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- Thus, **orthogonality in time-domain is equivalent to mirrored frequency-domain responses**.
- Example 1: Design an FIR filter ($g[n]$) of order $M = 1$ to allow for frequencies above 10 KHz to pass through. The input signal to be filtered was sampled at 40000 samples per second.
- Example 2: Design an FIR filter ($g[n]$) of order $M = 3$ to allow for frequencies above 1 KHz to pass through. The input signal to be filtered was sampled at 10000 samples per second.
- Example 3: Verify, for the example 1, that the low-pass and high-pass filters involved in the calculations are orthogonal, i.e., the dot product is $h[n] \cdot g[n] = 0$.
- Example 4: Verify, for the example 2, that the low-pass and high-pass filters involved in the calculations are orthogonal, i.e., the dot product is $h[n] \cdot g[n] = 0$.

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- Before studying the other basic filter functions, i.e., band-pass and band-stop, it is important to comment on an important tool which allows for a signal to be converted from the time-domain to the frequency-domain. **Fourier Transform** is the most popular transformation used to do so but, first, let us take a look at another tool: the **Z-Transform**.
- To simplify, we may say that **Z-Transform** is a generalized tool whereas the Fourier Transform corresponds to one of its specific counterparts.
- The **Z-Transform** of an N-sample long time-domain signal $x[n]$, which is used to handle **only discrete-time signals**, is defined as

$$Z(x[n]) = X[z] = \sum_{k=0}^{N-1} x_k \cdot z^{-k} \qquad ,$$

where z is a complex variable of the format $a + bj$, with $\{a, b \in \Re\}$, and $j = \sqrt{-1}$. Letting $z = e^{j\omega} = cos(\omega) + j \cdot sin(\omega)$ forces the Z-Transform to become the Fourier Transform.

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- Example 1: Letting $x[n] = \{1, 5, -2, 4\}$, calculate $X[z]$.
- Example 2: Letting $H[z] = 2 + 3z^{-2} - 6z^{-3}$ be the Z-transform of $h[n]$, what is the latter signal?
- Example 3: Letting $h[n] = \{\frac{1}{2}, \frac{1}{2}\}$, calculate $H[z]$, i.e., the Z-transform of $h[n]$, which corresponds to the system transfer function. Take advantage of the frequency-domain representation just obtained to plot, for a few points, the system frequency response.
- Example 4: Repeat the previous example for $g[n] = \{\frac{1}{2}, -\frac{1}{2}\}$. What is your conclusion?
- **Today's Short Test (ST4)**: Design an FIR filter ($g[n]$) of order $M = 5$ to allow for frequencies above 1500 Hz to pass through, assuming that the input signal to be filtered ($x[n]$) was sampled at 24000 samples per second. Then, obtain the system transfer function, i.e., the Z-Transform of $g[n]$.

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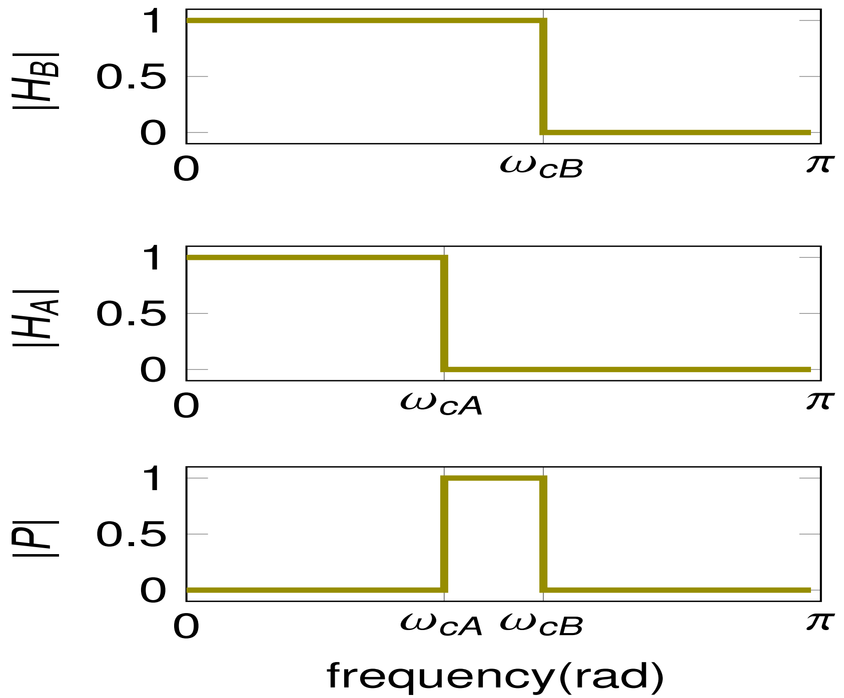
- **Band-pass FIR filters**: A band-pass filter $p[n]$, i.e., a filter which allows for just a specific frequency band to pass through, is, **in the frequency domain**, the subtraction of a low-pass filter with cutoff frequency ω_{cA} from another low-pass filter with cutoff frequency ω_{cB}, as follows.



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- However, we usually want the filter to be specified in time-domain in such a way that it allows for the application of convolution to filter the input signal. Thus, we need the previous subtraction performed in the frequency-domain to be equivalently performed in the time-domain. Let us investigate that equivalence:

$$\begin{aligned}
P[z] &= H_B[z] - H_A[z] \\
&= \left(h_{0,B} + h_{1,B}z^{-1} + h_{2,B}z^{-2} + ...\right) - \left(h_{0,A} + h_{1,A}z^{-1} + h_{2,A}z^{-2} + ...\right) \\
&= h_{0,B} + h_{1,B}z^{-1} + h_{2,B}z^{-2} + ... - h_{0,A} - h_{1,A}z^{-1} - h_{2,A}z^{-2} - ... \\
&= \left(h_{0,B} - h_{0,A}\right) + \left(h_{1,B} - h_{1,A}\right)z^{-1} + \left(h_{2,B} - h_{2,A}\right)z^{-2} + ...
\end{aligned}$$

Then, going back to the time domain by using the Inverse Z-Transform, we get: