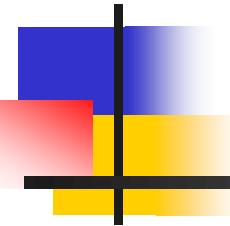
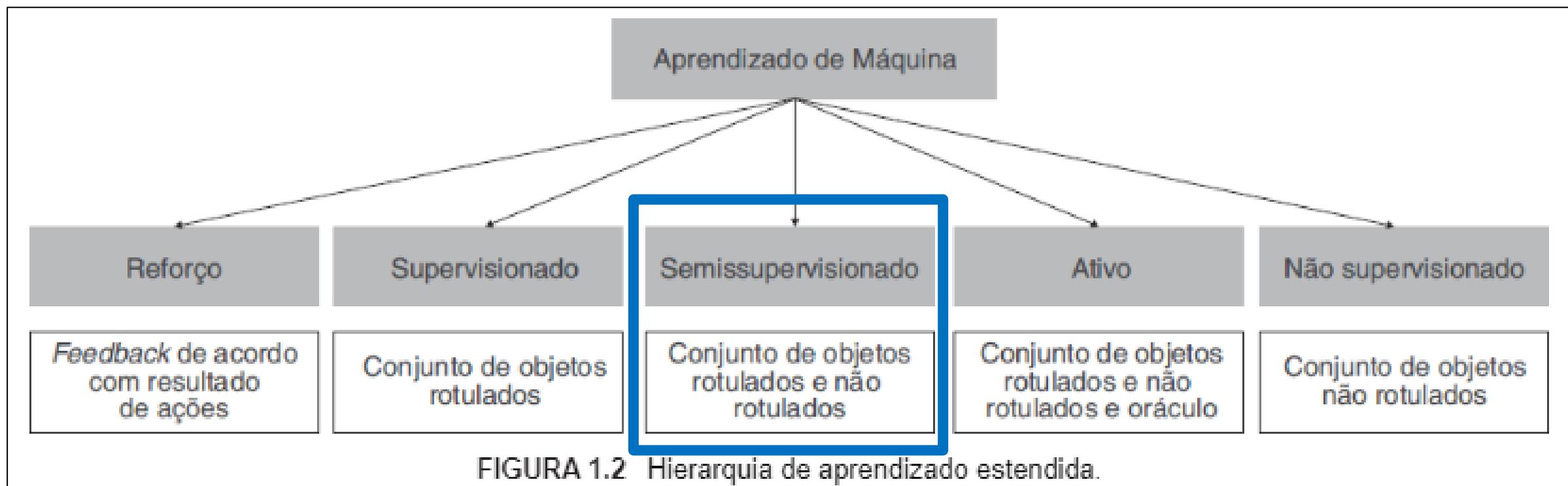
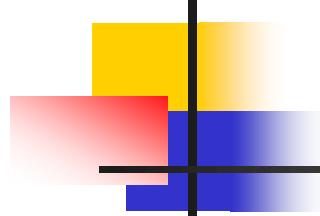


# Aprendizado Semissupervisionado



# Introdução



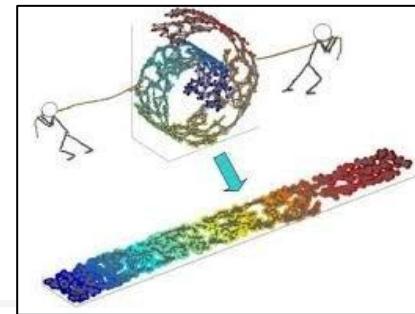


# Introdução

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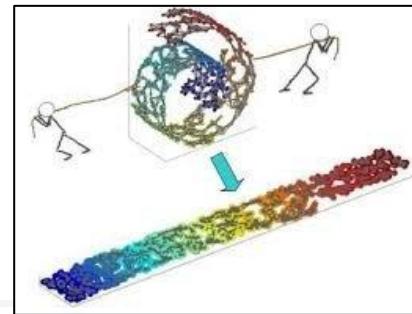
- In many applications, labeled data is expensive and hard to acquire
- On the other hand, unlabeled data is often available
- Semi-supervised learning will be useful whenever there are far more unlabeled data than labeled
  - It turns out that unlabeled data can be used to significantly improve the accuracy of many mining algorithms

# Introdução



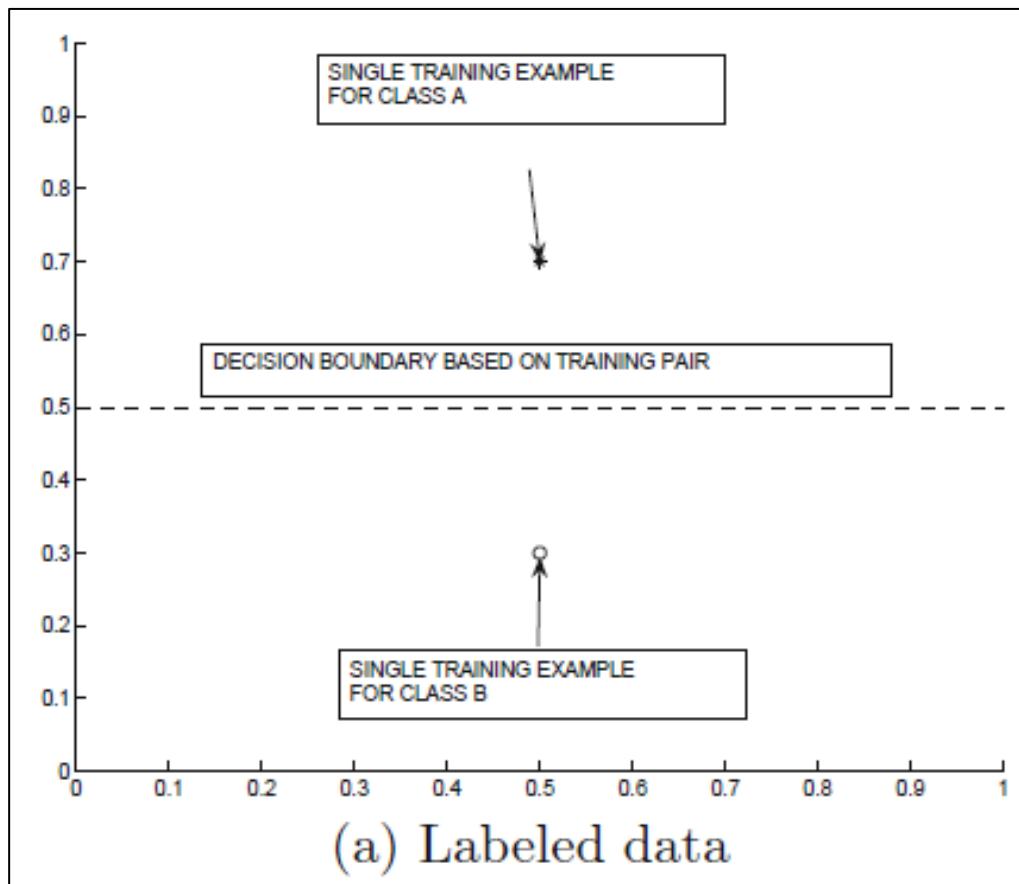
- Unlabeled data is useful because of the following two reasons:
  - Unlabeled data can be used to estimate the low-dimensional manifold structure of the data
    - Mesmo sem saber as classes, os dados sem rótulo ajudam o modelo a entender o formato, i.e., a estrutura natural do espaço de dados
      - Imagine que cada ponto de dado é uma estrela no céu. Você ainda não sabe quais são “constelações” (rótulos), mas consegue ver como as estrelas se agrupam naturalmente em desenhos. Esses agrupamentos formam a estrutura (ou manifold) onde os dados vivem
        - Com os dados não rotulados, o modelo descobre essa forma — “o mapa” do espaço de dados
        - Com os dados rotulados, ele coloca nomes nos pedaços desse mapa

# Introdução



- Unlabeled data is useful because of the following two reasons (cont.):
  - Unlabeled data can be used to estimate the joint probability distribution of features, useful for indirectly relating feature values to labels
    - Os dados não rotulados ajudam o modelo a entender como as variáveis se relacionam entre si — mesmo sem saber as classes
      - Imagine que você observa centenas de frutas, sem saber o nome delas. Você nota padrões: (i) frutas mais vermelhas costumam ser menores; (ii) frutas mais amarelas são mais compridas; etc.
      - Mesmo sem rótulo ("maçã", "banana", etc.), você já entende como as características se combinam. Depois, quando aparece pouca informação rotulada, o modelo já sabe como as características se distribuem, e isso facilita ligar padrões a rótulos

# Introdução



Portions of this decision boundary are in regions of the space where almost no feature values are available

Therefore, the decision boundaries in these regions may not reflect the class behavior of unseen test instances

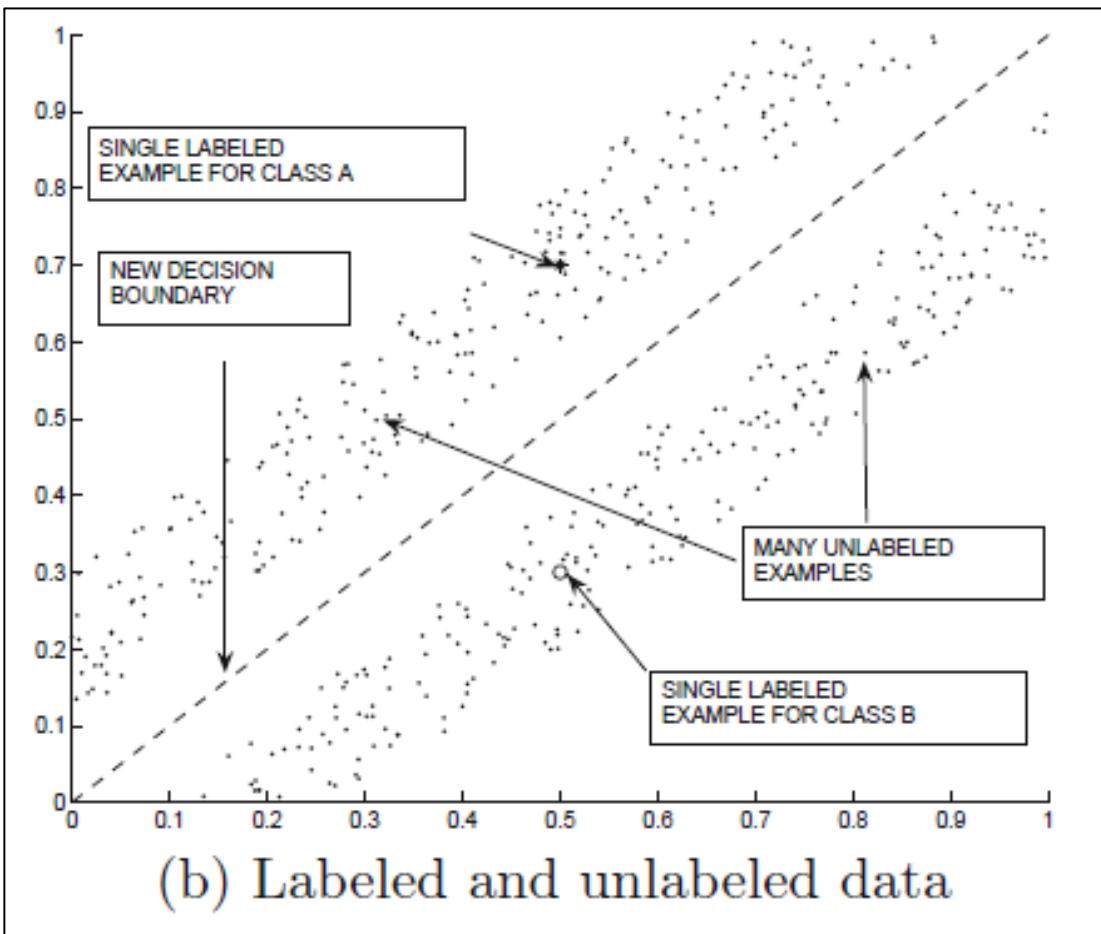
# Introdução

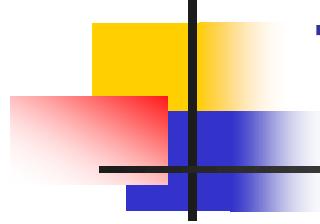
Suppose that a large number of unlabeled examples are added to the training data

It becomes immediately evident that the data is distributed along two manifolds, each of which contains one of the training examples

A key assumption here is that the class variables are likely to vary smoothly over dense regions of the space, but it may vary significantly over sparse regions of the space

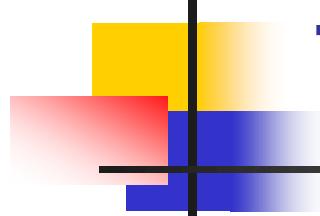
Leads to a new decision boundary that takes the underlying feature correlations into account in addition to the labeled instances





# Transduction x Induction

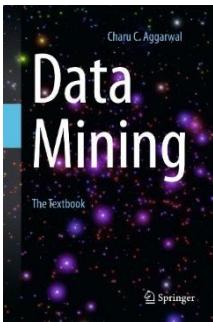
- An inductive method infers a function  $f : X \rightarrow Y$  on the entire space  $X$ , and afterward returns the evaluations  $f(x_i)$  at the test points



# Transduction x Induction

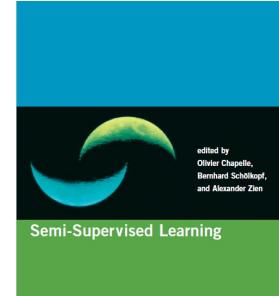
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- Transduction consists of directly estimating the finite set of test labels, i.e., a function  $f : X_u \rightarrow Y$  only defined on the test set
  - All test instances need to be specified at the time of constructing the training model
    - New out-of-sample instances cannot be classified after the model has been constructed
  - Transduction is not the same as SSL: some semi-supervised algorithms are transductive, but others are inductive



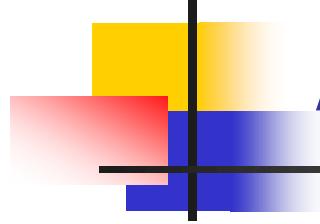
# Techniques

- Meta-algorithms
  - Uses any existing classification algorithm as a subroutine, and leverage it to incorporate the impact of unlabeled data
  - Self-training, Co-training
- Variations of Classification Algorithms
  - Modifications are incorporated in specific classifiers to account for the impact of unlabeled data
  - Semisupervised Bayes Classification with EM [2], TSVM [3], Graph-Based [4]



# Techniques

- For semi-supervised learning to work, certain assumptions will have to hold
  - Smoothness assumption [1]
    - If two points  $x_1, x_2$  in a high-density region are close, then so should be the corresponding outputs  $y_1, y_2$
  - Cluster assumption [2]
  - Low density separation assumption [3]
  - Manifold assumption [4]

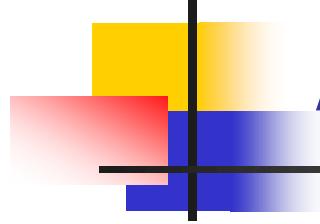


# Assumptions

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- **Cluster assumption [2]**

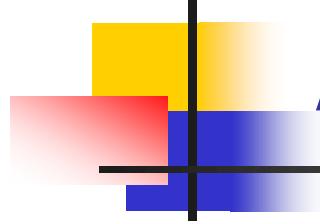
- If points are in the same cluster, they are likely to be of the same class
- Algorithm: Generative Models
  - Semisupervised Bayes Classification with EM



# Assumptions

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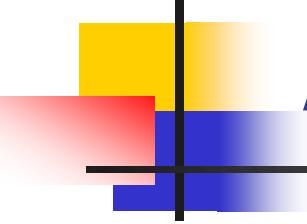
- **Low density separation assumption [3]**
  - The decision boundary should lie in a low-density region
    - Although equivalent with assumption [2], they can inspire different algorithms
  - Algorithm: TSVM (Transductive SVM)



# Assumptions

## ■ **Manifold assumption [4]**

- The (high-dimensional) data lie (roughly) on a low-dimensional manifold
  - Curse of dimensionality = pairwise distances tend to become more similar, and thus less expressive
  - Ensures more appropriate distances and, thus, “if two points  $x_1, x_2$  are close, then so should be the corresponding outputs  $y_1, y_2$ ”  
(smoothness assumption of supervised learning)



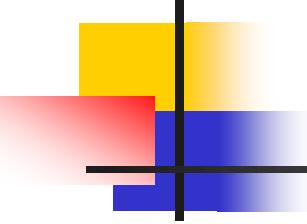
# Assumptions

[https://en.wikipedia.org/wiki/Distance\\_\(graph\\_theory\)](https://en.wikipedia.org/wiki/Distance_(graph_theory))

The **distance** between two vertices in a graph is the number of edges in a shortest path (also called a **graph geodesic**) connecting them

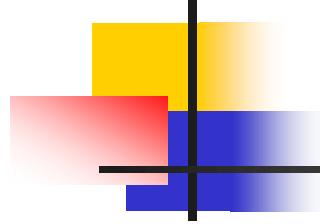
## ■ Manifold assumption [4]

- The most active area of research in semi-supervised learning has been in graph-based methods
- Graph methods can be argued to build on the manifold assumption
  - If the distance of two points is computed by minimizing the path distance over all paths connecting the two points, this can be seen as an approximation of the geodesic distance of the two points with respect to the manifold of data points



# Meta-algorithms: Self-training

- The self-training procedure can use any existing classification algorithm A as input
- The classifier A is used to incrementally assign labels to unlabeled examples for which it has the most confident prediction
- As input, the self-training procedure uses the initial labeled set L, the unlabeled set U, and a user-defined parameter k that may sometimes be set to 1

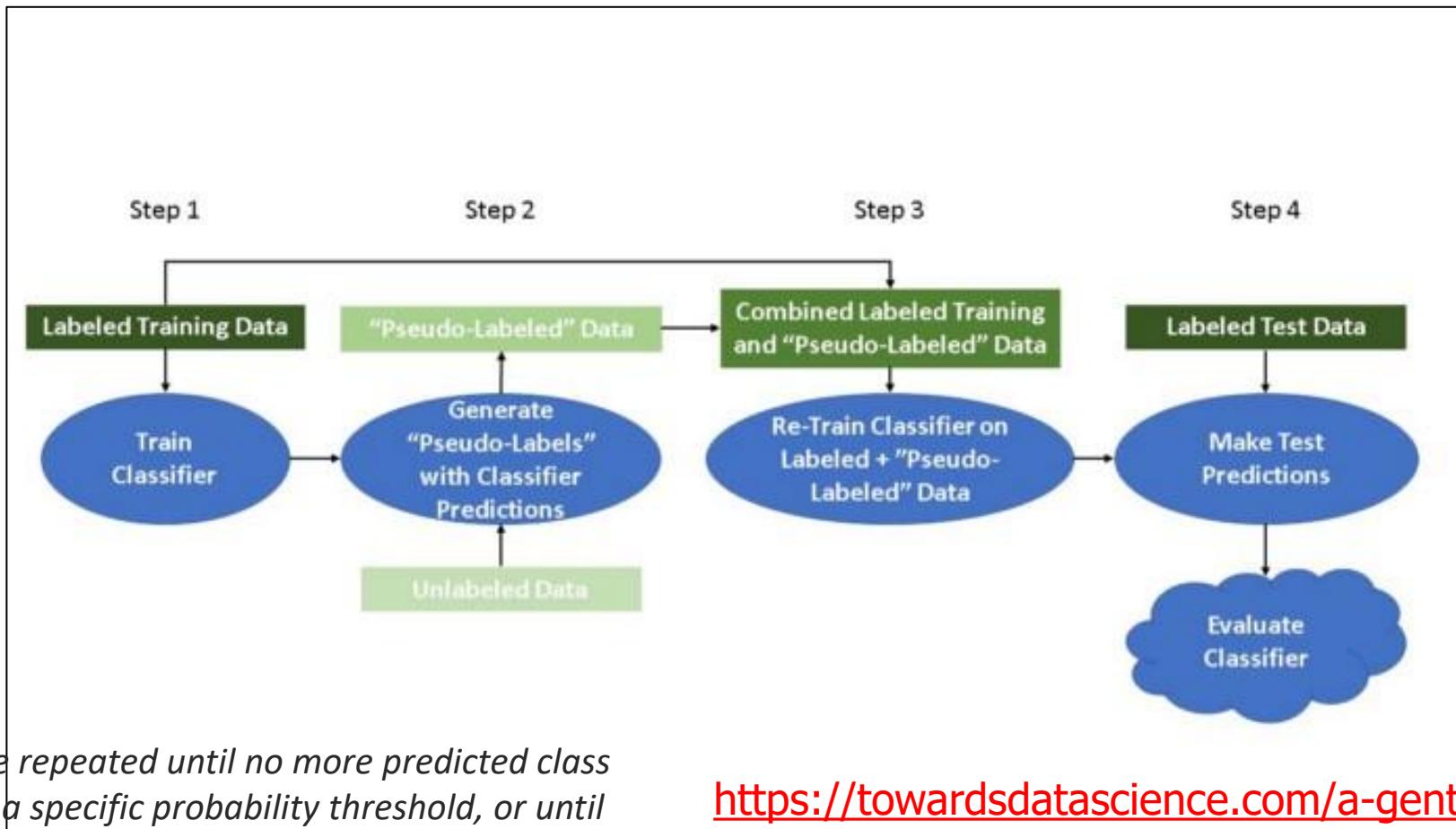


# Meta-algorithms: Self-training

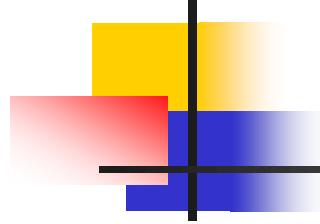
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- The self-training procedure iteratively uses the following steps:
  - (a) use algorithm A on the current labeled set L to identify the k instances in the unlabeled data U for which the classifier A is the most confident
  - (b) assign labels to the k most confidently predicted instances and add them to L. Remove these instances from U

# Meta-algorithms: Self-training



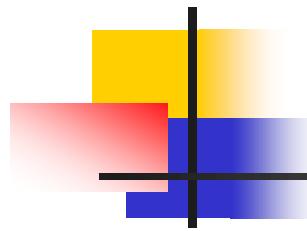
<https://towardsdatascience.com/a-gentle-introduction-to-self-training-and-semi-supervised-learning-ceee73178b38>



# Meta-algorithms: Self-training

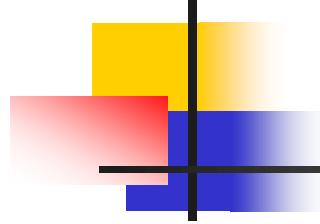
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- The major drawback of self-training is that the addition of predicted labels to the training data can lead to propagation of errors in the presence of noise



# Meta-algorithms: Co-training

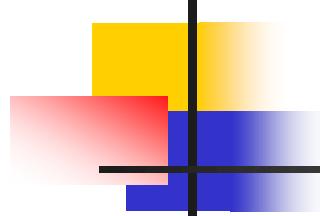
- In Co-training, it is assumed that the feature set can be partitioned into two disjoint groups  $F_1$  and  $F_2$ , such that each of them is sufficient to learn the target classification function
- It is important to select the two feature subsets so that they are as independent from one another as possible



# Meta-algorithms: Co-training

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- Two classifiers are constructed, such that one classifier is constructed on each of these groups
- These classifiers are not allowed to interact with one another directly for prediction of unlabeled examples though they are used to build up training sets for each other
- This is the reason that the approach is referred to as Co-training



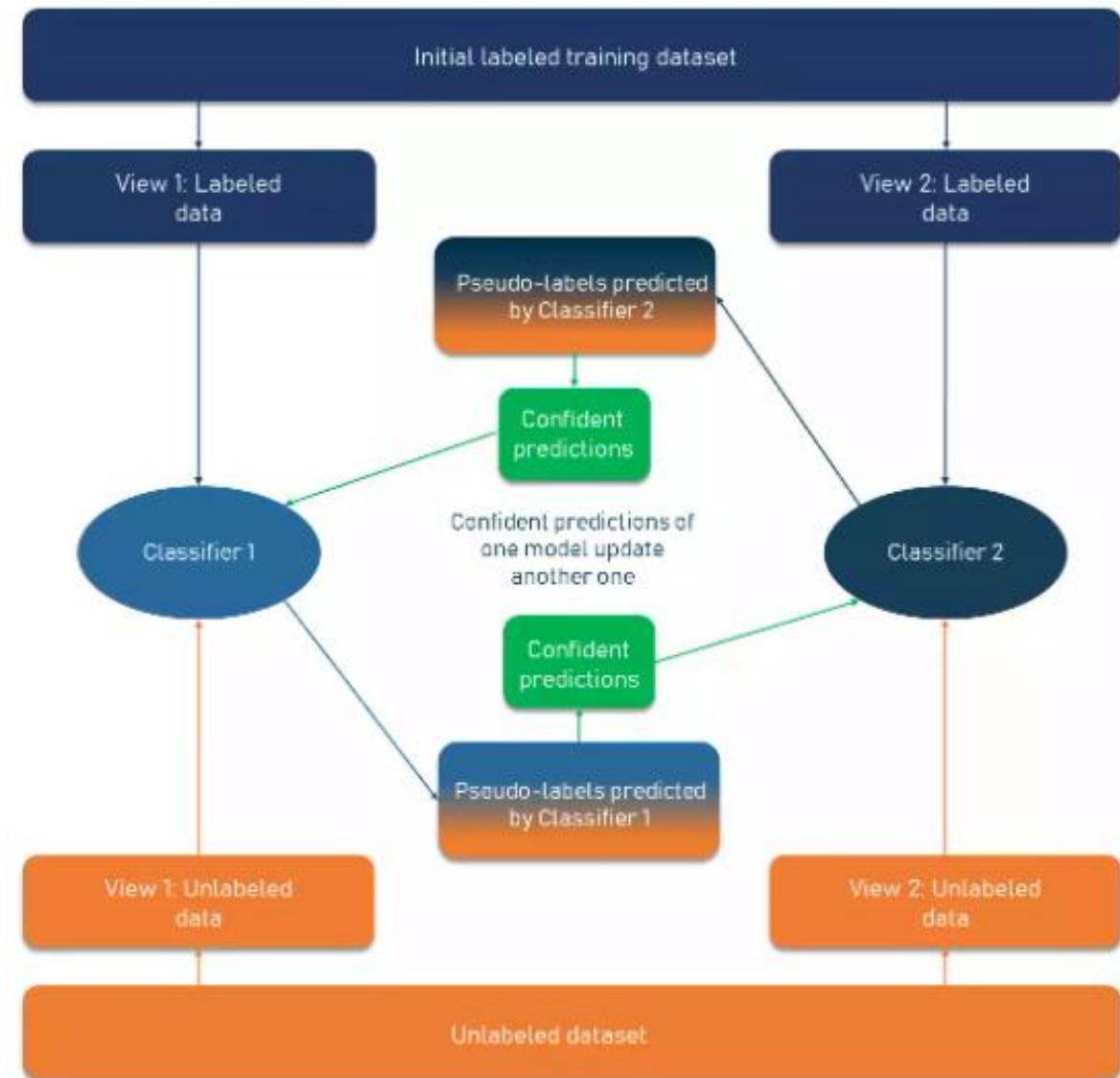
# Meta-algorithms: Co-training

- The Co-training procedure iteratively uses the following steps:
  - (a) train classifier  $A_1$  using labeled set  $L_1$ , and add  $k$  most confidently predicted instances from unlabeled set  $U - L_2$  to training data set  $L_2$  for classifier  $A_2$
  - (b) train classifier  $A_2$  using labeled set  $L_2$ , and add  $k$  most confidently predicted instances from unlabeled set  $U - L_1$  to training data set  $L_1$  for classifier  $A_1$

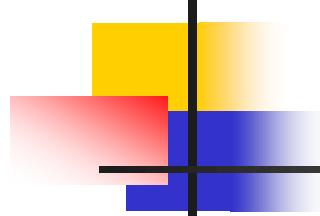
The sets  $L_1$  and  $L_2$  are initialized to the available labeled data  $L$ . As different examples from the initially unlabeled set  $U$  are added to  $L_1$  and  $L_2$ , respectively, the training instances in  $L_1$  and  $L_2$  may vary from one another.

# Meta-algorithms: Co-training

SEMI-SUPERVISED CO-TRAINING METHOD

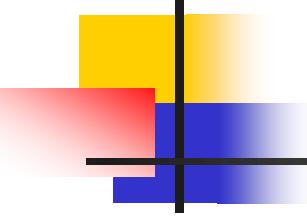


<https://www.altexsoft.com/blog/semi-supervised-learning/>



# Meta-algorithms: Co-training

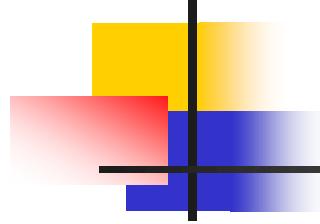
- At the end, the two classifiers are then retrained with the expanded training data sets and, therefore, two classifiers are returned
  - Can be used to label not only the unlabeled data set  $U$ , but also unseen test instances
- For an unseen test instance, each classifier may be used to determine the class label scores
- The score for a test instance is determined by combining the scores of the two classifiers
  - For example, if the Bayes method is used as the base classifier, then the product of the posterior probabilities returned by the two classifiers may be used



# Meta-algorithms: Co-training

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- The Co-training approach is more robust to noise because of the disjoint feature sets used by the two algorithms
  - An important assumption is that of conditional independence of the features in the two sets with respect to a particular class
  - The intuition is that instances generated by one classifier appear to be randomly distributed to the other, and vice-versa



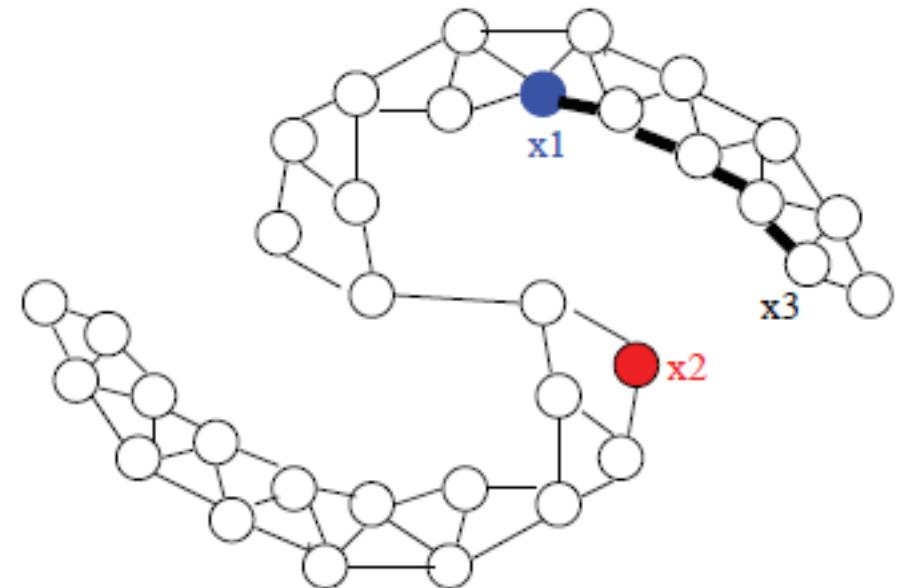
# Graph-Based

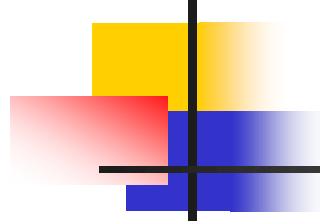
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- One advantage of this approach is that it can be used for semisupervised classification of arbitrary data types, as long as a distance function is available for quantifying proximity between data objects

# Graph-Based

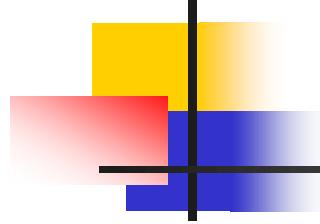
- Label Propagation
  - Vértices similares devem possuir a mesma classe
  - Dois passos:
    - Construção do grafo
    - Propagação dos rótulos





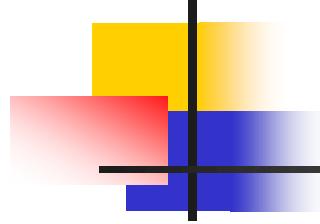
# Graph-Based Label Propagation

- Construção do Grafo
  - $\varepsilon$ -vizinhança
  - $\kappa$ -vizinhos mais próximos ( $\kappa$ NN)



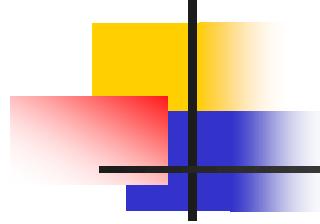
# Graph-Based Label Propagation

- Construção do Grafo
  - **$\varepsilon$ -vizinhança**
    - Dois vértices  $x_i$  e  $x_j$  são conectados se a distância entre os mesmos é menor do que  $\varepsilon$  (threshold)
    - Em geral, não é muito utilizada, pois na maioria das vezes gera-se uma rede desconexa
      - Alguns vértices não possuirão ligação com vértices rotulados



# Graph-Based Label Propagation

- Construção do Grafo
  - **$\kappa$ -vizinhos mais próximos ( $\kappa$ NN)**
    - Dois vértices  $x_i$  e  $x_j$  são conectados se  $x_j$  está entre os  $\kappa$  vizinhos mais próximos de  $x_i$ 
      - A matriz de adjacência pode não ser simétrica, pois  $x_j$  ser vizinho de  $x_i$  não implica em  $x_i$  ser vizinho de  $x_j$ 
        - $\kappa$ NN mútuo
        - $\kappa$ NN simétrico



# Graph-Based Label Propagation

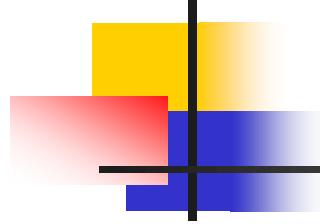
- Construção do Grafo
  - **$\kappa$ -vizinhos mais próximos ( $\kappa$ NN)**
    - $\kappa$ NN mútuo
      - Verificam-se os casos em que existe uma conexão entre  $x_i$  e  $x_j$  e não existe uma conexão entre  $x_j$  e  $x_i$  e adiciona-se uma conexão entre  $x_j$  e  $x_i$
    - $\kappa$ NN simétrico
      - Verificam-se os casos em que existe uma conexão entre  $x_i$  e  $x_j$  e não existe uma conexão entre  $x_j$  e  $x_i$  e remove-se a conexão entre  $x_j$  e  $x_i$

# Graph-Based Label Propagation

[https://en.wikipedia.org/wiki/Radial\\_basis\\_function\\_kernel](https://en.wikipedia.org/wiki/Radial_basis_function_kernel)  
[https://scikit-learn.org/stable/modules/semi\\_supervised.html](https://scikit-learn.org/stable/modules/semi_supervised.html)

- Construção do Grafo
  - Existem métodos que alteram o peso das conexões
    - Visam ponderar os valores das conexões entre dois vértices
      - Kernel RBF (medida de similaridade [0, 1])
        - Aplica uma função Gaussiana no peso, ponderando seu valor de acordo com a abertura  $\sigma$  da Gaussiana

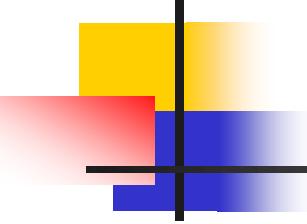
$$K(\mathbf{x}, \mathbf{x}') = \exp\left(-\frac{\|\mathbf{x} - \mathbf{x}'\|^2}{2\sigma^2}\right) \quad \left| \quad \begin{aligned} K(\mathbf{x}, \mathbf{x}') &= \exp(-\gamma \|\mathbf{x} - \mathbf{x}'\|^2) \\ \gamma &= \frac{1}{2\sigma^2} \end{aligned} \right.$$



# Graph-Based Label Propagation

Learning from labeled and unlabeled data with label propagation  
<https://pages.cs.wisc.edu/~jerryzhu/pub/CMU-CALD-02-107.pdf>

- Setup
  - $(x_1, \dots, y_1) \dots (x_l, \dots, y_l)$  = labeled data,  $Y_L = \{y_1, \dots, y_l\}$  = class labels
    - Assume that the number of classes  $C$  is known and that all classes are present in the labeled data
  - $(x_{l+1}, \dots, y_{l+1}) \dots (x_{l+u}, \dots, y_{l+u})$  = unlabeled data,  $Y_U = \{y_{l+1}, \dots, y_{l+u}\}$  are unknown,  $u \gg l$
  - $X = \{x_1, \dots, x_{l+u}\}$
- Problem
  - Estimate  $Y_U$  from  $X$  and  $Y_L$



# Graph-Based Label Propagation

Learning from labeled and unlabeled data with label propagation  
<https://pages.cs.wisc.edu/~jerryzhu/pub/CMU-CALD-02-107.pdf>

- Construção do Grafo
  - Create a fully connected graph where the nodes are all data points – labeled+unlabeled
  - $W_{ij}$  é computada via distância euclidiana
  - $W_{ij}$  é normalizada via Kernel RBF

# Graph-Based Label Propagation

# Learning from labeled and unlabeled data with label propagation

<https://pages.cs.wisc.edu/~jerryzhu/pub/CMU-CALD-02-107.pdf>

## ■ Propagação

- All nodes have soft labels = a distribution over labels
    - Thus, labels of a node are propagate to all nodes through the edges

Matriz T (n X n)			

$$(I + u)$$

$$T_{ij} = P(j \rightarrow i) = \frac{w_{ij}}{\sum_{k=1}^{l+u} w_{kj}}$$

$T$  = probabilistic transition matrix  
 $T_{ij}$  = probability to jump from node  $j$  to  $i$

# Graph-Based Label Propagation

Learning from labeled and unlabeled data with label propagation  
<https://pages.cs.wisc.edu/~jerryzhu/pub/CMU-CALD-02-107.pdf>

- Propagação
  - All nodes have soft labels = a distribution over labels
    - Thus, labels of a node are propagate to all nodes through the edges

Matriz Y (n X C)			
(l + u)			C

Y = label matrix

Each row represents the label probability distribution of node  $x_i$   
Initialization of rows corresponding to unlabeled data points is random

# Graph-Based Label Propagation

Learning from labeled and unlabeled data with label propagation  
<https://pages.cs.wisc.edu/~jerryzhu/pub/CMU-CALD-02-107.pdf>

## ■ Algoritmo

1. Propagate  $Y \leftarrow TY$
2. Row-normalize  $Y$ .
3. Clamp the labeled data. Repeat from step 1 until  $Y$  converges.



It is important to maintain labels sources from labeled data

<https://machinelearningmastery.com/semi-supervised-learning-with-label-propagation/>

# Graph-Based Label Propagation

- Existem vários algoritmos

$$\min_{f: f(\mathbf{x}) \in \{-1, 1\}} \infty \sum_{i=1}^l (y_i - f(\mathbf{x}_i))^2 + \sum_{i,j=1}^{l+u} w_{ij} (f(\mathbf{x}_i) - f(\mathbf{x}_j))^2,$$

If  $\mathbf{x}_i$  and  $\mathbf{x}_j$  are not connected, then  $w_{ij} = 0$   
If the edge exists and is not cut, then  $f(\mathbf{x}_i) - f(\mathbf{x}_j) = 0$

## Introduction to Semi-Supervised Learning [ref.]

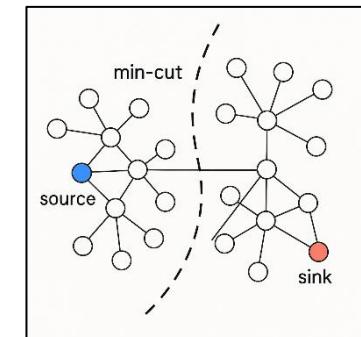
Separar os dados em dois conjuntos de rótulo, de modo a cortar arestas de menor peso total. Isso significa que ele procura evitar separar pontos muito similares, respeitando a ideia de suavidade local (smoothness).

$$\sum_{i,j: f(\mathbf{x}_i) \neq f(\mathbf{x}_j)} w_{ij}.$$

The “cut size” is measured by the sum of the weights on the edges defining the cut

## MinCut

- > duas classes ( $\{+1, -1\}$ )
- > objetivo é encontrar o menor conjunto de arestas cuja remoção isole totalmente as duas classes



# Graph-Based Label Propagation

- Existem vários algoritmos

$$\min_{f: f(\mathbf{x}) \in \mathbb{R}}$$

$$\infty \sum_{i=1}^l (y_i - f(\mathbf{x}_i))^2 + \sum_{i,j=1}^{l+u} w_{ij} (f(\mathbf{x}_i) - f(\mathbf{x}_j))^2.$$

Now,  $f$  produces real values

$$\begin{aligned} & \min_{f: f(\mathbf{x}) \in \mathbb{R}} \sum_{i,j=1}^{l+u} w_{ij} (f(\mathbf{x}_i) - f(\mathbf{x}_j))^2 \\ & \text{subject to } f(\mathbf{x}_i) = y_i \text{ for } i = 1 \dots l. \end{aligned}$$

## GFHF (Gaussian Fields and Harmonic Functions)

-> The value assigned to each unlabeled vertex is the weighted average of its neighbors' values

$$\begin{aligned} f(\mathbf{x}_i) &= y_i, \quad i = 1 \dots l \\ f(\mathbf{x}_j) &= \frac{\sum_{k=1}^{l+u} w_{jk} f(\mathbf{x}_k)}{\sum_{k=1}^{l+u} w_{jk}}, \quad j = l + 1 \dots l + u. \end{aligned}$$

# Graph-Based Label Propagation

- Existem vários algoritmos

**Learning with local and global consistency**  
[https://proceedings.neurips.cc/paper\\_files/paper/2003/file/87682805257e619d49b8e0dfdc14affa-Paper.pdf](https://proceedings.neurips.cc/paper_files/paper/2003/file/87682805257e619d49b8e0dfdc14affa-Paper.pdf)

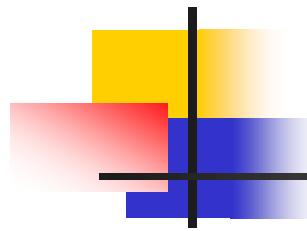
[https://scikit-learn.org/stable/modules/semi\\_supervised.html](https://scikit-learn.org/stable/modules/semi_supervised.html)

$$F = \mu \sum_{x_i \in L} (f_i - y_i)^2 + \frac{1}{2} \sum_{x_i, x_j \in D} w_{ij} \left\| \frac{f_i}{\sqrt{d_{ii}}} - \frac{f_j}{\sqrt{d_{jj}}} \right\|^2$$

**LLGC (Learning with Local and Global Consistency)**  
->  $d_{ii}$  = grau do vértice i

Permite que os rótulos originais se alterem

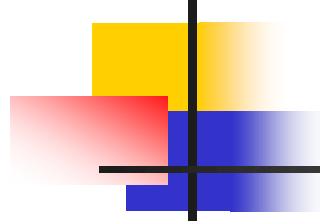
Evita que vértices com um alto grau não dominem a rede, i.e., diminuem sua influência na propagação das classes



# Demais Referências

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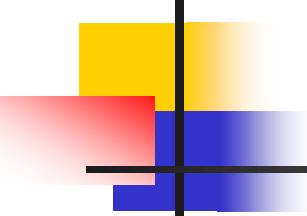
- A survey on semi-supervised learning
  - <https://link.springer.com/article/10.1007/s10994-019-05855-6>
- Introduction to Semi-Supervised Learning
  - <https://link.springer.com/book/10.1007/978-3-031-01548-9>
- Pós-processamento de regras de associação via redes e propagação de rótulos
  - <https://teses.usp.br/teses/disponiveis/55/55134/tde-14102016-165710/pt-br.php>



# Implementações

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- scikit-learn
  - [https://scikit-learn.org/stable/modules/semi\\_supervised.html](https://scikit-learn.org/stable/modules/semi_supervised.html)
- NetworkX
  - <https://networkx.org/>
  - [https://networkx.org/documentation/stable/reference/algorithms/node\\_classification.html](https://networkx.org/documentation/stable/reference/algorithms/node_classification.html)



# Papers

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- Graph-based Semi-supervised Learning: A Comprehensive Review
  - <https://arxiv.org/abs/2102.13303>
- A Survey on Deep Semi-supervised Learning
  - <https://arxiv.org/abs/2103.00550>
- A Survey of Data-Efficient Graph Learning
  - <https://arxiv.org/abs/2402.00447>