

StatisticalInferenceW4

Me

23/10/2021

```
#Load packages  
library(dplyr, warn.conflicts = F)
```

```
## Warning: package 'dplyr' was built under R version 4.0.5
```

```
library(ggplot2)
```

```
#Exponential function parameters
```

```
lambda <- 0.2
```

```
n <- 40
```

```
num.of.sim <- 1000
```

```
#set the seed
```

```
set.seed(119983)
```

```
#Create a 1000x40 matrix containing the results of the simulation
```

```
sim.distrib <- matrix(data=rexp(n * num.of.sim, lambda), nrow=num.of.sim)
```

```
###Sample mean vs theoretical mean
```

we compute the means and store the results in a dataframe which is what the dplyr and ggplot2 packages take as input and it's also the typical datastructure in R. So we create a dataframe, *sim_mns*.

Here we want to compare the theoretical mean for an exponential distribution, given by $\mu = 1/\lambda = 5$, to the mean of our simulated distribution.

```
#compute the mean for each of the 1000 simulations(rows)
```

```
sim_mns <- data.frame(means=apply(sim.distrib, 1, mean))
```

```
sim<-sim_mns
```

```
#Convert dataframe to tbl_df object for more convenient printing
```

```
sim_mns <- tbl_df(sim_mns)
```

```
## Warning: 'tbl_df()' was deprecated in dplyr 1.0.0.
```

```
## Please use 'tibble::as_tibble()' instead.
```

```
#compute the mean of the simulated means
```

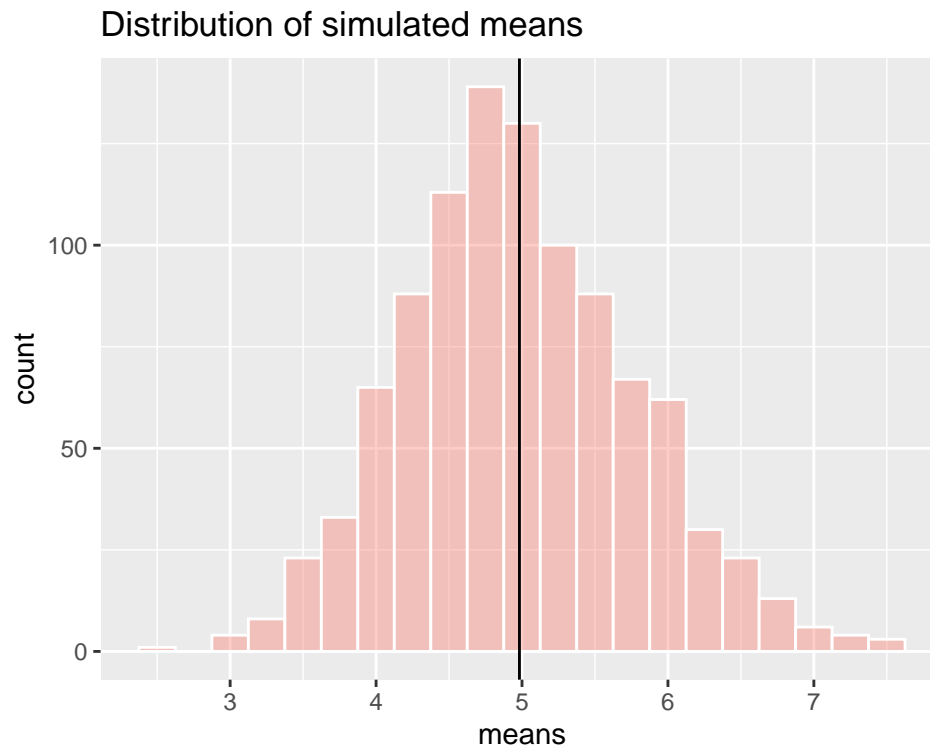
```
(mean_sim <- sim_mns %>% summarize(simulated.mean = mean(means)) %>% unlist())
```

```
## simulated.mean
```

```
## 4.982365
```

```
#Sample Mean Distribution Plot
```

```
sim_mns %>%
  ggplot(aes(x = means) ) + geom_histogram(alpha=0.4, binwidth= .25, fill = "salmon", col = "white") +
  geom_vline(xintercept = mean_sim, color="black", size = 0.5) +
  ggtitle("Distribution of simulated means")
```

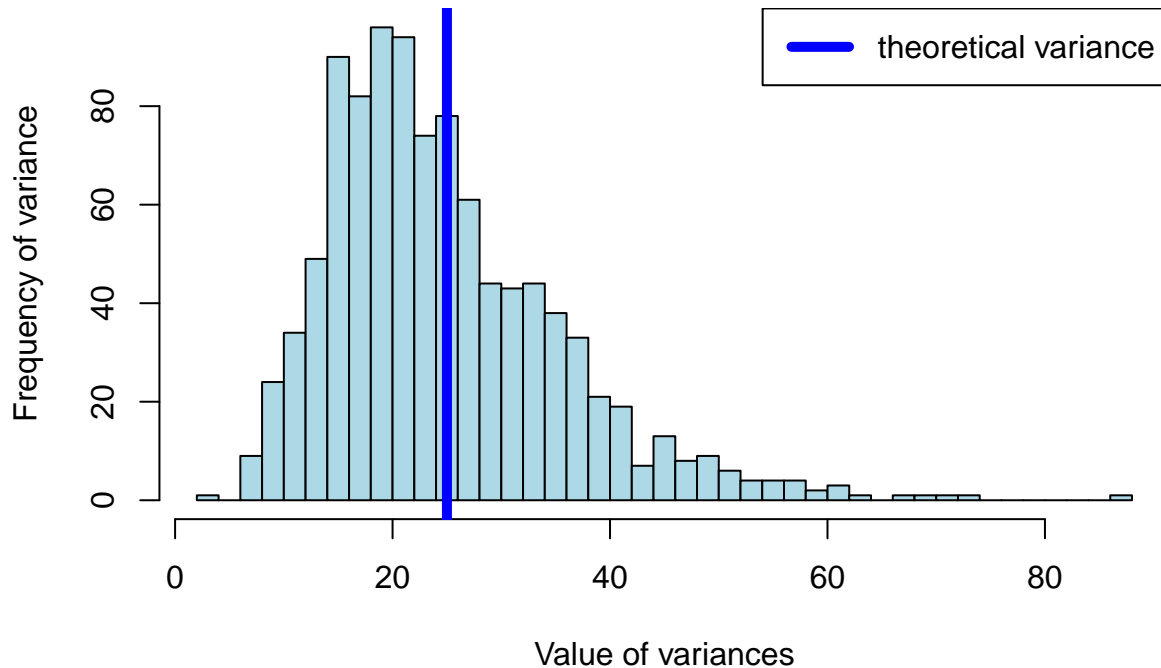


From the plot we can see that the sample mean(the vertical line) is **4.982365** and is very close to the theoretical mean $1/\lambda = 5$

###Sample Variance VS Theoretical Variance

```
Var.distrib <- apply(sim.distrib, 1, var)
hist(Var.distrib, breaks = 50, main = "The distribution of 1000 variance of 40 random exponentials", xlab = "variance", ylab = "count")
abline(v = (1/lambda)^2, lty = 1, lwd = 5, col = "blue")
legend("topright", lty = 1, lwd = 5, col = "blue", legend = "theoretical variance")
```

The distribution of 1000 variance of 40 random exponentials



```
#Compute the variance of the sample means
sd.samp <- sim_mns %>% select(means) %>% unlist() %>% sd()
(var.samp <- sd.samp ^ 2)
```

```
## [1] 0.628253
```

```
#Theoretical variance of the exponential distribution
(((1/lambda))/sqrt(40))^2
```

```
## [1] 0.625
```

The sample variance and the theoretical variance are very close, 0.628253 and 0.625, respectively.

Normality of the Distribution

From the Central limit theorem we know that the distribution of averages of normalized variables becomes that of a standard normal distribution as the sample size increases.

```
par(mfrow = c(3, 1))
hist(sim.distrib, breaks = 50, main = "Distribution of exponentials with lambda equals to 0.2", xlab = "Value of exponentials")
hist(sim.distrib, breaks = 50, main = "The distribution of 1000 averages of 40 random exponentials", xlab = "Value of averages")
simN <- rnorm(1000, mean = mean(sim.distrib), sd = sd(sim.distrib))
hist(simN, breaks = 50, main = "A normal distribution with theoretical mean and sd of the exponentials", xlab = "Value of normal distribution")
```

