Mathematics for Data Science - 1
Practice Assignment
Week 10

1 MULTIPLE CHOICE QUESTIONS:

1. Suppose $R = \{(1,3),(3,4),(4,5)\}$ is a relation on the set $\{1,3,4,5,7\}$. Which of the following represents the transitive closure of R? [Ans: c]

   (a) $\{(1,3),(3,4),(1,4),(4,5),(3,5),(5,1)\}$
   (b) $\{(1,3),(3,4),(4,5),(3,5),(1,5),(4,3)\}$
   (c) $\{(1,3),(3,4),(4,5),(3,5),(1,5),(1,4)\}$
   (d) $\{(1,3),(3,1),(3,4),(4,3),(4,5),(5,4)\}$

Given $\Rightarrow$ $R = \{(1,3),(3,4),(4,5)\}$

$\Rightarrow$ for Transitive closure if $(1,3)$ and $(3,4)$ are in relation $(1,4)$ should also be in Relation.

$\Rightarrow$ similarly $(3,4)$ and $(4,5) \in R$

$(3,5) \in R$

$\Rightarrow$ $R = \{(1,3)(3,4),(4,5),(1,4),(3,5)\}$

Now $(1,4)$ and $(4,5) \in R$

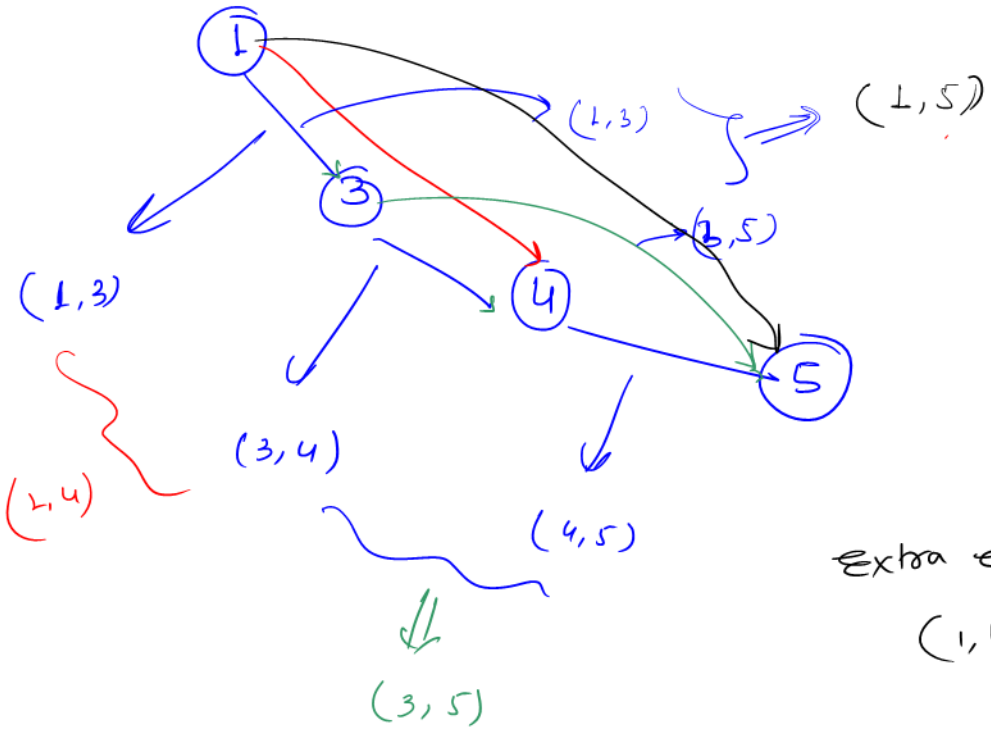
$(1,5) \in R$

Therefore Transitive closure of R will be:

$\{(1,3),(3,5),(4,5),(1,4),(3,5),(1,5)\}$

Second Method: If we solve using graphs

(1, 3), (3, 4), (4, 5)



extra edges:

(1, 4), (3, 5), (1, 5)

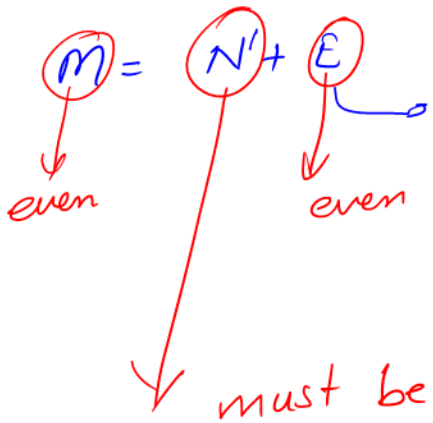
2. An undirected graph G has 31 vertices. The sum of the degrees of all the vertices in G is M. The number of vertices of odd degree in G is N. Which of these values are possible for M and N? [Ans: c]

(a) $M = 98,\ N = 11$
(b) $M = 103,\ N = 10$
(c) $M = 98,\ N = 10$
(d) $M = 103,\ N = 11$

⇒ M is sum of degrees of all vertices therefore M should be even.

⇒ Let E be the sum of degrees of all vertices having even degrees, so E also be even.

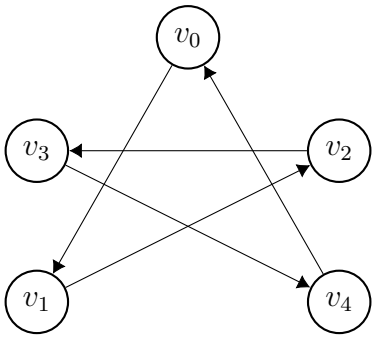
⇒ Let N′ be the sum of all vertices having odd degrees then we can write as



Now if N′ is even which is sum of odd degree vertices therefore N (No. of odd degree vertices) should be even.

Only option (c) satisfies the condition.

3. Consider the graph given below



If A is the adjacency matrix of G, then which of the following represents A^2? [Ans: b]

(a)

	v_0	v_1	v_2	v_3	v_4
v_0	0	1	0	0	0
v_1	0	0	1	0	0
v_2	0	0	0	1	0
v_3	0	0	0	0	1
v_4	1	0	0	0	0

(b)

	v_0	v_1	v_2	v_3	v_4
v_0	0	0	1	0	0
v_1	0	0	0	1	0
v_2	0	0	0	0	1
v_3	1	0	0	0	0
v_4	0	1	0	0	0

(c)

	v_0	v_1	v_2	v_3	v_4
v_0	0	1	0	0	1
v_1	1	0	1	0	0
v_2	0	1	0	1	0
v_3	0	0	1	0	1
v_4	1	0	0	1	0

(d)

	v_0	v_1	v_2	v_3	v_4
v_0	0	0	1	1	0
v_1	0	0	0	1	1
v_2	1	0	0	0	1
v_3	1	1	0	0	0
v_4	0	1	1	0	0

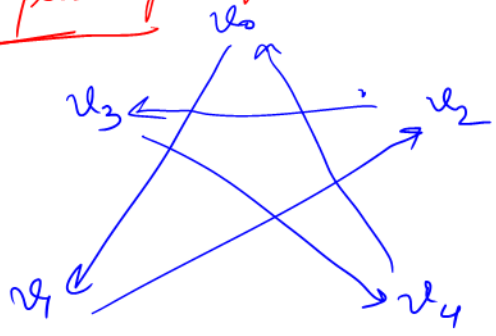
$\Rightarrow$ we can find first A and then A^2

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$A^2 = A \times A$.

Second Method:

A^2 represents the path of Length 2.



① vertices have path of length 2 from v_0 ⇒

$v_0 \to v_4 \to v_2$

which means only v_2 is reachable in path of length 2 from v_0.

② v_1 ⇒ $v_1 \to v_2 \to v_3$, $v_1 \to v_3$ } only v_3

③ v_2 ⇒ $v_2 \to v_3 \to v_4 \Rightarrow v_2 \to v_4$ } only v_4

④ v_3 ⇒ $v_3 \to v_4 \to v_0$ } only v_0.

⑤ v_4 ⇒ $v_4 \to v_0 \to v_1$ } only v_1

Therefore

$A^2 =$

	v_0	v_1	v_2	v_3	v_4
v_0			1		
v_1				1	
v_2					1
v_3	1				
v_4		1			

2 MUTIPLE SELECT QUESTIONS:

4. Suppose $G = (V, E)$ is a directed graph, where $V = \{1, 2, 3, 4, 6, 7, 12\}$. There is an edge from a to b ($a \neq b$), that is, $(a, b) \in E$ if and only if $a|b$ (a divides b). [Ans: a,b,d]

   Which of the following can be a topological sorting of the graph G ?

   (a) $1, 2, 4, 3, 6, 12, 7$

   (b) $1, 7, 2, 4, 3, 6, 12$

   (c) $7, 1, 2, 4, 3, 6, 12$

   (d) $1, 2, 3, 4, 7, 6, 12$

If $V = \{1, 2, 3, 4, 6, 7, 12\}$

$\Rightarrow$ 1 devides all other elements therefore there will edges from 1 to all other elements.

$\Rightarrow$ 2 devides 4, 6, and 12 so there will be edges from 2 to 4, 6 and 12 respectively.

$\Rightarrow$ 3 devides 6 and 12

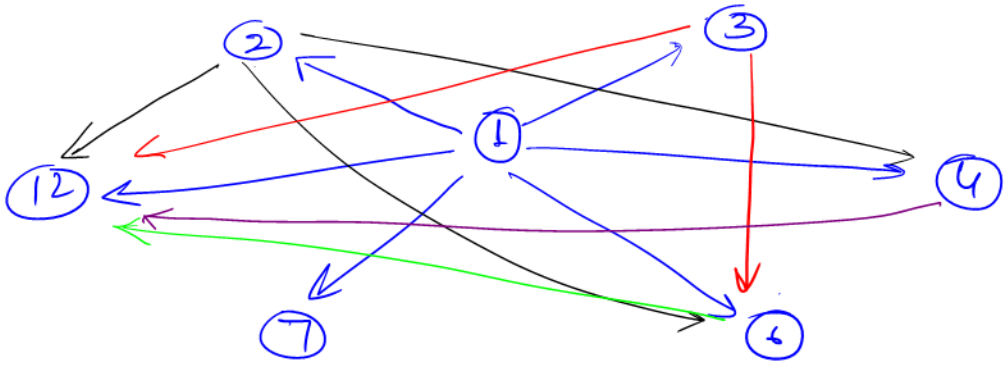
$\Rightarrow$ 4 devides only 12

$\Rightarrow$ 6 devides only 12

$\Rightarrow$ 7 and 12 will not devide any of the elements

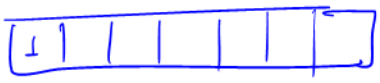
So no edges from 7 and 12.

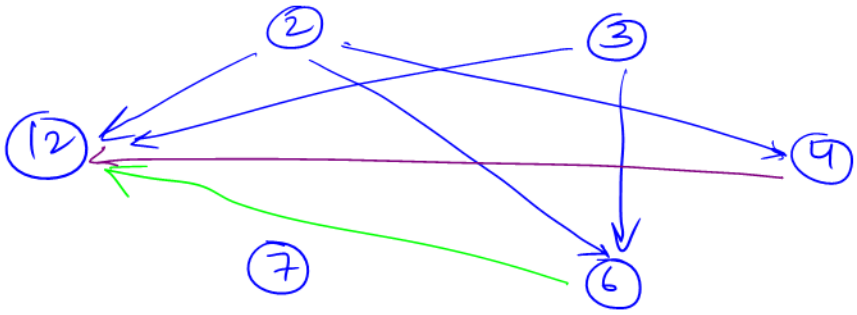
Therefore if represent as graph:



Now ① has indegree zero and if we remove ①.

Topological order.



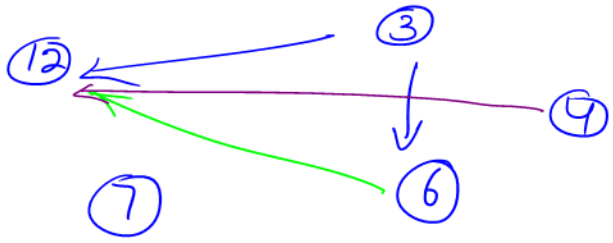


Now can remove any vertex among 7, 2, and 3. Therefore we will match with options. In option ⓒ the sequence starts with 7 which is not possible.

Now we can check along option ⓐ –

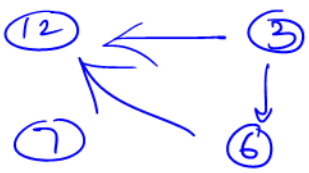
1 2 4 3 6 12 7

removing ②

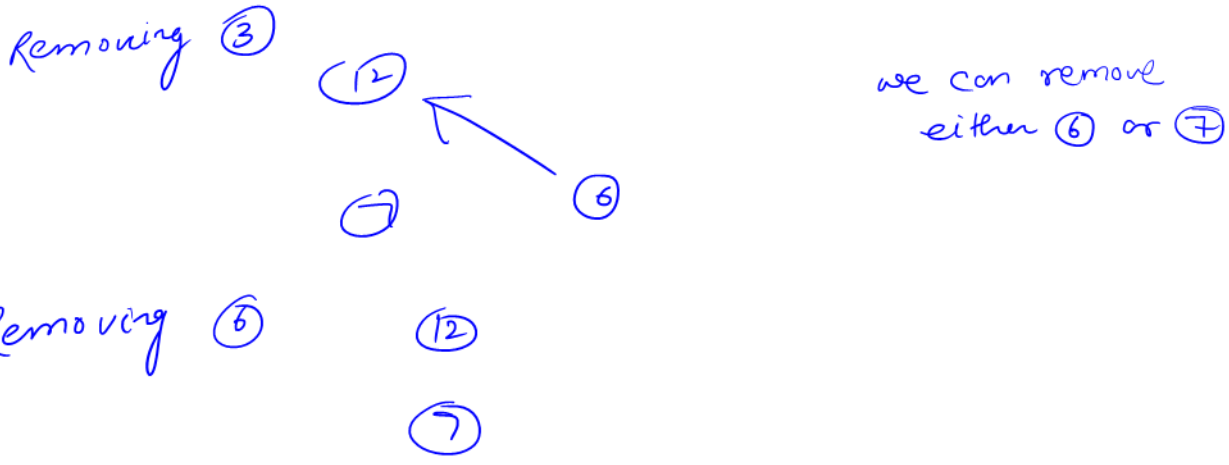


we can remove either 3 or 4 or 7

removing ④



we can remove either 7 or 3



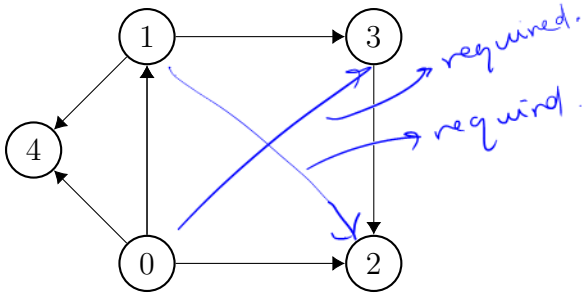
Removing 6

Then order 1 2 4 3 6 7 12 is possible.

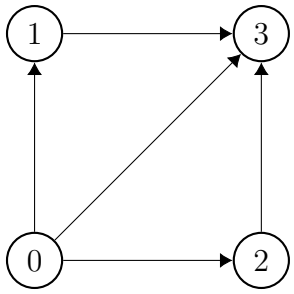
We can also check for option (b) and (d) similarly.

5. Which of the following graphs represent its own transitive closure? [Ans: b,c]

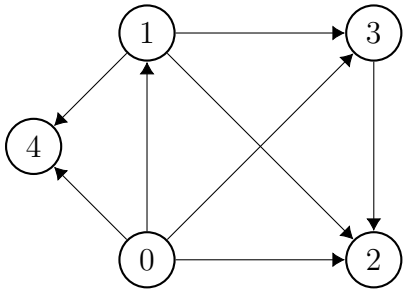
(a)



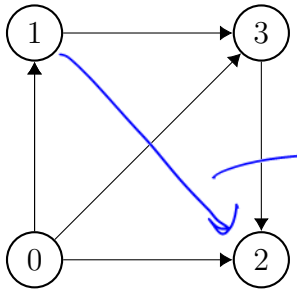
(b)



(c)



(d)



option (a) (1, 3) and (3, 2) are in R.
Therefor (1, 2) should also be in R which is not there.

option (b)

(0, 1) and (1, 3) is in R and therefore (0, 3) should be in R which there.

(0, 2) and (2, 3) ∈ R and (0, 3) also in R.

There fore option (b) is correct.

option (c):

The required edges in option (a) to make transitive are presented in option (c).

6. Which of the followings options are correct? [Ans: b,d]

(a) If G is a graph with n vertices then length of a path in G is bounded by $n-2$.

(b) If G is a directed graph, then the sum of the in-degrees of all the vertices is equal to the sum of out-degrees of all the vertices.

(c) $A = \begin{matrix} \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix}\begin{matrix} \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{matrix}$ can represent the adjacency matrix of an undirected graph G.

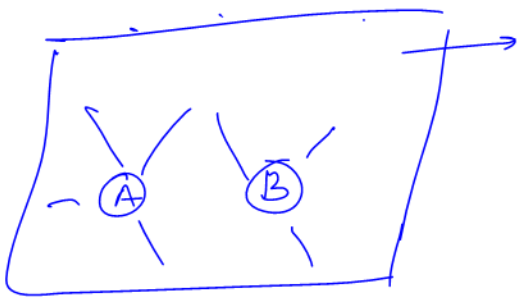
(d) If G is an undirected graph with exactly two vertices of odd degree, then those two vertices are connected in G.

option (a) ⇒ bound is $(n-1)$ so incorrect.

option (b) every edge contributes to one outdegree and one indegree in a directed graph. Therefore correct.

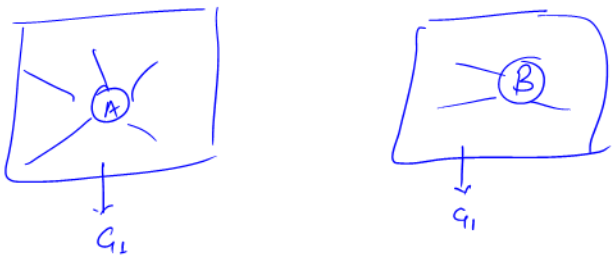
option (c) for undirected graph A_{ij} should be equal to A_{ji}. Here $A_{23} \neq A_{32}$.

Option (d) Let a graph G which have only 2 odd degree vertex.



A connected graph G.

Now if we think that G is not connected then we will get minimum two separate graph let us say G_1 and G_2 and each graph will have one vertex with odd degree.



Now G_1 is also a graph and it should have the properties of graph like sum of degrees of all vertices should be even.

But if see in G_1 the sum of all vertices will be odd which is not possible. Therefore the graph G should be connected.

7. Which of the following options are correct? [Ans: b,c]

(a) If $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, then $A^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

(b) If $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, then $A^n = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ for all $n > 2$.

} can be calculated

(c) If A is a 3×3 matrix and $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then $AI = IA = A$.

(d) If A is a 3×3 matrix and $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then $AI \neq A$.

Soln: Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow AI = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$= A$.

Now $IA = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = A$ Therefore $AI = IA = A$.

Therefore option (C) is correct and option (D) is wrong.

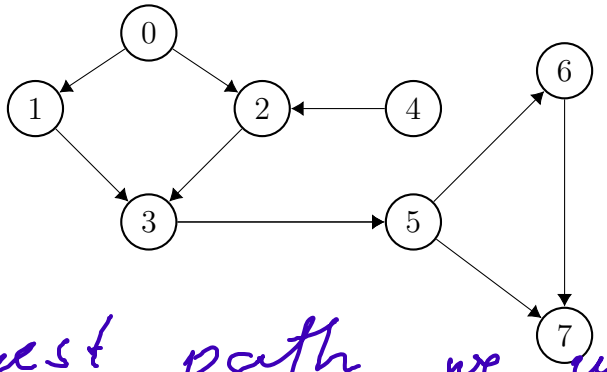
3 NUMERICAL ANSWER TYPE:

8. Let G be an undirected graph with 8 vertices and all vertices have degree 4. How many edges are there in the graph G? [Ans: 16]

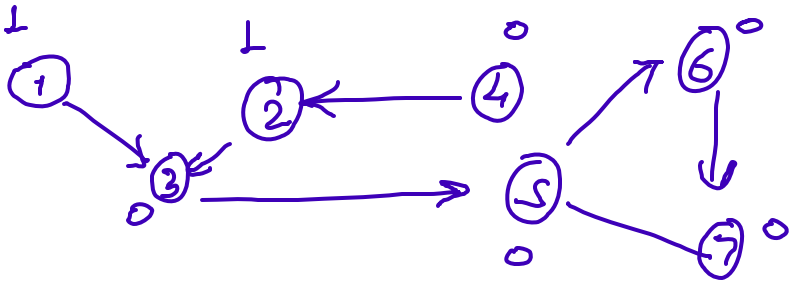
If there are 8 vertices and each vertex has 4 degree then the sum of all degrees of all vertices = 8 × 4 = 32

As we know the number of edges is half of the sum of degrees of all vertices, therefore 16 edges.

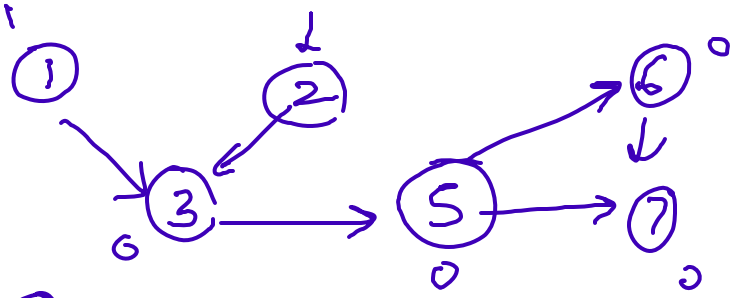
9. The longest path of the below DAG contains x edges. Find x [Answer: 5]



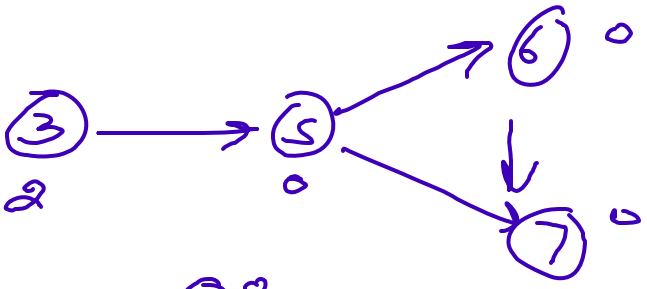
So for longest path we will first remove 0.



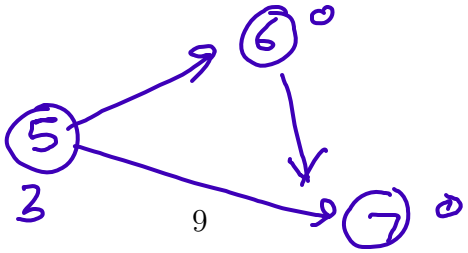
remove 4



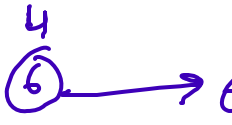
remove 1 and 2



remove 3



remove 5



remove 6 → L.P = 5

Ans.