - (-7)) + 0(0 - 3)\ |$$

$$a = |\ -21\ | = 21$$

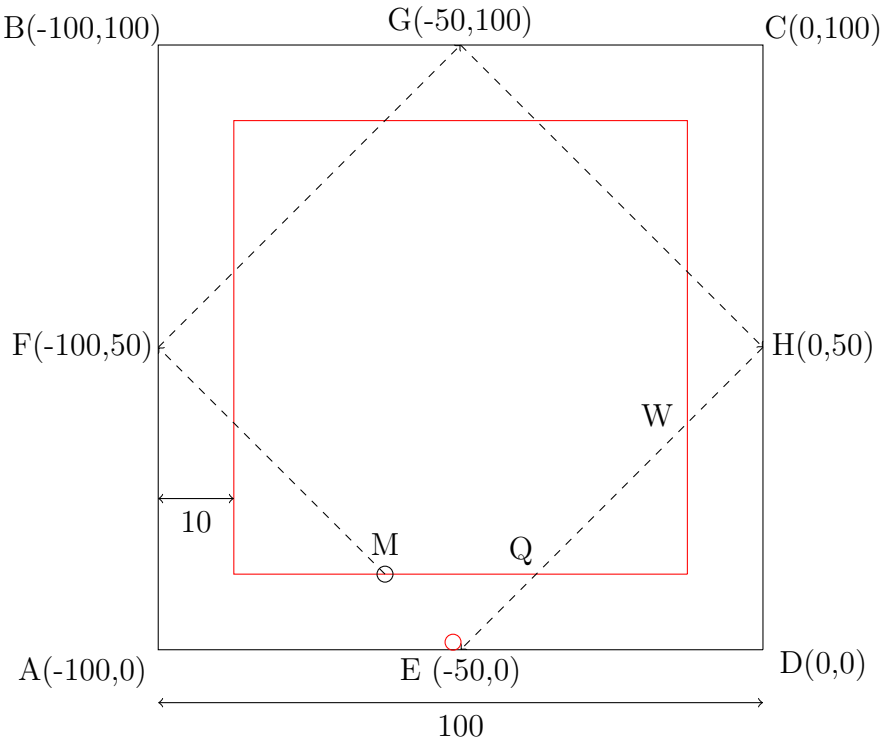
Therefore, area of geometry $ABDC = 2 \times 21 = 42$ square units

4. A carom board is a square board with a symmetrical design as shown in the diagram below. E, F, G, and H are the midpoints of AD, BA, CB, and DC respectively. Anand wants to pocket his last coin which is at E. Taking the laws of reflection to be applicable, he strikes from the point M, so that the striker reflects at F, then at G, then at H and finally hits the coin at E. Choosing D to be the origin (right direction is $+ve\, X-axis$ and upward direction is +ve Y-axis), choose the correct options.



- ◯ M does not lie on the line segment EF.
- ◯ **The coordinate of point M is $(-60, 10)$.**
- ◯ The coordinate of point M is $(-40, 10)$.
- ◯ **The slope of MF is -1.**
- ◯ **FG and GH are perpendicular line segments.**
- ◯ Equation of GH will be $y + x = 150$.
- ◯ **Segment HE intersects the inner square at $(-10, 40)$.**

Solution:
Considering corner D of the carom board as origin in coordinate system.
Then coordinates of the corners will be:$D(0,0), C(0,100), B(-100,100), A(-100,0)$.
Therfore coordinates of mid points on carom will be:
$H(0,50), G(-50,100), F(-100,50), E(-50,0)$ as shown in below Figure.



Option (a) is incorrect because point M comes in the path of FE as per the law of reflection.
Option (b): Correct
Because using the point E(-50,0), F(-100,50), we can form a equation of line in two-point form.

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\implies y - (0) = \frac{50 - 0}{-100 - (-50)}(x - (-50)) \implies y = -x - 50$$

Since, $Y-$coordinate of M is 10 (given: Carom symmetry). On solving the above equation with $y = 10$:

$$10 = -x - 50$$
$$x = -60$$

Therefore the coordinate of M is (-60,10)

Option (c): Incorrect, because coordinate of M is (-60,10) as explained above.

Option (d): Correct, because using the point M(-60,10), F(-100,50)

$$\text{Slope of MF} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{50 - (10))}{-100 - (-60)}$$

$$\text{Slope of MF} = -1$$

Option (e): Correct
Because the slopes of FG and GH can be found using the points F(-100,50), G(-50,100), H(0,50).

$$\text{Slope of FG } (m_1) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{100 - 50}{-50 - (-100)}$$

$$\text{Slope of FG } (m_1) = 1$$

$$\text{Slope of GH } (m_2) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{50 - 100}{0 - (-50)}$$

$$\text{Slope of GH } (m_2) = -1$$

Since, $(m_1)\times(m_2) = -1$, therefore they are perpendicular to each other.

Option (f): Incorrect
Equation of line segment GH can be found using the slope-intercept form.
Where slope (m) is -1 (found in above option) and Y-intercept (c) is 50 at point H.

$$\text{Equation of line in slope-intercept form is: } y = mx + c$$
$$y = -x + 50 \implies \boldsymbol{y + x = 50}$$

Option (g): Correct
Clearly from the above figure there are two points lets say Q and W where the line segment HE intersects the inner square.
Equation of a line segment HE can be obtained in two-points form, using the coordinates H(0,50), E(-50,0).

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\implies y - (50) = \frac{0 - 50}{-50 - 0}(x - 0) \implies y = x + 50$$

$Y-$coordinate of Q is 10. Therefore, the other coordinate can be found by substituting in equation of line segment HE.

$$10 = x + 50$$
$$x = -40$$
$$\text{Q(-40,10)}$$

Similarly $X-$coordinate of W is -10 and the other coordinate can be obtained in same way.

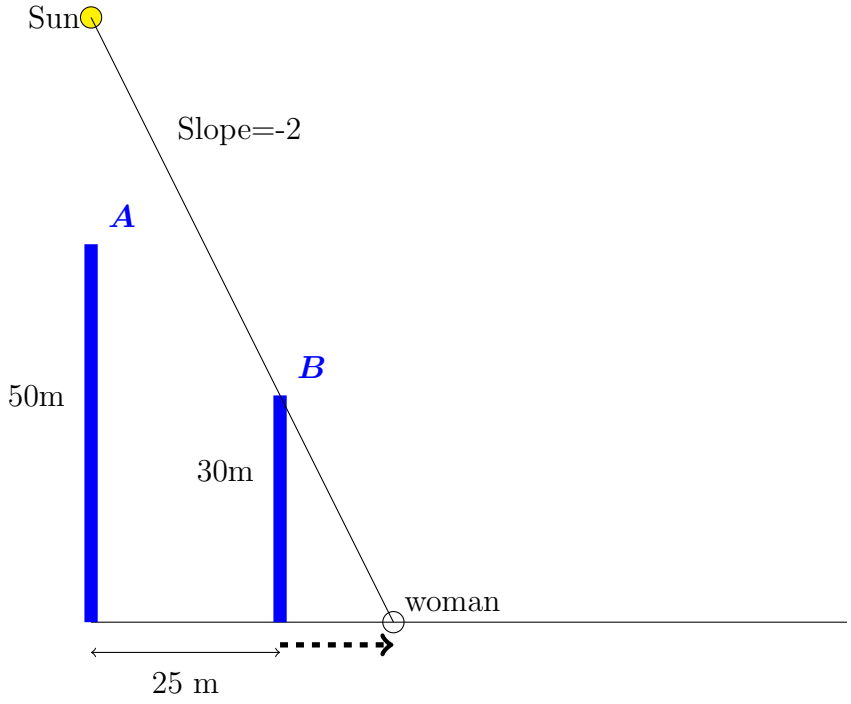
$$y = -10 + 50$$
$$x = 40$$
W(-10,40)

Therefore, W(-10,40) is one of the point in the inner square where the line segment HE intersects.

3 Numerical Answer Type (NAT):

1. Two buildings A and B are as shown in the diagram. A woman walks out of building B to get some sunshine. How much minimum distance in metres will she have to walk in the right direction of building B, according to the situations given below? Assume the right direction of buildings as positive and ground as $X-axis$.

   (a) The first sunray not blocked by B has a slope of -2. [Ans: 15]



Solution:

Let us consider the sunray as a line and the foot of the building B as origin in the coordinate plane.

Then, the coordinate $B(0,30)$ forms the $Y-$intercept and the slope of the sunray is given as -2.

Therefore, a equation of line in a slope intercept form can be established:

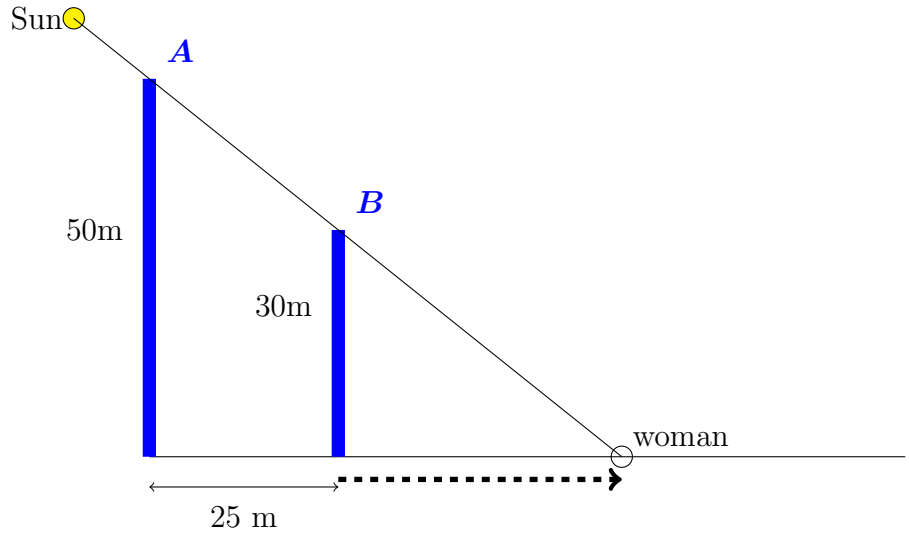
y represents the height of building and x representing the minimum distance travelled by women to get sunshine from the foot of building B.

$$y = mx + c, \quad (\text{ c} = 30\text{m}, Y-\text{intercept})$$
$$y = -2x + 30$$

On solving the above equation with $y = 0$:

$$2x = 30$$
$$x = 15m$$

(b) The first sunray not blocked by A is also the first sunray not blocked by B. [Ans: 37.5]



Solution:
Let us again consider the sunray as a line and this time foot of the building A as origin in the coordinate plane.
Then, the coordinate $A(0,50)$, and the coordinate of B is $(25,30)$.
Therefore, a equation of line in two point form can be established:
y represents the height of building and x representing the minimum distance travelled by women to get sunshine from the foot of building A.

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

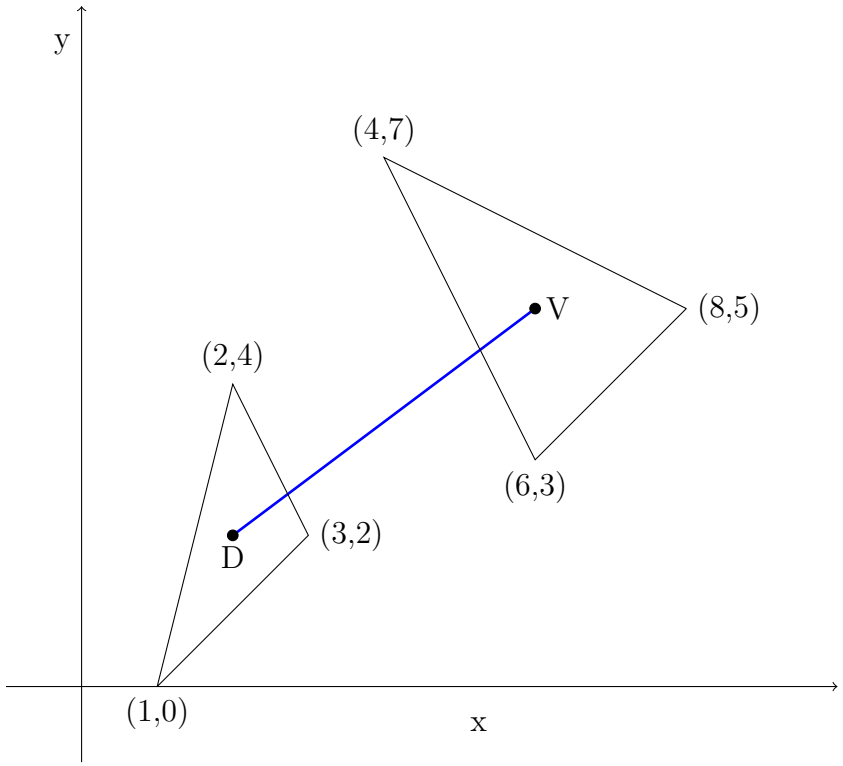
$$\implies y - 50 = \frac{30 - 50}{25 - 0}(x - 0) \implies y = -0.8x + 50$$

On solving the above equation with $y = 0$:

$$0.8x = 50$$
$$x = 62.5m$$

Therefore, women has to travel 62.5 - 25 = **37.5m** from the foot of building B

2. Veeru (V) and his horse Dhanno (D) are running towards each other from their locations which are the centroids of two different triangles shown in figure below. The x - coordinate x_c of the centroid of a triangle can be found using the formula $x_c = \frac{x_1+x_2+x_3}{3}$, where x_1, x_2, x_3 are the x-coordinates of the vertices of the triangle. Corresponding formula can be applied for the y-coordinate, too. Then answer the following questions.



(a) If Veeru and Dhanno follow the path DV then what will $tan\,\theta$ be, if θ is the angle which DV makes with the horizontal? [Ans = $\frac{3}{4}$]

Solution:

Using the formula for the centroid of a triangle (i.e. $x_c = \frac{x_1+x_2+x_3}{3}, y_c = \frac{y_1+y_2+y_3}{3}$), the coordinates of D and V can be given as:

$$D \equiv (x_c = \tfrac{x_1+x_2+x_3}{3}, y_c = \tfrac{y_1+y_2+y_3}{3}) \implies D(\tfrac{1+2+3}{3}, \tfrac{0+4+2}{3})$$

$$\implies D(2,2)$$

$$V \equiv (x_c = \tfrac{x_1+x_2+x_3}{3}, y_c = \tfrac{y_1+y_2+y_3}{3}) \implies V(\tfrac{6+4+8}{3}, \tfrac{3+7+5}{3})$$

$$\implies V(6,5)$$

Now, the slope of line segment DV:

$tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{5-2}{6-2}$

$= \mathbf{\frac{3}{4}}$

(b) If Veeru can run two units per minute and Dhanno can run three units per minute and they meet at a point M on line segment DV, what will be the ordinate of M? [Ans = $\frac{19}{5}$]

Solution:
The distance between the points D and V can be obtained by distance formula:

$$DV = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$= \sqrt{(5-2)^2 + (6-2)^2} \implies \sqrt{3^2 + 5^2}$$

$$DV = 5 \text{ units}$$

Distance travelled by Dhanno and Veeru in 1 min are 3 units and 2 units respectively which accounts for 5 units.
Since they meet at M so we can consider the travelled distance as the ratio in which the line segment DV is divided.
Therefore,

$$\text{Ordinate } (y) = \frac{m \times y_2 + n \times y_1}{m + n}$$

$$y = \frac{3 \times 5 + 2 \times 2}{3+2}$$

$\boldsymbol{y = \frac{19}{5}}$