Week - 5
Practice Assignment
Algebra of polynomials
Mathematics for Data Science - 1

1 Multiple Choice Questions (MCQ):

1. Let x be the number of years since the year 2000 (i.e., $x = 0$ denotes the year 2000). The total amount of profit (in ₹) on books in a shop is given by the function $T(x) = 5x^3 + 3x + 1$. The shop sells books of four languages English, Bengali, Hindi, and Tamil. The profits from selling English and Bengali books are given by $E(x) = 3x^3 - 5x^2 + x$ and $B(x) = x^2 + 4x + 5$ respectively. The profit from selling Hindi and Tamil books are found to be the same.

   (a) Which of the following polynomial functions represents the profit from selling Tamil books?

   - ○ $2x^3 + 4x^2 - 2x - 4$
   - ○ $x^3 - 2x^2 - x + 2$
   - ○ $\boldsymbol{x^3 + 2x^2 - x - 2}$
   - ○ $2x^3 - 4x^2 - 2x + 4$

   (b) In which year was the profit from Hindi books zero?

   - ○ **2001**
   - ○ 2002
   - ○ 2004
   - ○ 2010

Solution:

(a) The total profit from selling English and Bengali books is= $E(x) + B(x) = (3x^3 - 5x^2 + x) + (x^2 + 4x + 5) = 3x^3 - 4x^2 + 5x + 5$. Hence the total profit from selling Hindi and Tamil books is= $T(x) - (3x^3 - 4x^2 + 5x + 5) = 5x^3 + 3x + 1 - 3x^3 + 4x^2 - 5x - 5 = 2x^3 + 4x^2 - 2x - 4$.
As the profit from selling Hindi and Tamil books are found to be the same, the profit from selling Tamil books is= $\frac{1}{2}(2x^3 + 4x^2 - 2x - 4) = x^3 + 2x^2 - x - 2$

(b) Profit from selling Hindi books (which is same as the profit from selling Tamil books) is $x^3 + 2x^2 - x - 2$.

$$x^3 + 2x^2 - x - 2 = x^2(x + 2) - 1(x + 2) = (x + 2)(x^2 - 1) = (x + 2)(x + 1)(x - 1)$$

So the profit will be zero if $(x + 2)(x + 1)(x - 1) = 0$, i.e., at $x = -2, -1, 1$ the profit can be 0. But in this context, x cannot be negative. So $x = 1$ is the only possibility. Hence in the year 2001 the profit from Hindi books was zero.

2. Find the quadratic polynomial which when divided by x, $x-1$, and $x+1$ gives the remainders 7, 14, and 8 respectively.

- ◯ $4x^2 - 3x + 7$
- ◯ $x^2 + 7x + 7$
- ◯ $7x^2 + x + 7$
- ◯ $\mathbf{4x^2 + 3x + 7}$

Solution: Let the quadratic polynomial which is satisfying the given condition be $p(x) = ax^2 + bx + c$.
When it is divided by x the remainder is 7. It implies that if we substitute $x = 0$ in $p(x)$ we will get 7, i.e., $p(0) = 7$. Similarly we have $p(1) = 14$ and $p(-1) = 8$.
Hence we have the following equations:

$$\begin{aligned}
p(0) &= a(0)^2 + b(0) + c \\
&= c \\
&= 7 \\
p(1) &= a.(1)^2 + b.1 + c \\
&= a + b + c \\
&= 14 \\
p(-1) &= a(-1)^2 + b(-1) + c \\
&= a - b + c \\
&= 8
\end{aligned}$$

So, we have $c = 7$, and substituting c in the second and third equation we get, $a + b = 7$, and $a - b = 1$. By solving these two equations we get $a = 4$ and $b = 3$.
Hence the quadratic polynomial is $4x^2 + 3x + 7$.

3. Box A has length x unit, breadth $(x+1)$ unit, and height $(x+2)$ unit. Box B has length $(x+1)$ unit, breadth $(x+1)$ unit, and height $(x+2)$ unit. There are two more boxes C and D of cubic shape with side x unit. The total volume of A and B is y cubic unit more than the total volume of C and D. Find y in terms of x.

   - ◯ $x^3 + 7x^2 + 7x + 2$
   - ◯ $\mathbf{7x^2 + 7x + 2}$
   - ◯ $7x^2 - 7x - 2$
   - ◯ $x^3 + 7x^2 - 7x - 2$

   Solution: The volume of box A is $x(x+1)(x+2) = x^3 + 3x^2 + 2x$ cubic unit.
   The volume of box B is $(x+1)(x+1)(x+2) = (x^2+2x+1)(x+2) = x^3 + 4x^2 + 5x + 2$ cubic unit.
   The volume of box C and D is x^3 cubic unit each. So the total volume of A and B is $2x^3 + 7x^2 + 7x + 2$ and the total volume of C and D is $2x^3$.
   Hence $y = (2x^3 + 7x^2 + 7x + 2) - 2x^3 = 7x^2 + 7x + 2$.

4. The population of a bacteria culture in laboratory conditions is known to be a function of time of the form $p(t) = at^5 + bt^2 + c$, where p represents the population (in lakhs) and t represents the time (in minutes). Suppose a student conducts an experiment to determine the coefficients a, b, and c in the formula and obtains the following data:

- $p(0) = 3$
- $p(1) = 5$
- $p(2) = 39$

Which of the following options is correct?

○ $p(t) = 3t^5 - t^2 + 3$

○ $p(t) = 4t^5 - 2t^2 + 3$

○ $\boldsymbol{p(t) = t^5 + t^2 + 3}$

○ $p(t) = 39t^5 + 5t^2 + 3$

Solution: Given that, $p(t) = at^5 + bt^2 + c$.
$p(0) = c = 3$
$p(1) = a + b + c = 5$, putting $c = 3$, we get $a + b = 2$.
$p(2) = a(2)^5 + b(2)^2 + c = 32a + 4b + c = 39$, substituting $c = 3$, we get $32a + 4b = 36$, implies, $8a + b = 9$ (cancelling 4 from both sides)
By solving these two equations we get $a = 1$, and $b = 1$.
Hence, $p(t) = t^5 + t^2 + 3$.

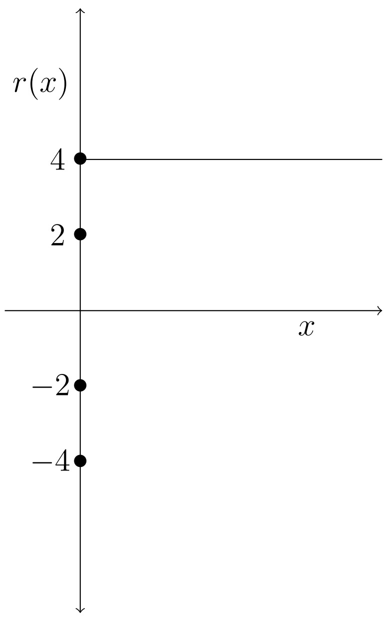
5. If the polynomials $x^3 + ax^2 + 5x + 7$ and $x^3 + 2x^2 + 3x + 2a$ leave the same remainder when divided by $(x - 2)$, then the value of a is:

- ◯ $\frac{3}{2}$
- ◯ $\boldsymbol{-\frac{3}{2}}$
- ◯ $\frac{5}{2}$
- ◯ $-\frac{5}{2}$

Solution: Given that both the polynomials leave same remainder when divided by $(x - 2)$. By substituting $x = 2$ both the polynomial should have same value.
By substituting $x = 2$ in $x^3 + ax^2 + 5x + 7$, we get $8 + 4a + 10 + 7 = 4a + 25$.
By substituting $x = 2$ in $x^3 + 2x^2 + 3x + 2a$, we get $8 + 8 + 6 + 2a = 2a + 22$.
So we have,

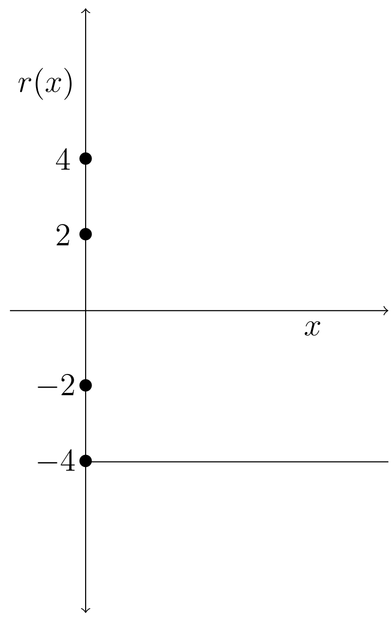
$$4a + 25 = 2a + 22$$
$$2a = -3$$
$$a = -\frac{3}{2}$$

6. Let $r(x)$ be a polynomial function which is obtained as the remainder after dividing the polynomial $2x^3 + x^2 - 5$ by the polynomial $2x - 3$. Choose the correct option which represents the polynomial $r(x)$ most appropriately.



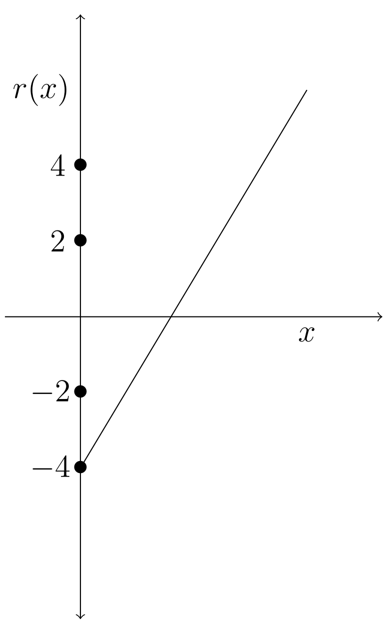
◯ **Option A**

Fig P-6.2



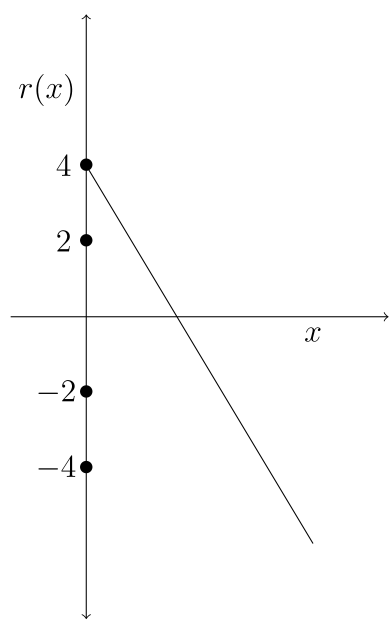
◯ Option B

Fig P-6.3



◯ Option C

Fig P-6.4



◯ Option D

Fig P-6.5

Solution We get 4 as the remainder if $2x^3 + x^2 - 5$ is divided by the polynomial $2x - 3$.

$$2x^3 + x^2 - 5 = (2x - 3)(x^2 + 2x + 3) + 4$$

Hence $r(x) = 4$, which is a constant polynomial. Hence, the first option is the correct.

2 Multiple Select Questions (MSQ):

7. By dividing a polynomial $p(x)$ with another polynomial $q(x)$ we get $h(x)$ as the quotient and $r(x)$ as the remainder.

   (a) The maximum degree of $r(x)$ can be,

   - ◯ $deg\ p(x)$
   - ◯ $deg\ (p(x)) - 1$
   - ◯ $deg\ q(x)$
   - ◯ $\boldsymbol{deg\ (q(x)) - 1}$

   (b) If $deg\ p(x) < deg\ q(x)$, then choose the set of correct answers:

   - ◯ $\boldsymbol{h(x) = 0}$
   - ◯ $deg\ h(x) = deg\ q(x)$
   - ◯ $deg\ r(x) = deg\ q(x)$
   - ◯ $\boldsymbol{deg\ r(x) = deg\ p(x)}$

Solution:

(a) The degree of the remainder $r(x)$ should be strictly less than the degree of the polynomial $q(x)$. So the maximum degree of $r(x)$ is $deg\ (q(x)) - 1$.

(b) If $deg\ p(x) < deg\ q(x)$, then quotient will be zero polynomial, hence $deg\ h(x) = 0$. The remainder will be $p(x)$ itself. So $deg\ r(x) = deg\ p(x)$.

3 Numerical Answer Type (NAT):

8. An open box can be made from a piece of cardboard of length $7x$ unit and breadth $5x$ unit, by cutting squares of side x unit out of the corners of the rectangular cardboard, then folding up the sides as shown in the Figure P-6.1.

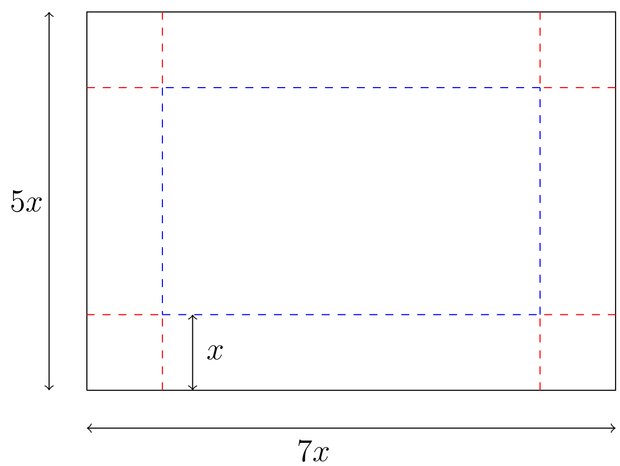


Figure P-6.1

(a) What will be the coefficient of x^3 in the polynomial representing the volume of the box? [Answer:15]

(b) What will be the coefficient of x^2 in the polynomial representing the volume of the box? [Answer:0]

Solution: As the sides of the piece of the cardboard has been cut out, the length of the box made will be $7x - (x + x) = 5x$ unit and the breadth of the box made will be $5x - (x + x) = 3x$ unit, and the height will be x unit.
Hence the volume of the box will be $5x \times 3x \times x = 15x^3$ cubic unit.

(a) The coefficient of x^3 in the polynomial representing the volume of the box is 15.

(b) The coefficient of x^2 in the polynomial representing the volume of the box is 0.