Lecture 19

$\Delta = cx^{**} - cx^{*}$.

Lemma $\forall \epsilon > 0$, $\Pr[\Delta \le \epsilon] \le 2n\epsilon\phi$

[isolation lemma].

Proof:

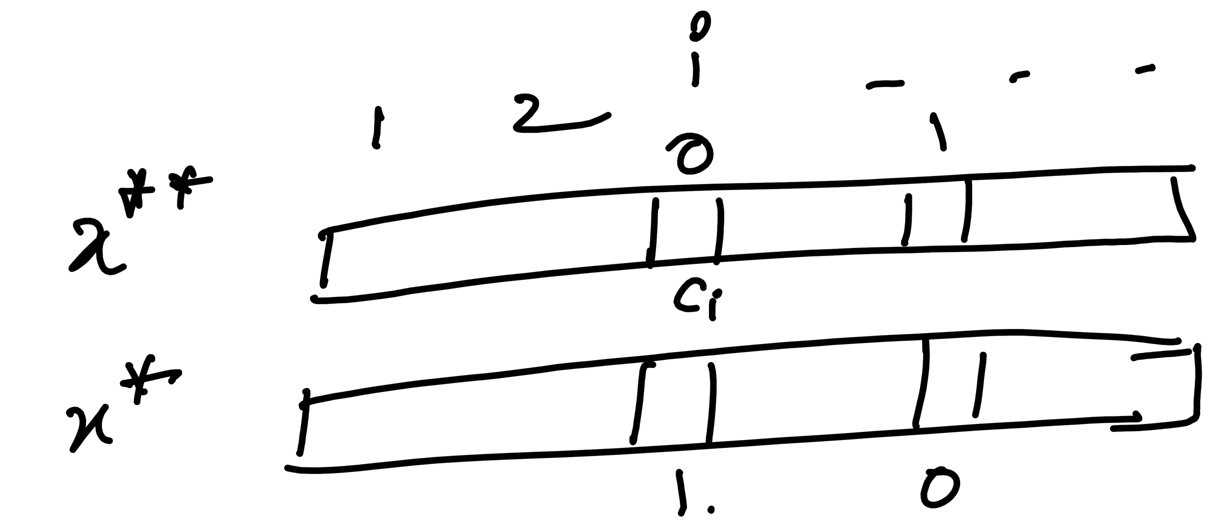
Π I $c_1, \dots, c_n$ $n\phi$ pm

$\min \; c_1x_1 + \dots + c_nx_n$

subj $x \in S$.

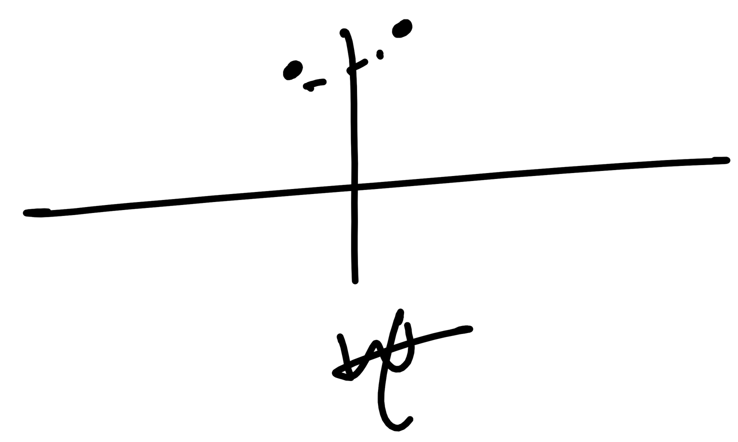
$$x^{*} = \arg\min \{ cx \mid x \in S \}$$

$$x^{**} = \arg\min \{ cx \mid x \in S \setminus \{x^*\} \}.$$



$$\Delta = cx^{**} - cx^{*}$$

$$= \sum_{\substack{i:\, x^{**}_i = 1 \\ \& \; x^*_i = 0}} c_i - \sum_{\substack{j:\, x^*_j = 1 \,\& \\ x^{**}_j = 0}} c_j$$



To define $\Delta_1, \dots, \Delta_n$

$$x^{i=0} = \arg\min \{ cx \mid x \in S \text{ and } x_i = 0 \}.$$

$$x^{i=1} = \arg\min \{ cx \mid x \in S \;\&\; x_i = 1 \}.$$

$$\Delta_i = | cx^{i=0} - cx^{i=1} |$$

Claim $\exists i$ s.t. $\Delta = \Delta_i$

Pf:- Let i be any index s.t. $x^*_i \neq x^{**}_i$

Suppose $x^*_i = 0$ & $x^{**}_i = 1$

then $x^* = x^{i=0}$ & $x^{**} = x^{i=1}$

Case 2: $x^*_i = 1$ & $x^{**}_i = 0$

then $x^* = x^{i=1}$ & $x^{**} = x^{i=0}$

$\therefore$ $\Delta = \Delta_i$ for this index.

$$\Pr[\Delta \le \epsilon] = \Pr[\exists i \;\; \Delta_i \le \epsilon] \le \underline{\underline{\sum_{i=1}^{n} \Pr[\Delta_i \le \epsilon]}}$$

To bound $\Pr[\Delta_i \le \epsilon]$. $\qquad c_1 \cdot c_i \cdot c_n$

Suppose all c_j except c_i are drawn.

then $\Pr[\Delta_i \le \epsilon] = \Pr[|k - c_i| \le \epsilon]$

$= \Pr[c_i$ lies in some interval of length atmost $2\epsilon] \le 2\epsilon\phi$

$\therefore \Pr[\Delta \le \epsilon] \le \underline{\underline{2n\phi\epsilon}}$ $\square$.

Binary optimization

Strongly NP hard $\rightarrow$ no smoothed poly time exact algo

Pseudo linear time $\rightarrow$ smoothed poly time algorithm (zero error)

Pseudo poly time $\rightarrow$ ''

Network Flows

- Successive Shortest path algo

APSP

$O(n^3)$

$O(n^2)$

0.1

$n^{3-\epsilon}$

$\Downarrow$

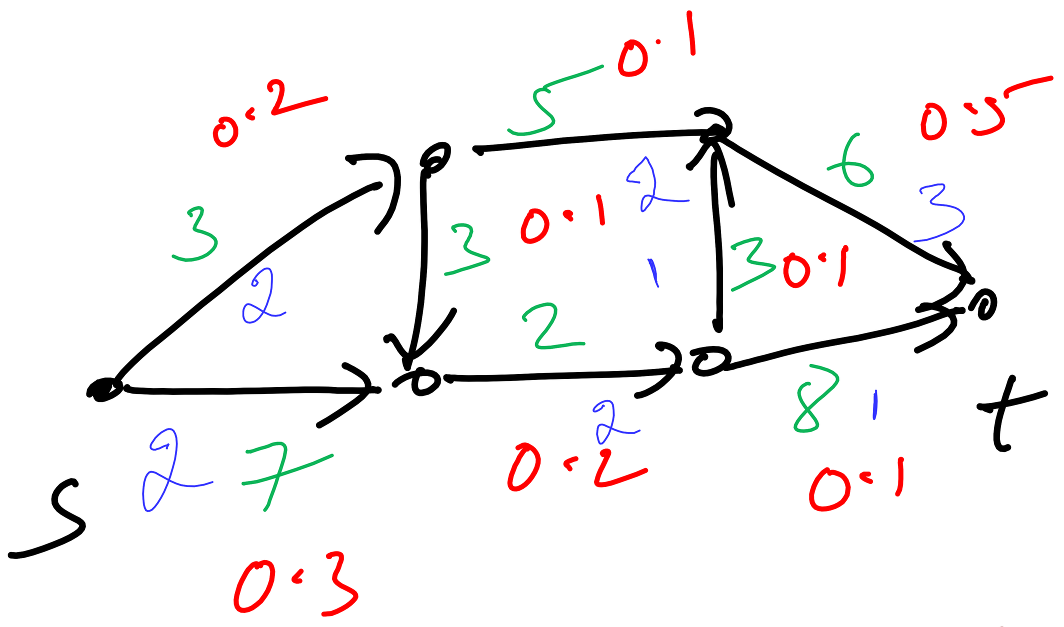
NP vs P

A network $G=(V,E)$ is a directed graph.

s - designated source $t \neq s$ is a designated terminal

Given: edge capacities

$u: E \to \mathbb{R}^{\geq 0}$



and edge costs

$c: E \to [0,1]$.

Cost: $0.2\times 2 + 0.3\times 2$
$+ 2\times 0.2 + 2\times 0.1$
$+ 1\times 0.1 + 1\times .1$
$+ 3\times 0.5$

Objective: assign flows. $f: E \to \mathbb{R}^{\geq 0}$

s.t. $|f|$ is maximum.

s.t.

(1) $\forall e \in E$, $\quad 0 \leq f(e) \leq u(e)$

(2) Conservation $\forall u \notin \{s,t\}$, $\displaystyle\sum_{w:(w,u)\in E} f(w,u) = \sum_{v:(u,v)\in E} f(u,v)$.

net flow at u is zero

$$|f| = \underbrace{\sum_{u:(s,u)\in E} f(s,u)}_{\text{net outflow at } s} = \underbrace{\sum_{v:(v,t)\in E} f(v,t)}_{\text{net inflow at } t}$$

minimum cost flow problem. Compute a maximum flow with minimum cost.

for a flow f, cost of f

$$c(f) = \sum_{e\in E} f(e)\cdot c(e)$$