Objective

The goal of this assignment is to design, implement, train, and evaluate a Conditional Random Field (CRF) model for Named Entity Recognition (NER). This involves both theoretical derivation and practical coding. By the end of this assignment, you should have a complete understanding of how CRFs work and how they can be applied to sequence labeling tasks like NER.

Detailed Instructions

1. Derive the Mathematical Formulation of a Linear-Chain CRF

Objective: Understand the theoretical foundation of CRFs. What to Do:

- Start by explaining what Conditional Random Fields are and why they are used for sequence labeling tasks.
- Derive the conditional probability distribution formula for a linearchain CRF. Explain how it models the probability of a sequence of labels given a sequence of inputs.
- Define key components such as:
 - **Feature functions:** Functions that capture relationships between input data, labels, and transitions.
 - Weights: Parameters that determine the importance of each feature.
 - Partition function: A normalization factor that ensures probabilities sum to 1.
- Explain inference in CRFs using algorithms like the Viterbi algorithm for finding the most likely sequence of labels.

2. Implement the CRF Model Using Python

Objective: Build a linear-chain CRF model from scratch using Python libraries like NumPy and SciPy. What to Do:

- Write code to represent key components of a CRF:
 - Feature extraction: Extract features from input data (e.g., words, POS tags, capitalization).
 - Parameter initialization: Initialize weights for feature functions.
 - Log-likelihood computation: Implement the log-likelihood function and its gradient.
 - Inference: Use dynamic programming (e.g., Viterbi algorithm) to find the most probable label sequence.
- Avoid using pre-built CRF libraries. Instead, focus on implementing the core logic yourself.

3. Train the Model on a Provided Dataset

Objective: Train your CRF model using real-world labeled data. What to Do:

- Use the dataset provided on the piazza.
- Optimize model parameters using numerical optimization techniques such as gradient descent or L-BFGS. Use libraries like SciPy for optimization if needed.

4. Evaluate the Model's Performance

Objective: Measure how well your model performs on unseen data. What to Do:

- Evaluate your model on the test set using standard metrics:
 - **Precision:** The proportion of correctly predicted entities out of all predicted entities.
 - **Recall:** The proportion of correctly predicted entities out of all actual entities in the data.
 - **F1-score:** The harmonic mean of precision and recall. This provides a balanced evaluation metric.
- Provide a detailed analysis of your results:

- Highlight which entity types (e.g., PERSON, LOCATION) perform well and which do not.
- Discuss possible reasons for errors or misclassifications.

5. Write a Report

Summarize your work in a report that includes:

- 1. An introduction explaining Named Entity Recognition and Conditional Random Fields.
- 2. A detailed explanation of your mathematical derivations.
- 3. A description of your implementation approach, including challenges faced and how you overcame them.
- 4. Results from training and evaluation, along with insights gained from analyzing your model's performance.

Expected Deliverables

You are expected to submit:

- 1. A Python script or notebook containing:
 - (a) Your implementation of the CRF model from scratch.
 - (b) Code for training and evaluating the model on the dataset.
- 2. A report summarizing your work as described above.

Additional Notes

- 1. Ensure that your code is well-documented with comments explaining each step.
- 2. If you use any external resources or libraries beyond NumPy/SciPy, cite them in your report.
- 3. Pay attention to computational efficiency when implementing algorithms like Viterbi or gradient computation.

Example Input/Output

To clarify expectations, here's an example:

Input: A sentence represented as tokens:

```
["Barack", "Obama", "visited", "India", "last", "week"]
```

Output: Predicted entity tags for each token:

```
[("Barack", "B-PER"), ("Obama", "I-PER"), ("visited", "O"), ("India", "B-LOC"), ("last", "O"), ("week", "O")]
```

Good luck!