IITM-CS4410 : Topics in Algorithmic Combinatorics and Graph Theory Given on: Feb 18, 20:00

Problem Set #2 Due on : Mar 05 , 23:55 Evaluation due: Mar 20, 17:00

- Turn in your solutions electronically at the institute moodle (courses.iitm.ac.in) page. Those who are not familiar with latex, may submit handwritten solutions to the instructor or TAs. Use LaTeX.
- You are expected to obtain the solutions independently.
- Any form of plagiarism will be reported to the Institute Disciplinary committee
- You are discouraged from using any source (internet, textbooks etc) for obtaining the solution. In case referred to, it should be indicated.
- Kindly use this latex file to type in your solutions.
- Late submission penalty: 10% of the earned points with will be deducted if the submission is made within the first two days of the deadline. 20% of the earned points with will be deducted if the submission is made after the first two days and within the first four days the deadline. No submissions will be accepted after four days.

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1. (10 points) A planar graph G is called *outer planar* if and only if G has a planar embedding such that every vertex lies on the boundary of the outer face.
   (a) (4 points) We know that the number of edges in a planar graphs is bounded by $3n - 6$. What is the graph is outer planar? Can you give a better bound on the number of edges? If not, obtain a sequence $G = (G_n)_{n \geq 0}$ of outer planar graphs such that G_n has n vertices and $3n - 6$ edges. Prove your answers.
   (b) (3 points) Show that if G is outer planar, then every minor of G is outer planar.
   (c) (3 points) Using the above, show that an outer planar graph cannot have a K_4 as a minor.
   (d) (**Bonus question: 5 points**) Show that a graph is outer planar if and only if it does not have a K_4 or a $K_{2,3}$ as a minor.
2. (5 points) Let G_1 and G_2 be two graphs such that $|V(G_1) \cap V(G_2)| \leq 1$. Show that $\mathcal{C}(G)$ has a sparse cycle basis if and only if both $\mathcal{C}(G_1)$ and $\mathcal{C}(G_2)$ have one.
3. (5 points) Show that $K_{1,3}$ is extremal without a path of length three.
4. (5 points) Show that $t_{r-1}(n) \leq \frac{1}{2}n^2\frac{r-2}{r-1}$.
5. (5 points) Without using Turán's theorem, show that the maximum number of edges in a triangle free graph is at most $n^2/4$.