29.04.

$$j(y) = \text{"}\delta(y \in \Sigma_{\text{FreezeOut}})\text{"}\ \phi_{SF}(r(\alpha), \tau(\alpha))$$

← superfluid

$$\phi(x) = \int d^4y\, G_{ret}(x,y)\, j(y)$$

$$= \underbrace{\int_0^{2\pi} d\varphi \int_0^{\infty} d\eta \int_0^{\pi} d\alpha\, r(\alpha)}_{=\int_{\Sigma_{\text{FreezeOut}}} d^3y} \left[\Theta(x^0-y^0)\left(D(x-y) - D(y-x)\right)\right]\Big|_{\Sigma_{\text{FreezeOut}}} \cdot \phi_{SF}(r(\alpha), \tau(\alpha))$$

$$D(x-y) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega_{\vec p}} e^{-i(\omega_{\vec p}(x^0-y^0) - \vec p(\vec x - \vec y))}$$

$$e^{-i(\omega t - \vec p \vec y)}\Big|_{\Sigma} = e^{-i(\omega \tau(\alpha)\cosh\eta - p_1 y_1 - p_2 y_2 - p_3 y_3)}$$

$$= e^{-i\left(\tau(\alpha)(\omega\cosh\eta - p_3\sinh\eta) - r(\alpha)(p_1\cos\varphi + p_2\sin\varphi)\right)}$$

$$p_1 = p_\perp \cos\psi, \quad p_2 = p_\perp \sin\psi$$

$$= e^{-i\left(\tau(\alpha)(\omega\cosh\eta - p_3\sinh\eta) - r(\alpha) p_\perp \cos(\varphi - \psi)\right)}$$

30.04.

$$\int \frac{d^4p_{kart}}{(2\pi)^4} \delta(p^2 - m^2)\Theta(p_0) f(p_\mu) = \int \frac{dp_0}{2\pi} \int \frac{d^3p_{kart}}{(2\pi)^3} \frac{1}{2\omega_{\vec p}} \left(\delta(p_0 - \omega_{\vec p}) + \cancel{\delta(p_0 + \omega_{\vec p})}\right) \Theta(p_0) f(p_\mu) = \frac{1}{2\pi} \int \frac{d^3p_{cart}}{(2\pi)^3} \frac{1}{2\omega_{\vec p}} f(p_\mu)\Big|_{p_0 = \omega_{\vec p}}$$

$$\int \frac{d^4p_{kart}}{(2\pi)^4} \underbrace{\delta(p^2 - m^2)}_{=\delta(m_T^2 - p_T^2)}\Theta(p_0) f(p_\mu) = \frac{1}{(2\pi)^4} \int_0^{\infty} dp_T \int_0^{\infty} dm_T \int_{-\infty}^{\infty} d\eta_p \int_0^{2\pi} d\varphi_p\, m_T p_T \underbrace{\delta(p_T^2 - m_T^2)}_{=\frac{1}{2m_T}(\delta(p_T - m_T) + \cancel{\delta(p_T + m_T)})} f(p_\mu)$$

assume $\partial_{\varphi_p} f(p_\mu) = 0$

$$= \frac{1}{(2\pi)^3} \int_0^{\infty} dm_T \underbrace{\int_{-\infty}^{\infty} d\eta_p}_{=2\int_0^{\infty} d\eta_p} \frac{m_T}{2} f(p_\mu)\Big|_{p_T = m_T}$$

$$\begin{cases} p_x = p_T \cos\varphi_p \\ p_y = p_T \sin\varphi_p \\ p_z = m_T \sinh\eta_p \\ p_0 = m_T \cosh\eta_p \end{cases} \qquad \begin{cases} p_T = \sqrt{p_x^2 + p_y^2} \\ \varphi_p = \arctan\left(\frac{p_y}{p_x}\right) \\ m_T = \sqrt{p_0^2 - p_z^2} \\ \eta_p = \operatorname{arctanh}\left(\frac{p_z}{p_0}\right) \end{cases} \qquad \left|\frac{\partial(m_T, \eta_p, p_T, \varphi_p)}{\partial(p_0, p_x, p_y, p_z)}\right| = \frac{1}{m_T} \cdot \frac{1}{p_T}$$

$$\Rightarrow \boxed{\frac{1}{2\pi} \int \frac{d^3p_{cart}}{(2\pi)^3} \frac{1}{2\omega_{\vec p}} f(p_\mu)\Big|_{p_0 = \omega = \omega_{\vec p}} = \frac{1}{(2\pi)^3} \int_0^{\infty} dm_T \int_{-\infty}^{\infty} d\eta_p \frac{m_T}{2} f(p_\mu)\Big|_{p_T = m_T}}$$

$$N = \int \frac{d^3p}{(2\pi)^3} n(\vec p) = \frac{1}{2\pi} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega_{\vec p}} \cdot 2\pi \cdot 2\omega_{\vec p}\, n(\vec p) = \frac{1}{(2\pi)^3} \int_0^{\infty} dm_T \int_{-\infty}^{\infty} d\eta_p \frac{m_T}{2} n(\vec p)\Big|_{p_T = m_T} \cdot 2\omega_{\vec p} \cdot 2\pi$$

$$n(\vec p) = (2\pi)^3 \frac{d^3N}{dp_x dp_y dp_z} = (2\pi)^3 \cdot \frac{1}{2\pi m_T \omega_{\vec p}} \frac{d^2N}{dm_T\, d\eta_p}$$

$$\boxed{\omega_{\vec p} \frac{d^3N}{dp_x dp_y dp_z} = \frac{d^2N}{2\pi\, m_T\, dm_T\, d\eta_p}}$$

For complex scalar

$$n(\vec x) = \phi(\vec x)\phi^*(\vec x)$$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \phi(\vec p) e^{-i\vec p\vec x}\ \phi^*(\vec q) e^{i\vec q\vec x}$$

$$N = \int d^3\vec x\, n(\vec x)$$

$$= \int \frac{d^3p}{(2\pi)^3} \underbrace{\phi(\vec p)\phi^*(\vec p)}_{=n(\vec p)}$$

$$n(\vec x) = \frac{d^3N}{d^3x}$$

$$n(\vec p) = (2\pi)^3 \frac{d^3N}{d^3p}$$

$$\phi(t, \vec p) = \int d^3x\, e^{i\vec p\vec x}\, \phi(\underbrace{t, \vec x}_{=x})$$

$$= \int d^3x\, e^{i\vec p\vec x} \int d^4y\, G(x,y)\, j(y)$$

$$= \int \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_{\vec q}} e^{-i(\omega_{\vec q}(x^0-y^0) - \vec q(\vec x - \vec y))}$$

$$= \int d^3x \int d^4y\, e^{i\vec p\vec x}\, j(y) \left[\Theta(x^0-y^0)\left(D(x-y) - D(y-x)\right)\right]\Big|_{\Sigma_{\text{FreezeOut}}}$$

$$= \int d^3x\, d^4y \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_{\vec q}} \Theta(x^0-y^0) \left(e^{-i(\omega_{\vec q}(x^0-y^0) + \vec q\vec y)} \underbrace{e^{-i(-\vec q\vec x - \vec p\vec x)}}_{\to (2\pi)^3\delta^{(3)}(\vec p + \vec q)} - e^{-i(\omega_{\vec q}(y^0-x^0) - \vec q\vec y)} \underbrace{e^{-i(+\vec q\vec x - \vec p\vec x)}}_{\to (2\pi)^3\delta^{(3)}(\vec p - \vec q)}\right)$$

$$= \int d^4y \frac{1}{(2\pi)^3} \frac{1}{2\omega_{\vec p}} \Theta(x^0-y^0) \left[e^{-i(\omega_{\vec p}(x^0-y^0) + \vec p\vec y)} - e^{-i(\omega_{\vec p}(y^0-x^0) - \vec p\vec y)}\right.$$