IITM-CS4410: Topics in Algoritmic Combinatorics and Grapth Theory Given on: Jan 29, 17:00

Problem Set #1 Due on : Feb 13 , 23:55 Evaluation due: Feb 25, 17:00

• Turn in your solutions electronically at the institute moodle (courses.iitm.ac.in) page. Those who are not familiar with latex, may submit handwritten solutions to the instructor or TAs. Use LaTeX.

- You are expected to obtain the solutions independently.
- Any form of plagiarism will be reported to the Institute Disciplinary committee
- You are discouraged from using any source (internet, textbooks etc) for obtaining the solution. In case referred to, it should be indicated.
- Kindly use this latex file to type in your solutions.
- 1. (6 points) Recall the definition of a cut and a bond in a graph. Show that a cut in a connected graph G is a bond if and only if the corresponding bipartition of V(G) are connected in G.
- 2. (6 points) Show that the cycle space of a graph is spanned by its induced cycles.
- 3. (8 points) Let G be a connected graph and T be a spanning tree of G. For an edge $e \in G \setminus T$, let C_e be the fundamental cycle of e with respect to T and for an edge $f \in T$ let D_f denote the fundamental cut of f with respect to T. Show that $e \in D_f \iff f \in C_e$
- 4. (10 points) Suppose $k \geq 2$ and G be a k-connected graph. Show that for any $S \subseteq V$ with |S| = k, G has a cycle containing all of the vertices in S.