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$$p[n] = \Big\{(h_{0,B} - h_{0,A}), (h_{1,B} - h_{1,A}), (h_{2,B} - h_{2,A}), \ldots\Big\} \qquad .$$

- Consequently, the time-domain band-pass filter is just the subtraction of the time-domain low-pass filter with cutoff frequency ω_{cA} from the time-domain low-pass filter with cutoff frequency ω_{cB}.
- Example: Design a band-pass filter of order $M = 3$ that allows for frequencies in the range 2000 Hz $\sim$ 3000 Hz to pass through, cutting-off all the others outside that range. The input signal to be filtered was sampled at 10000 samples per second.
- **Band-stop FIR filters**: A band-stop filter $q[n]$, i.e., a filter which cuts-off just a specific frequency band allowing all the others to pass through, is, **in the frequency domain**, the sum of a low-pass filter with cutoff frequency ω_{cA} and a high-pass filter with cutoff frequency ω_{cB}, as follows.

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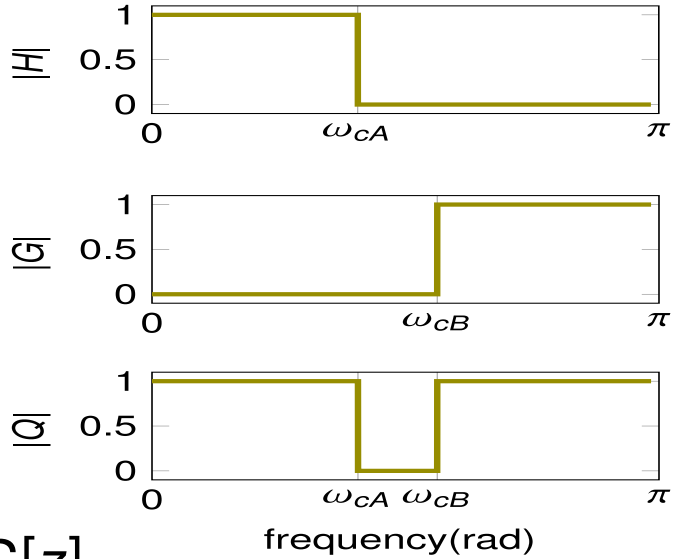

$$
\begin{aligned}
Q[z] &= H[z] + G[z] \\
&= \Big(h_0 + h_1 z^{-1} + h_2 z^{-2} + \ldots\Big) + \Big(g_0 + g_1 z^{-1} + g_2 z^{-2} + \ldots\Big) \\
&= h_0 + h_1 z^{-1} + h_2 z^{-2} + \ldots + g_0 + g_1 z^{-1} + g_2 z^{-2} + \ldots \\
&= \Big(h_0 + g_0\Big) + \Big(h_1 + g_1\Big) z^{-1} + \Big(h_2 + g_2\Big) z^{-2} + \ldots
\end{aligned}
$$

Then, going back to the time domain by using the Inverse Z-Transform, we get:

$$q[n] = \Big\{(h_0 + g_0), (h_1 + g_1), (h_2 + g_2), \ldots\Big\}.$$

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- Consequently, the time-domain band-stop filter is just the sum of the time-domain low-pass filter with cutoff frequency ω_{cA} and the time-domain high-pass filter with cutoff frequency ω_{cB}.
- Example: Design a band-stop filter of order $M = 3$ that allows for frequencies outside the range 1000 Hz $\sim$ 3000 Hz to pass through, cutting-off those within that range. The input signal to be filtered was sampled at 10000 samples per second.
- Remark: interestingly, all the filters we have designed are either symmetric or anti-symmetric. There is a special, and **very important**, reason for that! To understand it, we need, first, another concept: the signal phase.
- From a time-domain signal we can, by using the Z-Transform and subsequently replacing z with $e^{j\omega}$, get a frequency-domain representation. From this representation, which corresponds to the Fourier Transform of the time-domain signal, two results can be obtained: the signal module and the signal phase.

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- The module is used to check the amplitude of specific frequencies whereas the phase is adopted to measure the corresponding time shifting of those frequencies.
- Filters with either a symmetric or an anti-symmetric impulse response exhibits a linear phase response. Consequently, all the remaining frequencies after the filtering procedure are **equally shifted** in time. This is, for some applications, **very important**.
- Example: Assuming that the impulse response of a filter is $x[n] = \{2, 3, 3, 2\}$, calculate its Z-Transform and comment on the corresponding amplitude and phase responses.
- **Today's Short Test (ST5)**: Design an FIR filter ($q[n]$) of order $M = 5$ to cut-off frequencies within the range 2500 Hz $\sim$ 3500 Hz, allowing for all the others to pass through. Assume that the input signal to be filtered ($x[n]$) was sampled at 10000 samples per second.