Week - 1

Solutions for Practice Assignment-2 Mathematics for Data Science - 1

1 Multiple Select Questions (MSQ):

- 1. (a) $(3+3\sqrt{5})(2-2\sqrt{5}) = (6+6\sqrt{5}-6\sqrt{5}-30) = -24$. Hence this is a rational number.
 - (b) $\frac{\sqrt{64}}{\sqrt{25}} = \frac{8}{5}$. This is a rational number.
 - (c) $\frac{(3+\sqrt{5})}{(3-\sqrt{5})} = \frac{(3+\sqrt{5})(3-\sqrt{5})}{3^2-5} = \frac{(9+6\sqrt{5}+5)}{4} = \frac{7}{2} + \frac{3\sqrt{5}}{2}$. It is an irrational number as $\sqrt{5}$ is irrational.
 - (d) $\sqrt[3]{2} = 2^{\frac{1}{3}}$ is also an irrational number.
- 2. According to given question,
 - $R = \{(\text{Satyajit}, \text{Kalyani}), (\text{Satyajit}, \text{Nalini}), (\text{Kalyani}, \text{Satyajit}), (\text{Nalini}, \text{Satyajit})\}$ $S = \{(\text{Satyajit}, \text{Sukumar}), (\text{Sukumar}, \text{Upendra})\}$ A relation R on a set A is said to be reflexive if $(a, a) \in R$ for all $a \in A$. R is called symmetric if $(a, b) \in R$ implies $(b, a) \in R$, and R is called transitive if (a, b) and (b, c) is in R also implies $(a, c) \in R$.
 - (a) Hence (Satyajit, Sukumar) \in S but (Sukumar, Satyajit) \notin S.
 - (b) (Kalyani, Satyajit) and (Satyajit, Nalini) are in R, but (Kalyani, Nalini) is not in R. Hence, R is not a transitive relation.
 - (c) Also, by observing the elements of S, (Sukumar, Upendra) $\in S$ and (Satyajit, Sukumar) $\in S$ but (Satyajit, Upendra) $\notin S$.
 - (d) (Satyajit, Sukhalata) is neither in R nor in S.
- 3. Suppose $x \in A$, then $x \in A \cup B$. Now it is given that $A \cup B = A \cap B$. Hence $x \in A \cap B$. Hence $x \in B$. Hence every element of A must be in B. Hence, $A \subseteq B$. Now, let $x \in B$. Hence, $x \in A \cup B$. As $A \cup B = A \cap B$, $x \in A \cap B$. Hence $x \in A$. Which implies that every element of B must be in A. Hence $B \subseteq A$. Hence we have A = B.
- 4. $R = \{(A, B) | A \subseteq B\}$
 - (a) For any set A, we have $A \subseteq A$. Hence $(A, A) \in R$. Hence R is reflexive. Moreover, If $(A, B) \in R$ and $(B, C) \in R$, then we have $A \subseteq B \subseteq C$. Hence $A \subseteq C$. Which implies that $(A, C) \in R$. Hence R is transitive.
 - (b) Let $A = \{1, 2\}$ and $B = \{1, 2, 3, 4\}$. Hence $A \subseteq B$, but the converse is not true. Hence $(A, B) \in R$, does not imply $(B, A) \in R$. Hence R is not symmetric.
 - (c) If $A \subseteq B$, and $B \subseteq A$, then A = B. Hence if $(A, B) \in R$ and $A \neq B$, then $(B, A) \notin R$. Hence R is anti-symmetric.
 - (d) As R is transitive, the last option is not correct.

- 5. $R = \{(A, B) \mid \text{ Both } A \text{ and } B \text{ live in the same city } \}.$
 - (a) A and B live in the same city is same as saying B and A live in the same city. Hence $(A, B) \in R$, implies $(B, A) \in R$.
 - (b) As it is symmetric, clearly it is not anti-symmetric.
 - (c) Suppose $(A, B) \in R$ and $(B, C) \in R$. Hence A and B live in same city, and B and C also live in the same city. Hence A and C live in the same city. Hence $(A, C) \in R$. Hence R is transitive.
 - (d) Clearly $(A, A) \in R$, for all A. Hence R is reflexive.
- 6. Elements in set $X = \{100, 200, 300, 400, 500, 600, ...\}$ Elements in set $Y = \{25, 50, 75, 100, 125, 150, 175, 200, ...\}$ Elements in set $Z = \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100, ...\}$
 - A contains the elements of X which are not included in $(Y \cup Z)$
 - B contains the elements of X which are not included in $(X \cup Z)$
 - C contains the elements of Z which are not included in $(X \cup Y)$
 - D represents the set of natural numbers which are divisible by both 25 and 100, but are not perfect squares
 - \bullet E represents the set of natural numbers which are divisible by 25 and are perfect squares but are not divisible by 100
 - F represents the set of natural numbers which are divisible by 100 and are perfect squares, but are not divisible by 25
 - G represents the set of natural numbers which are divisible by 25, 100 and are perfect squares
 - (a) 625 is a perfect square and divisible by 25, but not divisible by 100. Hence it is in E.
 - (b) 2500 is a perfect square which is divisible by both 100 and 25. Hence it is an element of G. Hence G is not an empty set.
 - (c) 200 is divisible by 100 and 25, but not a perfect square. Hence it is not in G.
 - (d) As we have seen earlier 2500 is in G, not in F.
 - (e) As we have seen earlier 2500 is in G.
 - (f) 25 is a perfect square, but it is not divisible by 100. Hence it is not in F.
 - (g) Any number which is divisible by 100 must be divisible by 25. Hence there cannot be any element which is in X but not included in Y and as a result in $Y \cup Z$. Hence A is empty. For the similar reason F is also empty.
- 7. (a) (University, Institute) has the letters 'i', 'e', 'u' in common. Therefore they are in both R and S.
 - (b) (Science, Literature) has only 'e' and 'i' in common, hence they are in R but not in S.

- (c) (Movies, Technology) has only 'o' and 'e' in common, hence they are not in S
- (d) The pair (Company, Literature) is neither in R, nor in S, as they have only one letter in common.

2 Multiple Choice Questions (MCQ):

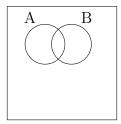
- 8. The codomain for the function is nothing but the set of integers between 50 and 70.
 - (a) Hasan and Rahul has same weight. Hence the function is not one to one.
 - (b) There is no student in the list whose weight is 60, but 60 is in the codomain. Hence the function is not onto. The range of the function is $\{51, 54, 55, 62\}$, which is not same as the codoamin of the function.
 - (c) Clearly the function is neither one to one nor onto.
 - (d) A function is called bijective if it is both one to one and onto, which is not the case here. Hence the function is not bijective.
- 9. Possible subsets of two elements = $\{(a, b), (a, c), (a, d), (b, c), (b, d), (c, d)\}$ Number of subsets = 6.
- 10. Elements of $A = \{3, 6, 9, 12\}$ Elements of $B = \{4, 8, 12\}$

Elements of $C = \{5, 10\}$

The element 12 is common in both sets A and B. There is no common element between B and C.

Hence diagram A is accurate representation.

11. $(B \cup A)^c$ is the region out side both the circles. Hence it does not have any element common with A. Hence $A \cap (B \cup A)^c$ is empty set.



12. As even numbered students have taken Physics, odd numbered students have chances of taking none of the subjects. Apart from this, in this set there should not be any student whose number is divisible by either 5 or 7.

Hence the elements are $\{1, 3, 9, 11, 13, 17, 19, 23, 27, 29, 31, 33, 37, 39, 41, 43, 47, 51, 53, 57, 59, 61, 67, 69, 71, 73, 79, 81, 83, 87, 89, 93, 97, 99, 101, 103, 107, 109, 111, 113, 117\}$

Hence the cardinality of this set is 41.

Another procedure:

Number of students who have taken physics is 60.

Number students who have taken chemistry is 24.

Number of students who have taken maths is 17.

The students who have taken both physics and chemistry are the students who have been numbered as multiples of both 2 and 5, i.e. multiples of 10. Hence the number of students who have taken both physics and chemistry is 12.

The students who have taken both physics and maths are the students who have been numbered as multiples of both 2 and 7, i.e. multiples of 14. Hence the number of students who have taken both physics and chemistry is 8.

The students who have taken both maths and chemistry are the students who have been numbered as multiples of both 7 and 5, i.e. multiples of 35. Hence the number of students who have taken both physics and chemistry is 3.

The students who have taken all the subjects are the students who have been numbered as the multiples of all the three numbers 2, 5 and 7, i.e. multiples of 70. Hence the number of students who have taken all the three subjects is 1.

Hence the number of students who have not taken at least one subject is= 60 + 24 + 17 - 12 - 8 - 3 + 1 = 79. Hence the number of students who have not taken any subject is= 120 - 79 = 41.

13. It is clear that f(1) = f(3) = f(-1) = 0 as f(x) = 0, for all odd integer x. So f is not one to one. Moreover, for any integer y, f(2y) = y as 2y is always an even integer. Hence f is onto.

Hence f is onto but not one to one.