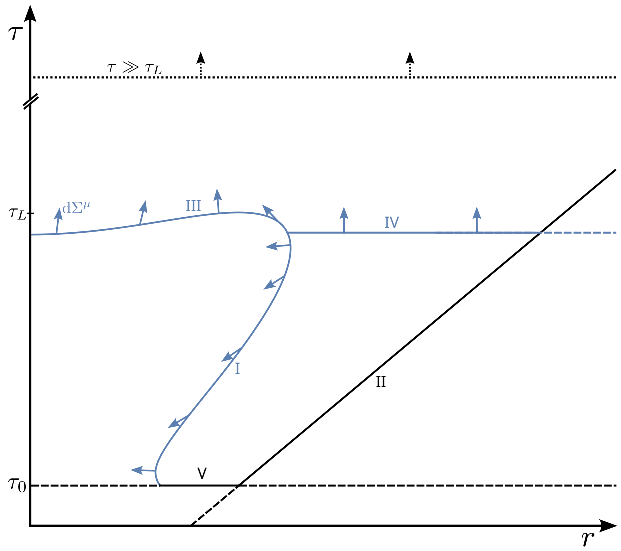

FIG. 1: Illustrative cartoon of a freeze-out surface. The curves I and III define the constant temperature hypersurface. Curve IV is one possible extension of III to form a Cauchy surface. The black line II is the outward-going part of the lightcone originating from the outmost point of the fireball at its creation time τ_0. Curve V indicates the corona of the fireball at its creation time. The lifetime of the fireball is indicated by τ_L. The arrows indicate the direction of the normal vectors $n^\mu \propto \mathrm{d}\Sigma^\mu$.

We now discuss how the conservation laws in Equation 6 and Gauss's theorem allow to relate the integrations over two different Cauchy surfaces with equivalent results. This enables a choice more convenient for calculating and modeling the freeze-out's complicated dynamics.

Let us consider a simplified version of a freeze-out, depicted in Figure 1. The fireball lies in the region enclosed by curves I and III; the free streaming phase is assumed to be everywhere outside these curves. The curve V indicates the region of a possible corona at the creation of the fireball where the particle density might be non-vanishing but is assumed to be very small. The black line II is the light cone originating from the outermost point of the fireball and its corona at the creation time τ_0 of the fireball. Since most of the energy of the collision is deposited inside the fireball, we assume that the contributions from the corona to the final hadron spectra are negligible. The space-time region on the right of this light cone line is causally disconnected from the evolution of the fireball. Therefore we assume the distribution function to be zero on the light cone and in the region outside. We will show the equivalence of the integration in the asymptotic regime close to the detector at some large time $\tau \gg \tau_L$, with τ_L being the lifetime of the fireball, and on the freeze-out surface (I+III), in two steps: First, we consider a space-time volume Ω enclosed by the time axis, the hypersurface at constant large time $\tau \gg \tau_L$, the light cone, and the hypersurface III+IV. Inside this space-time volume, the hadrons are free-streaming. Therefore we can write

$$\int_\Omega \nabla_\mu J_p^\mu = 0. \tag{7}$$

Applying Gauss's theorem and using that $f(x,p) = 0$ on the time axis and the light cone, we can write

$$\int_{\Sigma(\tau\gg\tau_L)} \mathrm{d}\Sigma_\mu J_p^\mu - \int_{\Sigma_{\mathrm{IV}}\cup\Sigma_{\mathrm{III}}} \mathrm{d}\Sigma_\mu J_p^\mu = 0. \tag{8}$$

Note that in both integrals over Cauchy surfaces we take the normal vector to be future oriented. Making use of Gauss's theorem a second time for the space-time volume enclosed by I, II, and IV, lets us write

$$\int_{\Sigma(\tau\gg\tau_L)} \mathrm{d}\Sigma_\mu J_p^\mu = \int_{\Sigma_{\mathrm{I}}} \mathrm{d}\Sigma_\mu J_p^\mu + \int_{\Sigma_{\mathrm{III}}} \mathrm{d}\Sigma_\mu J_p^\mu. \tag{9}$$

Here the normal vector $n^\mu \propto \mathrm{d}\Sigma^\mu$ is still future oriented in region III and oriented to the inside (as illustrated in Figure 1), in region I.

We want to emphasize two important points at this step of the calculation: Considering the first in-between step given in Equation 8, we want to stress that the integrand of the second integration is strictly positive. This originates again from the integration being over a Cauchy hypersurface, which corresponds to an interpretation of integrating a particle density to obtain the particle number. A second important point to make here is that this interpretation breaks down when considering the second step

$$E_p \frac{\mathrm{d}N}{\mathrm{d}^3p} = \int_{\Sigma_{\mathrm{III}}} \mathrm{d}\Sigma_\mu J_p^\mu + \int_{\Sigma_{\mathrm{I}}} \mathrm{d}\Sigma_\mu J_p^\mu, \tag{10}$$

since the hypersurface, I is no longer part of a Cauchy surface. Vividly the interpretation breaks down because surface I is no longer one instant of time, which means that the integrand cannot be seen as density anymore. Note that in Equation 10 we took the orientation of the hypersurface I to point inwards. This follows the standard prescription for regions of spacetime and leads to a normal vector that is continuous at the intersection point between regions I and III, where the normal vector is light-like. Moreover, one can combine the surfaces Σ_{III} and Σ_{I} so that

$$E_p \frac{\mathrm{d}N}{\mathrm{d}^3p} = \int_{T=const.} \mathrm{d}\Sigma_\mu J_p^\mu. \tag{11}$$

The same reasoning can be applied to the two-particle correlation function, and with little additional effort for any n-particle correlation. For simplicity, we will only demonstrate this for the two-particle correlation function. The two-particle correlation function that can be measured experimentally is defined as

$$C_2(p,k) = E_p E_k \left\langle \frac{dN}{d^3p} \frac{dN}{d^3k} \right\rangle. \tag{12}$$