

8: More Closure Properties

Scribe: Piyush Mohite

More Closure Properties

A tuple (S, \bullet, e) is known as a *monoid*, if:

- S is a set.
- \bullet : is *associative*, i.e., $\forall u, v, w \in S, (w \bullet u) \bullet v = w \bullet (u \bullet v)$
- $e \in S$ is the identity element, i.e., $\forall w \in S, e \bullet w = w \bullet e = w$

Eg: (Σ^*, \bullet, e) is a monoid.

Eg: $(\mathbb{N}, +, 0)$ is a monoid. [Also commutative.]

Eg: $(\{e, a\}, \bullet, e)$ is *NOT* a monoid. [Because, if we consider ' a ' and ' a ', we get the concatenation $a \bullet a = aa$, which is not a part of the set $\{e, a\}$, since it has to be $SxS \rightarrow S$]