CS6100

Lecture 9

Knapsack Problem.

n- items labelled $1, \dots, n$.

$p_1, \dots p_n \in \mathbb{R}_{\geq 0}$ $\quad$ p_i - profit for ith item.

$w_1, \dots w_n \in \mathbb{R}_{\geq 0}$ $\quad$ w_i - weight of the ith item.

W - capacity.

Goal: obtain $x_1, \dots x_n \in \{0,1\}$ s.t

maximize $p_1x_1 + \dots + p_nx_n$

subject to $w_1x_1 + \dots + w_nx_n \leq W$.

$x_1, \dots x_n \in \{0,1\}$.

$x = (x_1, \dots x_n) \in \{0,1\}^n$.

$p = (p_1, \dots p_n) \in \mathbb{R}^n_{\geq 0}$

$w = (w_1, \dots w_n) \in \mathbb{R}^n_{\geq 0}$

max. $p \cdot x^T$

s.t $w x^T \leq W$.

& $x \in \{0,1\}^n$

Pareto optimality.

0 1 0 0 1

0 0 1 1 1

Let x & $y \in \{0,1\}^n$

We say that solution y dominates x if.

① $p y^T \geq p x^T$ (ie y has at least as much profit as x)

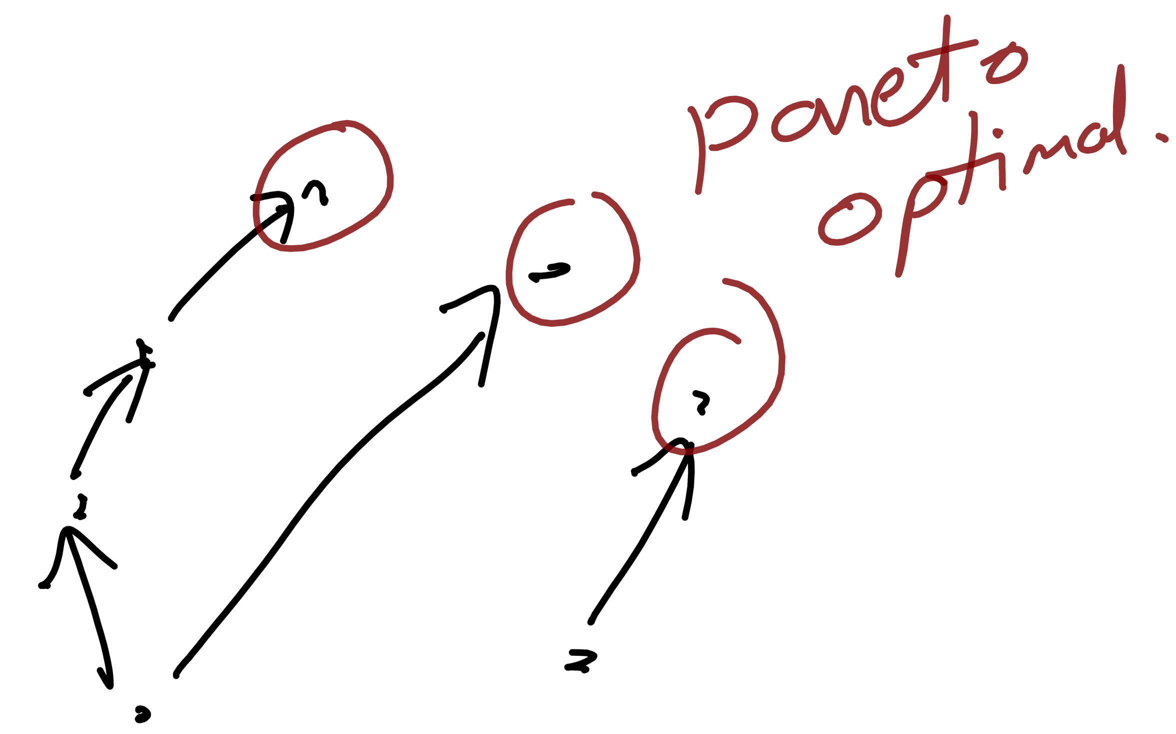
&

② $w y^T \leq w x^T$

and at least one of these is a strict inequality.

Defn: A solution y is said to be "pareto optimal" if it is not dominated by any other solution z.

soln



Lemma :- There is always an optimal solution which is also pareto optimal.