Class: Digital Speech Processing
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Final Exam

1. Designing an FIR band-stop filter of order M=8 and cutoff frequencies 2000Hz and 4000Hz, assuming the filtered signal was sampled at 22050 s/s, with normalized gain of 0dB in the pass band and obtaining the difference equation for the designed filter:

   Considering the sample rate of the signal, the maximum frequency that can be contained in it is 22050/2 = 11025Hz.

   We will design two filters, a low-pass (A) and a high-pass (B) filter, and combine them to arrive at the desired band-stop filter. To design the high pass filter we will first design a low-pass filter with the same cutoff frequency and complement its frequency response with an all-pass filter.

   Finding the cutoff frequency coefficients for the filters:

$$\omega_{cA} = \frac{\pi \,.\, 2000}{11025} = \frac{80\pi}{441}$$

$$\omega_{cB} = \frac{\pi \,.\, 4000}{11025} = \frac{160\pi}{441}$$

   Designing Filter A:

$$a[n] = \frac{sin\left(\omega_c\left(n - \frac{M}{2}\right)\right)}{\pi\left(n - \frac{M}{2}\right)}$$

$$a[n] = \frac{sin\left(\frac{80\pi}{441}\left(n - \frac{8}{2}\right)\right)}{\pi\left(n - \frac{8}{2}\right)} = \frac{sin\left(\frac{80\pi}{441}(n-4)\right)}{\pi(n-4)}$$

   For n=0:

$$a[0] = a[8] = \frac{sin\left(\frac{80\pi}{441}(-4)\right)}{\pi(-4)} = 0,0604$$

   For n=1:

$$a[1] = a[7] = \frac{sin\left(\frac{80\pi}{441}(1-4)\right)}{\pi(1-4)} = 0,1051$$

For n=2 we have a special case:

$$a[2] = a[6] = \frac{sin\left(\frac{80\pi}{441}(2-4)\right)}{\pi(2-4)} = 0,1446$$

For n=3:

$$a[3] = a[5] = \frac{sin\left(\frac{80\pi}{441}(3-4)\right)}{\pi(3-4)} = 0,1717$$

For n=4:

$$a[4] = \frac{sin\left(\frac{80\pi}{441}(4-4)\right)}{\pi(4-4)} \cong \frac{\frac{80\pi}{441}(4-4)}{\pi(4-4)} = \frac{80}{441} = 0,1814$$

Now, putting everything together we have the following low pass filter:

$$a[.] = \{0,0604\ ,\ 0,1051\ ,\ 0,1446\ ,\ 0,1717\ ,\ 0,1814\ ,\ 0,1717\ ,\ 0,1446\ ,\ 0,1051\ ,\ 0,0604\}$$

Designing Filter B:

$$b[n] = \frac{sin\left(\pi\left(n-\frac{M}{2}\right)\right)}{\pi\left(n-\frac{M}{2}\right)} - \frac{sin\left(\omega_c\left(n-\frac{M}{2}\right)\right)}{\pi\left(n-\frac{M}{2}\right)}$$

$$b[n] = \frac{sin\left(\pi\left(n-\frac{8}{2}\right)\right)}{\pi\left(n-\frac{8}{2}\right)} - \frac{sin\left(\frac{160\pi}{441}\left(n-\frac{8}{2}\right)\right)}{\pi\left(n-\frac{8}{2}\right)} = \frac{sin(\pi(n-4))}{\pi(n-4)} - \frac{sin\left(\frac{160\pi}{441}(n-4)\right)}{\pi(n-4)}$$

For n=0:

$$b[0] = b[8] = \frac{sin(\pi(-4))}{\pi(-4)} - \frac{sin\left(\frac{160\pi}{441}(-4)\right)}{\pi(-4)} = 0,0786$$

For n=1:

$$b[1] = b[7] = \frac{sin(\pi(1-4))}{\pi(1-4)} - \frac{sin\left(\frac{160\pi}{441}(1-4)\right)}{\pi(1-4)} = 0,0291$$

For n=2:

$$b[2] = b[6] = \frac{sin(\pi(2-4))}{\pi(2-4)} - \frac{sin\left(\frac{160\pi}{441}(2-4)\right)}{\pi(2-4)} = -0,1208$$

For n=3:

$$b[3] = b[5] = \frac{sin(\pi(3-4))}{\pi(3-4)} - \frac{sin\left(\frac{160\pi}{441}(3-4)\right)}{\pi(3-4)} = -0,2892$$

For n=4:

$$b[4] \cong \frac{160\pi}{441} = 1,1398$$

Now, putting everything together we have the following high pass filter:

$$b[.] = \{0,0786,\ 0,0291,-0,1208,-0,2892,\ 1,1398,-0,2892,-0,1208,\ 0,0291,\ 0,0786\}$$

Now we find both filter's transfer functions using the Z transform and add them together to discover the desired band-stop filter coefficients:

$$G[Z] = \sum_{k=0}^{N-1} g_k .\ z^{-k}$$

$$A[Z] = 0,0604 + 0,1051z^{-1} + 0,1446z^{-2} + 0,1717z^{-3} + 0,1814z^{-4} + 0,1717z^{-5} + 0,1446z^{-6} + 0,1051z^{-7} + 0,0604z^{-8}$$

$$B[Z] = 0,0786 + 0,0291z^{-1} - 0,1208z^{-2} - 0,2892z^{-3} + 1,1398z^{-4} - 0,2892z^{-5} - 0,1208z^{-6} + 0,0291z^{-7} + 0,0786z^{-8}$$

$Q[Z\} = A[Z] + B[Z]$

$$Q[Z] = 0,1390 + 0,1342z^{-1} + 0,0238z^{-2} - 0,1175z^{-3} + 1,3212z^{-4} - 0,1175z^{-5} + 0,0238z^{-6} + 0,1342z^{-7} + 0,1390z^{-8}$$

Now to normalize a band stop filter to 0dBs, let's check if the coefficients add up to 1:

$0,1390 + 0,1342 + 0,0238 - 0,1175 + 1,3212 - 0,1175 + 0,0238 + 0,1342 + 0,1390 = 1,6802$

In order for the filter to have a gain of 0dB in the pass band, the coefficients must to add up to a total of 1, therefore, we must divide them by the total, giving us:

$$Q[Z] = 0,0827 + 0,0799z^{-1} + 0,0142z^{-2} - 0,0699z^{-3} + 0,7863z^{-4} - 0,0699z^{-5} + 0,0142z^{-6} + 0,0799z^{-7} + 0,0827z^{-8}$$

Finally, to obtain the differential equation y[n] for filtering the signal x[n] with q[n]:

$Y[Z] = X[Z].Q[Z]$

$Y[Z] = 0,0827X[Z] + 0,0799z^{-1}X[Z] + 0,0142z^{-2}X[Z] - 0,0699z^{-3}X[Z] + 0,7863z^{-4}X[Z] - 0,0699z^{-5}X[Z] + 0,0142z^{-6}X[Z] + 0,0799z^{-7}X[Z] + 0,0827z^{-8}X[Z]$

using the inverse Z transform property:

$z^{-k}X[Z] \rightarrow x[n - k]$

We get the following difference equation:

$$y[n] = 0,0827x[n] + 0,0799x[n-1] + 0,0142x[n-2] - 0,0699x[n-3] + 0,7863x[n-4] - 0,0699x[n-5] + 0,0142x[n-6] + 0,0799x[n-7] + 0,0827x[n-8]$$

2. Designing and writing the difference equation for an FIR filter with the following specifications:

$0,98 \leq \left|H(e^{j\omega})\right| \leq 1,02$ in the range of $0 \leq \omega \leq 0,1\pi$
$\left|H(e^{j\omega})\right| \leq 0,06$ in the range of $0,4\pi \leq \omega \leq \pi$

Finding the smallest fluctuation:

$\Delta_{H1} = 1,02 - 0,98 = 0,04$
$\Delta_{H2} = 0,06$
Therefore Δ_{H1} is more restrictive

Finding the window function:

$20log(0.04) \ = \ -27,9588dB$

From this we can conclude the Hanning window will be sufficient for our needs.

Our filter has a transition band of:

$\Delta_t \ = \ \frac{0,4\pi - 0,1\pi}{2\pi} = \ 0,15Hz$

Finding the filter order:

$M \ = \ \frac{3,1}{\Delta t} \ = \ \frac{3.1}{0.15} \ = \ 20,66667$

Rounding up to the next even number gives us:

$M \ = \ 22$

Designing the low-pass filter H:

Approximating the cutoff frequency:

$\omega_c \ = \ 0,4\pi \ - \ 0,1\pi \ = \ 0,3\pi$

$$h[n] \ = \ \frac{sin\left(\omega_c\left(n-\frac{M}{2}\right)\right)}{\pi\left(n-\frac{M}{2}\right)}\left(\frac{1}{2} - \frac{1}{2}cos(2\pi\frac{n}{M})\right)$$

$$h[n] \ = \ \frac{sin(0,3\pi(n-11))}{\pi(n-11)}(0,5 \ - \ 0,5cos(2\pi\frac{n}{22}))$$

For n=0 to n=22:

$h[0] \ = h[22] \ = \ 0$
$h[1] \ = h[21] \ = \ 0$
$h[2] \ = h[20] \ = \ 2,2711x10^{-3}$
$h[3] \ = h[19] \ = \ 6,5303x10^{-3}$
$h[4] \ = h[18] \ = \ 4,1073x10^{-3}$
$h[5] \ = h[17] \ =- \ 1,3373x10^{-2}$
$h[6] \ = h[16] \ =- \ 3,6361x10^{-2}$
$h[7] \ = h[15] \ =- \ 3,3103x10^{-2}$
$h[8] \ = h[14] \ = \ 2,7130x10^{-2}$
$h[9] \ = h[13] \ = \ 1,3935x10^{-1}$
$h[10] \ = h[12] \ = \ 2,5230x10^{-1}$

$h[11] \ = 3x10^{-1}$

Now, putting everything together we have the following low pass filter:

$$h[.] \ = \ \{0, 0, \ 2, 2711x10^{-3}, \ 6, 5303x10^{-3}, \ 4, 1073x10^{-3}, - \ 1, 3373x10^{-2}, - \ 3, 6361x10^{-2}, - \ 3, 3103x10^{-2}, \ 2, 7130x10^{-2}, 1, 3935x10^{-1}, \ 2, 5230x10^{-1}, \ 3x10^{-1}, 2, 5230x10^{-1}, \ 1, 3935x10^{-1}, \ 2, 7130x10^{-2}, - \ 3, 3103x10^{-2}, - \ 3, 6361x10^{-2}, - \ 1, 3373x10^{-2}, \ 4, 1073x10^{-3}, \ 6, 5303x10^{-3}, \ 2, 2711x10^{-3}, 0, 0\}$$

Now we will normalize the filter to a gain of 0dB in the pass band, by checking if the coefficients add up to 1, and if not, divide them by the total sum. Firstly, checking the total sum value:

$$\sum_{n=0}^{M=22} h[n] \ = \ 0, 9977034$$

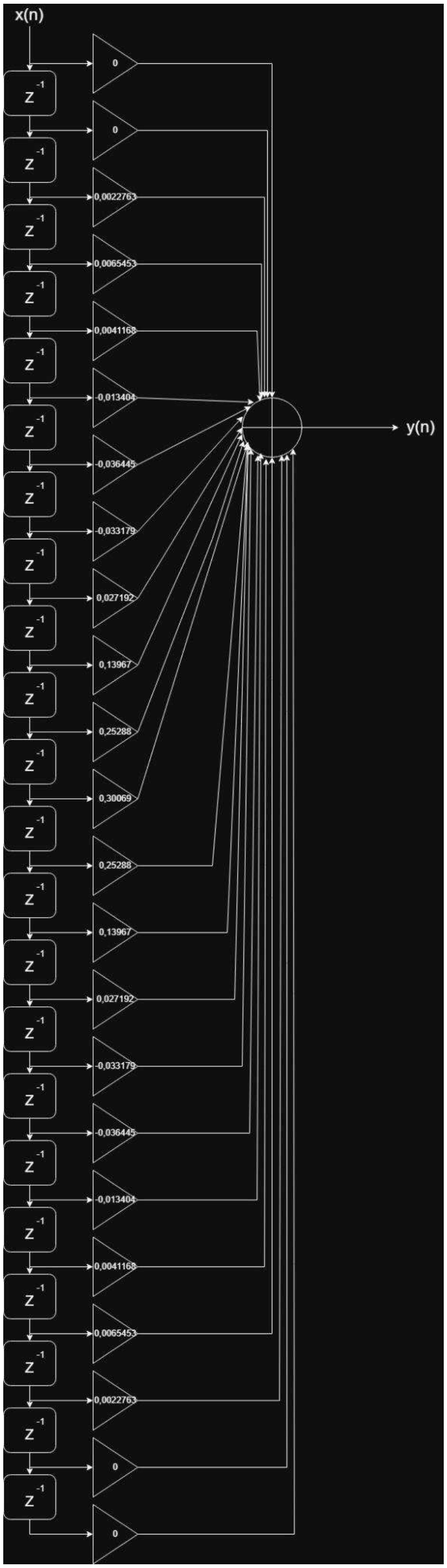
To have the normalized filter, we will divide the coefficients by the value above, giving us the filter:

$$h[.] \ = \ \{0, 0, \ 2, 2763x10^{-3}, \ 6, 5453x10^{-3}, \ 4, 1168x10^{-3}, - \ 1, 3404x10^{-2}, - \ 3, 6445x10^{-2}, - \ 3, 3179x10^{-2}, \ 2, 7192x10^{-2}, 1, 3967x10^{-1}, \ 2, 5288x10^{-1}, \ 3, 0069x10^{-1}, 2, 5288x10^{-1}, \ 1, 3967x10^{-1}, \ 2, 7192x10^{-2}, - \ 3, 3179x10^{-2}, - \ 3, 6445x10^{-2}, - \ 1, 3404x10^{-2}, \ 4, 1168x10^{-3}, \ 6, 5453x10^{-3}, \ 2, 2763x10^{-3}, 0, 0\}$$

Finally, to obtain the differential equation y[n] for filtering the signal x[n] with q[n] we transform them into the Z domain, multiply X[Z] by H[Z] and apply the inverse Z transform, giving us:

$$Y[Z] \ = \ 0x[n] \ + \ 0x[n \ - \ 1] \ + \ 2, 2763.10^{-3}x[n \ - \ 2] \ + \ 6, 5453.10^{-3}x[n \ - \ 3] \ + 4, 1168.10^{-3}x[n \ - \ 4] \ - \ 1, 3404.10^{-2}x[n \ - \ 5] \ - \ 3, 6445.10^{-2}x[n \ - \ 6] - \ 3, 3179.10^{-2}x[n \ - \ 7] \ + \ 2, 7192.10^{-2}x[n \ - \ 8] \ + \ 1, 3967.10^{-1}x[n \ - \ 9] + \ 2, 5288.10^{-1}x[n \ - \ 10] \ + \ 3, 0069.10^{-1}x[n \ - \ 11] \ + \ 2, 5288.10^{-1}x[n \ - \ 12] + \ 1, 3967.10^{-1}x[n \ - \ 13] \ + \ 2, 7192.10^{-2}x[n \ - \ 14] \ - \ 3, 3179.10^{-2}x[n \ - \ 15] - \ 3, 6445.10^{-2}x[n \ - \ 16] \ - \ 1, 3404.10^{-2}x[n \ - \ 17] \ + \ 4, 1168.10^{-3}x[n \ - \ 18] + \ 6, 5453.10^{-3}x[n \ - \ 19] \ + \ 2, 2763.10^{-3}x[n \ - \ 20] \ + \ 0x[n \ - \ 21] \ + \ 0x[n \ - \ 22]$$

Now we can draw the block diagram for this filter:



x(n)
z^{-1}
0
0
0,0022763
0,0065453
0,0041168
-0,013404
-0,036445
-0,033179
0,027192
0,13967
0,25288
0,30069
0,25288
0,13967
0,027192
-0,033179
-0,036445
-0,013404
0,0041168
0,0065453
0,0022763
0
0
y(n)