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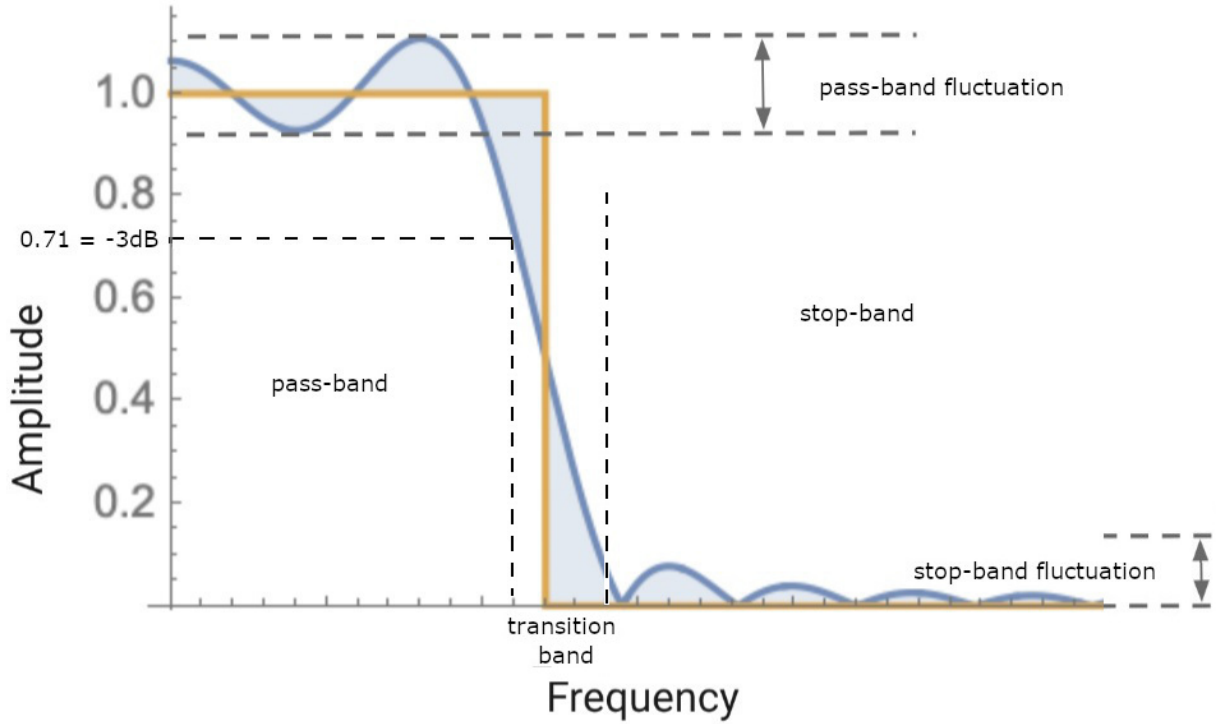
- FIR filter design by windows: the technique we have been using to design FIR filters is called *window method*. This is because we, indeed, truncate the sinc function by covering the part we are interested in with a rectangular window. This produces an effect known as Gibbs Effect, as shown in the Figure below.



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- To get a better frequency response, attenuating the Gibbs effect, we can use different window functions ($w[n]$) to cover the sinc function:

window	**equation**	**order**	**att.**
rectangular	$w[n] = 1, (0 \leqslant n \leqslant M)$	$M = \frac{0.9}{\Delta_t}$	-21dB
Barlett	$w[n] = \begin{cases} \frac{2n}{M}, (0 \leqslant n \leqslant \frac{M}{2}) \\ 2 - \frac{2n}{M}, (\frac{M}{2} + 1 \leqslant n \leqslant M) \end{cases}$	$M = \frac{3.0}{\Delta_t}$	-25dB
Hanning	$w[n] = \frac{1}{2} - \frac{1}{2}\cos(2\pi\frac{n}{M})$	$M = \frac{3.1}{\Delta_t}$	-44dB
Hamming	$w[n] = 0.54 - 0.46 \cdot \cos(2\pi\frac{n}{M})$	$M = \frac{3.3}{\Delta_t}$	-53dB
Blackman	$w[n] = 0.42 - 0.5 \cdot \cos(2\pi\frac{n}{M}) + 0.08 \cdot \cos(4\pi\frac{n}{M})$	$M = \frac{5.5}{\Delta_t}$	-74dB

(*): att. = band attenuation; Δ_t = transition band in Hertz.

Thus, the procedure used to cover the sinc function consists of just a multiplication: $h[n] \leftarrow h[n] \cdot w[n]$, where the window $w[n]$ has just the same length of the filter impulse response. In addition to attenuate the Gibbs effect, the window function can also be used as a parameter to specify the proper filter length for a certain application.

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- Remark: windows, such as Hamming, are also used in signal and speech processing applications that are not related to filtering. Usually, to avoid undesired fluctuations in frequency-domain, time-domain signals are multiplied by a window before its Fourier Transform is performed, for instance.
- Example: Design an FIR filter with the following specifications:
  - $0.95 \leqslant |H(e^{j\omega})| \leqslant 1.05$, in the range $0 \leqslant \omega \leqslant 0.19\pi$
  - $|H(e^{j\omega})| \leqslant 0.01$, in the range $0.21\pi \leqslant \omega \leqslant \pi$
- **Today's Short Test (ST7)**: Design an FIR filter with the following specifications:
  - $0.99 \leqslant |H(e^{j\omega})| \leqslant 1.01$, in the range $0 \leqslant \omega \leqslant 0.15\pi$
  - $|H(e^{j\omega})| \leqslant 0.06$, in the range $0.45\pi \leqslant \omega \leqslant \pi$

  Then, normalize the windowed filter and write down the difference equation to implement the filter you have just designed in a computer-based application.

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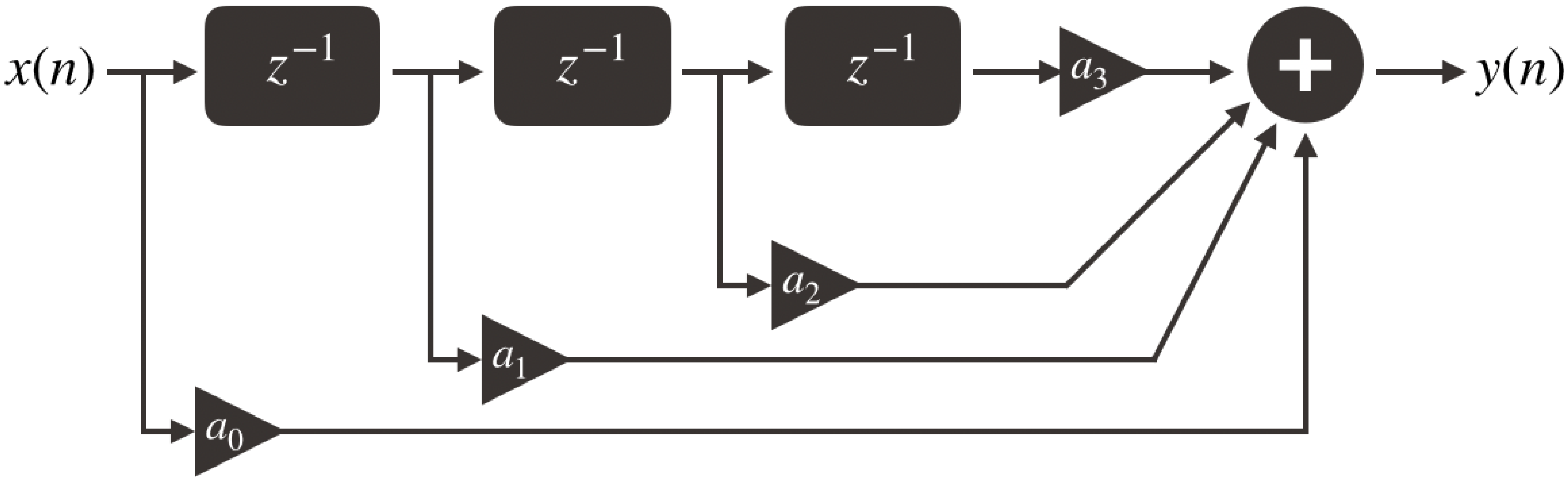
- Difference equations may be depicted graphically by means of a **block diagram**. Here is an example with generic coefficients:



$$y[n] = a_0 x[n] + a_1 x[n-1] + a_2 x[n-2] + a_3 x[n-3]$$

- Numerical Example: draw the block diagram to the difference equation $y[n] = 3x[n] + 4x[n-1] - 5x[n-2] + 2x[n-3]$.