DIGITAL SIGNAL PROCESSING

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FINAL PROJECT

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INTRODUCTION

The aim of the project is to extract two audio information signals from a modulated signal (*signal_028.wav*) of the form:

$$y(nT) = (x_1(nT) + A_1)\cos(2\pi f_1 nT) + (x_2(nT) + A_2)\cos(2\pi f_2 nT)$$

Characteristics of the signal:

- Sampling Frequency: F_s = 96000 Hz
- Useful Frequency Band: [20 8000] Hz
- Carriers at Frequencies f_1 and f_2, with 10000 Hz ≤ f_1 ≤ f_2 ≤ 38000 Hz and f_2 - f_1 ≥ 17000 Hz, with amplitude A_1 and A_2
- Duration of the signal: 42 s

ELABORATION OF THE SIGNAL

First, I analysed the whole signal, plotting the original signal and computing the DFT in order to find the peaks in the spectrum, corresponding to the frequencies of the two carriers.

The obtained results are reported below.

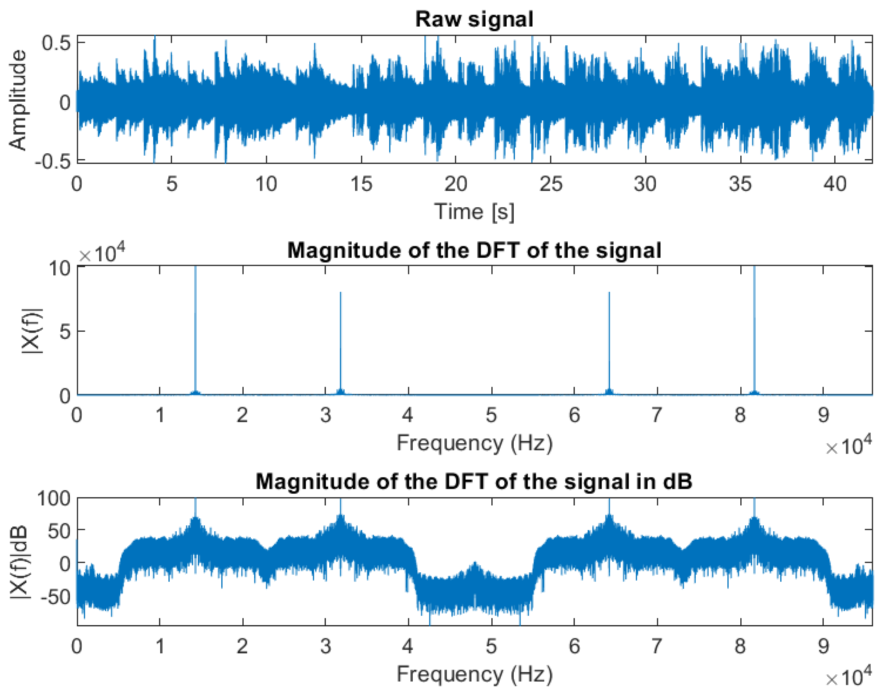

Figure 1 - Original signal and its DFT

The peaks, which represent the frequencies of the two carriers, are at f_1 = 14300 Hz and f_2 = 31800 Hz.

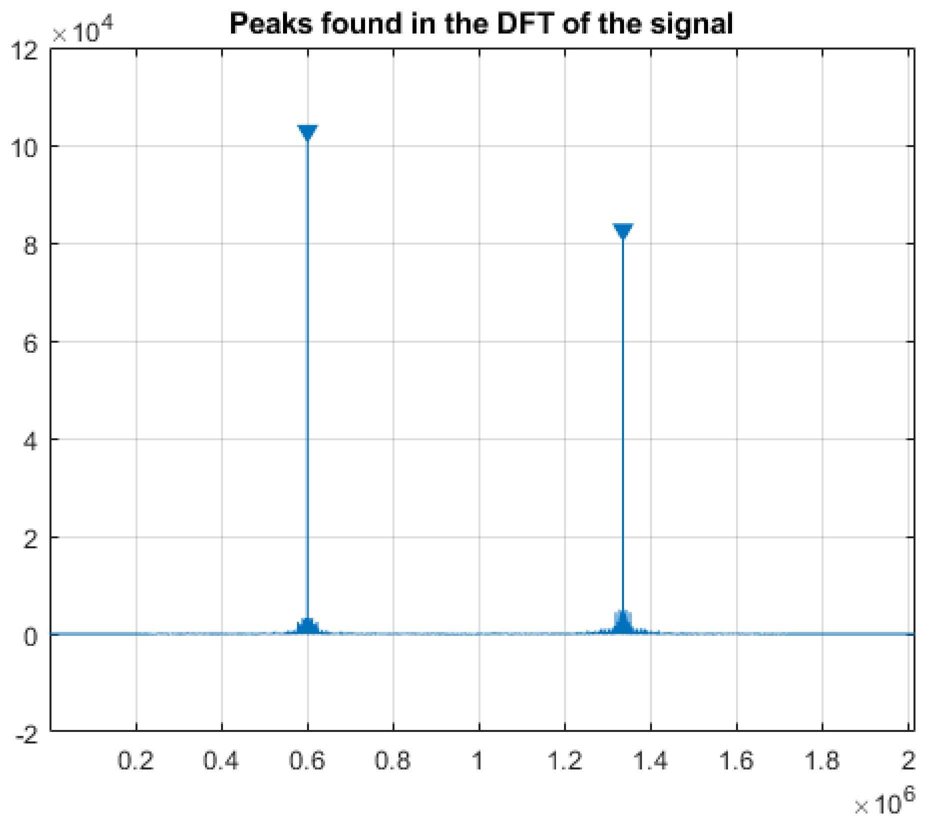

Figure 2 – Peaks found in the DFT (from 0 to Fs/2) of the signal (x-axis: number of samples of half the DFT) -

After finding the peaks, I estimated the amplitude of the two carriers. Since the amplitude of the samples computed by the FFT algorithm in MATLAB is equal to

$$\frac{A \cdot n}{2}$$

where A is the amplitude of the original signal and n the number of FFT points, I obtained the amplitudes of the carriers inverting the formula, dividing the double of the amplitudes of the peaks found by the number of samples of the frequencies vector.

The values I found are:

- A_1 = 0,05
- A_2 = 0,04

EXTRACTION OF THE TWO CARRIES

The extraction of the two carriers is done by filtering the original signal with two filters. These filters are designed such that they have a very narrow bandwidth centred at the frequency of each carrier.

To do this I designed two second order IIR filters with $\Delta f_{3dB} = 10$ Hz. Therefore, I got: $r = 1 - \frac{\pi\, \Delta f_{3dB}}{F_s}$.

The coefficients were computed with the following formulas (second order IIR filter with two zeros at the numerator):

- $b_0 = 1 - r$
- $a_1 = 2\cos(2\pi \frac{f_0}{F_s})$
- $a_2 = -r^2$

with f_0 = f_1 , f_2 . The plot of the frequency response for each filter is plotted below: we can see the peaks at the desired frequencies and the zeros at 0, $^{F_s}/_2$ and F_s.

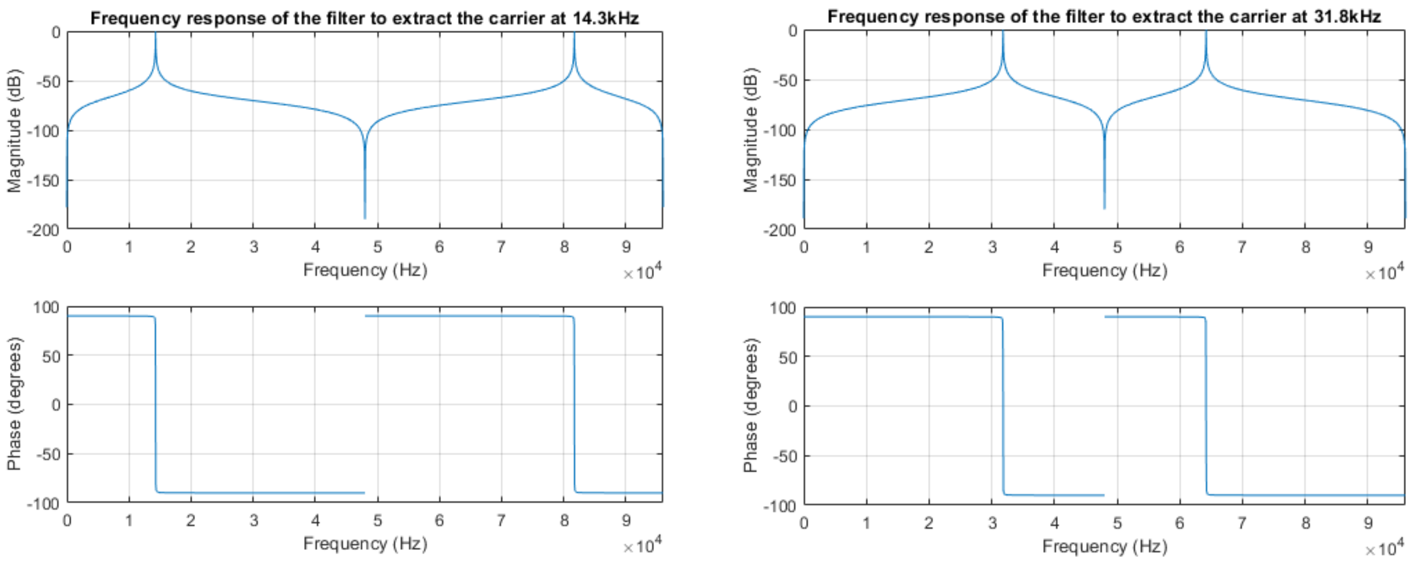

Figure 3 - Second Order IIR filters at 14.3 kHz and 31.8 kHz -

DEMODULATION OF THE TWO SIGNALS

For the demodulation of the two information signals I multiplied the original modulated signal by the product of each carrier with the reciprocal of its own amplitude, in order to get an audible output at the end of the procedure.

To avoid audible distortions in the demodulated signals I filtered them with the cascade of a low-pass filter and of a high-pass notch IIR filter.

The first filter used is a linear-phase IIR filter, with the following parameters:

- Attenuation at the stop-band of $(\delta_s)_{dB} = 80$ dB
- Pass-band ripple of $(\delta_p)_{dB} = 3$ dB
- Cutoff frequencies: $f_p = 8000$ Hz and $f_s = 9000$ Hz
- Order N = 446, computed with the following formula: $N = -\frac{10\log(\delta_s\,\delta_p)+13}{14{,}6\,\frac{f_s - f_p}{F_s}}$

The next image shows the frequency response obtained. We can notice the requested attenuation and cutoff frequencies, as well as the linear phase.

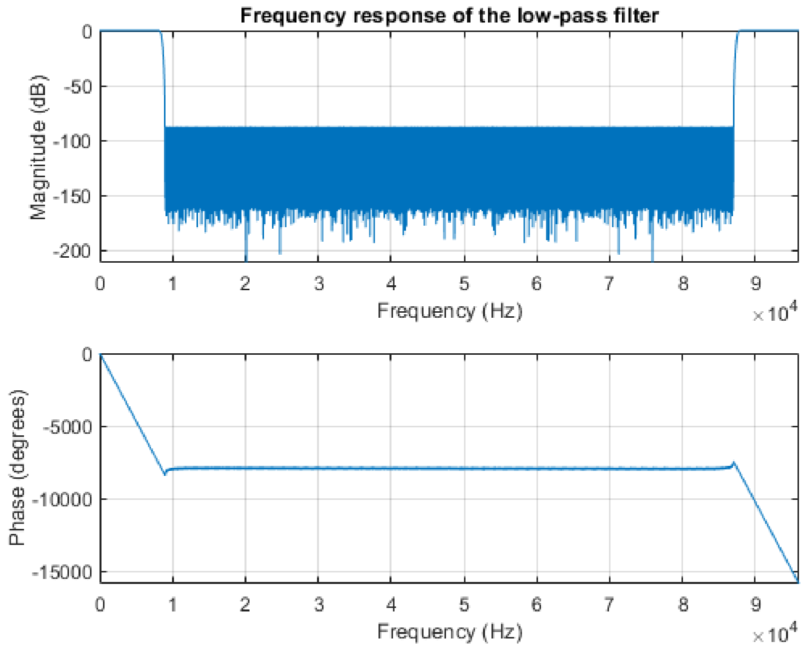

Figure 4 - Frequency response of the low-pass linear-phase FIR filter -

The second filter used is a high-pass notch IIR filter with a bandwidth of $\Delta f_{3dB} = 40$ Hz ($r = 1 - \frac{\pi\,\Delta f_{3dB}}{F_s}$) centred at frequency $f_0 = 0$ Hz, in order to reject the DC component due to the carrier. The following coefficients were computed:

- $b_1 = -2\cos(2\pi\frac{f_0}{F_s})$
- $a_1 = -2\,r\cos(2\pi\frac{f_0}{F_s})$
- $a_2 = -r^2$

The plot of the frequency response shows a filter with a very narrow bandwidth that goes to zero at frequency zero, as requested.

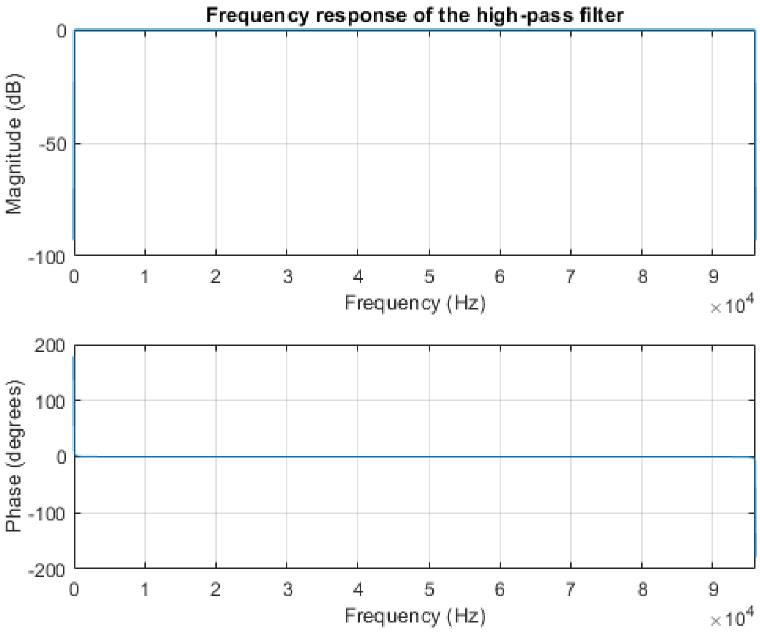

Figure 5 - Frequency response of the high-pass notch IIR filter –

CONCLUSIONS

The spectrum of the two demodulated signals is reported in Figure 6. We can notice frequency components up until 8 kHz, as expected from the filtering done before. There are no other undesired components at higher frequencies.

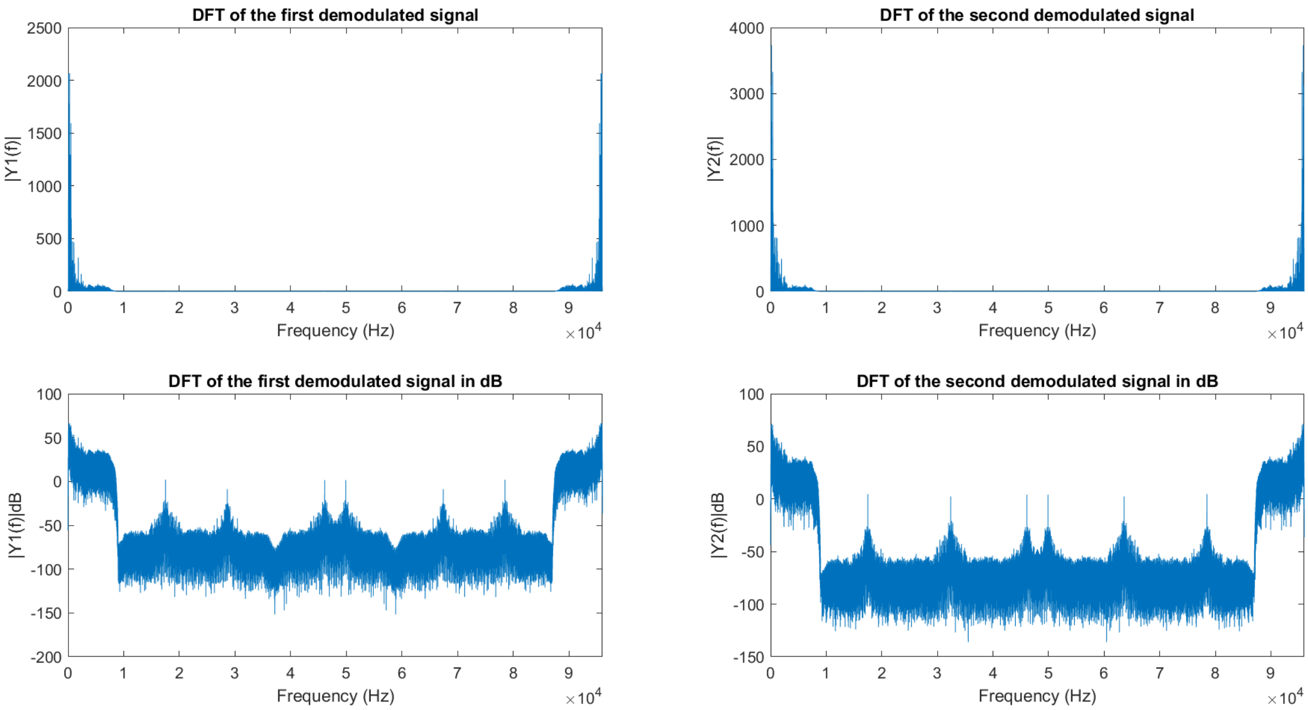

Figure 6 - DFT of the two demodulated signals -

The stereo audio file generated from the MATLAB procedure is without audible distortions and now understandable and listenable.

Here I report the output of the MATLAB procedure.

```
1 Original signal with Sampling Frequency = 96000 Hz
2 Computation of the DFT of the original signal
3 First peak found at frequency = 14300 Hz
  Second peak found at frequency = 31800 Hz
4 Estimated amplitude of the first carrier = 0.05
  Estimated amplitude of the second carrier = 0.04
5 Extraction of the two carriers with second order IIR filters
    SPECIFICATIONS OF THE 1st SECOND ORDER IIR FILTER
    3dB Bandwidth = 10 Hz
    Center frequency = 14300 Hz
    SPECIFICATIONS OF THE 2nd SECOND ORDER IIR FILTER
    3dB Bandwidth = 10 Hz
    Center frequency = 31800 Hz
6 Demodulation of the two information signals
7 Filtering of the demodulated signal to reject audible distortions
    SPECIFICATIONS OF THE LOW-PASS LINEAR-PHASE FIR FILTER
    Pass-band frequency = 8000 Hz
    Stop-band frequency = 9000 Hz
    Desired amplitude of pass-band = 1
    Desired amplitude of stop-band = 0
    Attenuation at the stop-band = 80 dB
    Pass-band ripple = 3 dB
    SPECIFICATIONS OF THE HIGH-PASS NOTCH IIR FILTER
    Center frequency = 0 Hz
    3dB Bandwidth = 40 Hz
8 Computation of the spectrum of the two demodulated signals
9 Writing of the stereo audio file containing the two output signals
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