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Joseph E. Yukich

Probability Theory of Classical Euclidean Optimization Problems

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Author

Joseph E. Yukich
Department of Mathematics
Lehigh University
Bethlehem, PA 18015, USA
e-mail: jey0@lehigh.edu

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To Rahul, Nicholas, and Arati

PREFACE

This monograph aims to develop the probability theory of solutions to the classic problems in Euclidean combinatorial optimization, computational geometry, and operations research. These problems are naturally associated with graphs and our chief goal is to describe the almost sure (a.s.) behavior of the total edge length of these graphs. We do this by formulating a general approach which describes the total edge length behavior of graphs on random point sets in Euclidean space.

Many random Euclidean graphs, especially those motivated by some of the classic problems of discrete mathematics, have intrinsic similarities including self-similarity, subadditivity, and superadditivity. We use these similarities to prove laws of large numbers, rates of convergence, and large deviation principles for the solutions of the classic problems in combinatorial optimization. We prove limit theorems for the length of the shortest tour on a random sample, the minimal length of a tree spanned by a random sample, and the length of a minimal Euclidean matching on a random sample. The general tools and methods used to analyze these archetypical problems may also be used to study the stochastic behavior of geometric location problems, Steiner minimal spanning tree problems, and semi-matching problems.

The approach is not limited to problems in combinatorial optimization, but also provides the asymptotics for the total edge lengths of some of the fundamental graphs in computational geometry, including the k nearest neighbors graph. It is anticipated that the approach treats the lengths of the Voronoi and Delaunay tessellations of a random sample. The approach also yields asymptotics for graphs occurring in minimal surfaces, including the length of the minimal triangulation of a random sample as well as the area of the minimal tetrahedralization of a random sample, sometimes known as the probabilistic Plateau problem.

While the main goal is to develop a general structure describing the limit behavior of solutions to problems in combinatorial optimization, operations research, and computational geometry, there are also some secondary goals. First, we want to survey the remarkable progress in the field, much of which originates in the work of Steele, Rhee, and Talagrand. Along the way we describe some of the main open problems.

Second, we introduce a set of tools which have universal interest and which have applications beyond those mentioned here. The chief tool involves the simultaneous use of geometric subadditivity and superadditivity in nearly all problems. Additional useful tools include isoperimetry and martingale inequalities.

Most of the tools and methods come from probability and combinatorics. This monograph, which is essentially self-contained, may be read by both probabilists and combinatorialists. I have tried to make the monograph accessible to both sets of researchers and hope that it will benefit graph theorists and theoretical computer scientists. The close connection to problems of statistical mechanics may interest researchers in that field as well.

There is considerable overlap between the existing literature and this monograph. However, some of the foundational results, including Theorems 4.1 and 4.3, are new. The analysis of partitioning heuristics (Chapter 5), large deviations, and the applications of isoperimetry (Chapter 6) are also new. In Chapter 7 the generalized umbrella theorem for Euclidean functionals on $\mathbb{R}^d$ has not appeared before. Several of the applications in Chapter 8 are new, particularly the applications to the k nearest neighbors problem, the many traveling salesman problem, and the geometric location problem.

I owe a considerable debt of gratitude to Michel Talagrand, who at the outset encouraged me to write this monograph. His continued support, inspiration, and suggestions were invaluable. I am especially happy to thank Michael Steele for insightful suggestions and conversations covering a span of several years. His comments on a preliminary version of this book have improved the exposition in more ways than one. Numerous researchers and colleagues have provided valuable comments and it is a pleasure to acknowledge the assistance of Michael Aizenman, Amir Dembo, Vladimir Dobrić, Bennett Eisenberg, Wei-Min Huang, Garth Isaak, David Johnson, Kuntal McElroy, Kate McGivney, Charles Redmond, WanSoo Rhee, Peter Shor, and Ofer Zeitouni. In particular, Sungchul Lee and Daniel Rose read through most chapters in their entirety, checking for accuracy. They are due special thanks.

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