

Charged Leptons

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1.1 Overview

The theme of the “Snowmass on the Mississippi” exercise can be simply summed up as “How do we find New Physics?”. The Intensity Frontier answer evokes the power and reach of virtual processes in both finding evidence for New Physics and constraining its properties. Experiments in the lepton sector of the Intensity Frontier, by searching for rare decay processes involving lepton flavor violation and CP -violation, and by making precision measurements of quantities whose value is extremely well-predicted in the Standard Model, can advance our understanding of the most basic features of the Standard Model for which we currently have no rationale. Why are there three lepton families? Since lepton flavor conservation is violated in the neutrino sector, is it violated in the charged lepton sector as well? Why are the patterns of lepton and quark flavor mixing so different?

Charged leptons are unique in several ways:

- They directly probe the couplings of new particles to leptons. This is unique in that the current energy frontier machine, the CERN Large Hadron Collider (LHC), is a hadron collider. It is very effective for probing the quark sector, but is significantly more limited in the lepton sector.
- Very precise measurements and sensitive searches can be made at a level that is difficult to achieve in other sectors.
- They can be studied using a diverse set of independent processes. The combination of these studies can provide additional insights into the structure of the lepton sector.
- Hadronic uncertainties in the Standard Model predictions are either insignificant, or in the case of muon $g - 2$, are controlled using independent data sets.
- There are many cases, particularly charged lepton flavor violation (CLFV), where any signal would be an indisputable discovery of New Physics.

There are many important charged lepton observables that are best studied using electrons, most notably the search for an electron electric dipole moment (EDM). In most cases, these experiments are performed using outer-shell or shared electrons in either atoms or molecules. These topics are covered in detail by the Nucleons/Nuclei/Atoms working group; we refer the reader to the that chapter of this document.

The program of studies of charged leptons is diverse, encompassing highly optimized, single-purpose experiments that focus on near-forbidden interactions of muons and multi-purpose experiments that take advantage of the large τ -pair production cross section at B factories. Very large improvements in sensitivity are possible in the immediate future, and even larger sensitivity gains can be made at Project X. New experiments can probe rare processes at rates five orders of magnitude more sensitive than current bounds. These improvements will be a significant part of the program to understand new short-distance dynamics or new ultra-weak interactions.

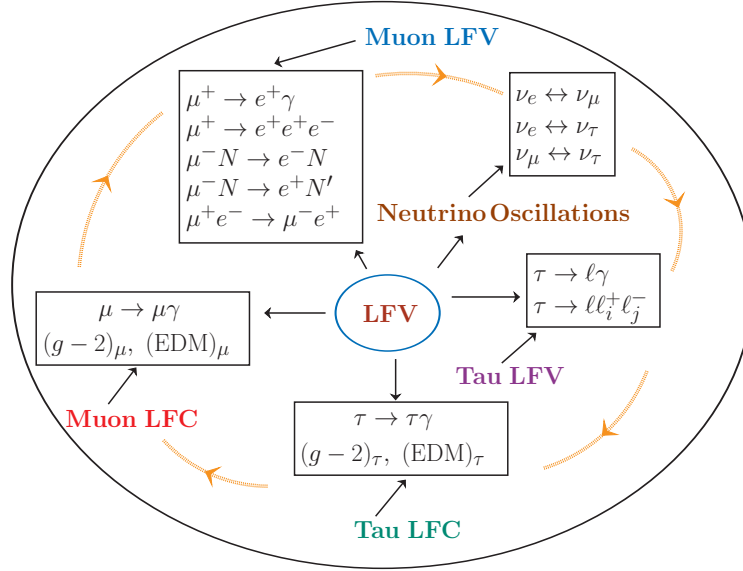


Figure 1-1. Interconnection between various lepton flavor violating and lepton flavor conserving processes.

The charged lepton sector is an integral part of the broader intensity frontier program and provides a vital link to the energy frontier. In the same way that the results of each charged lepton experiment are much more sensitive to New Physics when taken together, charged lepton sector results as a whole are more powerful in concert with other Intensity and Energy Frontier experiments. In particular, there are three domains where such a combination is a crucial probe of New Physics. First, since neutrinos and charged leptons form a natural doublet, one would expect any new physics effects in neutrinos to also be seen in charged lepton experiments with sufficient sensitivity. Second, any complete theory of flavor generation and the observed matter-antimatter asymmetry in the universe must relate flavor and CP (or T) violation in the heavy quark, neutrino, and charged lepton sectors. Third, any theory that predicts new particles or interactions at the LHC must also account for the virtual effects of those particles on decays and interactions of charged leptons and heavy quarks. Thus, the major expansion in the study of charged leptons now underway is a natural extension of the successful heavy quark, neutrino and energy frontier programs of the previous decades.

A fourth domain is the probe of new ultra-weak, low energy interactions, referred to collectively as hidden or dark sectors. Here, charged lepton experiments overlap with a wide variety of experiments at the intensity, cosmic, and energy frontiers. A large experimental program is now under way to directly probe for new hidden sectors, particularly in regions of parameter space consistent with the muon $g-2$ anomaly. This program is covered in detail in the “New, Light, Weakly-Coupled Particles” chapter of this report.

Fig. 1-1 schematically depicts the interconnection between various flavor-conserving and -violating processes in the lepton sector. In an underlying theory, neutrino flavor oscillations, charged lepton flavor violation, the anomalous magnetic moments, and permanent electric dipole moments are all related. Each experimental avenue we pursue allows us to uncover further attributes of the underlying theory.

There are many important physical observables potentially sensitive to New Physics effects in charged lepton processes. Below, they are split into flavor violating observables and flavor conserving observables such as $g-2$, EDMs, and parity violation measurements. Tau decays offer a unique opportunity to simultaneously study flavor-conserving, flavor-violating, CP -violating, and T -violating effects and are discussed in their own section below.

1.2 Flavor-Violating Processes

1.2.1 Theory Overview

1.2.1.1 CLFV Decays in Specific New Physics Models

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The current limit on the $\mu^+ \rightarrow e^+\gamma$ branching fraction is 5.7×10^{-13} at 90% confidence level from MEG at PSI [1], using 3.6×10^{14} stopped muons, from data taken in 2009–2011. Their sensitivity is dominated by accidental background, which is related to the muon stop rate R_μ and various experimental resolutions [3]:

$$N_{\text{acc}} \propto R_\mu^2 \times \Delta E_\gamma^2 \times \Delta P_e \times \Delta \Theta_{e\gamma}^2 \times \Delta t_{e\gamma} \times T, \quad (1.1)$$

where $\Delta t_{e\gamma}$, ΔP_e , ΔE_γ , and $\Delta \Theta_{e\gamma}$ are the resolutions of detector timing, positron momentum, photon energy, and positron-photon angle, respectively, and T is total data acquisition time. MEG will continue taking data through 2013. They expect to approximately double their published dataset [?].

The MEG upgrade plans to improve the experiment sensitivity by a factor of 10. They will increase the intensity of the surface beam, and use a thinner or active stopping target. The detector upgrade includes a larger drift chamber with thinner wires and smaller cells, an improved timing counter, and a larger LXe calorimeter with SiPM readout. Data-taking is planned in the years 2016–2019.

The photon energy resolution is a limiting factor in a $\mu^+ \rightarrow e^+\gamma$ search. A pair spectrometer, based on reconstruction of e^+e^- pair tracks produced in a thin converter can provide improved photon energy resolution, at a sacrifice in efficiency. Even though only a small fraction of photons will convert, the much higher power beam at Project X can compensate for the loss of statistics [2]. The thickness of the converter affects the energy resolution due to multiple scattering. A detailed study is thus required to prove that this approach does indeed provide an overall improvement, as well as to optimize the converter thickness and to study the utility of making the converter active.

We have conducted an initial study of this concept using a fast simulation tool (FastSim) originally developed for the SuperB experiment using the BABAR software framework and analysis tools. FastSim allows us

to model detector components as two-dimensional shells of simple geometries. Particle scattering, energy loss, secondary particle production (due to Compton scattering, Bremsstrahlung, conversion, EM or hadron showers, *etc.*) are simulated at the intersection of particles with detector shells. Tracks are reconstructed with a Kalman filter into piece-wise trajectories.

The FastSim model consists of a thin aluminum stopping target and a six-layer cylindrical silicon detector. A 0.56 mm thick lead ($10\% X_0$) half cylinder covering $0-\pi$ in azimuthal angle at $R = 80$ mm serves as the photon converter. The target consists of two cones connected at their base; each cone is 30 mm high, 5 mm in radius, and $50\ \mu\text{m}$ thick. Two silicon detector cylinders are placed close the target for better vertexing resolution; two layers are placed just outside the Pb converter, and two layers a few cm away. The layout is shown in Fig. 1-2. The entire detector is placed in a 1 T solenoidal magnetic field.

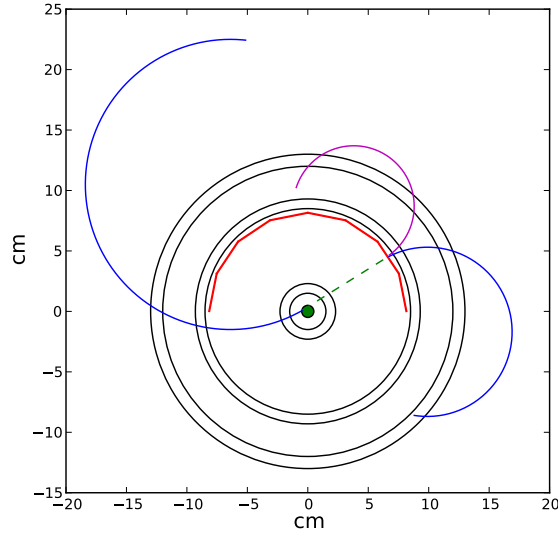


Figure 1-2. Schematic drawing (x-y view) of the $\mu \rightarrow e\gamma$ detector used in the FastSim model.

We generate muons at rest and let them decay via $\mu^+ \rightarrow e^+\gamma$ to study the reconstruction efficiency and resolution. Approximately 1.3% of generated signal events are well-reconstructed, passing quality and fiducial selections criteria. The photon energy resolution is approximately 200 keV, similar to the positron momentum resolution, which corresponds to 0.37% for 52.8 MeV photons. This is a great improvement compared to the 1.7%–2.4% resolution of the current MEG and the 1.0%–1.1% resolution goal of the MEG upgrade. The muon candidate mass resolution is 340 keV (85% Gaussian core). Figure 1-3 shows the photon energy and muon candidate mass resolutions. The positron energy resolution is better than that of MEG, but not as good as what is expected in the MEG upgrade. Angular resolution is similar to the current MEG.

Using a converted photon to increase the $\mu^+ \rightarrow e^+\gamma$ detection sensitivity thus appears to be a promising approach. Further studies are needed to quantify the requirements to improve upon the MEG upgrade sensitivity by an order of magnitude or more..

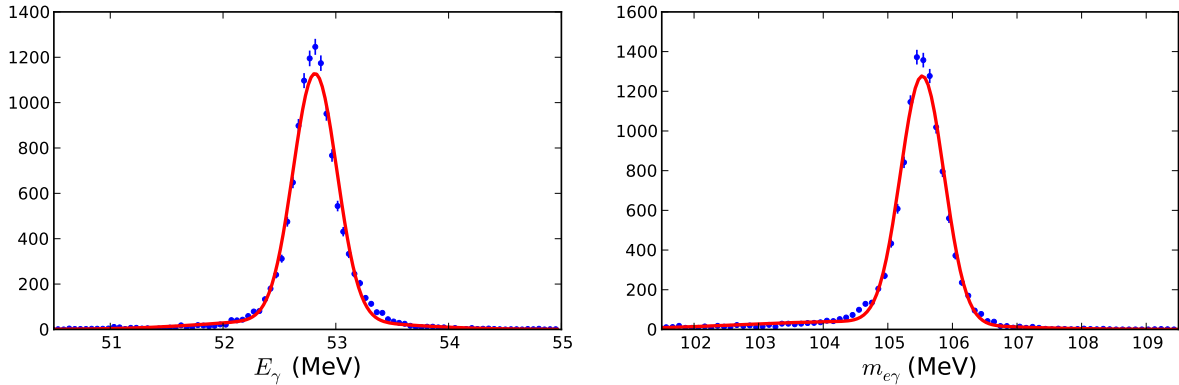


Figure 1-3. Photon energy and muon candidate mass resolutions in $\mu^+ \rightarrow e^+ \gamma$ FastSim study. Fitted curve is a double-Gaussian distribution.

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1.2.2.2.4 $\mu \rightarrow 3e$

1.2.2.2.5 Muonium \rightarrow anti-muonium

1.2.3 Tau Experimental Overview

In contrast to muon CLFV searches, in which a single dedicated experiment is required for a given decay, τ lepton CLFV searches are conducted using the large data sets collected in comprehensive e^+e^- or hadron collider experiments. The relative theoretical parameter reach of μ and τ decay experiments is model-dependent, and thus comparisons of limits or observations in the two cases can serve to distinguish between models. Tests with taus can be more powerful on an event-by-event basis than those using muons, since the large τ mass greatly decreases Glashow-Iliopoulos-Maiani (GIM) suppression, correspondingly increasing new physics partial widths (typically by a factor of ≥ 500 in $\mathcal{B}(\tau \rightarrow \mu\gamma)$ or $e\gamma$ *vs.* $\mathcal{B}(\mu \rightarrow \gamma\gamma)$). The difficulty is that one can make 10^{11} muons per second, while the samples from *BABAR* and *Belle* together total $\sim 10^{10}$ events. The new generation of super B or τ/c factories, [?] promise to extend the experimental reach in τ decays to levels that sensitively probe new physics in the lepton sector. Since CLFV is severely suppressed in the Standard Model, CLFV τ decays are especially clean probes for new physics effects. Super flavor factories can search for CLFV decays at a sensitivity that directly confronts many models of new physics. The super flavor factories can access τ CLFV decay rates two orders of magnitude smaller than current limits for the cleanest channels (*e.g.*, $\tau \rightarrow 3\ell$), and one order of magnitude smaller for other modes that have irreducible backgrounds, such as $\tau \rightarrow \ell\gamma$.

Polarized beams at an e^+e^- collider can provide further experimental advantages. Belle II at SuperKEKB will not have a polarized beam, but both the proposed BINP and Tor Vergata τ/c factories will have polarized electron beams. Polarization of the taus thus produced provides several advantages. It allows reduction of backgrounds in certain CLFV decay modes, as well as providing sensitive new observables that increase precision in other important measurements, including searches for CP violation in τ production and decay, the measurement of $g-2$ of the τ , and the search for a τ EDM. Preliminary studies indicate that polarization improves the sensitivity on these quantities by a factor of two to three. Should the CLFV decay $\tau \rightarrow 3\ell$ be found, a study of the Dalitz plot of the polarized τ decay can determine the Lorentz structure of the CLFV coupling.

The provision of polarization requires a polarized electron gun, a lattice that supports transverse polarization at the desired CM energy, a means of interchanging transverse polarization in the ring and longitudinal polarization at the interaction point and a means of monitoring the polarization, typically a Compton polarimeter to monitor the backscattering of circularly polarized laser light. Provision of a polarized positron beam is difficult and expensive; it is generally also regarded as unnecessary, as most of the advantages of polarization for the measurements cited above can be accomplished with a single polarized beam.

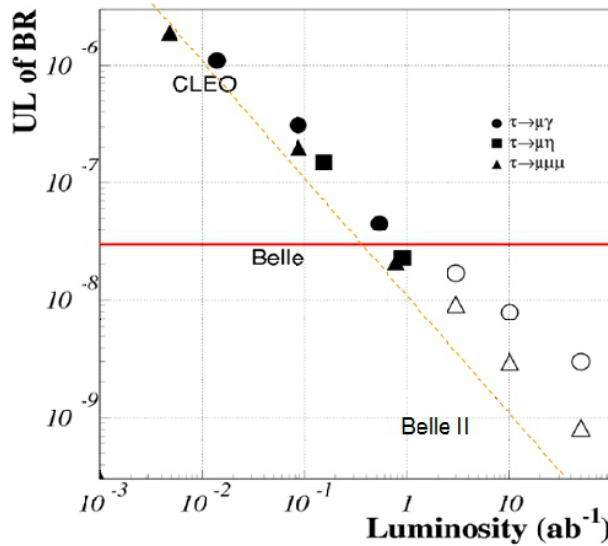


Figure 1-4. Extrapolation of the 90% upper limit sensitivity of Belle-II (open symbols) from existing limits (filled symbols). For $\tau \rightarrow \mu\gamma$, which has irreducible backgrounds, the limit scales as $1/\sqrt{\int \mathcal{L} dt}$. For $\tau \rightarrow \mu\mu\mu$, which is essentially background-free, the limit scales as $1/\int \mathcal{L} dt$.

The sensitivity of τ CLFV searches at SuperKEKB has been estimated by extrapolating from current CLEO, Belle and BABAR limits (see Figure ??). The optimization of search sensitivities depends on the size of the sample as well as on the sources of background. For SuperKEKB-B, the extrapolation for the (largely background-free) $\tau \rightarrow \ell\ell\ell$ modes assumes $1/\mathcal{L}$ scaling up to 5 ab^{-1} ; that for $\tau \rightarrow \ell\gamma$ modes scales as $1/\sqrt{\mathcal{L}}$. The expected sensitivities for several modes are shown for the Belle II experiment in Table ?? [?].

These CLFV sensitivities directly confront a large variety of new physics models. Of particular interest is the correlation between τ CLFV branching ratios such as $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$, as well as the correlation with $\mu \rightarrow e\gamma$ and the $\mu \rightarrow e$ conversion rate, all of which are diagnostic of particular models. A polarized electron beam potentially allows the possibility of determining the helicity structure of CLFV couplings from Dalitz plot analyses of, for example, $\tau \rightarrow 3\ell$ decays.

The experimental situation at a τ/c factory is somewhat different. The luminosity of the proposed projects is $10^{35}\text{cm}^{-2}\text{s}^{-1}$, a factor of eight below the eventual SuperKEKB luminosity. The τ production cross section is, however, larger: $\sigma_{\tau\bar{\tau}}(3.77\text{ GeV})/\sigma_{\tau\bar{\tau}}(10.58\text{ GeV}) = 3$, and both have a polarized electron beam. In addition, while a Super B factory is likely to spend the bulk of its running time at the $\Upsilon(4S)$, a τ/c factory will take data more evenly through the accessible energy range. A study for the BINP machine, with 1.5 ab^{-1} at 3.686 GeV , 3.5 ab^{-1} at 3.770 GeV , and 2.0 ab^{-1} at 4.170 GeV , corresponding to 2.5×10^{10} produced τ pairs, quotes a 90% confidence level limit on $\mathcal{B}(\tau \rightarrow \mu\gamma) = (3.3 \times 10^{-10})$, provided the detector has μ/π rejection of a factor of 30. This is nearly an order of magnitude improvement over the SuperKEKB expectation at 50 ab^{-1} .

The experimental discrepancy with the Standard Model prediction for the muon anomalous magnetic moment heightens interest in the possibility of measuring $g - 2$ of the τ lepton using angular distributions in τ -pair production. This can be done at a super flavor factory, with or without electron polarization. With polarized taus one can access new observables that are estimated by Bernab   *et al.*[?] to increase the sensitivity to $g - 2$ by a factor of three, to $\sim 2 \times 10^{-6}$ with 80% electron polarization, which could allow a measurement of the Standard Model moment to a precision of several percent with a data sample of 75 ab^{-1} .

Observation of a τ EDM would be evidence of T violation. T -odd observables can be isolated by the study of τ angular distributions using unpolarized beams. Having a polarized electron beam allows these investigations to be done using the decay products of individual polarized taus. The upper-limit sensitivity for the real part of the τ EDM has been estimated to be to be $|\mathcal{R}e d_\gamma| \simeq 3 \times 10^{-19}\text{ e} \cdot \text{cm}$ with 50 ab^{-1} at Belle II and $|\mathcal{R}e d_\gamma| \simeq 7 \times 10^{-20}\text{ e} \cdot \text{cm}$ with 75 ab^{-1} at SuperB[?].

A CP -violating asymmetry in τ decay would be manifest evidence for physics beyond the Standard Model. BABAR has recently published a 3σ asymmetry in $\tau \rightarrow \pi K_S^0 (\geq 0\pi^0)$ decay[?]. The super flavor factories have the sensitivity to definitely confirm or refute this measurement, and, further, provide access to new CP -odd observables that increase the sensitivity in the search for a CP asymmetry to the level of $\sim 10^{-3}$.

Table 1-1. Expected 90% CL upper limits on $\tau \rightarrow \mu\gamma$, $\tau \rightarrow \mu\mu\mu$, and $\tau \rightarrow \mu\eta$ with 5 ab^{-1} and 50 ab^{-1} data sets from Belle II and Super KEKB.

Process	5 ab^{-1}	50 ab^{-1}
$\text{BR}(\tau \rightarrow \mu\gamma)$	10×10^{-9}	3×10^{-9}
$\text{BR}(\tau \rightarrow \mu\mu\mu)$	3×10^{-9}	1×10^{-9}
$\text{BR}(\tau \rightarrow \mu\eta)$	5×10^{-9}	2×10^{-9}

1.2.3.1 Super B Factory

1.2.3.2 τ -charm Factory

1.2.3.3 Leptoquark ($e \rightarrow \tau$) search

1.3 Flavor-Conserving Processes

1.3.1 Theory overview

1.3.1.1 Muon $g - 2$ and EDM

1.3.1.2 τ $g - 2$ and EDM

1.3.2 Muon $g - 2$ and EDM Experiments

1.4 Dipole Moments

The muon provides a unique opportunity to explore the properties of a second-generation particle with great precision. Several muon properties make these measurements possible. It has a long lifetime of $\simeq 2.2 \mu\text{s}$, it is produced in the weak decay $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$ providing copious numbers of polarized muons, and the weak decay $\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e$ is self-analyzing providing information on the muon spin direction at the time of decay.

In his famous paper on the relativistic theory of the electron, Dirac[?] obtained the correct magnetic moment for the electron, and he also mentioned the possibility of an electric dipole moment, which like the magnetic dipole moment, would be directed along the electron spin direction. The magnetic dipole (MDM) and electric dipole (EDM) moments are given by

$$\vec{\mu} = g \left(\frac{Qe}{2m} \right) \vec{s}, \quad \vec{d} = \eta \left(\frac{Qe}{2mc} \right) \vec{s}, \quad (1.2)$$

where $Q = \pm 1$ and $e > 0$. Dirac theory predicts $g \equiv 2$, but radiative corrections dominated by the lowest-order (mass-independent) Schwinger contribution $a_{e,\mu,\tau} = \alpha/(2\pi)$ [?] make it necessary to write the magnetic moment as

$$\mu = (1 + a) \frac{Qe\hbar}{2m} \quad \text{with} \quad a = \frac{g - 2}{2}. \quad (1.3)$$

The muon played an important role in our discovery of the generation structure of the Standard Model (SM) when experiments at the Nevis cyclotron showed that g_μ was consistent with 2 [?]. Subsequent experiments at Nevis and CERN showed that $a_\mu \simeq \alpha/(2\pi)$ [?, ?], implying that in a magnetic field, the muon behaves like a heavy electron. The SM value of the muon anomaly is now known to better than half a part per million (ppm), and has been measured to a similar precision [?].

The quantity η in Eq. ?? is analogous to the g -value for the magnetic dipole moment. An EDM violates both P and T symmetries [?, ?, ?], and since C is conserved, CP is violated as well. Thus searches for EDMs provide an important tool in our quest to find non-Standard Model CP violation.

The measured value of the muon anomalous magnetic moment is in apparent disagreement with the expected value based on the SM. The BNL E821 experiment finds [?]

$$a_\mu(\text{Expt}) = 116\,592\,089(54)(33) \times 10^{-11}, \quad (1.4)$$

where $a_\mu = (g-2)/2$ is the muon anomaly, and the uncertainties are statistical and systematic, respectively. This can be compared with the SM prediction [?, ?]

$$a_\mu(\text{SM}) = 116\,591\,802(42)(26)(02) \times 10^{-11}, \quad (1.5)$$

where the uncertainties are from the $\mathcal{O}(\alpha^2)$ hadronic vacuum polarization (HVP) contribution, $\mathcal{O}(\alpha^3)$ hadronic contributions (including hadronic light-by-light (HLbL) scattering), and all others (pure QED, including a 5-loop estimate [?], and electroweak, including 2-loops [?]). The hadronic contributions dominate the uncertainty in $a_\mu(\text{SM})$. The discrepancy between the measurement and the SM stands at

$$\Delta a_\mu = 287(80) \times 10^{-11} \quad (1.6)$$

(3.6 standard deviations (σ)), when based on the $e^+e^- \rightarrow \text{hadrons}$ analysis for the HVP contribution [?]. When the HVP analysis is complemented by $\tau \rightarrow \text{hadrons}$, the discrepancy is reduced to 2.4σ [?]. However, a recent re-analysis, employing effective field theory techniques, of the τ data [?] shows virtual agreement with the e^+e^- -based analysis, which would solidify the current discrepancy at the 3.6σ level. Δa_μ is large, roughly two times the EW contribution [?], indicating potentially large new physics contributions.

The anomalous magnetic moment of the muon is sensitive to contributions from a wide range of physics beyond the standard model. It will continue to place stringent restrictions on all of the models, both present and yet to be written down. If physics beyond the standard model is discovered at the LHC or other experiments, a_μ will constitute an indispensable tool to discriminate between very different types of new physics, especially since it is highly sensitive to parameters which are difficult to measure at the LHC. If no new phenomena are found elsewhere, then it represents one of the few ways to probe physics beyond the standard model. In either case, it will play an essential and complementary role in the quest to understand physics beyond the standard model at the TeV scale.

The muon magnetic moment has a special role because it is sensitive to a large class of models related and unrelated to electroweak symmetry breaking and because it combines several properties in a unique way: it is a flavor- and CP-conserving, chirality-flipping and loop-induced quantity. In contrast, many high-energy collider observables at the LHC and a future linear collider are chirality-conserving, and many other low-energy precision observables are CP- or flavor-violating. These unique properties might be the reason why the muon ($g-2$) is the only among the mentioned observables which shows a significant deviation between the experimental value and the SM prediction. Furthermore, while $g-2$ is sensitive to leptonic couplings, b - or K -physics more naturally probe the hadronic couplings of new physics. If charged lepton-flavor violation exists, observables such as $\mu \rightarrow e$ conversion can only determine a combination of the strength of lepton-flavor violation and the mass scale of new physics. In that case, $g-2$ can help to disentangle the nature of the new physics.

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Unravelling the existence and the properties of such new physics requires experimental information complementary to the LHC. The muon ($g-2$), together with searches for charged lepton flavor violation, electric dipole moments, and rare decays, belongs to a class of complementary low-energy experiments.

In fact, The role of $g-2$ as a discriminator between very different standard model extensions is well illustrated by a relation stressed by Czarnecki and Marciano [?]. It holds in a wide range of models as a result of the

chirality-flipping nature of both $g - 2$ and the muon mass: If a new physics model with a mass scale Λ contributes to the muon mass $\delta m_\mu(\text{N.P.})$, it also contributes to a_μ , and the two contributions are related as

$$a_\mu(\text{N.P.}) = \mathcal{O}(1) \times \left(\frac{m_\mu}{\Lambda}\right)^2 \times \left(\frac{\delta m_\mu(\text{N.P.})}{m_\mu}\right). \quad (1.7)$$

The ratio $C(\text{N.P.}) \equiv \delta m_\mu(\text{N.P.})/m_\mu$ cannot be larger than unity unless there is fine-tuning in the muon mass. Hence a first consequence of this relation is that new physics can explain the currently observed deviation (??) only if Λ is at the few-TeV scale or smaller.

In many models, the ratio C arises from one- or even two-loop diagrams, and is then suppressed by factors like $\alpha/4\pi$ or $(\alpha/4\pi)^2$. Hence, even for a given Λ , the contributions to a_μ are highly model dependent.

It is instructive to classify new physics models as follows:

- Models with $C(\text{N.P.}) \simeq 1$: Such models are of interest since the muon mass is essentially generated by radiative effects at some scale Λ . A variety of such models have been discussed in [?], including extended technicolor or generic models with naturally vanishing bare muon mass. For examples of radiative muon mass generation within supersymmetry, see e.g. [?, ?]. In these models the new physics contribution to a_μ can be very large,

$$a_\mu(\Lambda) \simeq \frac{m_\mu^2}{\Lambda^2} \simeq 1100 \times 10^{-11} \left(\frac{1 \text{ TeV}}{\Lambda}\right)^2. \quad (1.8)$$

and the difference Eq. (??) can be used to place a lower limit on the new physics mass scale, which is in the few TeV range [?, ?].

- Models with $C(\text{N.P.}) = \mathcal{O}(\alpha/4\pi)$: Such a loop suppression happens in many models with new weakly interacting particles like Z' or W' , little Higgs or certain extra dimension models. As examples, the contributions to a_μ in a model with $\delta = 1$ (or 2) universal extra dimensions (UED) [?] and the Littlest Higgs model with T-parity (LHT) [?] are given by with $|S_{KK}| \lesssim 1$ [?]. A difference as large as Eq. (??) is very hard to accommodate unless the mass scale is very small, of the order of M_Z , which however is often excluded e.g. by LEP measurements. So typically these models predict very small contributions to a_μ and will be disfavored if the current deviation will be confirmed by the new a_μ measurement.

Exceptions are provided by models where new particles interact with muons but are otherwise hidden from searches. An example is the model with a new gauge boson associated to a gauged lepton number $L_\mu - L_\tau$ [?], where a gauge boson mass of $\mathcal{O}(100 \text{ GeV})$ and large a_μ are viable.

- Models with intermediate values for $C(\text{N.P.})$ and mass scales around the weak scale: In such models, contributions to a_μ could be as large as Eq. (??) or even larger, or smaller, depending on the details of the model. This implies that a more precise a_μ -measurement will have significant impact on such models and can even be used to measure model parameters. Supersymmetric (SUSY) models are the best known examples, so muon $g - 2$ would have substantial sensitivity to SUSY particles. Compared to generic perturbative models, supersymmetry provides an enhancement to $C(\text{SUSY}) = \mathcal{O}(\tan \beta \times \alpha/4\pi)$ and to $a_\mu(\text{SUSY})$ by a factor $\tan \beta$ (the ratio of the vacuum expectation values of the two Higgs fields). Typical SUSY diagrams for the magnetic dipole moment, the electric dipole moment, and the lepton-number violating conversion process $\mu \rightarrow e$ in the field of a nucleus are shown pictorially in Fig. ???. The shown diagrams contain the SUSY partners of the muon, electron and the SM $U(1)_Y$ gauge boson, $\tilde{\mu}$, \tilde{e} , \tilde{B} . The full SUSY contributions involve also the SUSY partners to the neutrinos and all SM gauge

and Higgs bosons. In a model with SUSY masses equal to Λ the SUSY contribution to a_μ is given by [?]

$$a_\mu(\text{SUSY}) \simeq \text{sgn}(\mu) 130 \times 10^{-11} \tan \beta \left(\frac{100 \text{ GeV}}{\Lambda} \right)^2 \quad (1.9)$$

which indicates the dependence on $\tan \beta$, and the SUSY mass scale, as well as the sign of the SUSY μ -parameter. The formula still approximately applies even if only the smuon and chargino masses are of the order Λ but e.g. squarks and gluinos are much heavier. However the SUSY contributions to a_μ depend strongly on the details of mass splittings between the weakly interacting SUSY particles. Thus muon $g - 2$ is sensitive to SUSY models with SUSY masses in the few hundred GeV range, and it will help to measure SUSY parameters.

There are also non-supersymmetric models with similar enhancements. For instance, lepton flavor mixing can help. An example is provided in Ref. [?] by a model with two Higgs doublets and four generations, which can accommodate large Δa_μ without violating constraints on lepton flavor violation. In variants of Randall-Sundrum models [?, ?, ?] and large extra dimension models [?], large contributions to a_μ might be possible from exchange of Kaluza-Klein gravitons, but the theoretical evaluation is difficult because of cutoff dependences. A recent evaluation of the non-graviton contributions in Randall-Sundrum models, however, obtained a very small result [?].

Further examples include scenarios of unparticle physics [?, ?] (here a more precise a_μ -measurement would constrain the unparticle scale dimension and effective couplings), generic models with a hidden sector at the weak scale [?] or a model with the discrete flavor symmetry group T' and Higgs triplets [?] (here a more precise a_μ -measurement would constrain hidden sector/Higgs triplet masses and couplings), or the model proposed in Ref. [?], which implements the idea that neutrino masses, leptogenesis and the deviation in a_μ all originate from dark matter particles. In the latter model, new leptons and scalar particles are predicted, and a_μ provides significant constraints on the masses and Yukawa couplings of the new particles.

The following types of new physics scenarios are quite different from the ones above:

- Models with extended Higgs sector but without the $\tan \beta$ -enhancement of SUSY models. Among these models are the usual two-Higgs-doublet models. The one-loop contribution of the extra Higgs states to a_μ is suppressed by two additional powers of the muon Yukawa coupling, corresponding to $a_\mu(\text{N.P.}) \propto m_\mu^4/\Lambda^4$ at the one-loop level. Two-loop effects from Barr-Zee diagrams can be larger [?], but typically the contributions to a_μ are negligible in these models.
- Models with additional light particles with masses below the GeV-scale, generically called dark sector models: Examples are provided by the models of Refs. [?, ?], where additional light neutral gauge bosons can affect electromagnetic interactions. Such models are intriguing since they completely decouple $g - 2$ from the physics of EWSB, and since they are hidden from collider searches at LEP or LHC (see however Refs. [?, ?] for studies of possible effects at dedicated low-energy colliders and in Higgs decays at the LHC). They can lead to contributions to a_μ which are of the same order as the deviation in Eq. (??). Hence the new $g - 2$ measurement will provide an important test of such models.

To summarize: many well-motivated models can accommodate larger contributions to a_μ — if any of these are realized $g - 2$ can be used to constrain model parameters; many well-motivated new physics models give tiny contributions to a_μ and would be disfavored if the more precise $g - 2$ measurement confirms the current deviation. There are also examples of models which lead to similar LHC signatures but which can be distinguished using $g - 2$.

1.4.1 Muon $g - 2$: Experiment

Measurements of the magnetic and electric dipole moments make use of the torque on a dipole in an external field, $\vec{\tau} = \vec{\mu} \times \vec{B} + \vec{d} \times \vec{E}$. All muon MDM experiments except the original Nevis ones used polarized muons in flight, and measured the rate at which the spin turns relative to the momentum, $\vec{\omega}_a = \vec{\omega}_S - \vec{\omega}_C$, when a beam of polarized muons is injected into a magnetic field. The resulting frequency, assuming that $\vec{\beta} \cdot \vec{B} = 0$, is given by [?, ?]

$$\vec{\omega}_{a\eta} = \vec{\omega}_a + \vec{\omega}_\eta = -\frac{Qe}{m} \left[a\vec{B} + \left(a - \left(\frac{m}{p} \right)^2 \right) \frac{\vec{\beta} \times \vec{E}}{c} \right] - \eta \frac{Qe}{2m} \left[\frac{\vec{E}}{c} + \vec{\beta} \times \vec{B} \right]. \quad (1.10)$$

Important features of this equation are the motional magnetic and electric fields: $\vec{\beta} \times \vec{E}$ and $\vec{\beta} \times \vec{B}$.

The E821 Collaboration working at the Brookhaven AGS used an electric quadrupole field to provide vertical focusing in the storage ring, and shimmed the magnetic field to 1 ppm uniformity on average. The storage ring was operated at the “ $g - 2$ ” momentum, $p_{g-2} = 3.094$ GeV/c, ($\gamma_{g-2} = 29.3$), so that $a_\mu = (m/p)^2$ and the electric field did not contribute to ω_a . They obtained[?]

$$a_\mu^{(\text{E821})} = 116\,592\,089(63) \times 10^{-11} \quad (0.54 \text{ ppm}) \quad (1.11)$$

The final uncertainty of 0.54 ppm consists of a 0.46 ppm statistical component and a 0.28 ppm systematic component.

The present limit on the EDM also comes from E821 [?]

$$d_\mu = (0.1 \pm 0.9) \times 10^{-19} e \cdot \text{cm}; \quad |d_\mu| < 1.9 \times 10^{-19} e \cdot \text{cm} \quad (95\% \text{ C.L.}), \quad (1.12)$$

so the EDM contribution to the precession is very small. In the muon $g - 2$ experiments, the motional electric field dominates the ω_η term, which means that $\vec{\omega}_a$ and $\vec{\omega}_\eta$ are orthogonal. The presence of an EDM in the $g - 2$ momentum experiments has two effects: the measured frequency is the quadrature sum of the two frequencies, $\omega = \sqrt{\omega_a^2 + \omega_\eta^2}$, and the EDM causes a tipping of the plane of precession, by an angle $\delta = \tan^{-1}[\eta\beta/(2a_\mu)]$. This tipping results in an up-down oscillation of the decay positrons relative to the midplane of the storage ring with frequency ω_a *out of phase by $\pi/2$* with the a_μ precession.

1.4.1.1 Future $g - 2$ Experiments

The E989 collaboration at Fermilab will move the E821 muon storage ring to Fermilab, and will use the $g - 2$ momentum technique to measure $a_{\mu+}$. New detectors and electronics, and a beam handling scheme that increases the stored muon rate per hour by a factor of 6 over E821 will be implemented. The goal is at least 21 times the statistics of E821, and a factor of four overall uncertainty reduction, with equal systematic and statistical uncertainties of ± 0.1 ppm.

The scope of Project X includes 50-200kW of beam power at 8 GeV, about three to fifteen times the beam power of E989. This large step in beam power could be used to measure $g - 2$ for negative muons, and provide muon beams with lower emittance thereby reducing experimental systematics.

Given the high impact of the E821 result and the crucial role the value of $g - 2$ plays in interpreting energy frontier results, it is imperative to have a second measurement with at least equal precision but with a complementary approach to the measurement. An alternate approach planned for J-PARC [?] uses a much

lower muon energy, and does not use the $g - 2$ momentum technique. A surface muon beam produced by the low energy Booster is brought to rest in an aerogel target, where muonium (the μ^+e^- atom) is formed. The muonium is ionized by a powerful laser which produces a very slow muon beam with extremely small emittance. This low emittance beam is then accelerated by a linac to 300 MeV, and injected into a ~ 1 m diameter solenoidal magnet with point to point uniformity of ± 1 ppm, approximately 100 times better than at the Brookhaven experiment. The average uniformity is expected to be known to better than 0.1 ppm. The decays are detected by a full volume tracker consisting of an array of silicon detectors. This provides time, energy, and decay angle information for every positron, maximizing the sensitivity to separate the $g - 2$ and EDM precession frequencies. The expected $g - 2$ sensitivity is comparable to the Fermilab experiment but will have very different systematic uncertainties and the combined results from the two experiments should bring the precision to below the 100 ppb level.

1.4.2 Muon $g - 2$: Expected Improvements in the predicted value

(((((This can be shortened, updated, and add a table that has the expected improvements)))))

The QED and electroweak contributions to $g - 2$ can be calculated to from first principles and are regarded as robust. The two dominant QCD contributions are hadronic vacuum polarization (HVP) and hadronic light-by-light contribution (HLbL). The HVP contribution to a_μ can be determined from the cross-section for $e^+e^- \rightarrow$ hadrons (and over a certain energy range, by $\tau \rightarrow$ hadrons) and a dispersion relation. It can also be computed from purely first principles using lattice QCD to calculate the HVP directly [?]. The two methods are complementary and can be used to check each other. The current best uncertainty comes from the first method,

$$a_\mu(\text{HVP}) = (692.3 \pm 4.2) \times 10^{-10}, \quad (1.13)$$

or about 0.61% [?] when only e^+e^- data are used. If τ data are included, $a_\mu = 701.5 \pm 4.7 \times 10^{-10}$, or 0.67% (but see [?] for the analysis that brings the τ into good agreement with e^+e^-). In the next 3-5 years the uncertainty on $a_\mu(\text{HVP})$ is expected to drop by roughly a factor of 2, relying on new results from BABAR, Belle, BES, and VEPP2000. The lattice calculations presently have an uncertainty of about 5% [?, ?, ?, ?], which is expected to decrease to 1-2% in the next 3-5 years [?]. At the one-percent level contributions from the charm and so-called disconnected diagrams (right panel, Fig. ??) enter. Both are currently under investigation.

The hadronic-light-by-light (HLbL) scattering amplitude shown in Fig. ?? is much more challenging. The contribution to $g - 2$,

$$a_\mu(\text{HLbL}) = 105(26) \times 10^{-11}, \quad (1.14)$$

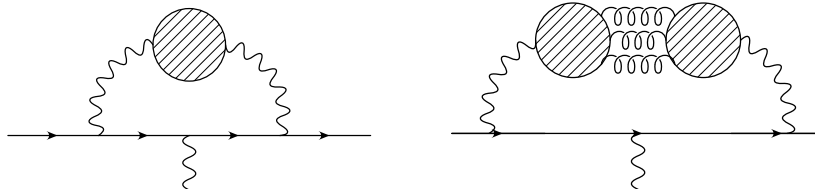


Figure 1-5. Hadronic vacuum polarization diagrams contributing to the SM muon anomaly. The horizontal lines represent the muon. (Left panel) The blob formed by the quark-antiquark loop represents all possible hadronic intermediate states. (Right panel) Disconnected quark line contribution. The quark loops are connected by gluons.

is not well known. It is based on the size of various hadronic contributions estimated in several different models [?]. Its uncertainty, though less than that in $a_\mu(\text{HVP})$ by about a factor of two, seems harder to reduce and is expected to be the dominant uncertainty as the HVP uncertainty is reduced. Finding a new approach, such as lattice QCD, in which uncertainties are systematically improvable, is crucial for making greatest use of the next round of experiments. With this in mind, a workshop was recently convened at the Institute for Nuclear Theory [?]. Workshop participants discussed how models, lattice QCD, and data-driven methods could be exploited to reduce the uncertainty on $a_\mu(\text{HLbL})$. The outcome of this workshop is that a SM calculation of the HLbL contribution with a total uncertainty of 10%, or less, can be achieved within five years. A detailed discussion of the computation of $a_\mu(\text{HLbL})$ in lattice QCD is given in the USQCD Collaboration white paper on $g - 2$ [?].

There are two methods, using the lattice framework, under investigation. The conventional one, analogous to the HVP calculation, is to calculate the correlation function of four electromagnetic currents for the quarks in pure QCD, one for each possible, independent, momentum configuration (there are V^2), fit the resulting function of discrete momenta to a smooth function and insert it into the two-loop QED integrals. The resulting four-Lorentz-index hadronic tensor has 32 independent contractions. For these reasons, the calculation is computationally demanding. An intermediate but useful step is to calculate the four-point correlation function at well chosen values of the vertex momenta to partially check model calculations.

A second method is to compute the entire amplitude on the lattice, including the muon, in a combined QED+QCD gauge field [?, ?, ?]. The method has passed several non-trivial tests. First, it has been successfully checked against perturbation theory in pure QED. Large finite volume effects (the photons are long range) appear manageable. Preliminary calculations in full QED+QCD, at unphysical quark and muon mass and momentum transfer q^2 , show a statistically significant result. The method requires a non-perturbative subtraction of leading order in α contributions which has been checked by varying the strength of the electric charge in the calculations and observing the expected scaling, before and after the subtraction. Disconnected contributions like the one shown in the right panel of Fig. ?? have not been included yet, but will be once the simpler first diagram (left panel, same figure) is fully under control. Calculations on a larger volume with smaller masses are in progress.

In addition to these direct approaches, there is other ongoing work on lattice-QCD calculations that check or supplement the model calculations. For example, it is well-known that the pion pole (namely, $\gamma\gamma^* \rightarrow \pi^0 \rightarrow \gamma^*\gamma^*$) provides the largest contribution to the QCD blob in Fig. ?. Just as experiments are being mounted to examine this physics (*e.g.*, PrimEx at JLab and KLOE at LNF), several groups [?, ?, ?] are using lattice QCD to compute the amplitudes for $\pi^0 \rightarrow \gamma\gamma^*$ and $\pi^0 \rightarrow \gamma^*\gamma^*$ (with one or two virtual photons).

If the SM and experiment central values do not change while both experiment and theory uncertainties are reduced, the discrepancy between the two becomes irresistible. The improvement expected from E989 (0.14 PPM) by itself improves Δa_μ to 5σ . A simultaneous decrease in the HLbL uncertainty to 10% from the

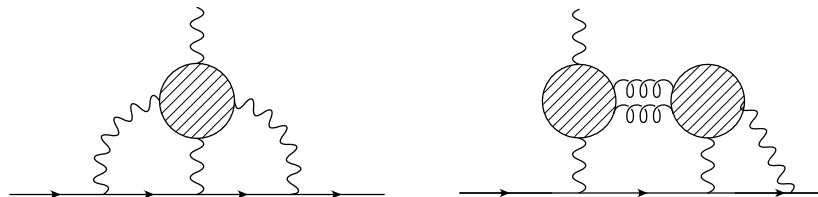


Figure 1-6. Hadronic light-by-light scattering diagrams contributing to the SM muon anomaly. The horizontal lines represent the muon. (Left panel) The blob formed by the quark loop represents all possible hadronic intermediate states. (Right panel) One of the disconnected quark line contributions. The quark loops are connected by gluons.

current 25% pushes it to 6σ , and finally, reducing the uncertainty on the HVP contribution by a factor of two increases it to 9σ . Such a large and clear difference between experiment and the Standard Model for the muon $g - 2$ will be extremely discriminating between new physics scenarios responsible for this discrepancy and will significantly leverage results from the energy frontier being explored at the LHC.

1.4.3 Storage Ring EDM experiments

At the magic momentum, the equation for the spin precession frequency of a charged particle in a storage ring is given by

$$\vec{\omega}_{a\eta} = \vec{\omega}_a + \vec{\omega}_\eta = -a \frac{Qe}{m} \vec{B} - \eta \frac{Qe}{2m} \left[\frac{\vec{E}}{c} + \vec{\beta} \times \vec{B} \right]. \quad (1.15)$$

The precession frequency is by far dominated by the $a = (g - 2)/2$ term. The key to extracting sensitivity to the EDM term η is to find ways of reducing or eliminating the motion due to the magnetic term a .

The first method is to use a magnetic storage ring such as the Muon $g-2$ experiment to extract a limit on the muon EDM. In the muon rest frame, the muon sees a strong motional electric field pointing towards the center of the ring adding a small horizontal component to the precession frequency vector that tilts the rotation plane. For a positive EDM, when the spin is pointing into the ring it will have a negative vertical component and when the spin is pointing to the outside of the ring it will have a positive vertical component. Since the positrons are emitted along the spin direction, this asymmetry maps into the positron decay angle. Since the asymmetry is maximized when the spin are perpendicular, the angular asymmetry is 90 degrees out of phase with the $g - 2$ precession frequency. Searches for this asymmetry have been used to set limits on the muon EDM both at the CERN and Brookhaven $g - 2$ experiments.

A number of the E989 detector stations will be instrumented with straw chambers to measure the decay positron tracks. With this instrumentation, a simultaneous EDM measurement can be made during the a_μ data collection, improving on the E821 muon EDM [?] limit by up to two orders of magnitude down to $\sim 10^{-21} e \cdot \text{cm}$. The J-PARC muon $g - 2$ proposal also will have decay angle information for all tracks and expects a similar improvement.

To go beyond this level for the muon, will require a dedicated EDM experiment that uses the “frozen spin” method [?, ?]. The idea is to operate a muon storage ring off of the $g - 2$ momentum and to use a radial electric field to cancel the ω_a term in Eq. ??, the $g - 2$ precession. The electric field needed to freeze the spin is $E \simeq aBc\beta\gamma^2$. Once the spin is frozen, the EDM will cause a steadily increasing out-of-plane motion of the spin vector. One stores polarized muons in a ring with detectors above and below the storage region and forms the asymmetry (up - down)/(up + down). To reach a sensitivity of $10^{-24} e \cdot \text{cm}$ would require $\sim 4 \times 10^{16}$ recorded events [?]. Preliminary discussions have begun on a frozen spin experiment using the 1000 kW beam power available at the Project X 3 GeV rare process campus.

An alternative method is to remove the $g - 2$ precession frequency completely by removing the magnet and using an electrostatic storage ring. This still requires the particle to be at the magic momentum to cancel the motional magnetic field. For these experiments, counter rotating beams are used to cancel the dominant systematic effects associated with stray magnetic fields. This idea has been studied in detail for the proton and deuteron with projected sensitivities approaching 10^{-30} using fairly large storage rings and proton momenta of 700 MeV.

For the electron, the magic momentum is 15 MeV. The smaller momentum would allow for a much smaller storage ring. Initial studies indicate that sensitivities up to $10^{-27} e \cdot \text{cm}$ can be achieved which would be

competitive with current limits and would be the best limits for a bare fermion. Furthermore, this would act as a much smaller test bed for the proton storage ring EDM experiment and would help demonstrate that the systematic uncertainties could be controlled.

((((((((Add a few paragraphs about the details of the electron proposal)))))))

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1.4.3.1 Future $g - 2$ Experiments

1.4.3.2 Future Muon EDM Experiments

1.4.4 Free electron EDM Experiment

1.4.5 τ $g - 2$ and EDM Experiments

1.4.6 Parity-Violating Experiments

1.5 Summary